VIGNAN INSTITUTE OF TECHNOLOGY AND SCIENCE

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Course: **B. Tech** Branch: **Common to All** Year: **I** & Semester: **I**

Subject: **Matrices and Calculus (UNIT-1)**

S.NO		QUESTIONS	Marks	COS	POS	BTL
1.	a)	Define Rank of the matrix.	[1M]	1	1,2,3	1
	b)	Find the Value of k such that the rank of $\begin{bmatrix}1 & 2 & 3\\ 2 & k & 7\\ 3 & 6 & 10\end{bmatrix}$ is 2.	[1M]	1	1,2,3	1
	c)	Find the rank of matrix A= $\begin{bmatrix}0 & 1 & -3 & -1\\ 1 & 0 & 1 & 1\\ 3 & 1 & 0 & 2\\ 1 & 1 & -2 & 0\end{bmatrix}$ by reducing to echelon form.	[5M]	1	1,2,3	1,5
	d)	Discuss for what values of λ, μ the simultaneous equations x + y + z = 6, x +2y + 3z = 10, x + 2y + λz = μ have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.	[5M]	1	1,2,3	1,6
2.	a)	Define echelon form	[1M]	1	1,2,3	1
	b)	Find the rank of matrix A= $\begin{bmatrix}2 & 3 & 7\\ 3 & -2 & 4\\ 1 & -3 & -1\end{bmatrix}$ by reducing to echelon form	[1M]	1	1,2,3	1
	c)	Find the rank of $\begin{bmatrix}1 & 4 & 3 & -2 & 1\\ -2 & -3 & -1 & 4 & 3\\ -1 & 6 & 7 & 2 & 9\\ -3 & 3 & 6 & 6 & 12\end{bmatrix}$ reducing to echelon form	[5M]	1	1,2,3	1
	d)	Find whether the following systems of equations are consistent. If so solve them. x + 2y + 2z =2 ; 3x – 2y - z = 5; 2x – 5y + 3z = - 4 ; x + 4y + 6z = 0	[5M]	1	1,2,3	1,3

3.	a)	Solve the system of equations $2x + 3y = 0$, $3x-2y = 0$.	[1M]	1	1,2,3	1,6
	b)	Define Normal form	[1M]	1	1,2,3	1,2
	c)	Find the rank of matrix A $=\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$ by reducing it to canonical form.	[5M]	1	1,2,3	1,2
	d)	Find whether the following equations are consistent, if so solve them $x + y + 2z = 4$, $2x - y + 3z = 9$, $3x - y - z = 2$.	[5M]	1	1,2,3	1,2
4.	a)	Define Homogeneous system of equations.	[1M]	1	1,2,3	1,2
	b)	Define matrix with example.	[1M]	1	1,2,3	1,6
	c)	Determine whether the following equations will have a non-trivial solution if so solve them. $4x + 2y + z + 3w = 0$, $6x + 3y + 4z + 7w = 0$, $2x + y + w = 0$.	[5M]	1	1,2,3	1,6
	d)	Use the Gauss Elimination method to solve $x +2y-3z = 9$, $2x- y + z = 0$, $4x- y + z = 4$.	[5M]	1	1,2,3	1.3
5	a)	When given system of equations are consistent and it has unique solution?	[1M]	1	1,2,3	1
	b)	Define a diagonal matrix.	[1M]	1	1,2,3	1
	c)	Find the rank of matrix A $=\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ by reducing it to normal form.	[5M]	1	1,2,3	1,5
	d)	Find the values of 'a' and 'b' for which the equations $x + y + z = 3$, $x + 2y + 3z = 6$, $x + 9y + az = b$ have (i) No Solution (ii) A unique solution (iii) Infinite number of solutions.	[5M]	1	1,2,3	
6	a)	Find the inverse of the matrix $\begin{bmatrix} 1 & 3 \\ 1 & 6 \end{bmatrix}$	[1M]	1	1,2,3	1,6
	b)	When given system of equations are	[1M]	1	1,2,3	1

		consistent and it has infinite number of solutions?				
	c)	Compute the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 2 & 3 & 3 & 3 \end{bmatrix}$ by using elementary operations.	[5M]	1	1,2,3	1,5
	d)	Solve the system of equations $x + 3y - 2z = 0$; $2x - y + 4z = 0$; $x - 11y + 14z = 0$.	[5M]	1	1,2,3	1,6
7	a)	When given system of equations are inconsistent and it has no solution ?	[1M]	1	1,2,3	1,6
	b)	When given system of equations are consistent and it has non-trivial solution?	[1M]	1	1,2,3	1,6
	c)	Solve the system of equations $3x + y + 2z = 3$, $2x - 3y - z = -3$, $x + 2y + z = 4$ using Gauss elimination method.	[5M]	1	1,2,3	1,6
	d)	Solve the system of equations $26x_1 + 2x_2 + 2x_3 = 12.6$, $3x_1 + 27x_2 + x_3 = -14.3$, $2x_1 + 3x_2 + 17x_3 = 6.0$ using Gauss Siedel method.	[5M]	1	1,2,3	1,6
8	a)	Find the value of "k" such that the rank of $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & k & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$ is 2.	[1M]	1	1,2,3	1,5
	b)	When given system of equations are consistent and it has unique solution?	[1M]	1	1,2,3	1,6
	c)	Solve the system of equations $2x + 3y + 2z = 0$; $2x + y + 5z = 0$; $2x + 11y + 15z = 0$.	[5M]	1	1,2,3	1,6
	d)	Solve the system of equations $5x_1 - 2x_2 + 3x_3 = -1$, $-3x_1 + 9x_2 + x_3 = 2$, $2x_1 - x_2 - 7x_3 = 3$ using Gauss Siedel method.	[5M]	1	1,2,3	1,6

Extra IMP questions: (1 MARKS):-

1) Find the value of "x" such that the matrix A is singular where A = $\begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -(1+x) \end{bmatrix}$.

2) Find the value of A^2- 4A + 9I, where A = $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$.

3) Find the inverse of the matrix $A = \begin{bmatrix} a+ib & c+id \\ -c+id & a-ib \end{bmatrix}$, if $a^2+b^2+c^2+d^2=1$.

4) Find the rank of the matrix A = $\begin{bmatrix} 1 & 0 & 5 \\ 2 & 1 & 6 \\ 3 & 4 & 0 \end{bmatrix}$.

5) Find the inverse of the matrix diag[a, b, c]. Where a ≠ 0, b ≠ 0, c ≠ 0.

Extra IMP questions: (5 MARKS):-

1) Find the rank of matrix A = $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ by reducing it to normal form.

2) Apply rank test to find whether the following system has any solution other than x = y = z = w = 0, x + 2y + 3z + 4w = 0, 5x + 6y +8z + w =0, 8x + 3y + 7z + 2w = 0.

3) Solve by Gauss Elimination method

5x – y – 2z = 142, x – 3y – z = –30, 2x – y - 3z = –5.

4) Find the inverse of the matrix A by using Gauss-Jordan method A= $\begin{bmatrix} 1 & 0 & 5 \\ 2 & 1 & 6 \\ 3 & 4 & 0 \end{bmatrix}$.

