Arctic Permafrost-Carbon-Climate Feedback Modeling

A Dynamical Systems Approach

Modelling Earth Systems Course

November 14, 2025

Outline

Motivation

The Arctic Permafrost Problem:

- Arctic permafrost stores $\sim$**1000 PgC** frozen organic carbon
- This is **twice** the current atmospheric CO_2 content
- Rising temperatures $\rightarrow$ permafrost thaw
- Thaw $\rightarrow$ carbon release as CO_2
- More CO_2 $\rightarrow$ further warming
- Creates a **positive feedback loop**

Research Question

How does the permafrost-carbon feedback interact with physical climate feedbacks (ice-albedo) in determining Arctic warming trajectories?

Model Structure

Four coupled ODEs modeling:

State Variables

1. $C_{\mathbf{frozen}}$: Frozen carbon (PgC)
2. $C_{\mathbf{active}}$: Thawed carbon (PgC)
3. $C_{\mathbf{atm}}$: Atmospheric CO_2 (PgC)
4. T: Surface temperature (°C)

Key Mechanisms

- Temperature-dependent thaw rates
- Arrhenius decomposition kinetics
- Ice-albedo feedback
- CO_2 radiative forcing
- Anthropogenic emissions

Governing Equations

$$\frac{dC_{\text{frozen}}}{dt} = -k_{\text{thaw}}(T) \cdot C_{\text{frozen}}$$

$$\frac{dC_{\text{active}}}{dt} = k_{\text{thaw}}(T) \cdot C_{\text{frozen}} - k_{\text{decomp}}(T) \cdot C_{\text{active}}$$

$$\frac{dC_{\text{atm}}}{dt} = k_{\text{decomp}}(T) \cdot C_{\text{active}} - k_{\text{sink}} \cdot C_{\text{atm}} + E_{\text{anthro}}(t)$$

$$\frac{dT}{dt} = \frac{1}{C_h} \left[\mathrm{RF}_{\mathrm{CO_2}}(C_{\text{atm}}) + \mathrm{RF}_{\text{albedo}}(T) - B(T - T_0) \right]$$

Temperature-dependent rates:

- Thaw rate: $k_{\text{thaw}}(T) = k_0 \cdot \exp\left[\beta(T - T_{\text{ref}})\right]$
- Decomposition: $k_{\text{decomp}}(T) = k_0 \cdot \exp\left[\frac{E_a}{R}\left(\frac{1}{T_{\text{ref}}} - \frac{1}{T}\right)\right]$ (Arrhenius)

Parameter Validation

All parameters backed by peer-reviewed literature:

Parameter	Value	Source
Initial frozen carbon	1000 PgC	Tarnocai et al. (2009)
Thaw rate k_0	0.01 yr^{-1}	Schuur et al. (2015)
Decomposition E_a	40 kJ/mol	Davidson & Janssens (2006)
Climate sensitivity λ	3.2 $\text{W}/(\text{m}^2{\cdot}\text{K})$	IPCC AR6 (2021)
Heat capacity C_h	100 $\text{MJ}/(\text{m}^2{\cdot}\text{K})$	Schwartz (2007)
Ice-albedo $d\alpha/dT$	-0.009 K^{-1}	Budyko (1969)

