A Template for Theses and Dissertations

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by

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ABSTRACT

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DEDICATION

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A double backslash can be used to force line breaks if desired.

ACKNOWLEDGMENTS

Insert your acknowledgments here or comment out this line.

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z - Redshift
H_0 - Hubble Constant
h - Dimensional Hubble Parameter
Λ - Cosmological Constant
Ω - Density Parameter
ρ_c - Critical Density
D_L - Luminosity Distance
σ_v - Dispersion Velocity
N'' - N Arcseconds
r_s - Scale Radius
r_{200} - Virial Radius
M_{200} - Virial Mass
Mpc - Mega Parsecs
α - Einsteins's Deflection Angle
ψ - Deflection Potential
θ_E - Einstein Angle
β - True Angular Position of the Source
θ - Observed Position of the Source
ξ - Impact Parameter
μ - Magnification
e_T - Tangential Ellipticity
e_X - Cross Ellipticity
γ - Shear
g - Reduced Shear
κ - Convergence
Σ_c - Critical Surface Mass Density
q - Ratio of Minor Axis to Major Axis of an Ellipse i.e. b/a
I - Intensity
R_e - Radius of an Isophote Containing the Half Of the Total Luminosity

LIST OF ACRONYMS

ACS   Advanced Camera for Surveys
CCD   Charge-coupled Device
CDM   Cold Dark Matter
CMB   Cosmic Microwave Background
DE   Dark Energy
DM   Dark Matter
DMstack   Data Management stack pipeline of LSST
FWHM   Full Width Half Maximum
GR   General Relativity
HST   Hubble Space Telescope
IMCAT   Image and Catalog Manipulation Software
LSS   Large Scale Structure
LSST   Large Synoptic Survey Telescope
PSF   Point Spread Function
SED   Spectral Energy Distribution
UDF   Ultra Deep Field
WCS   World Coordinate System
WFC3   Wide Field Camera 3 of LSST

WFIRST   Wide-Field Infra-red Survey Telescope

1 INTRODUCTION

1.1   Introduction

Gravitational Lensing provides us a way to see how dark matter along with visible matter is distributed in the universe. This theory of gravitational lensing is supported by Einstein's general theory of relativity which predicts the deflection of light in a gravitational field if any massive object is present there ([Har03]).

1.1.1   Einstein's Deflection Angle
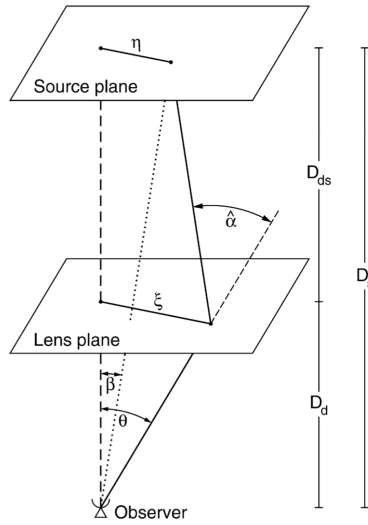
Figure 1.1: Simple sketch to gravitational lensing [BS01]

General Relativity predicts that when the beam of light passes through near the massive objects, the rays of light undergo deflection from their original path. This angle of deflection was predicted by Einstein's theory of General Relativity. According to this theory if the massive object of mass M is located at a perpendicular distance ξ (called **impact parameter**) from the line of sight of source and the observer then the deflection caused by that mass is given by [Sch07]

$$\hat{\alpha} = \frac{4GM}{c^2\xi}.$$

(1.1)

This equation (1.1) is valid only when the angle $\hat{\alpha} << 1$. In case of gravitational lensing, the product of mass of the deflector and the Gravitational Constant is always the much smaller than the squared of velocity of light, thus making the deflection angle very small.

For quantitative purpose, we can calculate the value of deflection angle for distant stars appearing near to the solar limb by setting the mass $M = M_\odot$ and radius $R = R_\odot$ in the above equation to obtain the angle of deflection $\hat{\alpha} = 1.74''$. This value was tested in famous solar eclipse experiment in May 29, 1991 which was conceived by Sir Frank Watson Dyson, Astronomer Royal of Britain in 1917 and led by Sir Arthur Stanley Eddington two years later in 1919 ([DED20]). The experiment was designed to test following hypotheses:

- The light path is uninfluenced by gravitation.

- The law of gravitation will follow Newtonian law and will produce 0".87 apparent displacement.
  The classical calculation for deflection angle was devised by Soldner in 1801 using Newtonian theory:
  $$\frac{2GM_\odot}{v^2 R_\odot} \tag{1.2}$$
  where Solar mass $M_\odot = 1.989 * 10^{30} kg$ and Solar radius $R_\odot = 6.96 * 10^8 \ m$ and plugging the values gives deflection of 0.875 arc seconds.

- The course of ray of light will follow Einsteins generalized relativity and lead to apparent displacement of 1.74". Note: in 1915 Einstein used general relativity and derived a formula for gravitational deflection as

$$\frac{4GM_\odot}{c^2 R_\odot} \tag{1.3}$$

The experiment concluded that the results were close to the Einsteins predicted deflection angle and thus supporting the gravitational lensing theory.
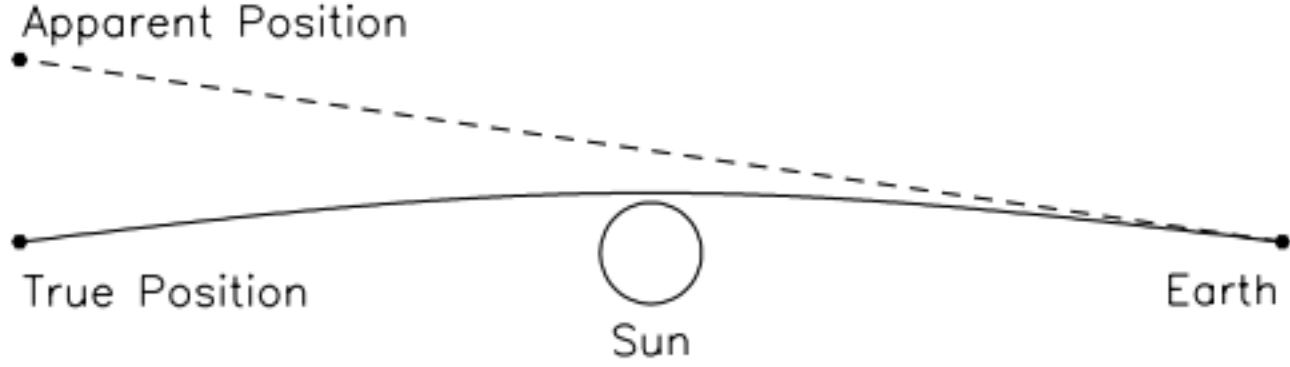
Figure 1.2: Solar deflection angle