5) Show that the only real number for which the system x + 2y + 3z = λx;

3x + y + 2z = λy ; 2x + 3y + z = λz has non-zero solution is 6 and solve them, when λ = 6.

6) Find the values of 'p' and 'q' for which the equations

$2x + 3y + 5z = 9$, $7x + 3y + 2z = 8$, $2x + 3y + pz = q$ have

(i) No Solution (ii) a unique solution (iii) Infinite number of solutions.

7) Reduce the matrix A to normal form and hence find its rank

$$A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$$

8) Show that the equations $3x + 4y + 5z = a$, $4x + 5y + 6z = b$

and $5x + 6y + 7z = c$ do not have a solution unless $a + c = 2b$.

9) Express the following system in matrix form and solve by Gauss Elimination method. $2x_1 + x_2 + 2x_3 + x_4 = 6$;

$6x_1 - 6x_2 + 6x_3 + 12x_4 = 36$;

$4x_1 + 3x_2 + 3x_3 - 3x_4 = -1$;

$2x_1 + 2x_2 - x_3 + x_4 = 10$.

10) Solve the system of equations $2x + y + z = 5$, $3x + 5y + 2z = 15$, $2x + y + 4z = 8$ using Gauss Siedel method.

11) Express the following system in matrix form and solve by Gauss Elimination method.

$x_1 - x_2 + x_3 + x_4 = 2$; $x_1 + x_2 - x_3 + x_4 = -4$;

$x_1 + x_2 + x_3 - x_4 = 4$; $x_1 + x_2 + x_3 + x_4 = 0$.

12) Determine b such that the system of homogeneous equations $2x + y + 2z = 0$, $x + y + 3z = 0$, $4x + 3y + bz = 0$ has trivial and non trivial solutions. Find the Non trivial solution.

13) Find the inverse of the matrix A using elementary operations

$$A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$$

Objective-PART-A (Choose the Correct Answer)

1) The Rank of a null matrix is – [ ]

$a)$0 b) 1 c) 2 d) n

2) The rank of a unit matrix of order "n" is– [ ]

$a)$0 b) 1 c) 2 d) n

3) The rank of the non-singular matrix of order "n" is --- [ ]

$a)$0 b) 1 c) n d) none

4) The rank of a matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ is – [ ]

$a)$0 b) 1 c) 2 d) n

5) The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is – [ ]

$a)$0 b) 1 c) 3 d) 2

6) The system of equations AX = B is said to be inconsistent if --- [ ]

a) $\rho(A) = \rho(A/B)$ b) $\rho(A) \neq \rho(A/B)$

c) $\rho(A) = \rho(A/B) < n$ d) $\rho(A) = \rho(A/B) = n$

7) The system of equations AX = B is said to have Unique if --- [ ]

a) $\rho(A) = \rho(A/B)$ b) $\rho(A) \neq \rho(A/B)$

c) $\rho(A) = \rho(A/B) < n$ d) $\rho(A) = \rho(A/B) = n$

8) The system of equations AX = B is said to have infinite solutions if--- [ ]

$a)\rho(A) = \rho(A/B)$ b) $\rho(A) \neq \rho(A/B)$

c) $\rho(A) = \rho(A/B) < n$ d) $\rho(A) = \rho(A/B) = n$

9) Which of the following method is known as Diagonally dominant system [ ]

$a)Gauss - elemination\ method$ b) $Gauss - jordan\ method$

c) $Gauss - Seidal\ method$ d) none

10) The system of equations AX = B is said to be Trivial solutions if --- [ ]

$a)$ $\rho(A) = n$ b) $\rho(A) \neq n$

c) $\rho(A) < n$ d) $none$

Objective- PART-B (Fill in the Blanks)

11) The rank of a unit matrix of order '9' is -----------------------

12) The rank of the matrix A = $\begin{bmatrix} 5 & 10 & 10 \\ 1 & 0 & 2 \\ 3 & 6 & 6 \end{bmatrix}$ is ------------------

13) $If\ A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$, Find A^{-1} = -----------------------

14) If A = $\begin{bmatrix} a & c & b \\ b & a & c \\ c & b & a \end{bmatrix}$ is singular matrix, then $a^3 + b^3 + c^3$ =---------------------

15) The maximum value of the rank of a 4×5 matrix is ----------------------

16) The rank of the matrix $\begin{bmatrix} k & -1 & 0 \\ 0 & k & -1 \\ -1 & 0 & k \end{bmatrix}$ is 2. For k = -----------------

17) The System of equations Ax = B is said to be Non-Homogeneous then ---------------------

18) The rank of a singular matrix of order 3 is ------------------------

19) The rank of 3×3 matrix whose all elements are equal to 2 is ------------------

20) If $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ $then\ x = - - -and\ y =$ --------------

Answers

Objective

1) A
2) D
3) C
4) B
5) D
6) B
7) D
8) C
9) C
10) A

Fill in the blanks:

11) 9

12) 2

13) $\begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$

14) $3abc$

15) 4

16) 1

17) $B \neq 0$

18) ≤ 2

19) 1

20) $X = 4 \; and \; Y = -5.$

21)

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Course: **B. Tech** Branch: **Common to All** Year: **I** **&**Semester: **I**

Subject: **Matrices and Calculus (UNIT-2)**

S.NO		**QUESTIONS**	**Marks**	**COS**	**POS**	**BTL**
1	a)	Define latent roots and latent vectors.	[1M]	2	1-6	1,4
	b)	Find the quadratic form corresponding to the matrix A = $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$.	[1M]	2	1-6	4,5,6
	c)	Determine the Eigen values and Eigen vectors of the following matrix. $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$	[5M]	2	1-6	4,6
	d)	Verify Cayley –Hamilton theorem of the matrix A = $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and also find its inverse.	[5M]	2	1-6	5,6
2.	a)	Find the quadratic form corresponding to the matrix $diag[\lambda_1, \lambda_2, \lambda_3 \dots \dots . \lambda_n]$.	[1M]	2	1-6	1,5,6
	b)	Find the Product of the Eigen values of matrix $\begin{bmatrix} 2 & 3 & -2 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$.	[1M]	2	1-6	1,6
	c)	Diagonalize the matrix A = $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.	[5M]	2	1-6	1,5
	d)	Reduce the following quadratic form to canonical form by orthogonal transformation and also find the (i)rank (ii) index (iii) nature (iv) signature. where $Q = 2xy +$	[5M]	2	1-6	1,6

