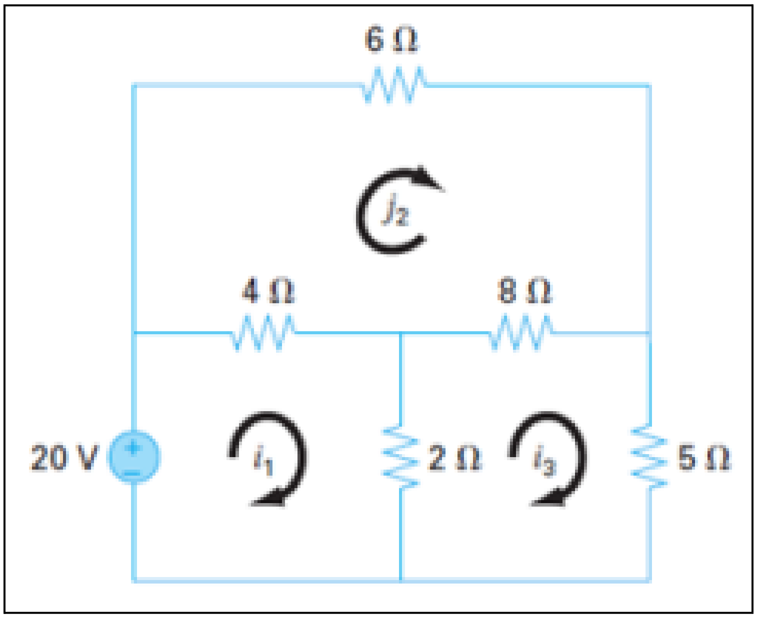
Problem 3

Using Kirchhoff's voltage law to derive a set of equations for calculating currents for the circuit shown below. Solve the resulting system of equations numerically.



Description of Problem:

The problem involves using Kirchhoff's voltage law (KVL) to derive a set of equations for calculating currents in a given electrical circuit. The circuit's components and connections are depicted in the diagram. After obtaining the system of equations, the goal is to solve it numerically using the Gaussian-Seidel method. This method iteratively finds solutions for a system of linear equations.

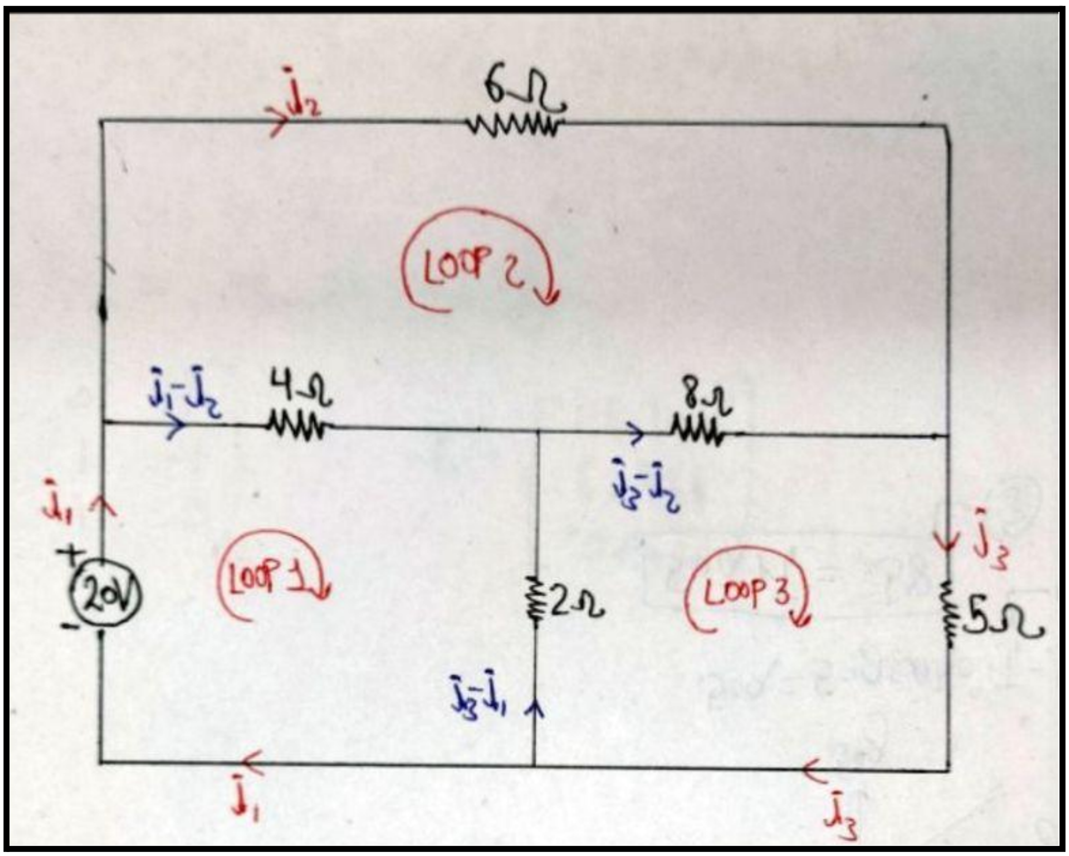
Kirchhoff's Voltage Law (KVL):

