ASTR 4201/5201 (Fall 2016)

Homework #6

Due: Friday, October 7 in class

The questions labeled G1 is required for graduate students. Please remember to show your work. **Note:** You will need the handout from Wednesday's (Sep. 28) lecture (also available on blackboard).

1. The data in the table below come from two locations in the interior of a single main-sequence stellar model:

eq 10.68 p.316
eq 10.89 p.322

r	$M(r)$	$L(r)$	$T(r)$	$\rho(r)$	$\bar{\kappa}$
$0.242R_\odot$	$0.199M_\odot$	$340L_\odot$	2.52×10^7 K	1.88×10^4 kg m^{-3}	0.044 m^2 kg^{-1}
$0.670R_\odot$	$2.487M_\odot$	$528L_\odot$	1.47×10^7 K	6.91×10^3 kg m^{-3}	0.059 m^2 kg^{-1}

   (a) Is energy transport at these two locations in the star convective or radiative? You may assume that radiation pressure is negligible so that the gas behaves like a monatomic ideal gas with a mean molecular weight ($\bar{\mu}_m$) of 0.7 amu.

   (b) Explain your results in terms of the four situations where convection occurs that I listed in class and our discussion about the internal structures of main-sequence stars.

2. In class I used Sirius (the brightest star in the night sky) as my example of an astrometric binary. This was first noticed in 1844 by Bessel, the astronomer who also made the first parallax measurement of a star. He had trouble understanding his observations of Sirius because it had too much of a "wobble" for a simple parallax. Eventually it became clear that the system was a binary with a period of 50 yr and a total semimajor axis (of the reduced mass) of 7.61″. You may assume that this is in the plane of the sky (so ignore inclination) and that Sirius has a parallax measured by *Hipparcos* of 0.379″. We can now resolve the system into two components, Sirius A and the much fainter Sirius B, making this a visual binary. Assume that the ratio of the distances from the center of mass is $a_A/a_B = 0.466$.

107.4
p.200

$\frac{M_1}{M_2} = \frac{r_2}{r_1}$ $\quad P^2 = \frac{4\pi^2}{G(m_1+m_2)} a^3$ $\quad \alpha =$

   (a) Find the masses of both members of the system.

   (b) Sirius A has an absolute bolometric magnitude of 1.36 and that of Sirius B is 8.79. Determine their luminosities in terms of the Solar luminosity (and note the huge ratio between the two!).

   (c) Astronomers were shocked to learn that Sirius B is very blue, with $T_e \approx 25{,}200$ K. Estimate its radius and compare to those of the Sun and Earth.

   (d) Use this to compute the average density of Sirius B and compare to that of the Earth.

3. Solve the Lane-Emden equation for the case of n=0 by applying the appropriate boundary conditions to find $D_0(\xi)$. Our use of the dimensionless variable ξ and function D_0 obscures the physical significance of this solution. Find an expression for the actual density ρ in terms of the physical radius r. Does this seem like it describes a real star?

4. On Friday, we talked about using homology relationships to scale stellar structure from a known model of a star. Section 5 of the handout shows how to use the four equations of stellar structure combined with an assumption that the four variables (r, ρ, L, T) have power-law dependencies on the stellar mass M to arrive at a set of four coupled linear equations for the four exponents (e.g., $T \propto M^{\alpha_T}$). This is a linear algebra exercise that can either be solved by repeatedly eliminating a variable, or as an exercise in matrix multiplication (eqn. 32).

   (a) The handout leaves the energy generation equation in terms of two power-law dependencies, α and β. Insert the correct values for the CNO cycle (using the power-law approximation in the neighborhood of T=1.5×10^7 K) and solve the system of equations to get the four power-law exponents for (r, ρ, L, T) as functions of mass.

   (b) The trick of using linear algebra to solve the system of equations could make the coefficients resulting from this analysis appear to be almost random numbers. However, they do contain all of the input physics relating how various parameters scale with each other. To demonstrate this, use the virial theorem to find a scaling relationship to relate the average temperature T to the mass and radius. (Find the proportionality, don't worry about numerical coefficients because they don't matter for this problem.) Now write this relationship as an expression for α_T in terms of any other α parameters. Do your derived coefficients for the CNO cycle analysis obey this relationship?

   (c) As the handout states, the temperature in this approximation is some average internal T. If you want to compare to observations of stars, we only see the surface temperature, $T_{\text{eff}} \propto (L/R^2)^{1/4}$. Using the power-law dependencies from the previous section, find an expression for the dependence of luminosity on T_{eff}. Hint: using the expressions from 4a, eliminate the mass dependence to find a power-law relationship for the relationship between radius and luminosity. Insert this into the T_{eff} relationship to find T_{eff} as a function of L and convert that to $L \propto T_{\text{eff}}^{\alpha_{L,T}}$ and find $\alpha_{L,T}$.