		$2xz - 2yz$.				
3.	a)	Define spectral matrix.	[1M]	2	1-6	1,5
	b)	Find the Eigen values of $\begin{bmatrix} 1 & -2 & 0 \\ 0 & 2 & -4 \\ 0 & 0 & 3 \end{bmatrix}$	[1M]	2	1-6	4,5,6
	c)	Find the diagonal matrix orthogonally similar to the following real symmetric matrix. Also obtain the transforming matrix. A = $\begin{bmatrix} 7 & 4 & -4 \\ 4 & -8 & -1 \\ -4 & -1 & -8 \end{bmatrix}$.	[5M]	2	1-6	1,6
	d)	If λ is an eigen value of a non-singular matrix A then prove that λ^n is an eigen value of A^n.	[5M]	2	1-6	4,5,6
4.	a)	Express the following quadratic form to matrix notation $$2x^2 + 3y^2 - 5z^2 - 2xy + 6xz - 10yz.$$	[1M]	2	1-6	1,2
	b)	Define nature of the quadratic form.	[1M]	2	1-6	1,5,6
	c)	Using Cayley-Hamilton theorem find the inverse and A^4 of the matrix A = $\begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$.	[5M]	2	1-6	1,2,6
	d)	If λ is an eigen value of a non-singular matrix A then prove that $\frac{\lvert A\rvert}{\lambda}$ is an eigen value of adjA.	[5M]	2	1-6	4,5
5	a)	Define Modal matrix	[1M]	2	1-6	5,6
	b)	Find the quadratic form corresponding to the matrix A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{bmatrix}$.	[1M]	2	1-6	4,6
	c)	Find the diagonal matrix A = $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$ *and* also find (i) A^4 (ii) A^8.	[5M]	2	1-6	1,6

	d)	Show that the matrix satisfies Cayley Hamilton theorem and also find the value of the Matrix $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$, Where A = $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$	[5M]	2	1-6	1,4,
6	a)	Find the sum of the Eigen values of matrix $\begin{bmatrix} 2 & 3 & -2 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$	[1M]	2	1-6	1,5
	b)	Define Quadratic form	[1M]	2	1-6	1,4,6
	c)	Find the diagonal matrix orthogonally similar to the following real symmetric matrix. A = $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.	[5M]	2	1-6	1,5
	d)	Reduce the following quadratic form to canonical form by orthogonal transformation $Q = x^2 + 2y^2 + 2z^2 - 2xy + 2xz - 2yz$.	[5M]	2	1-6	1,6
7	a)	Write the matrix realting to the quadratic form $ax^2 + 2hxy + by^2$.	[1M]	2	1-6	1,4
	b)	If A = $\begin{bmatrix} 5 & 2 & 3 \\ 0 & -1 & 4 \\ 0 & 0 & 6 \end{bmatrix}$, find eigen values of A^T *and* A^{-1}.	[1M]	2	1-6	6
	c)	Verify Cayley Hamilton theorem for A = $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and hence find A^{-1} and (i) find the value of $B = A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$?. (ii) Find the eigen values of B, where $B = A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$.	[5M]	2	1-6	1,6
	d)	Reduce the following quadratic form to canonical form by orthogonal transformation $Q = 3x^2 + 5y^2 + 3z^2 -$	[5M]	2	1-6	4,6

		$2xy - 2yz + 2xz$.				
8	a)	Find the quadratic form corresponding to the matrix A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.	[1M]	2	1-6	5,6
	b)	State Cayley-Hamilton theorem and write its applications.	[1M]	2	1-6	1,6
	c)	For the matrix A = $\begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$ find the eigen values of $3A^3 + 5A^2 - 6A + 2I$.	[5M]	2	1-6	4,6
	d)	Find the rank, index, nature and signature of quadratic form $x^2 - 2y^2 + 3z^2 - 4yz + 6zx$.	[5M]	2	1-6	4,6

IMPORTANT QUESTIONS

UNIT-II

SHORT ANSWER QUESTIONS (1 MARK)

1. Find the eigen values and eigen vectors of $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.
2. Find the sum and product of the eigen values of the matrix

   A = $\begin{bmatrix} 2 & 5 & 7 \\ 1 & 4 & 6 \\ 2 & -2 & 3 \end{bmatrix}$
3. If λ is an eigen value of a non-singular matrix A then prove that λ^2 is an eigen value of A^2.
4. If λ is an eigen values of an orthogonal matrix then $1/\lambda$ is also its eigen value.
5. Define Algebraic multiplicity of a Characteristic root.
6. Define Geometric multiplicity of a Characteristic root.
7. Write the matrix realting to the quadratic form 2xy+2yz+2zx.

LONG ANSWER QUESTIONS (5 MARKS)

1. Find the eigen values and eigen vectors $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.
2. Diagonalize the matrix A = $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ and find A^4 using modal matrix P ?.
3. Find an orthogonal matrix that will diagonalize the real symmetric A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$. Also find the resulting diagonal matrix.
4. Reduce the quadratic form $2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx$ to canonical form by an orthogonal transformation and hence find rank, index, signature and the nature of the quadratic form.
5. Discuss the nature of the Quadratic form, Find its Rank, Index, and Signature. $x^2 + 4xy + 6xz - y^2 + 2yz + 4z^2$.

Objective (Choose the Correct Answer):

1) If 1,2,3 are the eigen values of A then the eigen values of A^{-1} are [ ]

$a)$1,2,3 b) $1,\frac{1}{2},\frac{1}{3}$ c) 1,4,9 d) 1,8,27

2) If 1,2,3 are the eigen values of A then the eigen values of A^T are [ ]

$a)$1,2,3 b) $1,\frac{1}{2},\frac{1}{3}$ c) 1,4,9 d) 1,8,27

3) If 1,2,3 are the eigen values of A then the eigen values of A^2 are [ ]

$a)$1,2,3 b) $1,\frac{1}{2},\frac{1}{3}$ c) 1,4,9 d) 1,8,27

4) If 1,2,3 are the eigen values of A then the eigen values of A^3 are [ ]

$a)$1,2,3 b) $1,\frac{1}{2},\frac{1}{3}$ c) 1,4,9 d) 1,8,27

5) Diagonal matrix (D) formula = ------------- [ ]

$a) P^{-1}AP$ b) PAP^{-1} c) PD^NP^{-1} d) PD^4P^{-1}

6) Find the value of $A^4 = -----$ [ ]

$a) P^{-1}AP$ b) PAP^{-1} c) PD^NP^{-1} d) PD^4P^{-1}

7) Quadratic form formula is ------------- [ ]

$a)XAX^T$ b) X^TAX c) $X = PY$ d) Y^TDY

8) Canonical form formula is ----------- [ ]

$a)XAX^T$ b) X^TAX c) $X = PY$ d) Y^TDY

9) Linear transformation formula is ------------- [ ]

$a)XAX^T$ b) X^TAX c) $X = PY$ d) Y^TDY

10) The eigen values of $A_{3\times3}$ are 2,8 and sum is 12 find the 3rd eigen value is ----------- [ ]

$a)0$ b) 8 c) 10 d) 2

Fill in the blanks:

11) Characteristic equation is ______________________________

12) If $X_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} and\ X_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$ are pairwise orthogonal then ________________

13) Every square matrix satisfies it's own_____________________________ equation.

14) Find the Eigen values of matrix $I_{3\times3}$ is -------------------

15) The eigen values of $\begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}$ are_____________________

16) The quadratic form corresponding to the matrix A=$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 2 & 3 & 2 \end{bmatrix}$ is________________

17) Write the matrix realting to the quadratic form $x_1^2 + 6x_1x_2 + 5x_2^2$.

18) All eigen values are positive then nature of the quadratic form ______________

19) Eigen values are λ = 1,2,0 , find signature ------------------

20) Define Index -----------------------

Answers

Objective

1) B
2) A
3) C
4) D
5) A
6) D
7) B
8) D
9) C
10) D

Fill in the blanks:

11) $|A - \lambda I| = 0$

12) $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

13) Characteristic equation

14) 1,1,1

15) 1,6

16) $x^2 + y^2 + z^2 + 4zx + 6yz.$

17) $\begin{bmatrix} 1 & 3 \\ 3 & 5 \end{bmatrix}$

18) Positive definite nature.

19) 2

20) Number of positive terms in canonical form (or) normal form (or) eigen values.

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Course: **B. Tech** Branch: **Common to All** Year: **I** **&** Semester: **I**

Subject: **Matrices and Calculus (UNIT-3)**

S.NO		QUESTIONS	Marks	COS	POS	BTL
1.	a)	State Lagrange 's Mean Value Theorem.	[1M]	3	1-6	5,6
	b)	Evaluate $\int_0^1 (1-x)^3 dx$ .using β *and* ρ *functions*	[1M]	3	1-6	6
	c)	Verify Rolle's Theorem for the functions $\log\left(\frac{x^2+ab}{x(a+b)}\right)$ in [a, b] , a > 0, b > 0.	[5M]	3	1-6	4,5,6
	d)	Prove that $\beta(m,n) = \frac{\gamma(m).\gamma(n)}{\gamma(m+n)}$	[5M]	3	6	5,6
2.	a)	State Cauchy's Mean Value Theorem.	[1M]	3	1-6	5,6
	b)	Evaluate $\int_0^\infty e^{-x} x^4 dx$	[1M]	3	1-6	6
	c)	Verify Rolle's theorem for the function f(x) = e^{-x} sin x on [0 , π].	[5M]	3	1-6	6
	d)	Find the Volume of solid generated by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (a > b) about major axis.	[5M]	3	1-6	5,6
3.	a)	State Rolles Mean Value Theorem.	[1M]	3	1-6	5,6
	b)	State Taylors theorem.	[1M]	3	1-6	5,6

	c)	If a < b P.T $\frac{b-a}{1+b^2} < \text{Tan}^{-1} b - \text{Tan}^{-1} a < \frac{b-a}{1+a^2}$ using Lagrange's Mean value Theorem. Deduce the following. i). $\frac{\pi}{4} + \frac{3}{25} < Tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$ ii). $\frac{5\pi+4}{20} < Tan^{-1} 2 < \frac{\pi+2}{4}$	[5M]	3	1-6	6
	d)	Find the Volume formed by the revolution of loop of the curve $y^2(a+x) = x^2(3a-x)$ about the x-axis.	[5M]	3	1-6	6
4.	a)	Give an example which not satisfies Rolle 's Theorem	[1M]	3	1-6	3,4
	b)	Evaluate $\Gamma\left(\frac{9}{2}\right)$	[1M]	3	1-6	6
	c)	Verify Cauchy's Mean value theorem for f(x) = e^x & g(x) = e^{-x} in [3,7] & find the value of c	[5M]	3	1-6	6
	d)	Prove that $\mathbf{log}(1+\mathbf{e}^{\mathbf{x}}) = \mathbf{log}\,2 + \frac{\mathbf{x}}{2} + \frac{\mathbf{x}^2}{8} + \frac{\mathbf{x}^4}{192} + \text{------} \&$ hence deduce that $\frac{\mathbf{e}^{\mathbf{x}}}{\mathbf{e}^{\mathbf{x}}+1} = \frac{1}{2} + \frac{\mathbf{x}}{4} - \frac{\mathbf{x}^3}{48} + \text{---}$	[5M]	3	1-6	6
5	a)	Give an example which not satisfies Lagrange's Mean Value Theorem.	[1M]	3	1-6	4,5
	b)	Find the volume of the sphere of radius is "a".	[1M]	3	1-6	6
	c)	Write Taylor's series for f(x) = $(1-x)^{5/2}$ with Lagrange's form of remainder upto 3 terms in the interval [0,1].	[5M]	3	1-6	4,5,6

	d)	Evaluate $\int_0^1 x^5(1-x^3)^{10}\,dx$	[5M]	3	1-6	6
6	a)	Define Beta function.	[1M]	3	1-6	1
	b)	Expand $f(x)=e^x$	[1M]	3	1-6	3,4
	c)	Expand $\mathbf{e^x \sin x}$ in powers of x	[5M]	3	1-6	3,6
	d)	S.T $\int_a^b (x-a)^m (b-x)^n\,dx = (b-a)^{m+n+1}\beta(m+1, n+1)$	[5M]	3	1-6	6
7	a)	Define Gamma function.	[1M]	3	1-6	2
	b)	Verify rolles theorem for $f(x) = Tanx$ in $[0,\ \pi]$.	[1M]	3	1-6	6
	c)	Find the area of the surface generated by revolving arc of the catenary $y = c\cosh\left(\frac{x}{c}\right)$ from x=0 to x=c about the x-axis.	[5M]	3	1-6	6
	d)	To P.T $\rho(n)\rho(1-n) = \frac{\pi}{\sin n\pi}$ and Also S.T $\rho\left(\frac{1}{2}\right) = \sqrt{\pi}$.	[5M]	3	1-6	6
8	a)	Establish relation between beta and gamma functions.	[1M]	3	1-6	5
	b)	Write surface area formula.	[1M]	3	1-6	2
	c)	Prove that $\frac{\pi}{3} - \frac{1}{5\sqrt{3}} > \cos^{-1}\frac{3}{5} > \frac{\pi}{3} - \frac{1}{8}$ using Lagrange's mean value theorem.	[5M]	3	1-6	6

	d)	Prove that $$\int_0^1 \frac{x^2}{\sqrt{(1-x^4)}}dx \times \int_0^1 \frac{1}{\sqrt{(1+x^4)}}dx = \frac{\pi}{4\sqrt{2}}.$$	[5M]	3	1-6	6

Extra IMP questions: (1 MARKS):-

1. $\rho\left(\frac{1}{2}\right) = \sqrt{\pi}.$

2. To P.T $\beta(m,n) = \beta(n,m)$..

3. Verify rolles theorem for $f(x) = 2x^3 + x^2 - 4x - 2$ in $\left[-\sqrt{3}, \sqrt{3}\right]$.

4. Verify rolles theorem for $f(x) = x^3$ in $[1,\ 3]$.

5. Expand Sin x in powers of "x".

6. To P.T $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$.

7. Find the value of $\rho\left(\frac{-7}{2}\right)$.

8. P.T $\int_0^{\pi/2} sin^2\theta\ cos^4\theta\ d\theta = \frac{\pi}{32}$.

9. P.T $\int_0^{\propto} \frac{x^8(1-x^6)}{(1+x)^{24}}dx = 0$ using β and ρ functions.

Extra IMP questions: (5 MARKS):-

1) Verify Rolle's theorem for the function $f(x) = (x-a)^m(x-b)^n$ where m,n are positive integers in [a,b].

2) Verify Rolle's theorem for the function $f(x) = e^x (\sin x - \cos x)$ on [π/4 , 5π /4]

3) Verify Lagrange's mean value theorem for f(x) = x (x-1) (x-2) in [0 , 1/2]

4) Verify Lagrange's mean value theorem for f(x) = (x-1)(x-2)(x-3) on [0,4]

5) Show that $x > 0,\ 1 + x < e^x < 1 = xe^x$.

6) Using Mean value theorem prove that Tan x > x in 0<x<π/2.

7) If a < b P.T $\frac{b-a}{\sqrt{1+a^2}} < \sin^{-1} b - \sin^{-1} a < \frac{b-a}{\sqrt{1+b^2}}$ using Lagrange's Mean value Theorem.

8) P.T $\frac{b-a}{b} < \log\left(\frac{b}{a}\right) < \frac{b-a}{a}$ for 0<a<b. Hence deduce that $\frac{1}{4} < \log\left(\frac{4}{3}\right) < \frac{1}{3}$.

9) Verify Rolle's theorem for f(x) = x (x+3) $e^{(-x/2)}$ on [-3 , 0]

10) Find c of Cauchy's mean value theorem for $f(x) = \sqrt{x}$ & $g(x) = \frac{1}{\sqrt{x}}$

11) Verify Cauchy's Mean value theorem for f(x) = $(1/x^2)$, g(x) = (1/x) on [a , b] ; a ,b > 0

12) Verify mean value theorem for f(x) and $f^1(x)$ in (1 ,e) given that f(x) = log x

13) Expand $\frac{\sin^{-1} x}{\sqrt{1-x^2}}$ in powers of x.

14) Evaluate $\int_0^1 x^m (\log x)^m dx$.

15) Evaluate $\int_0^\infty a^{-bx^2} dx$.

16) Prove that $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$

17) Evaluate $\int_0^{\pi/2} \sqrt{\operatorname{Tan}\theta}\, d\theta$

18) Evaluate $\int_0^\infty \frac{x^c}{c^x} dx$.

Objective-PART-A (Choose the Correct Answer)

1) The value of "c" of Rolles theorem for $f(x) = \frac{\sin x}{e^x}$ $in\ (0,\pi) is$ [ ]

$a)\ \pi$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

2) The value of "c" of Cauchys theorem for $f(x) = e^x$ and $g(x) = e^{-x} in\ [a,b] is$ [ ]

a) $\sqrt{ab}$ b) $\frac{a-b}{2}$ c) $\frac{a+b}{2}$ d) $\frac{2ab}{a+b}$

3) The value of "c" of Cauchys theorem for $f(x) = \sqrt{x}\ and\ g(x) = \frac{1}{\sqrt{x}} in\ [a, b]\ is$ [ ]

a) $\sqrt{ab}$ b) $\frac{a-b}{2}$ c) $\frac{a+b}{2}$ d) $\frac{2ab}{a+b}$

4) The value of "c" of Cauchys theorem for $f(x) = \sin x\ and\ g(x) = \cos x\ in\ \left[0, \frac{\pi}{2}\right]\ is$ [ ]

a) π b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

5) Find the Volume of solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a > b) about minor axis.

a) $\frac{2}{3}\pi a^2 b$ b) $\frac{2}{3}\pi a b^2$ c) $\frac{4}{3}\pi a^2 b$ d) $\frac{4}{3}\pi a b^2$ [ ]

6) If the circle $x^2 + y^2 = r^2$ is rotated about its diameter, the volume so generated is [ ]

a) $\frac{4}{3}\pi r^3$ b) $\frac{2}{3}\pi r^3$ c) $4\pi r^3$ d) $2\pi r^3$

7) The value of $\beta(1,2) + \beta(2,1)$ is [ ]

a) 0 b) 1 c) 2 d) 3

8) $\Gamma(n + 1) =$ __if n is positive real number [ ]

a) $n!$ b) $(n - 1)!$ c) $(n + 1)!$ d) n

9) $\Gamma(3/4)\, \Gamma(1/4) =$ [ ]

a) $\pi\sqrt{2}$ b) $\pi\sqrt{3}$ c) $\pi\sqrt{4}$ d) none

10) *The value of* $\Gamma\left(-\frac{1}{2}\right) = - - - - -$ [ ]

a) $-\sqrt{\pi}$ b) $-2\sqrt{\pi}$

c) $-3\sqrt{\pi}$ d) $2\sqrt{\pi}$

<u>Objective- PART-B (Fill in the Blanks)</u>

11) Geometrical interpretation of rolles theorem is ----------------------

12) The value of “c”=--------------- of lagranges mean value theorem for $f(x) = x^2\ in\ [1, 5]$

13) The value of “c” of Cauchys theorem for $f(x) = x^2\ and\ g(x) = x^3\ in\ [1,2]\ is$ --------------------

14) $The\ value\ of\ \Gamma(4) = -----$

15) Expand cos x in powers of “x” ------------------

16) The vaule of $\int_0^{\infty} e^{-x^2} dx$ = ------------------

17) The value of $\beta\left(\frac{1}{2}, \frac{1}{2}\right) =$ ---------------------

18) In terms of β function $\int_0^{\frac{\pi}{2}} cos^n\theta\ d\theta =$------------------------

19) Write the relation between beta and gamma functions -------------------

20) $The\ value\ of\ \Gamma\left(\frac{5}{2}\right) = -----$

Answers

Objective

1) B
2) C
3) A
4) B
5) C
6) A
7) B
8) A
9) A
10) B

Fill in the blanks:

11) There is at least one point on the curve, where the tangent is parallel to X-axis.

12) 3

13) $\frac{14}{9}$

14) 6

15) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$etc

16) $\frac{\sqrt{\pi}}{2}$

17) π

18) $\frac{1}{2}\beta\left(\frac{1}{2}, \frac{n+1}{2}\right)$

19) $\beta(m, n) = \frac{\gamma(m).\gamma(n)}{\gamma(m+n)}$

20) $\frac{3}{4}\sqrt{\pi}$

VIGNAN INSTITUTE OF TECHNOLOGY AND SCIENCE

Vignan Hills, Deshmukhi(V), Pochampally(M),
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Course: **B. Tech** Branch: **Common to All** Year: **I** **&** Semester: **I**

Subject: **Matrices and Calculus (UNIT-4)**

S.NO		QUESTIONS	Marks	COS	POS	BTL
1.	a)	Evalute $\lim_{\substack{x\to 1\\ y\to 2}} \frac{2x^2y}{x^2+y^2+1}$.	[1M]	4	1-6	6
	b)	If f (x, y) = $x^2 + y^2$, then $\frac{\partial^2 f}{\partial x \partial y}$.	[1M]	4	1-6	3,4
	c)	Find three positive members whose sum is 100 and whose product is maximum.	[5M]	4	1-6	5
	d)	If u = x + y + z ; uv = y + z ; uvw = z, then evaluate $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.	[5M]	4	1-6	6
2.	a)	Verify the continuity of $f(x,y) = \begin{cases} \frac{2xy}{x^2+y^2}, & if\ \ (x,y) \neq (0,0) \\ 0, & if\ \ (x,y) = (0,0) \end{cases}$	[1M]	4	1-6	6
	b)	Find the degree of the homogeneous functions Z = $\frac{\sqrt{x}+\sqrt{y}}{x+y}$.	[1M]	4	1-6	5
	c)	If $u = xy + yz + zx$, $v = x^2 + y^2 + z^2$, $w = x + y + z$ then show that the functions are functionally dependent and hence find the relation between them.	[5M]	4	1-6	6
	d)	Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.	[5M]	4	1-6	6
3.	a)	Find first and second order pattial derivatives of $ax^2 + 2hxy + by^2$.	[1M]	4	1-6	5
	b)	If u = $\tan^{-1}\frac{y}{x}$ then find x $\frac{\partial u}{\partial x}$ + y $\frac{\partial u}{\partial y}$.	[1M]	4	1-6	4
	c)	Find the extreme values of $f(x,y) = sinx.siny.\sin(x+y)$.	[5M]	4	1-6	5
	d)	If $x = e^r sec\theta$, $y = e^r tan\theta$ then prove that	[5M]	4	1-6	6