Kirchhoff's Voltage Law states that the algebraic sum of the electromotive forces (EMFs) and voltage drops around any closed loop in a circuit is zero. Using KVL, you can set up a system of equations based on the circuit components and connections.

Gaussian Seidel Method:

The Gaussian Seidel method is an iterative technique used to solve a system of linear equations. It starts with an initial guess for the unknowns and iteratively updates the values based on the equations. Each equation is solved for the current unknown using the most recent values of the other unknowns. This process is repeated until the solutions converge to a certain tolerance level.

Procedure:



- Apply KVL to each closed loop in the circuit, deriving equations that relate the currents and voltages.
  Applying KVL in loop 1,

$$20 - 4(i_1 - i_2) + 2(i_3 - i_1) = 0$$

$$\Rightarrow \quad 3i_1 - 2i_2 - i_3 = 10 \tag{1}$$

Applying KVL in loop 2,

$$-6i_2 + 8(i_3 - i_2) + 4(i_1 - i_2) = 0$$

$$\Rightarrow \quad 2i_1 - 9i_2 + 4i_3 = 0 \tag{2}$$

Applying KVL in loop 3,

$$-5i_3 - 2(i_3 - i_1) - 8(i_3 - i_2) = 0$$

$$\Rightarrow \quad 2i_1 + 8i_2 - 15i_3 = 0 \tag{3}$$

- By equation (1) we get, $i_1 = \frac{10+2i_2+i_3}{3}$

  By equation (2) we get, $i_2 = \frac{2i_1+4i_3}{9}$

  By equation (3) we get, $i_3 = \frac{2i_2+8i_2}{15}$

- Choose an initial guess for the currents and set a convergence criterion.
  Let the initial guess be $i_1 = i_2 = i_3 = 0$

  Convergence criterion

$$\varepsilon_a = \frac{i_{new} - i_{old}}{i_{new}} \times 100$$

$$\varepsilon_a < \varepsilon_s$$

  Taking $\varepsilon_s = 0.005\%$

- The solution we get after running the Gaussian Seidel algorithm is shown in the below table,

  $i_1 = 5.175659\,A$, $i_2 = 1.90943\,A$ and $i_3 = 1.708453\,A$

  The total number of iterations required were 16.

Iteration No.	i1	i2	i3	ea(%)
0	0.000000	0.000000	0.000000	-
1	3.333333	0.740741	0.839506	100.0
2	4.106996	1.285780	1.233349	31.053402
3	4.601636	1.570741	1.451280	14.635846
4	4.864254	1.725959	1.569078	7.299857
5	5.006998	1.810034	1.632952	3.802471
6	5.084340	1.855610	1.667570	2.017753
7	5.126263	1.880312	1.686335	1.081427
8	5.148986	1.893701	1.696506	0.582621
9	5.161303	1.900959	1.702018	0.314765
10	5.167979	1.904892	1.705006	0.170309
11	5.171597	1.907024	1.706626	0.092223
12	5.173558	1.908180	1.707504	0.049961
13	5.174621	1.908806	1.707980	0.027073
14	5.175197	1.909146	1.708237	0.014672
15	5.175510	1.909330	1.708377	0.007952
16	5.175679	1.909430	1.708453	0.00431