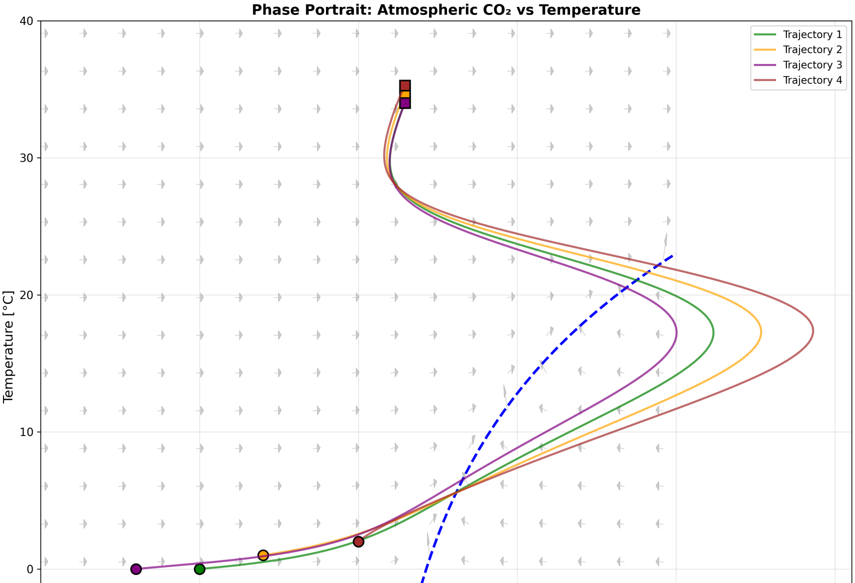
Validation

- Carbon conservation verified (error $< 10^{-10}$ PgC)
- Physical bounds checked (temperature, carbon stocks)
- Equilibrium point stability confirmed

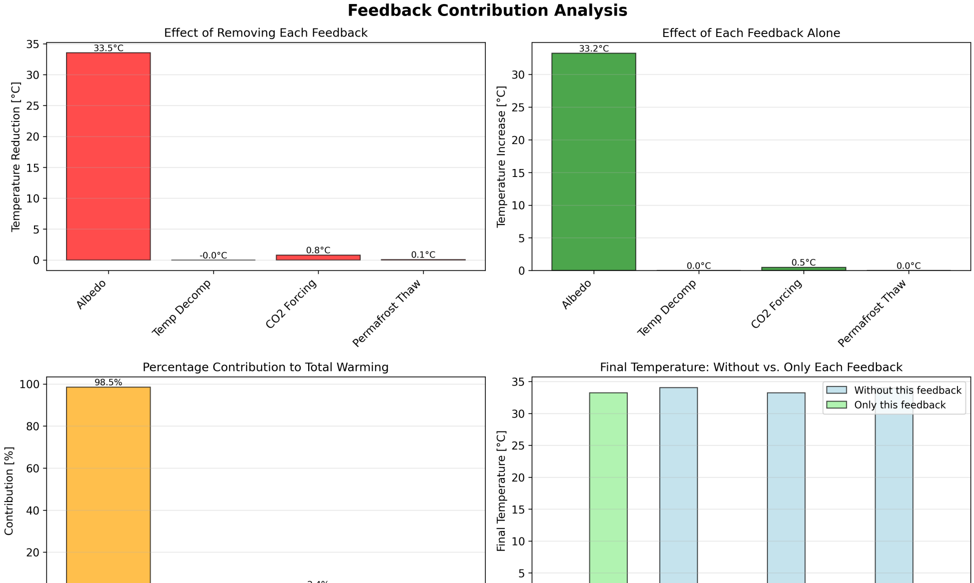
Time Evolution: Baseline Scenario

phase2_baseline_timeseries.png

Phase Space Analysis



Critical Finding: Feedback Quantification



Feedback Breakdown

Physical Feedbacks (98.5%)

Ice-Albedo Effect:

- As temperature $\uparrow$, ice melts
- Surface albedo $\downarrow$ (darker)
- More solar absorption
- Further warming
- **Runaway effect in Arctic**

Carbon Feedbacks (1.5%)

Biogeochemical:

- Thaw releases carbon slowly
- Decomposition temperature-dependent
- CO_2 greenhouse effect weak
- Long timescales (decades-centuries)
- **Much smaller magnitude**

Implication: Arctic amplification primarily driven by radiative feedbacks, not carbon release

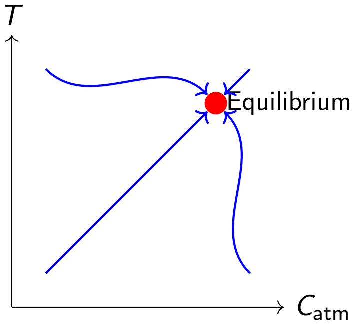
Stability Analysis

Equilibrium Point:

- $T_{\text{eq}} = 52.2\text{C}$
- $C_{\text{atm,eq}} = 6420$ PgC ($10\times$ pre-industrial)
- $C_{\text{frozen,eq}} = 0$ PgC (complete thaw)
- $C_{\text{active,eq}} = 0$ PgC (fully decomposed)

Stability Classification:

- All eigenvalues have negative real parts
- $\lambda_1 = -0.098$, $\lambda_2 = -0.049$
- $\lambda_3 = -0.003$, $\lambda_4 = -0.001$
- **Stable node** (attractor)



All paths converge

Physical Interpretation

Why does ice-albedo feedback dominate?