		$JJ^I = 1$.				
4.	a)	Define a Homogeneous function.	[1M]	4	1-6	4
	b)	If x = r cosθ ,y = r sinθ then find $\frac{\partial(r,\theta)}{\partial(x,y)}$	[1M]	4	1-6	5
	c)	Find the extreme values $u(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.	[5M]	4	1-6	6
	d)	Verify Euler's theorem for u = $\frac{x}{y} + \frac{y}{z} + \frac{z}{x}$.	[5M]	4	1-6	6
5	a)	Find the maximum and minimum values of the function $f(x) = x^5 + 3x^4 + 5$.	[1M]	4	1-6	5
	b)	State Euler's theorem on homogeneous function in x and y.	[1M]	4	1-6	6
	c)	A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction.	[5M]	4	1-6	6
	d)	If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then prove that $(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z})^2 u = \frac{-9}{(x+y+z)^2}$.	[5M]	4	1-6	6
6	a)	If $U = \frac{x^2y^2}{x+y}, then\ find\ x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y}$.	[1M]	4	1-6	6
	b)	Find the stationary points of $x^4 + y^4 - 2x^2 + 4xy - 2y^2$.	[1M]	4	1-6	5
	c)	If $x = r\cos\theta$ and $y = r\sin\theta$, show that $\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} = \frac{1}{r}\left[\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2\right]$.	[5M]	4	1-6	5,6
	d)	Find the minimum values of $(x^2 + y^2 + z^2)$, given that $xyz = a^3$	[5M]	4	1-6	6
7	a)	If $x = u(1+v),\ y = v(1+u)$ then find $\frac{\partial(x,y)}{\partial(u,v)}$.	[1M]	4	1-6	5
	b)	Define i) Saddle point and ii) Stationary point.	[1M]	4	1-6	2
	c)	Find the maximum and minimum distance of the points (3, 4, 12) from the sphere	[5M]	4	1-6	6

		$x^2+y^2+z^2=1.$				
	d)	If $u=\frac{yz}{x}, v=\frac{zx}{y}, w=\frac{xy}{z}, show\ that\ \frac{\partial(u,v,w)}{\partial(x,y,z)}=4$	[5M]	4	1-6	6
8	a)	Define functionally dependent.	[1M]	4	1-6	2
	b)	Write the sufficient condition's for the existence of maxima & minimam of f(x, y).	[1M]	4	1-6	6
	c)	If $f(x,y)=\log(x^2+y^2)+\tan^{-1}\left(\frac{y}{x}\right)$ then prove that $f_{xx}+f_{yy}=0.$	[5M]	4	1-6	6
	d)	Find the minimum value of $x^2+y^2+z^2$ Given x + y + z = 3a.	[5M]	4	1-6	6

Extra IMP questions: (1 MARKS):-

1. Define functionally Independent.
2. Define continuity.
3. Verify the continuity of $f(x,y)=\begin{cases}\frac{x^2-y^2}{x^2+y^2}, & if\ (x,y)\neq(0,0)\\ 0, & if\ (x,y)=(0,0)\end{cases}$
4. If u = e^x, find $\frac{\partial^2 u}{\partial y\partial x}$
5. If $x=\frac{u^2}{v}, y=\frac{v^2}{u}\ find\ \frac{\partial(u,v)}{\partial(x,y)}$.
6. If $x=uv, y=\frac{u}{v}\ then\ find\ \frac{\partial(x,y)}{\partial(u,v)}$.
7. If $u=log\left(\frac{x^2+y^2}{x+y}\right)$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=1$
8. If $u=Sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=tanu.$

Extra IMP questions: (5 MARKS):-

1. If u = x^2-2y ,v = x + y + z ,w = x - 2y + 3z then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

2. If $u = f(r)$ and $x = rcos\theta, y = rsin\theta$ $prove$ $that$ $\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$.

3. If $x = rsin\theta cos\emptyset, y = rsin\theta\ sin\emptyset, z = rcos\theta$ then find $\frac{\partial(x,y,z)}{\partial(r,\theta,\emptyset)}$.

4. Verify Euler's theorem for the function $xy + yz + zx$.

5. Verify Euler's theorem for the function $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$.

6. If $u = x + y + z$, $v = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$, $w = x^3 + y^3 + z^3 - 3xyz$ then show that the functions are functionally related.

7. If $u = \frac{x+y}{1-xy}$, $v = \tan^{-1} x + \tan^{-1} y$, then show that the functions are functionally dependent and find the relation between them.

8. Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.

9. Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third is maximum.

10. The temperature T at any point (x, y, z) in the space $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.

11. Find the maxima & minima of the function f(x) = x^3y^2 (1-x-y).

12. Find the maxima and minima of the function $f(x) = 2(x^2 - y^2) - x^4 + y^4$

13. Find the maxima & minima of the function $f(x) = x^4 + y^4 - 2x^2 - 2y^2 + 4xy$

14. Find the maximum & minimum distances of the point origin from the sphere $x^2 + y^2 + z^2 = 4$

Objective-PART-A (Choose the Correct Answer)

1) The degree of the homogeneous function $z = \frac{\sqrt{x}+\sqrt{y}}{x+y}$ is ---- [ ]

   a) $\frac{1}{2}$ b) 1 c) $\frac{-1}{2}$ d) -1

2) If $u = \frac{xy}{x+y}$ $then$ $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = ---$ [ ]

   a) 2u b) u c) 0 d) 1

3) The function $f(x.y)$ has maximum value for --------- [ ]

   a) $ln - m^2 = 0$ b) $ln - m^2 < 0$ c) $ln - m^2 > 0, l > 0$ d) $ln - m^2 > 0, l < 0$

4) The function $f(x.y)$ has minimum value for ------------ [ ]

   a) $ln - m^2 = 0$ b) $ln - m^2 < 0$ c) $ln - m^2 > 0, l > 0$ d) $ln - m^2 > 0, l < 0$

5) The function $f(x.y)$ is not an extreme value for ----------- [ ]

a) $ln - m^2 = 0$ b) $ln - m^2 < 0$ c) $ln - m^2 > 0, l > 0$ d) $ln - m^2 > 0, l < 0$

6) The function $f(x.y)$ has no conclusion for ----------- [ ]

a) $ln - m^2 = 0$ b) $ln - m^2 < 0$ c) $ln - m^2 > 0, l > 0$ d) $ln - m^2 > 0, l < 0$