This was the deflection due to point source. We can also define the deflection for the extended mass with certain surface mass density Σ as

$$\hat{\alpha}(\xi) = \frac{4G}{c^2} \int d^2\xi' \ \Sigma(\xi') \ \frac{\xi - \xi'}{|\xi - \xi'|^2} \tag{1.4}$$

Here, the *surface mass density* Σ is defined as,

$$\boxed{\Sigma(\xi) \equiv \int dr_3 \ \rho(\xi_1, \xi_2, r_3)} \quad \text{(Surface Mass Density)} \tag{1.5}$$

which is the mass density projected onto the lens plane, perpendicular to the light ray, and r_3 is the coordinate along the line of sight, and ξ_1, ξ_2 the other two perpendicular coordinates.

1.1.2 Lens equation

We consider a simplistic lensing model in which lens and the source objects are point objects. The observer observes the rays of light from the source at a distance D_s which

pass through the gravitational influence field of a massive object of mass M and distance D_d located perpendicularly ξ distance away from line of sight (also called impact parameter).

Let η denotes the true, two-dimensional position of the source in the source plane and β is the true angular position of the source. This means in absence of light deflection we would have,

$$\beta = \frac{\eta}{D_s}. \tag{1.6}$$

The relation between position ξ and θ is given by,

$$\theta = \frac{\xi}{D_d}. \tag{1.7}$$

This means θ is the observed position of the source on the sphere relative to the position of center of the lens which is the origin of the coordinate system with $\xi = 0$. Where, D_{ds} is the distance of the source plane from the lens plane.

Here we adopt the relation,

$$D_{ds} = D_s - D_d. \tag{1.8}$$

This relation holds true as long as the relevant distances are much smaller than the radius of the universe (c/H_0) and this is always the case for distances within our Galaxy and in Local Group. However, this relation no longer holds true for cosmological distances between source and lenses.

From the figure 1.1 , we can relate η with deflection angle α as

$$\eta = \frac{D_s}{D_d}\xi - D_{ds}\hat{\alpha}(\xi). \tag{1.9}$$

Using equation (1.6) we can write β as

$$\beta = \theta - \frac{D_{ds}}{D_s}\ \hat{\alpha}(D_d\theta). \tag{1.10}$$

In this equation (1.10) we have some factor multiplying the deflection angle, so we define *reduced deflection angle*

$$\boxed{\alpha(\theta) = \frac{D_{ds}}{D_s}\,\hat{\alpha}(D_d\theta)} \quad \text{(Reduced Deflection Angle)} \tag{1.11}$$

and then rewrite the lens equation (1.10) as

$$\beta = \theta - \alpha(\theta). \tag{1.12}$$

For a point mass object, using the equations (1.4) and (1.7) the equation of reduced deflection angle (1.11) becomes

$$\alpha(\theta) = \frac{D_{ds}}{D_s}\frac{4GM}{c^2 D_d\theta}. \tag{1.13}$$

1.1.3 Convergence κ and deflection potential ψ

Then convergence is defined as the ratio of the surface mass density of the lens and the critical surface mass density as like

$$\boxed{\kappa(\theta) \equiv \frac{\Sigma(D_d\theta)}{\Sigma_{cr}}} \quad \text{(Convergence)}. \tag{1.14}$$

Where, the critical surface mass density is given by

$$\Sigma_{cr} = \frac{c^2}{4\pi G}\frac{D_s}{D_d D_{ds}}. \tag{1.15}$$

The critical density depends on the redshift of source and lens. If the convergence $\kappa \geq 1$ i.e. surface density is less than critical surface density then we can see the multiple images of the source. If the value of κ is very large it is called "strong gravitational lensing" and if it is only slightly greater than one, it is called "weak gravitational lensing". So, the value of critical mass density plays the role in distinguishing weak vs. strong gravitational lensing.

Moreover, we can define scaled deflection angle in terms of convergence as

$$\alpha(\theta) = \frac{1}{\pi} \int d^2\theta\prime \; \kappa(\theta\prime) \; \frac{\theta - \theta\prime}{\theta - \theta\prime^2} \; .$$ (1.16)

and the *deflection potential* can be defined as

$$\psi(\theta) = \frac{1}{\pi} \int d^2\theta\prime \; \kappa(\theta\prime) \; ln\theta - \theta\prime.$$ (1.17)

1.1.4 Multiple Images

We can see the multiple images of the source at different places θ_i if the equation (1.12) holds true for different values of the deflection angles.
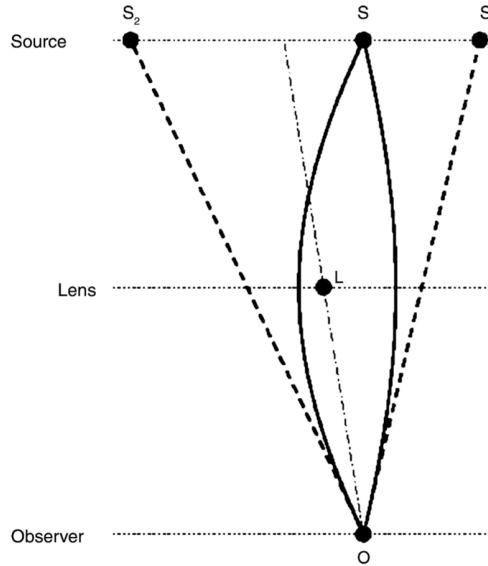
Figure 1.3: Multiple images of single source [Sch07]

This figure illustrates a typical situation in which there are two images S_1 and S_2 of the single source S which is lensed by the point massive object L.