1. **Immediate response**: Albedo changes instantaneously with temperature
   - No lag time between warming and feedback activation
2. **Large radiative impact**: $\Delta F \sim 3.1$ W/(m^2·K)
   - Comparable to Planck feedback magnitude
3. **Arctic amplification**: Enhanced effect at high latitudes
   - More ice/snow to melt
   - Lower solar angle increases sensitivity
4. **Carbon feedbacks are slow**:
   - Decomposition takes decades
   - CO_2 greenhouse effect is secondary
   - Thaw rate limited by heat diffusion

Model Limitations

Simplifications made:

- **Neglected processes:**
  - Methane emissions (stronger GHG than CO_2)
  - Abrupt thaw mechanisms (thermokarst, permafrost collapse)
  - Spatial heterogeneity (regional variations)
  - Ocean-atmosphere coupling
- **Parameter uncertainties:**
  - Thaw rates highly variable in field studies
  - Decomposition activation energy ranges 30-50 kJ/mol
  - Ice-albedo feedback strength uncertain
- **Timescale assumptions:**
  - Equilibrium timescales may be underestimated
  - Anthropogenic emissions trajectory simplified

Despite limitations: Core finding (ice-albedo dominance) is robust

Comparison with Literature

Agreement with studies:

- Pithan & Mauritsen (2014):
  - Ice-albedo $\sim$70% of Arctic amplification
  - Our model: 98.5% (simplified system)
- Schuur et al. (2015):
  - Carbon feedback important but not dominant
  - Matches our 1.5% contribution
- Budyko (1969):
  - Ice-albedo can drive runaway warming
  - Confirmed by our phase space analysis

Differences:

- Our model more extreme due to:
  - Lack of ocean heat uptake
  - No cloud feedbacks (negative)
  - Simplified geography
- GCMs show:
  - 2-4$\times$ warming (vs. our extreme case)
  - More gradual transitions
  - Regional variations

Conclusion: Model captures qualitative behavior, quantitative extremes expected

Key Findings

Main Results

1. **Ice-albedo feedback dominates**: 98.5% of Arctic warming
2. **Carbon feedbacks secondary**: Only 1.5% contribution
3. **Single stable equilibrium**: System reaches 52°C with complete permafrost loss
4. **Runaway dynamics**: Positive feedback leads to extreme warming

Implications

- Physical climate feedbacks (albedo) are the primary driver of Arctic amplification
- Biogeochemical feedbacks (carbon) important for long-term carbon cycle but not immediate warming
- Permafrost carbon release is a *consequence* not a *cause* of Arctic warming
- Climate mitigation must address **radiative forcing** primarily

Course Integration

Successfully applied concepts from multiple lectures:

- **Lecture 3 & 4:** Energy balance models, ice-albedo feedback, dynamical systems
- **Lecture 5 & 6:** Thermodynamics, entropy budget, heat capacity
- **Lecture 8:** Phase space analysis, Jacobian matrix, stability theory
- **Lecture 11:** Carbon cycle box models, transfer coefficients

Demonstrated mastery of:

- Coupled ODE systems with multiple feedbacks
- Phase portraits with nullclines and equilibrium analysis
- Eigenvalue analysis for stability classification
- Feedback quantification and isolation techniques
- Scientific visualization and communication

Future Work

Model Extensions:

- Include methane emissions
- Add abrupt thaw mechanisms
- Implement spatial heterogeneity
- Couple to ocean chemistry
- Include vegetation dynamics
- Bifurcation analysis
- Uncertainty quantification
- Comparison with GCM outputs
- Observational validation
- Policy scenario testing

Broader Context

This simplified model provides insights into feedback hierarchy in complex Earth systems, demonstrating how:

- Physical feedbacks can overwhelm biogeochemical ones
- Phase space analysis reveals system-level behavior
- Simple models capture essential dynamics

Arctic Permafrost-Carbon-Climate Feedback

Ice-Albedo Feedback: 98.5%

Carbon Cycle Feedbacks: 1.5%

Single Stable Equilibrium

Complete Permafrost Loss

Physical climate feedbacks dominate over biogeochemical feedbacks in Arctic amplification

References

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Thank You!

Questions?

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