7) If $u = xy,\ then\ x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = - - - - - -$ [ ]

a) u b) 2u c) 0 d) none

8) If $u = log(x^3 + y^3 + z^3 - 3xyz)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = - - -$ [ ]

a) 4u b) 3u c) 2u d) u

9) The stationary points of $x^4 + y^4 - 2x^2 + 4xy - 2y^2\ is$ [ ]

a) $(\sqrt{2}, -\sqrt{2})$ b) $(\sqrt{2}, \sqrt{2})$ c) $(-\sqrt{2}, \sqrt{2})$ $d)$ $(\sqrt{2},\ 0)$

10) If $u = e^{x/y}$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = - - -$ [ ]

a) 0 b) 1 c) 2 d) none

Objective- PART-B (Fill in the Blanks)

11) The degree of the homogeneous function $f = ax^2 + 2hxy + by^2$ is --------

12) If r^2=$x^2 + y^2 + z^2\ and\ u = r^m\ then\ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = - - - - -$

13) If $u = x^2y, v = xy^2\ \ then\ \frac{\partial(u,v)}{\partial(x,y)} = - - - - - - -$

14) The minimum value of $x^2 + y^2 + z^2\ given\ that\ x + y + z = 3a\ is - - - -$

15) The stationary points of $x^3y^2(1 - x - y)$ are ---------------

16) If $x = r\ cos\theta, y = r\ sin\theta, then\ \frac{\partial x}{\partial r} = - - - - - -and\ \frac{\partial y}{\partial \theta} = - - - - - - -$

17) The rectangular parallelepiped of maximum volume that can be inscribed in a sphere is a ----------

18) $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(u,v)} = - - - - - - -$

19) Two functions u and v are said to be functionally dependent if $\frac{\partial(u,v)}{\partial(x,y)} = - - - -$

20) If u is a homogeneous function of $x\ and\ y$ of degree $n\ then$

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = - - - - --$$

Answers

Objective

1) C
2) B
3) D
4) C
5) B
6) A
7) B
8) B
9) A
10) A

Fill in the blanks:

11) 2
12) $m(m+1)r^{m-2}$
13) $5x^2y^2$
14) $3a^2$
15) $(0,1)$
16) $cos\theta, rcos\theta$
17) $cube$

18) 1

19) 0

20) $n(n-1)u$.

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VIGNAN INSTITUTE OF TECHNOLOGY AND SCIENCE

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Course: **B. Tech** Branch: **Common to All** Year: **I** **&** Semester: **I**

Subject: **Matrices and Calculus (UNIT-5)**

S.NO		QUESTIONS	Marks	COS	POS	BTL
1.	a)	Evaluate $\int_0^1 \int_1^2 xydxdy$	[1M]	5	1-6	6
	b)	Evaluate $\int_0^1 \int_0^1 \int_0^1 dx\, dy\, dz$.	[1M]	5	1-6	6
	c)	By changing the order of integration evaluate $\int_0^1 \int_{x^2}^{2-x} xydxdy$.	[5M]	5	1-6	6
	d)	Change into polar coordinates and evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dydx$. Hence show that $\int_0^\propto e^{-x^2}\ dx = \frac{\sqrt{\pi}}{2}$.	[5M]	5	1-6	6
2.	a)	Evaluate $\int_1^2 \int_0^3 xy(1+x+y)\, dx\, dy$.	[1M]	5	1-6	6
	b)	Evaluate $\int_0^1 \int_1^2 \int_2^3 xyz\, dx\, dy\, dz$.	[1M]	5	1-6	6
	c)	By changing the order of integration evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} y^2 dxdy$.	[5M]	5	1-6	6
	d)	Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dzdydx$.	[5M]	5	1-6	6
3.	a)	Evaluate $\int_0^2 \int_0^x e^{x+y}\, dx\, dy$.	[1M]	5	1-6	6
	b)	Evaluate $\int_0^1 dx \int_0^2 dy \int_0^2 x^2yz\, dz$.	[1M]	5	1-6	6

	c)	Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.	[5M]	5	1-6	6
	d)	Evaluate $\iint r^3 dr d\theta$ over the area included between the circles r = 2sinθ and r = 4sinθ.	[5M]	5	1-6	6
4.	a)	Evaluate $\int_0^{\pi}\int_0^{a\cos\theta} dr d\theta$.	[1M]	5	1-6	6
	b)	Evaluate $\int_0^1\int_y^1\int_0^{1-x} x\, dz\, dy\, dx$.	[1M]	5	1-6	6
	c)	Evaluate by changing order of integration $\int_0^1\int_x^{\sqrt{2-x^2}} \frac{x^2}{\sqrt{x^2+y^2}} dydx$.	[5M]	5	1-6	6
	d)	Evaluate $\int_0^2\int_0^{\sqrt{2x-x^2}} (x^2+y^2) dxdy$ by changing into polar coordinates.	[5M]	5	1-6	6
5	a)	Write formula of Area in double integral (Cartesian form).	[1M]	5	1-6	6
	b)	Evaluate $\int_0^1\int_0^1\int_0^1 xyz\, dx\, dy\, dz$.	[1M]	5	1-6	6
	c)	Evaluate $\int_0^{\pi}\int_0^{a(1+\cos\theta)} r^2\cos\theta dr d\theta$.	[5M]	5	1-6	6
	d)	Find the area bounded between the parabolas $y^2 = 4ax \ and \ x^2 = 4ay$.	[5M]	5	1-6	6
6	a)	Write Area formula of polar form in double integral.	[1M]	5	1-6	5
	b)	Evaluate $\int_{-1}^1\int_{-2}^2\int_{-3}^3 dx\, dy\, dz$.	[1M]	5	1-6	6

	c)	Evaluate $\iint_R y\,dxdy$ where R is the region bounded by the parabolas $y^2 = 4x \; and x^2 = 4y$.	[5M]	5	1-6	6
	d)	Find the volume of the ellipsoid $x^2 + y^2 + z^2 = a^2$.	[5M]	5	1-6	5
7	a)	Write formula of Volume in double integral (Cartesian form).	[1M]	5	1-6	5
	b)	Evaluate $\int_1^3 \int_0^1 xy^2 dxdy$.	[1M]	5	1-6	6
	c)	Find the value of $\iint xydxdy$ taken over the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.	[5M]	5	1-6	6
	d)	Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$.	[5M]	5	1-6	6
8	a)	Write formula of Volume in triple integral.	[1M]	5	1-6	3
	b)	Evaluate $\int_1^2 \int_0^x (x + y^2) dydx$.	[1M]	5	1-6	6
	c)	Evaluate $\iint r^2 sin\theta dr d\theta$, over the cardiod r = a(1+cosθ)above the initial line.	[5M]	5	1-6	6
	d)	Evaluate $\iiint xy^2 z \; dx \; dy \; dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.	[5M]	5	1-6	6

Extra IMP questions: (1 MARKS):-

1. Evaluate $\int_{x=0}^{a}\int_{y=0}^{b}(x^2+y^2)dydx$.
2. Evaluate $\int_0^3\int_0^2(4-y)^2\,dy\,dx$.
3. Find the area of the circle using double integral.
4. Evaluate $\iint(x^2+y^2)dxdy$, over the region in the positive quadrant for which $x+y\le 1$.
5. Evaluate $\iint x^2\,dxdy$ over the region bounded by hyperbola $xy=4, y=0, x=1, x=4$.
6. Evaluate $\int_0^{\pi}\int_0^{a\,sin\theta} r\,dr\,d\theta$.
7. Evaluate $\int_0^{\frac{\pi}{2}}\int_0^{\propto}\frac{r\,dr\,d\theta}{(r^2+a^2)^2}$.
8. Evaluate $\iiint(x+y+z)dx\,dy\,dz$, taken over the volume bounded by the planes $x=0, x=1, y=0, y=1$ and $z=0, z=1$.

