Problem 5

A classic example of the Lotka-Volterra predator-prey model is the following pair of ODEs:

$$\frac{dx}{dt} = ax - bxy$$

$$\frac{dy}{dt} =- cy + dxy$$

where x and y denote the number of prey and predators respectively, a denotes the prey growth rate, c denotes the predator death rate, and band d denote the rates characterizing the effect of the predator-prey interaction on prey death and predator growth, respectively. Use the following parameter values and initial conditions:

$$a = 1.2,\ b = 0.6,\ c = 0.8,\ d = 0.3,\ x(0) = 2,\ y(0) = 1$$

With a step-size of 0.1, simulate from t = 0 to 20 using a) Euler's method, b) Heun's method, and c) midpoint method

Description of Problem:

The problem involves analyzing a mass-spring system consisting of three masses connected by four springs. The springs have different spring constants (k_1, k_2, k_3, k_4), and the masses are identical (m_1,m_2,m_3). The task is to determine the equations of motion for each mass using Newton's second law ($\sum F_x = ma$) and the free-body diagrams. These equations can then be expressed in matrix form, where the acceleration vector is related to the displacement vector using a matrix equation. Additionally, at a specific time when one of the accelerations is given, the matrix equation simplifies into a tridiagonal matrix system. The Tridiagonal Matrix Algorithm (TDMA) will be employed to solve for the displacements of each mass.

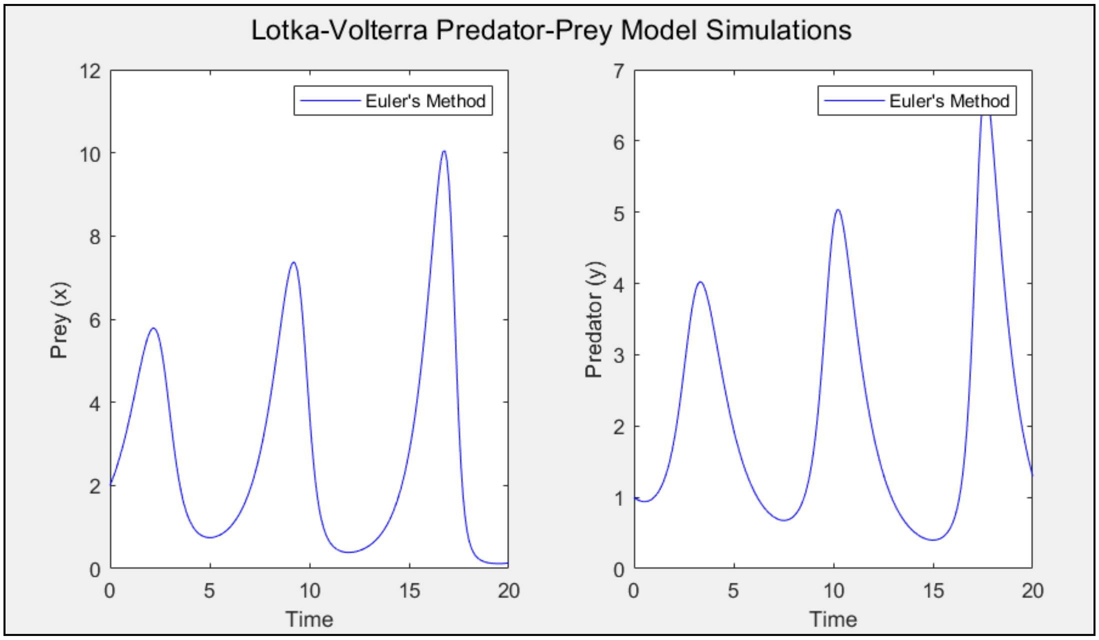
Procedure:

- **Euler's Method:**

  Step Size(dt) = h =0.1

  $x_{i+1} = x_i + (ax_i - bx_iy_i)h$

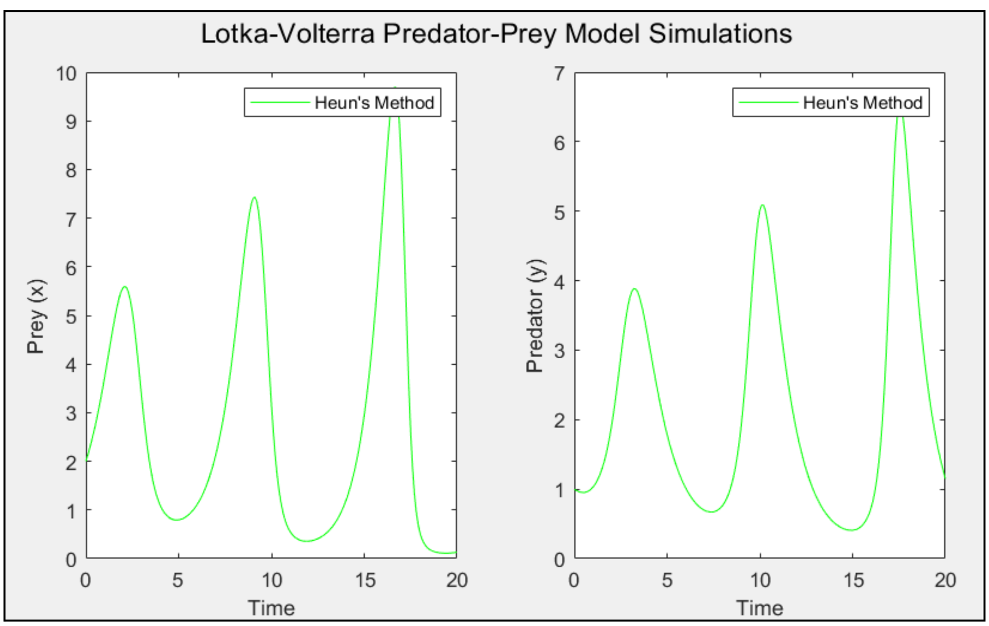
  $y_{i+1} = y_i + (-cy_i + dx_iy_i)$



➢ Heun's Method

$$x_{i+1} = x_i + h\left(\frac{f(x_i, y_i) + f(x_{i+1}, y^0_{i+1})}{2}\right)$$

$$y_{i+1} = y_i + h\left(\frac{f(x_i, y_i) + f(x_{i+1}, y^0_{i+1})}{2}\right)$$



➢ Euler's Method

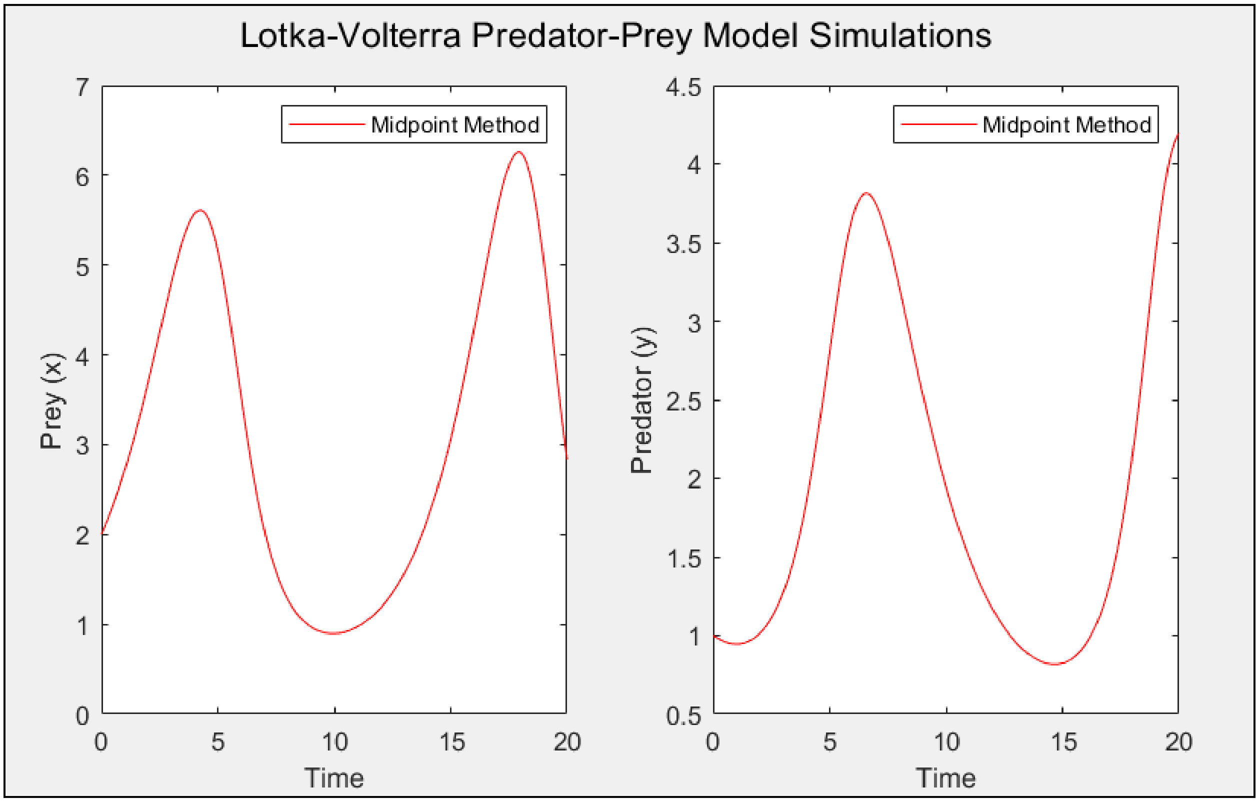
First use Euler's method to predict y at the midpoint of the interval $[x_i, x_{i+1}]$

I.e. $y_{i+1/2} = y_i + f(x_i, y_i)\frac{h}{2}$

Use $Q = f(x_{i+1/2}, y_{i+1/2})$

And , $x_{i+1} = x_i + Qh$

$y_{i+1} = y_i + Qh$



Final Plot for Comparision of Three methods: