Numerical Analysis of Quasi-One-Dimensional Nozzle Flow

Problem Statement

Understanding fluid flow behavior through convergent-divergent nozzles, also known as supersonic nozzles. These nozzles transform low-velocity inlet flow into supersonic flow by carefully designed changes in cross-sectional area, with a critical point at the throat where the velocity becomes sonic before accelerating to supersonic speeds in the divergent section.

Governing Equation

Continuity:

$$\rho\frac{\partial(\rho A)}{\partial t} + \rho A\frac{\partial V}{\partial x} + \rho V\frac{\partial A}{\partial x} + VA\frac{\partial \rho}{\partial x} = 0$$

Momentum:

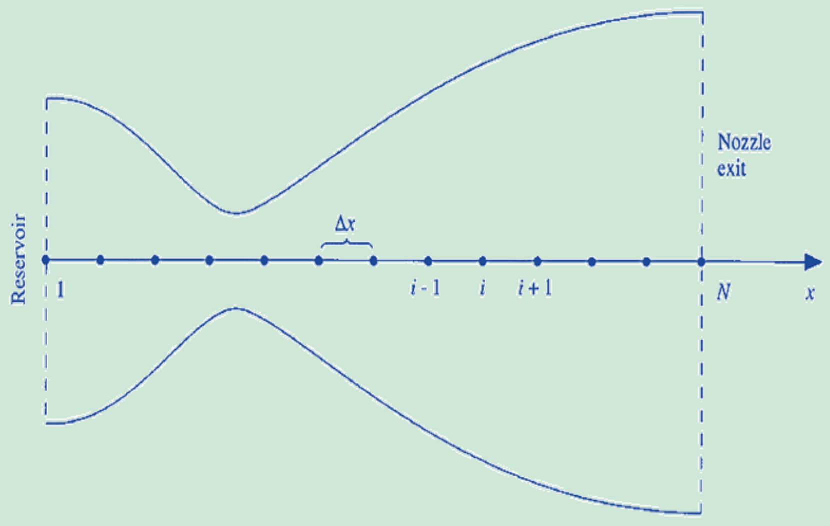
$$\rho\frac{\partial V}{\partial t} + \rho V\frac{\partial V}{\partial x} + \frac{\partial p}{\partial x} = 0$$

Energy:

$$\rho C_V\frac{\partial T}{\partial t} + \rho V C_V A\frac{\partial T}{\partial x} = -\rho RT\left(\frac{\partial V}{\partial x} + V\frac{\partial(lnA)}{\partial x}\right)$$

Numeric Solution

Grid Discretization



MacCormack Method

The MacCormack method is a widely used discretization scheme for solving hyperbolic PDE's.

- **Predictor step-** Provisional values are assigned through the PDE using forward difference scheme.
- **Corrector step-** The predicted values are then corrected using the provisional values and backward difference scheme.

$$\rho_i^{t+\Delta t} = \rho_i^t + \left(\frac{\partial \rho}{\partial t}\right)_{av}\Delta t$$

$$V_i^{t+\Delta t} = V_i^t + \left(\frac{\partial V}{\partial t}\right)_{av}\Delta t$$

$$T_i^{t+\Delta t} = T_i^t + \left(\frac{\partial T}{\partial t}\right)_{av}\Delta t$$

Time Step Calculation

In numerical analysis, choosing the right time step is vital for accurate and stable solutions in solving differential equations. Stability is paramount, setting the upper limit for the time step to prevent divergence. Striking the right balance between time step size and computational cost is essential for achieving the desired accuracy.

$$\Delta t = C\left(\frac{\Delta x}{a+V}\right)$$

Where, a is local speed of sound and V is local speed of fluid.

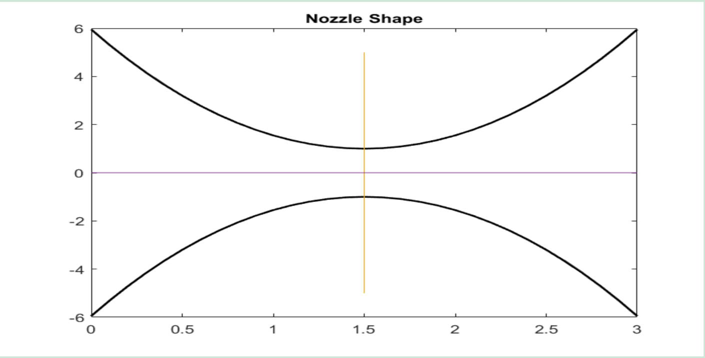
Assumptions

- **One-Dimensionality:** The flow field is assumed to vary only in one direction (e.g., along the x-axis), making it quasi-one-dimensional.
- **No Viscous Effects:** Viscous effects within the fluid, such as boundary layer development or shear stresses, are neglected.
- **Quasi-One-Dimensional Nozzle:** The geometry of the nozzle is assumed to be quasi-one-dimensional, which means that variations in the cross-sectional area occur primarily in one direction.
- **Steady State:** The analysis assumes that the flow parameters do not change with time, i.e., it's in a steady-state condition.
- **Perfect Gas:** The fluid is treated as an ideal gas, with constant specific heat ratios (γ) with temperature