Here, if we take direction of deflection angle as pointing towards the source, we can write the deflection angle for the point mass (1.13) as

$$\boxed{\alpha(\theta) \equiv \frac{4GM}{c^2} \frac{D_{ds}}{D_s D_d} \frac{\theta}{\theta^2}} \quad \text{(Reduced Deflection Angle)} \tag{1.18}$$

Now, we define *Einstein angle* as

$$\boxed{\theta_E = \sqrt{\frac{4GM}{c^2} \frac{D_{ds}}{D_s D_d}}} \quad \text{(Einstein Angle).} \tag{1.19}$$

Then, we can rewrite the equation (1.18) as

$$\beta = \theta - \theta_E{}^2 \frac{\theta}{\theta^2} \tag{1.20}$$

To solve this equation (1.20) we define two scaling factors

$$y = \frac{\beta}{\theta_E} \; ; \; x = \frac{\theta}{\theta_E} \tag{1.21}$$

Then we get

$$\mathbf{y} = \mathbf{x} - \frac{\mathbf{x}}{\mathbf{x}^2}. \tag{1.22}$$

This is a quadratic equation and the solutions are given by,

$$\mathbf{x} = \frac{1}{2}(\mathbf{y} \pm \sqrt{4 + \mathbf{y}^2}) \frac{\mathbf{y}}{\mathbf{y}}. \tag{1.23}$$

Following information can be drawn from the solution of lens equation:

- Except for the divergence condition $\theta \to 0$ any source at position y has two images.

- The two images are on the opposite sides of the source position when viewed from observer position.

- If the source is exactly behind the lens ($\mathbf{y} = 0$), we see the circular ring called *Einstein ring*.

- The Einstein ring has the angular diameter $2\theta_E$ and it gives the characteristic images separation.

1.1.5 Magnification μ and shear γ

The light rays in gravitational are not bent uniformly, the ones near to the lens are deviated more and the ones that are farther are bent in smaller proportion. This differential deflection gives rise to the distorted and magnified images of the source object.

Let $\mathbf{I}^s(\boldsymbol{\beta})$ be the surface-brightness distribution of the source, then following the conservation of total surface brightness the observed surface-brightness distribution in the lens plane is given by

$$I(\boldsymbol{\theta}) = \mathbf{I}^s\,[\boldsymbol{\beta}(\boldsymbol{\theta})]. \tag{1.24}$$

Now we expand the *true angular position* of the source $\boldsymbol{\beta}$ in terms of *observed angular position* of the source $\boldsymbol{\theta}$ using Taylor expansion around the central observed position $\boldsymbol{\theta_0}$ we get

$$\boldsymbol{\beta}(\boldsymbol{\theta}) = \boldsymbol{\beta}_0 + (\boldsymbol{\theta} - \boldsymbol{\theta}_0)\frac{\partial\boldsymbol{\beta}}{\partial\boldsymbol{\theta}} \tag{1.25}$$

Here, the term differential of $\boldsymbol{\beta}$ w.r.t. $\boldsymbol{\theta}$ is called *distortion matrix*

$$\boxed{A(\boldsymbol{\theta}) \equiv \frac{\partial\boldsymbol{\beta}}{\partial\boldsymbol{\theta}}} \quad \text{(Distortion Matrix)}. \tag{1.26}$$

Now we can write the observed surface brightness $I(\boldsymbol{\theta})$ in terms of distortion matrix as

$$\boxed{I(\boldsymbol{\theta}) = \boldsymbol{I}^s[\boldsymbol{\beta}_0 + A(\boldsymbol{\theta})(\boldsymbol{\theta} - \boldsymbol{\theta}_0)]} \quad \text{(Observed Surface Brightness)} \tag{1.27}$$

In terms of deflection potential ψ *Jacobian matrix A* can be written as

$$A(\boldsymbol{\theta}) = \frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}} \tag{1.28}$$

$$= (\delta_{ij} - \frac{\partial^2 \psi(\boldsymbol{\theta})}{\partial \theta_i \partial \theta_j})$$

$$= \begin{pmatrix} 1 - \psi, 11 & -\psi_{,12} \\ -\psi, 21 & 1 - \psi_{,22} . \end{pmatrix} \tag{1.29}$$

From these deflection potential terms, we define shear components

$$\gamma_1 = \frac{1}{2}(\psi_{,11} + \psi_{,22}) \tag{1.30}$$

$$\gamma_2 = \psi_{,12} \tag{1.31}$$

Here, the two components tensor γ is called *shear* and is given by

$$\boxed{\gamma \equiv \gamma_1 + i\gamma_2 = |\gamma|e^{2i\phi}} \quad \text{(Shear)} \tag{1.32}$$

Here, γ_1 and γ_2 are two components of shear as given in equation (1.30) and ϕ is the phase angle.