Extra IMP questions: (5 MARKS):-

1) Evaluate $\int_0^1\int_x^{\sqrt{x}}(x^2+y^2)\,dxdy$.
2) Evaluate $\int_0^1\int_0^{\sqrt{1+x^2}}\frac{dx\,dy}{1+x^2+y^2}$.
3) Evaluate $\int_0^{\pi}\int_0^{x^2}x(x^2+y^2)dxdy$.
4) Evaluate the following integral by transforming into polar coordinates.

   $\int_0^a\int_0^{\sqrt{a^2-x^2}}y\sqrt{x^2+y^2}\,dx\,dy$.
5) Evaluate the following integral by transforming into polar coordinates.

   $\int_0^a\int_0^{\sqrt{a^2-y^2}}(x^2+y^2)\,dx\,dy$.
6) Evaluate $\int_0^{4a}\int_{y^2/4a}^{y}\frac{x^2-y^2}{x^2+y^2}dx\,dy$ by changing to polar coordinates.
7) By changing into polar coordinates evaluate the integral $\int_0^{\propto}\int_0^{\propto}\frac{x^2}{(x^2+y^2)^{3/2}}\,dx\,dy$.
8) Find the limits after changing the order of integration for $\int_0^b\int_0^{a\sqrt{b^2-y^2}/b}xy\,dx\,dy$.

9) Change the order of integration and solve $\int_0^a \int_{x^2/a}^{2a-x} xy^2 \; dy\, dx$.

10) Find $\iint (x+y)^2 \, dxdy$ over the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

11) Evaluate $\iint (x^2+y^2)\, dxdy$ over the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

12) Find the value of $\iint (x+y)dxdy$ over the region in the positive quadrant bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

13) Evaluate $\int_0^{\frac{\pi}{4}} \int_0^{a\, sin\theta} \frac{r\, dr\, d\theta}{\sqrt{a^2-r^2}}$.

14) Evaluate $\iint r\, sin\theta \, dr\, d\theta$ over the cardioid $r = a(1-cos\theta)$ above the initial line.

15) Evaluate $\iint r\, sin\theta \, dr\, d\theta$ over the cardioid $r = a(1+cos\theta)$ above the initial line.

16) Find the area bounded by the curves $y = x, y = x^2$.

17) Find the area of the loop of the curve $r = a(1+cos\theta)$.

18) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} dxdydz$

19) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\; dxdydz$

20) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dx\, dy\, dz$.

21) Evaluate $\int_0^{log2} \int_0^x \int_0^{x+logy} e^{x+y+z}\, dz\, dy\, dx$.

22) Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z)dxdydz$.

23) **F**ind the volume of the solid bounded by the planes x = 0, y = 0, x + y + z = a and z = 0.

24) Evaluate $\iiint xyz\; dx\, dy\, dz$ taken through the positive octant of the sphere $x^2+y^2+z^2=a^2$.

25) Evaluate $\iiint (x^2+y^2+z^2)\; dx\, dy\, dz$ taken over the volume enclosed by the sphere $x^2+y^2+z^2=1$,

By transforming into spherical polar coordinates.

Objective-PART-A (Choose the Correct Answer)

1. $\int_0^1\int_0^1\int_0^1 e^{x+y+z} dxdydz =$ [ ]

(a) $(e-1)^2$ (b) e-1 (c) $(e-1)^3$ (d) e+1

2. $\int_0^2 \int_0^{x^2} x(x^2+y^2)dxdy =$ ---- [ ]

(a) $\frac{32}{3}$ (b) $\frac{64}{3}$ (c) $\frac{84}{3}$ (d) none

3. $\int_0^{\pi} \int_0^{a\cos\theta} r\sin\theta drd\theta. = ----$ [ ]

(a) $\frac{a^2}{2}$ (b) $\frac{a^2}{3}$ (c) $\frac{a^3}{3}$ (d) $\frac{a^3}{4}$

4. $\int_0^1 dx \int_0^x e^{y/x} dy =$ [ ]

(a) e-1 (b) $\frac{e-1}{2}$ (c) $\frac{e-1}{3}$ (d)none

5. $\iiint_V dv =$ [ ]

(a) $\iiint_V dxdydz$ (b) $\iiint_V dydxdz$ (c) $\iiint_V dzdxdy$ (d)All the above

6. The iterated integral with the order of integration reversed for $\int_0^1 \int_1^{e^x} dydx =$

(a) $\int_0^1 \int_1^{e^y} dxdy$ (b) $\int_1^e \int_{\log y}^1 dxdy$ (c) $\int_e^1 \int_1^{\log y} dxdy$ (d)none [ ]

7. $\int_{-1}^1 \int_{-2}^2 \int_{-3}^3 dxdydz =$ [ ]

(a) 24 (b) 36 (c) 48 (d)54

8. The volume of the tetrahedron bounded by x = 0,y = 0,z = 0 and x + y + z = 1 is

(a) 1/2 (b) 1/3 (c) 1/4 (d)1/6 [ ]

9. $\int_0^2 \int_0^x (x+y)dxdy =$ [ ]

(a) 1 (b) 2 (c) 3 (d)4

10. The area enclosed by the parabolas $x^2 = y$ and $y^2 = x$ [ ]

(a)1/3 Sq.units (b) 1/4 Sq.units (c) 1/2 Sq.units (d) 1/5 Sq.units

Fill in the blanks:

21) $\iint_A dxdy$ represents ______________

22) $\int_0^1 \int_1^2 xydydx$____________

23) $\int_0^a \int_0^{\sqrt{ay}} xydxdy =$ ___________________

24) $\int_1^0 \int_0^1 (x+y)dxdy =$_______________

25) $\int_0^1 \int_0^x e^{x+y} \; dydx =$ ________________

26) $\int_0^1 \int_x^{\sqrt{x}} xydxdy =$ _____________________

27) $\iiint_V dxdydz$ represents = __________________

28) $\int_0^1 \int_0^2 \int_1^2 x^2 yzdxdydz =$_______

29) The iterated integral for $\int_0^1 \int_x^{\sqrt{x}} f(x,y)dxdy$ after changing the order of integration is _____________________

30) The volume of the tetrahedron bounded by the surfaces x = 0, y = 0, z = 0 and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ is_______________.

Answers

Objective

1) C
2) B
3) C
4) B
5) D

6) B

7) C

8) D

9) D

10) A

Fill in the blanks:

11) Area of the surface

12) $\frac{3}{4}$

13) $a^{\frac{4}{6}}$

14) 1

15) $\frac{(e-1)^2}{2}$

16) $\frac{1}{24}$

17) $Volume$

18) 1

19) $\int_0^1 \int_{y^2}^{y} dxdy$

20) $\frac{abc}{6}$