Initial Conditions

$$\left.\begin{aligned} A &= 1 + 2.2(x-1.5)^2 \\ \rho &= 1 - 0.3146x \\ T &= 1 - 0.2314x \end{aligned}\right\} \textit{Initial conditions at } t=0$$

$$V = (0.1 + 1.09)T^{\frac{1}{2}}$$



Boundary Condition

Subsonic Boundary Inflow (M<1,Point=1)

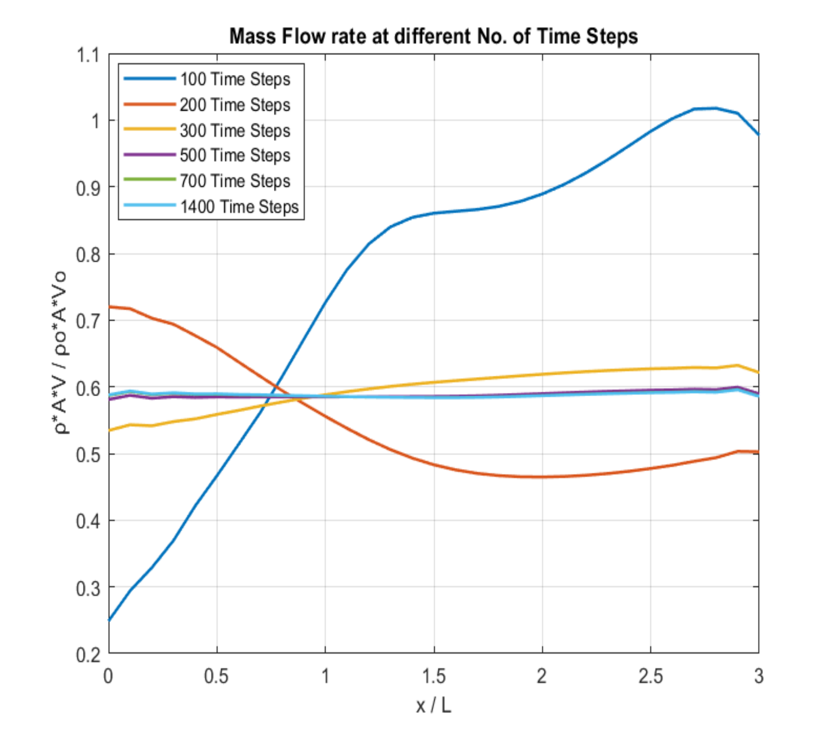
$$V_1 = 2V_2 - V_3$$
$$\rho_1 = 1$$
$$T_1 = 1$$

Supersonic Boundary Outflow (M>1,Point=1)

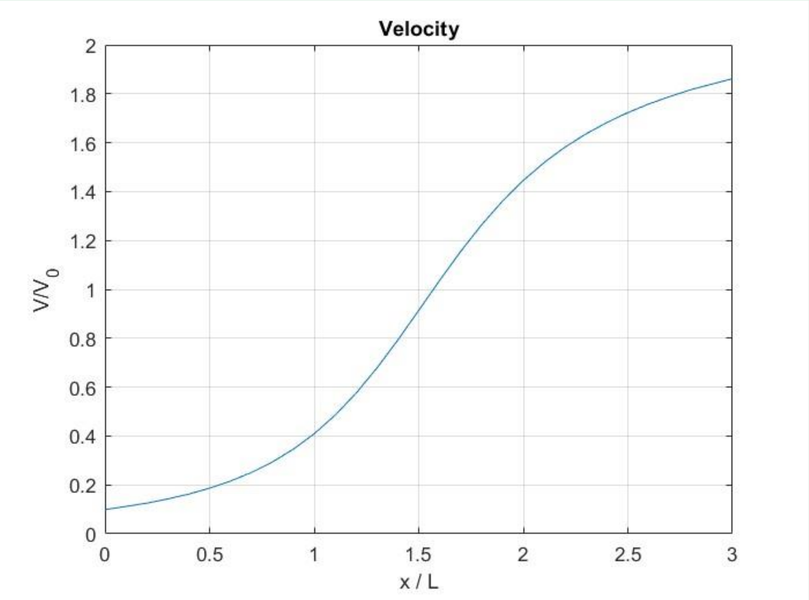
$$V_N' = 2V_{N-1}' - V_{N-2}'$$
$$\rho_N' = 2\rho_{N-1}' - \rho_{N-2}'$$
$$T_N' = 2T_{N-1}' - T_{N-2}'$$