The shear has two components γ_1 and γ_1 which can be expressed as

$$\gamma \equiv \gamma_1 + i\gamma_2 = |\gamma|e^{2i\phi}. \tag{1.33}$$

Also, in terms of deflection potential the shear components can be expressed as

$$\gamma_1 = \frac{1}{2}(\psi_{,11} - \psi_{,22}) \tag{1.34}$$

$$\gamma_2 = \psi_{,12}$$

Also, the convergence κ is related to the deflection potential through Poisson equation

$$\nabla^2 \psi(\boldsymbol{\theta}) = 2\kappa(\boldsymbol{\theta}). \tag{1.35}$$

In terms of matrix elements of deflection potential ψ we can write κ as

$$\boxed{\kappa = \frac{1}{2}(\psi_{,11} + \psi_{,22})} \quad \text{(Convergence)} \tag{1.36}$$

Now we can write the distortion matrix A in terms of shear components γ_1 and γ_2 and convergence κ as

$$A(\boldsymbol{\theta}) = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix} \qquad (1.37)$$

If we solve this matrix we get the eigenvalues

$$eig(A) = 1 - \kappa \pm |g|. \qquad (1.38)$$

The equation 1.27 has the shape of an ellipse. This means that if a circular galaxy is lensed then the resulting observed image will be an ellipse. The semi-major and semi-minor axes of the ellipse are given by

$$a = \frac{R}{1 - \kappa - |g|} \qquad (1.39)$$

$$b = \frac{R}{1 - \kappa + |g|} \qquad (1.40)$$

Here, R is the radius of circular source, a is the semi-major axis, b is semi-minor axis, and g is the reduced shear defined as

$$\boxed{g(\boldsymbol{\theta}) \equiv \frac{\gamma(\boldsymbol{\theta})}{1 - \kappa(\boldsymbol{\theta})}} \quad \text{(Reduced Shear).} \qquad (1.41)$$

The inverse of the determinant of the matrix α gives the magnification tensor. Then we define magnification as

$$\boxed{\mu(\boldsymbol{\theta}) \equiv \frac{1}{det|A|}} \quad \text{(Magnification).} \qquad (1.42)$$

1.1.6 Examples of Gravitational Lensing

The gravitational lensing phenomenon has been observed in numerous astrophysical studies. For example here we can see the famous arc of Abell 1689 cluster.

This example shows the gravitational due to one of the most massive galaxy clusters ever known called as "Abell 1689". These close up images shows Einstein ring (or arc)

(a) Einsteins ring (arc)   (b) Lensing

Figure 1.4: Gravitational Lensing in Abell 1689

due to strong lensing (left image) and lensed and magnified galaxies (right image). The cluster galaxies (yellow-white objects) are in Abell cluster and at distance 2.2 billion light-years away. The blue arcs are the distorted images of background galaxies which are again billions of light-years from Abell. The image source is NASA, N. Benitez, et al.

2 Bulge Disk Decomposition

2.1 Bulge Disk Decomposition

We have 201 monochromatic postage stamps of single galaxies without neighbors obtained from Hubble Space Telescope (HST) Deep Field (HDF) [WBD$^+$96] From these galaxy stamps we want to break each galaxies into bulge and disk parts. To do such a bulge disk decomposition we use a software called *Galfit* [1] to get the reasonable components of the base galaxy. Galfit is an image analysis algorithm that can model profiles of any given astronomical objects in the given fits image of the data sample. For example, if an fits image contains a galaxy, we can use galfit to fit the bulge and disk components to that galaxy.

Here, for the bulge part of the galaxy, we use the de Vaucouleurs profile in Galfit. The de Vaucouleurs profile describes how the surface brightness of a giant elliptical galaxy changes as a function of the radius R from the center of the galaxy.

Let R_e be the radius of an isophote containing the half of the total luminosity for a galaxy, then, for a de Voucouleurs profile, the surface brightness enclosed by the radius R in that galaxy is given by:

$$I(R) = I_e e^{-7.669[(R/R_e)^{1/4}-1]} \tag{2.1}$$

Galfit has this formula as a built-in feature to get the bulge component of the galaxy.

Similarly, to fit the disk profile to a galaxy, we use the profile called *exponential disk profile* in Galfit. This exponential disk profile is a special case of Sersic profile. In the Sersic profile, the total surface brightness enclosed by the radius R around the center of a galaxy is given by

$$ln \, I(R) = ln \, I_0 - kR^{1/n} \tag{2.2}$$

---

[1] https://users.obs.carnegiescience.edu/peng/work/galfit/galfit.html

where, I_0 is the central surface brightness at $R = 0$ and the parameter n is called *Sersic index* which determines the curviness of the Sersic profile and the parameter *k* determines the slope of the curve.

For the most of the spiral galaxies and dwarf elliptical galaxies, the Sersic index is close to 1. This case when the Sersic index n is equal to 1 is called exponential disk profile. In exponential disk profile, the surface brightness inside the radius R is given by

$$ln\ I(R) = ln\ I_0 - kR \qquad (2.3)$$

In this project we have 201 base galaxies obtained from the HST survey. As we know that some of the galaxies have both bulge and disk components and some do not have it. For the case, when the base galaxy has bulge-disk components, Galfit gives nice bulge and disk components, however, when the base galaxy itself does not have reasonable bulge-disk component, Galfit can not give bulge and disk components of that galaxy. In that case either Galfit fails to give two components or, gives bad parameter. We can see the whether the fitted parameters are good or bad in the log file created from the Galfit. If a parameter is enclosed by then, we can not trust the fitting, and we treat like the given galaxy does not have reasonable bulge-disk components.

Let's say we get two successful bulge and disk components of a base galaxy, then we choose devauc profile as the bulge and exponential profile as the disk image. On the other hand, if either the Galfit fails to produce two component fittings or gives non-reliable fitting parameters, then we make some assumptions how to choose bugle and disk components. If the single component fitting of *devauc profile* gives the better fit parameters than single component fitting of *expdisk profile* , we choose the base galaxy as the bulge component and we choose an empty image as the disk component. Similarly, if the single component *expdisk profile* gives better fitting than single component *devauc*

