Differential Equations: Final Project

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Problem 9

A motorboat weighs 32,000 lb and its motor provides a thrust of 5000 lb. Assue that the water resistance is 100 pounds for each foot per second of the speed v of the boat. Then

$$1000\frac{dv}{dt} = 5000 - 100v$$

If the boat starts from rest, what is the maximum velocity that it can attain?

Solution

We begin by simplifying the formula used in the problem

$$\begin{aligned} 1000\frac{dv}{dt} &= 5000 - 100v \\ 10\frac{dv}{dt} &= 50 - v \end{aligned}$$

We then integrate to find velocity as a function off time

$$\begin{aligned} \int \frac{dv}{(50-v)} &= \frac{1}{10}\int dt \\ -\ln(50-v) &= \frac{1}{10}t + C \\ \ln(50-v) &= -\frac{1}{10}t - C \\ 50 - v &= C_1 e^{-\frac{t}{10}} \\ v(t) &= 50 + C_2 e^{-\frac{t}{10}} \end{aligned}$$

Given that initial velocity is 0, we can then use our function to find C_2

$$\begin{aligned} v(0) &= 50 + C_2 e^{-\frac{0}{10}} \\ 0 &= 50 + C_2 e^0 C_2 = -50 \end{aligned}$$

Substituting back into our equation, we obtain

$$v(t) = 50 - 50e^{-\frac{t}{10}}$$

Finally, we examine the limit as t approaches infinity to find the maximum velocity

$$\lim_{t\to\infty} 50 - 50e^{-\frac{t}{10}} = 50 - 50(0) = 50$$

Therefore,

$$v_{max} \approx 50\frac{ft}{sec}$$