   (d) Use Figure 10.13 from Carroll & Ostlie (repeated below) and the fact that the slope of a line on a log-log plot tells you the power-law dependence between the two quantities to estimate the power-law relationship between L and T_{eff} in the plotted stellar models (over the range of stellar masses where CNO burning is appropriate) and compare to your previous result.

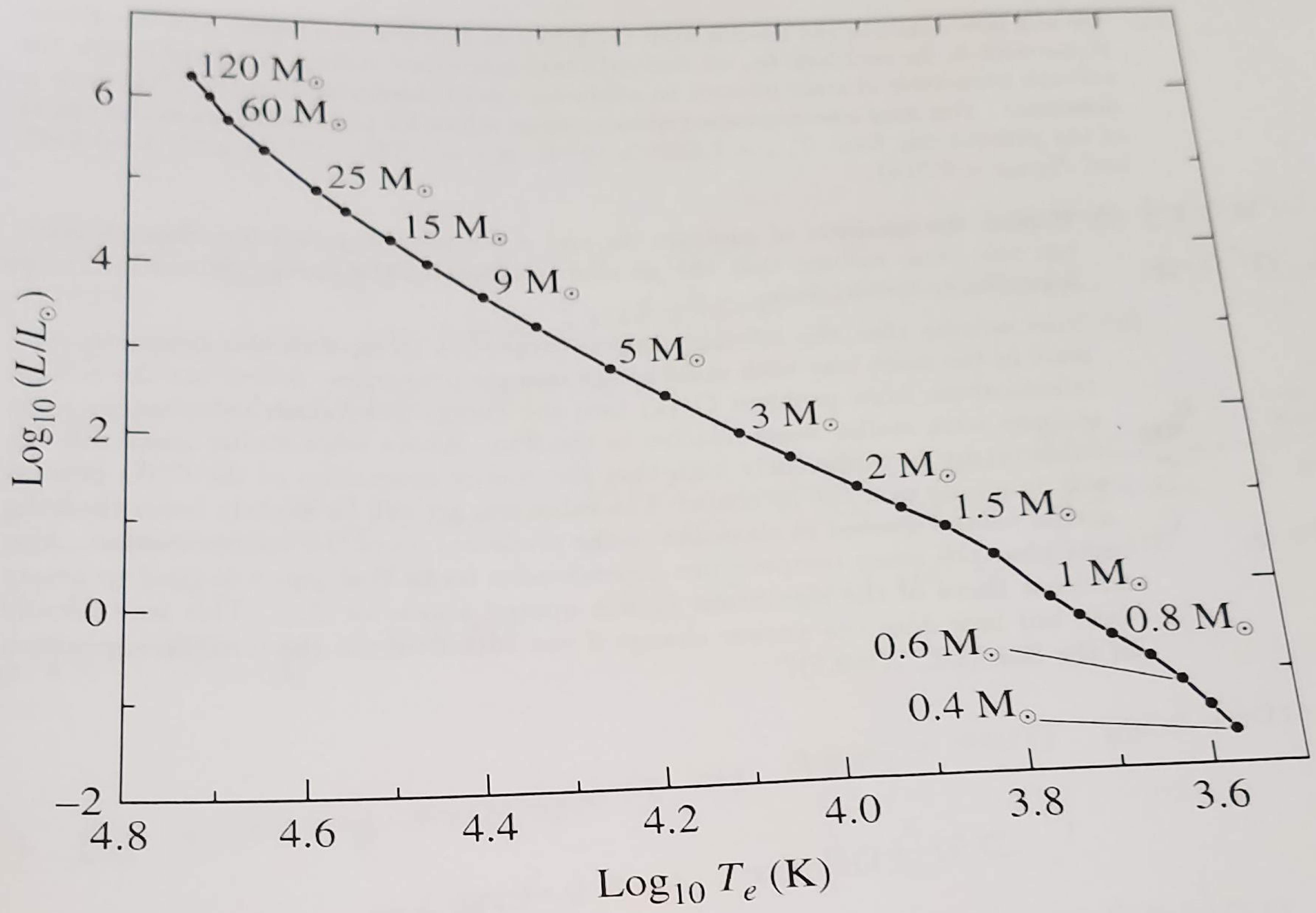

Figure 1: Figure 10.13 from Carroll & Ostlie. A set of models for the Zero Age Main Sequence (ZAMS).