fitting, we choose the base galaxy as the disk component and we choose an empty image as the bulge component. We may notice that, while doing the galaxy fitting to our 201 base galaxies, we found that most the galaxies gave better fit for the *devauc profile* than the *expdisk profile*. An example of input parameter file for *Galfit* is shown next page.

```
# Galfit Basics:

# command: galfit INPUT_PARAMETER_FILE

# outputs: a) fit.log, galfit.01

#          b) imgbolock.fits

#

# a) fit.log ==>  appended each time e.g. galfit.01, galfit.02

# b) imgblock.fits ==> it has 4 frames.

#                       0 is empty, 1 is input, 2 is model , 3 is residual.

#

#  command: ds9 -multiframe imgblock.fits

#  output : we can see 4 frames.

#

#

# command: galfit -o3 galfit.01 && rm -r galfit.01

# output : a) subcomps.fits

#

# a) subcomps.fits ==> it has two or more frames.

#                       0 is subcomps.fits, 1 is expdisk, 2 is devauc etc.

#

# INPUT_PARAMETER_FILE for galfit has two components:

# a) CONTROL PARAMETERS:  A-P   (these are compulsory)

# b) OBJECT PARAMETERS: 0-10 Z  (it should be at least one, e.g. devauc)

#

# a) CONTROL PARAMETERS

#        * These are fixed, not initial guesses.

#        * F: The row F is for masking
```

```
#              ic '1 0 %1 0 == ?'  INPUT_GALAXY  > mask.fits
#              If the bad pixel input file is a FITS image, all non-zero valued
#              pixels would be ignored during the fit. The pixel numbers where
#              value is 0 is only fitted.
#          * E: psf fine sampling factor is 2
#          * K: in my case plate scale is 0.03
#
# b) OBJECT PARAMETERS:
#          * These are initial guesses.
#          * Second column 1 means value not-fixed.
#          * Z: 0 fits the model, do not choose 1 while fitting.
#          * Better initial guess makes the simulation faster.
#            e.g. for f606w_gal0.fits  (from ds9 headers)
#            NAXIS1  =  601
#            NAXIS2  =  601
#
#            MAG    =  20.5874
#            RADIUS  =  17.703
#
#            PIXSCALE=  0.03 or 0.06 (first for 0-100,including)
#            MAG0    =  30 or 26.6611 or 26.78212 (first hundred, next f606, f814)
#
#   Main commands : ic '1 0 %1 0 == ?'  INPUT_GALAXY  > mask.fits
#                   rm -r imgblock.fits subcomps.fit ; galfit three_comps.sh
#                   galfit -o3 galfit.01 && rm -r galfit.01
#                   ds9 -cmap a -scale log -multiframe imgblock.fits subcomps.fits
```

```
#
# ic '1 0 %1 0 == ?'  /Users/poudel/jedisim/simdatabase/galaxies/
#                       f606w_gal107.fits  > mask.fits
# ds9 mask.fits /Users/poudel/jedisim/simdatabase/galaxies/f606w_gal107.fits &


# IMAGE and GALFIT CONTROL PARAMETERS
A) /Users/poudel/jedisim/simdatabase/galaxies_original/f814w_gal301.fits
B) imgblock.fits        # Output data image block
C) none                 # Sigma image name (made from data if blank or "none")
D) f814w_psf.fits # Input PSF image and (optional) diffusion kernel
E) 2                    # PSF fine sampling factor relative to data
F) mask.fits            # Bad pixel mask (FITS image or ASCII coord list)
G) none                 # File with parameter constraints (ASCII file)
H) 1 601 1 601          # Image region to fit (xmin xmax ymin ymax)
I) 200  200             # Size of the convolution box (x y)
J) 26.78212 # Magnitude photometric zeropoint
K) 0.06 0.06 # Plate scale (dx dy)    [arcsec per pixel]
O) regular              # Display type (regular, curses, both)
P) 0                    # Choose: 0=optimize, 1=model, 2=imgblock, 3=subcomps


# IMAGE and GALFIT OBJECT PARAMETERS
# Component number: 1
# Exponential function (concentration index n = 1)
# This gives disk profile.
0) expdisk              # Object type
1) 300.0 303.0 1 1          # position x, y        [pixel]
```

```
3) 22.1819 1           # total magnitude

4) 14.478 1            #     Rs                [Pixels]

9) 0.5          1       # axis ratio (b/a)

10) 100.0       1       # position angle (PA)  [Degrees: Up=0, Left=90]

Z) 0                    #  Skip this model in output image?  (yes=1, no=0)



# Component number: 2

# deVaucouleur function (concentration index n = 4)

# This gives the bulge profile.

0) devauc               # Object type

1) 300.0 303.0 1 1          # position x, y        [pixel]

3) 22.1819 1           # total magnitude

4) 14.478 1            #     R_e               [Pixels]

9) 0.5          1       # axis ratio (b/a)

10) 100.0        1       # position angle (PA)  [Degrees: Up=0, Left=90]

Z) 0                    #  Skip this model in output image?  (yes=1, no=0)
```

Another point to note is that, to use the Galfit program, we need to use a PSF image to convolve the original galaxy with the given PSF. The PSF image is different for different filter images of a given galaxy. That is, for F814W filter of a base galaxy *galaxy_f*814*w*0.*fits* we use the psf *psf_f*814*w.fits* and for the galaxy *galaxy_f*606*w*0.*fits* we use the psf *psf_f*606*w.fits*. For all the F814W filter images we use the same *psf_f*814*w.fits* and for all the F606W filter images we use the same *psf_f*066*w.fits*.

Here, in our project we use the F814W filter images taken from the HST ACS Wide Field Channel Camera. To create the relevant PSF, we use an on-line tool called *STScI TinyTim Web Application* [2].

The TinyTim web application needs some parameters to create a psf. For this project we chose the following parameters:

Table 2.1: Tiny Tim Parameters

Camera	ACS - Wide Field Channel
Chip	1
Pixel Position	301 301
Filter	F814w
Spectrumtype	Blackbody
Spectrumvalue	6000
PSF diameter	5.0 arcsec
Focus	0.0

---

[2] http://www.stsci.edu/hst/observatory/focus/TinyTim

3   GENERATING POINT SPREAD FUNCTION (PSF) FILES

3.1 Creating PSF Using SED file

In case of galaxy fitting software **Galfit**, we created the PSF needed using an on-line tool called **TinyTim**. The PSF was specially designed for HST ACS Wide Field Camera observations. Here, we again want to create a general purpose PSF using a flat SED. SED stands for "Spectral Energy Distribution" which is simply a table of flux and wavelength. In our galaxy simulation program **Jedisim**, we use the PSF created by a program called **PhoSim**. PhoSim is a photon simulator application which uses Monte Carlo codes to calculate the physics of the atmosphere and the telescope & camera optics. [3]