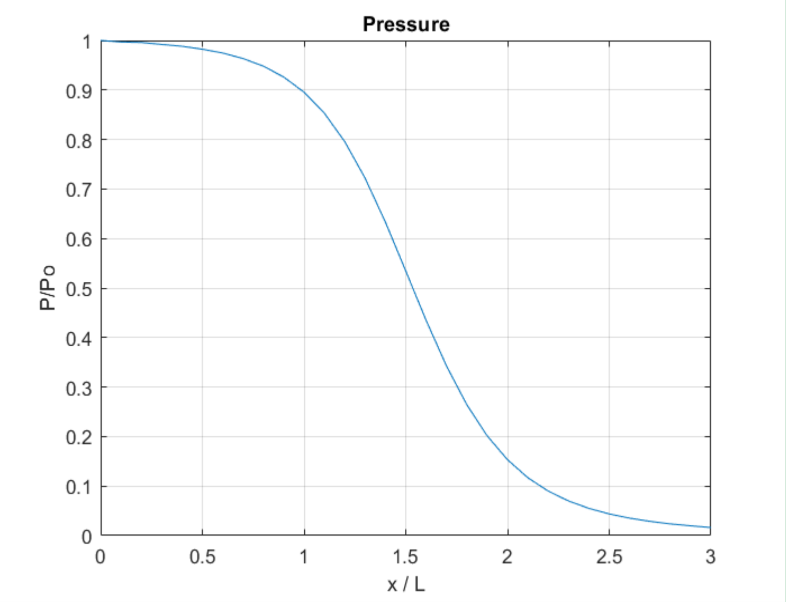
Results/Observation

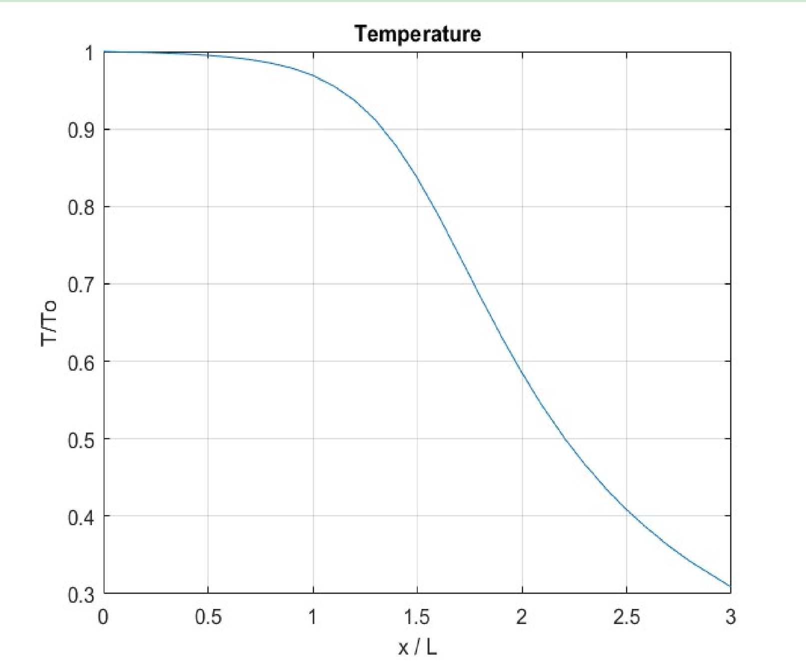
Our analysis confirms a direct relationship between the number of time steps used in the numerical solution and the achieved level of convergence. It is evident that employing a greater number of time steps results in a more precise and stable solution. This underscores the critical importance of judiciously choosing an appropriate time step for ensuring accurate and meaningful numerical simulations.

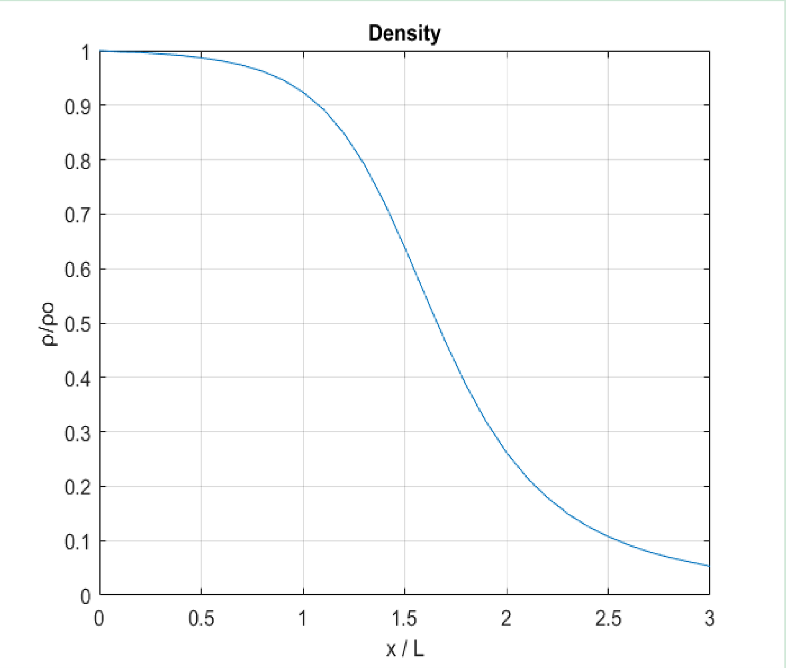


The four graphs below illustrate the spatial distribution of pressure ratio (P/Po), temperature ratio (T/To), velocity ratio (V/Vo), and density ratio (ρ/ρ0) normalized by the characteristic length (x/L) at the final time step. This visualization offers a pivotal snapshot of the different distributions and their respective magnitudes at this crucial point.









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