To run **PhoSim**, we need the following inputs:

- a SED file

- an instance catalog file

- a background file

3.1.1 SED File For PhoSim

Here, we are using flat SED file that comes with the package and is called *phosim/data/SEDs/sed_flat.txt* The plot of wavelength versus flux of flat SED is given below:

Here, the SED has flux values for wavelengths 300 to 1200 nanometers. We are particularly interested only for the wavelength range of LSST r band filter. Looking at the file *phosim/data/lsst/filter_2.txt* and choosing only the range of the filter for transmission >= 5% we get the range of 531 nm to 696 nm. From now on the range 531 nm to 696 nm will be called the broadband range. We split this broadband into 21 equal parts and call each part a narrowband. For example, narrowband0 is from wavelength

---

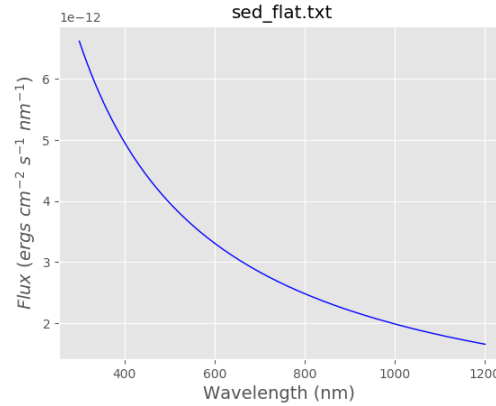[3] *https : //bitbucket.org/phosim/phosim_release/wiki/Home*

Figure 3.1: Flat SED

5310 Angstrom to 5388 Angstrom. From these wavelengths we create 21 narrowband SED files and one broadband SED file.

3.1.2 Background File for PhoSim

We have got SED file from SEDs directory recommended by PhoSim, now we need to choose the background. For the background we chose a simple background file as shown below:

```
zenith_v 1000.0
raydensity 0.0
pixelsize 1.5
saturation 0
blooming 0
chargesharing 0
```

Here we the zenith_v is the default parameter provided by the software. We have chosen raydensity as 0.0, this is also the default setting. We have used the pixelsize of 1.5 and removed saturation by using saturation parameters value set to 0. We also avoided

blooming and set the parameter to 0. Similarly, the effect of charge sharing between ions is also removed.

3.1.3 Instance Catalog For PhoSim

The third component needed is "instance catalog file". The important parameters in an instance catalog are SIM_SEED, SIM_VISTIME, and object. The output PSF file will depend on the initial random seed to the PhoSim software. For the simplicity, we choose this seed number to be 1000. Similarly we choose the simulation time to be 5 minutes (equal to 300 seconds) as the SIM_VISTIME parameter. The "object" parameter consist of multiple fields. For example, we have used:

```
object 0 0.0 0.0 24 ../../../Research/psf_creation_phosim/scripts/
narrowband_seds/narrowband0.sed 0 0 0 0 0 0 star none none
```

```
These fields corresponds to following settings:
object ID RA DEC MAG_NORM SED_NAME REDSHIFT GAMMA1 GAMMA2 KAPPA DELTA_RA DELTA_DI
```

```
The non-trivial components are magnitude of the star, name of the SED file used a
```

[4] An example of instance catalog file can be found in PhoSim software installation directory *phosim/examples/small_catalog*. An example of the instance catalog we used is given below:

```
Unrefracted_RA_deg 0
Unrefracted_Dec_deg 0
Unrefracted_Azimuth 0
Unrefracted_Altitude 89
```

[4] https://bitbucket.org/phosim/phosim_release/wiki/Instance%20Catalog

```
Slalib_date 1994/7/19/0.298822999997

Opsim_rotskypos 0

Opsim_rottelpos 0

Opsim_moondec -90

Opsim_moonra 180

Opsim_expmjd 49552.3

Opsim_moonalt -90

Opsim_sunalt -90

Opsim_filter 2

Opsim_dist2moon 180.0

Opsim_moonphase 10.0

Opsim_obshistid 99999999

Opsim_rawseeing 0.65

SIM_SEED  1000

SIM_MINSOURCE 1

SIM_TELCONFIG 0

SIM_CAMCONFIG 1

SIM_VISTIME 300

SIM_NSNAP 1

object 0 0.0 0.0 24 ../../../Research/psf_creation_phosim/scripts/
narrowband_seds/narrowband0.sed 0 0 0 0 0 0 star none none
```

3.1.4   PSF Generation

Once we have all three parts: a SED file, an instance catalog, and a background file, then we can use the program PhoSim to generate PSF files. The software PhoSim gives us

multiple outputs, out of which what we are interested in is only the electron image named as $lsst\_e\_99999999\_f2\_R22\_S11\_E000.fits.gz$. For a given SED, we unzip this file and take as the PSF file. Here for the broadband SED we got our PSF for the broadband settings. However, the program **Jedisim** needs 21 narrowband PSF images. To get these 21 narrowband PSF images, we feed the 21 narrowband SED files to the PhoSim and get the required PSF images. We take the middle narrowband PSF i.e. psf10.fits as the monochromatic PSF. We also normalize the total flux across all the PSF images. This means, we first calculate the total flux of psf10.fits and scale the flux of all other PSF images.

4   GALAXY SIMULATION

In this section we want to generate the realistic simulation for the upcoming ground based large telescope survey called Large Synoptic Survey Telescope (LSST). We have past data from the space based survey called "Hubble Space Telescope" (HST) using HST ACS Wide Field Camera. The HST data was cleaned and single galaxies were isolated. In previous chapter 2, we created the bulge and disk components of each of these base galaxies and we will be using these bulge and disk components instead of the original clean galaxies from the Hubble images.

4.0.1 Creation of Scaled Bulge, Disk, and Monochromatic Images

We have total 201 number of HST images, so, after bulge disk decomposition, we have 201 bulge images and 201 disk images. We have selected the cutoff redshift of 0.2. This means, we have 201 bulge.fits files for redshift 0.2 and 201 disk.fits files for redshift 0.2. Here, we want to simulate the galaxies at different redshift, let's say at redshift of 1.5. Then, we need to create 201 scaled_bulge.fits and 201 scaled_disk.fits with appropriate bulge to disk ratio. We also create 201 scaled_bulge_disk.fits for monochromatic study. We keep the "sum of flux of 201 bulge files" same as "sum of 201 scaled_bulge files" and "sum of flux of 201 disk files" same as "sum of flux of 201 scaled_disk files" but the ratio of any particular bulge and disk file is different. For example, the ratio of "flux of bulge_100.fits / flux of disk_100.fits" is different than the ratio of "flux of scaled_bulge_100.fits / flux of scaled_disk_100.fits".

For bulge and disk we have two different spectral energy distribution (SED) files. Each SED file has first column as wavelength and 12 more columns with flux at galaxy age 1Gyr to 12Gyr. In these original SED files, the wavelength separation is 5 Angstrom, but we need the wavelength separation of 1 Angstrom. So, I interpolate the bulge and disk SED files to get 1 Angstrom separation. Then we have bulge and disk SED files with wavelength range of 1000 Angstrom to 12,000 Angstrom with bin width of 1 Angstrom.

Now, we have interpolated bulge and disk sed files and bulge and disk fits files. From these, we can create scaled_bulge, scaled_disk, and scaled_bulge_disk files. For this, we need to find the bulge_factor (bf) and disk_factor (df). Then, then we can have scaled galaxies:

$$scaled\_bulge = bf * bulge.fits \tag{4.1}$$

$$scaled\_disk = df * disk.fits \tag{4.2}$$

To find bulge and disk factors, first we find fraction of bulge ratio and fraction of disk ratio as follows:

$$f_{ratioB} = \frac{\int_{\lambda 0}^{\lambda 20} f_{bz}(\lambda)d\lambda}{\int_{\lambda hst0}^{\lambda hst20} f_{bzcut}(\lambda)d\lambda} \tag{4.3}$$

$$f_{ratioD} = \frac{\int_{\lambda 0}^{\lambda 20} f_{dz}(\lambda)d\lambda}{\int_{\lambda hst0}^{\lambda hst20} f_{dzcut}(\lambda)d\lambda} \tag{4.4}$$

Here, f_{bz} is the flux column from the SED file according the redshift z for the bulge and f_{bzcut} is the flux column for cutout galaxy. Here, we have used the galaxy cutout redshift as $z_{cutout} = 0.2$. Similarly we have the flux columns for disk galaxies.

The wavelengths λ_0 and λ_{20} are the LSST R-band filter blue and red wavelengths. This range is 5520Å to 6910Å (Refer to:
$https : //www.lsst.org/about/camera/features$). We divide these wavelengths by a factor $(1 + z)$ to get the range 2208 to 2764 for the redshift of 1.5.

Similarly, for the HST the wavelengths are $\lambda_{hst0} = 7077.5$Å and $\lambda_{hst0} = 9588.5$Å after dividing by $1 + z = 1.2$ we get $\lambda_{hst0} = 5897.9$Å and $\lambda_{hst0} = 7990.4$Å. We can get more details about HST ACS/WFC filter at the website
$http : //www.stsci.edu/hst/acs/documents/handbooks/current/c05\_imaging2.html$.

Then, we get bulge factor and disk factor using the formula:

$$bf = \frac{F_b + F_d}{F_b * f_{ratioB} + F_d * f_{ratioD}} * f_{ratioB} \qquad (4.5)$$

$$bd = \frac{F_b + F_d}{F_b * f_{ratioB} + F_d * f_{ratioD}} * f_{ratioD} \qquad (4.6)$$

$$(4.7)$$

where, F_b is the flux of a bulge file (for example, a fitsfile, $simdatabase/bulge\_f8/f814w\_bulge0.fits$) and F_d is the flux of a disk file (for example, a fitsfile $simdatabase/disk\_f8/f814w\_disk0.fits$). For 201 bulge and disk files we have 201 bulge and disk factors.

After we get these bulge and disk factors we simply multiply them by the bulge.fits and disk.fits to get scaled_bulge.fits and scaled_disk.fits.

4.0.2 PSF Creation for Bulge, Disk, and Monochromatic Images

From the PHOSIM Software we have created 21 narrowband PSFs. Now we will use them to create PSF for scaled bulge, disk, and monochromatic images. The scaled PSF files are given by formula:

$$p_b = \frac{b0 * p0 + b1 * p1 + ... + b20 * p20}{b0 + b1 + ... + b20} \qquad (4.8)$$

$$p_d = \frac{d0 * p0 + d1 * p1 + ... + d20 * p20}{d0 + d1 + ... + d20} \qquad (4.9)$$

$$p_m = f_{rd}\, p_d + f_{rb}\, p_b \qquad (4.10)$$

Here, p_b, p_d, and p_m are PSF for bulge, disk, and monochromatic respectively. Also the quantities $b0, b1, ..., b20$ and $d0, d1, ..., d20$ are bulge and disk weights for 21 narrowbands. These quantities are the integrated flux in the given narrowbands. For example, for LSST R band filter the blue and red wavelength range is 2208 to 2764 Angstrom. We divide this range into 21 parts and integrate the flux in that range to get the bulge and disk factor for that range using SED file for bulge and disk.

4.0.3 Jedisim Simulations

Jedisim is a computer simulator that simulates realistic LSST images from HST images using various physics parameters. It was initially developed by Dr. Ian Dell'Antonio of Brown University (2014) and after that heavily expanded and maintained by Bhishan Poudel of Ohio University (2014-2018) with the help of Dr. Doug Clowe. This program takes in scaled bulge and scaled disk galaxies and finally will create chromatic and monochromatic lsst images. It will also create 90 degree rotated images for the chromatic and monochromatic lsst images. The **Jedisim** program itself consist of various sub programs, which I will describe briefly below.

4.0.3.1 Create the Catalogs for Jedisim

We use the subprogram **jedicatalog** to create the three catalog files need by Jedisim. The files are **catalog.txt**, **convolvedlist.txt**, and **distortedlist.txt**. The **catalog.txt** file contains various important quantities of a galaxy. Each row of catalog.txt file contains following parameters: galaxy_name, center_x, center_y, angle, redshift, pixscale, old_magnitude, old_radius, new_magnitude, new_radius, stamp_name, distorted_file_name. We will need these parameters to transform the galaxies.

In the jedicatalog program we specify a galaxy by six parameters magnitude, radius, image, redshift,position, and angle.

Magnitude In this simulation we have chosen the simulated galaxies magnitudes within range $22 \leq M \leq 28$. The galaxies are distributed with the power law,

$$P(M + dM) \propto 10^{BM} \tag{4.11}$$

where M is the magnitude and $B = 0.33 \ln 10$ is an empirical constant (refer to [**?**]). The magnitude zero-point is taken to be 30 throughout the simulations by convention

Radius The simulated galaxies have the magnitudes between 22 and 28. For each

magnitudes, we have a radius database for r50 radii. We choose a radius randomly

from that radius bin for the given magnitude.

Image The postage stamp image is chosen randomly from the list of r50 radii such that

the chosen r50 radius is larger than the radius of original galaxy. This makes sure

that images are always sized down and no information is artificially created by

scaling.

Redshift In this simulation we have chosen the fixed redshift of 1.5. However, we can

choose random redshifts for each magnitude bin from magnitude 22 to 28 is we opt

to vary the redshifts of galaxies. The redshift database was obtained from

ZCOSMOS database.

Position The position of center of the postage stamps are chosen randomly from the

range [301,40,660] . This range is taken to ensure that all 600 by 600 postage

stamps lie completely within the range [0,40,960]. In later simulation step, we will

trim the border by 480 pixels so as to ensure uniform distribution of galaxies with

typical edge effects.

Angle We chose the angle of orientation of a galaxy randomly between 0 to 360 degrees.

We should note that the orientation of galaxies has three degree of freedom, but

since we are dealing with 2D projections of galaxies, we can only make the

orientation random in one degree of freedom.

The **convolvedlist.txt** contains the names of files to be written after we convolve a galaxy

with a psf. A typical row of convolvedlist appears like this

$jedisim\_out/out0/convolved/convolved\_band\_0.fits$. When we convolve a large fitsfile

with a psf, due to the memory restrictions of computer instead of creating single large

convolved file we create 6 convolved bands and later combine them into a single large convolved galaxy.

The **distortedlist.txt** contains the names of the galaxies that will be after we distort them using Singular Isothermal Profile Lens. A typical row of convolvedlist appears like this *jedisim_out/out*0/*distorted_*0/*distorted_*0.*fits*. There are 12,420 rows and the last row is *jedisim_out/out*0/*distorted_*0/*distorted_*12419.*fits*.

The program **jedicatalog** will create

4.0.3.2   Transform the Galaxies

We transform the scaled bulge, scaled disk, and scaled bulge_disk files using **jeditransform**. This sub routine reads in the catalog file and various transforming physics parameters from that file and then transforms the galaxies. This program reads 201 bulge (or disk) galaxies and create 12,420 HST stamps.

4.0.3.3   Distort the Galaxies

Here, we use the Singular Isothermal Sphere (SIS) profile to lens the galaxies. We have chosen fixed position of the lens to be (6144,6144) and taken dispersion velocity $\sigma_v = 1000km/s$. In the singular isothermal profile the density is calculated as

$$\rho(r, \sigma_v) = \frac{\sigma_v^2}{2\pi G r^2} \tag{4.12}$$

where, G is the gravitational constant and r is radius in pixels. Since the total mass inside radius r diverges as r reaches to infinity, the SIS model is non-physical. However, when the profile is finitely bounded, it constitutes a possible physical distribution and can be used as a lens.

We may also use the Navarro-Frenk-White (NFW) profile which does not suffer from the divergence problem. The NFW profile is given by:

$$\rho(r, \rho_0, R_s) = \frac{\rho_0}{\frac{r}{R_s}(1 + \frac{r}{R_s})^2} \tag{4.13}$$

where, ρ_0, *and* R_s are the parameters dependent on the halo we use.

4.0.3.4 Convolve the Galaxies with the PSF

After we distort the galaxies, we convolve the big HST image with the PSF. For the bulge components we convolve the big HST galaxy with scaled bulge psf and for the disk components we convolve the big HST image with scaled disk psf.

4.0.3.5 Rescale the Galaxies from HST to LSST

Until now, we have been dealing with the HST images and HST psf images. Now, we scale down the pixels of HST to LSST using a routine **jedirescale**. After rescaling we go to the PIXSCALE 0.2 of LSST from the PIXSCALE of 0.06 of HST.

For bulge components, the output of jedirescale gives us lsst_bulge file and similarly disk components gives us lsst_disk fits files. Also, for the bulge_disk files we get lsst_monochromatic unnoised file.

4.0.3.6 Create Monochromatic LSST Image

From the **jedirescale** program if we feed the bulge_disk images as the input fits-files, we will get the lsst_bulge_disk fits-file as the output. We add the Poisson noise of mean noise 10 pixels to get the LSST monochromatic image. This is one of the main output of the **Jedisim** program.

4.0.3.7 Create Chromatic LSST Image

From the **jedirescale** program we get lsst_bulge and lsst_disk images. We combine them and add the Poisson noise of mean noise 10 pixels to get the LSST chromatic image. This is one of the main output of the **Jedisim** program.

4.0.3.8   Rotated galaxies output from Jedisim

If we run the **Jedisim** program for the normal case we will get two main outputs, namely, lsst.fits and lsst_mono.fits. But, the galaxies are randomly orientated in the universe and we may also want the 90 degree rotated versions of the galaxies. For, this purpose, the program Jedisim, will also gives us 90 degree rotated versions of the output files named as lsst90.fits and lsst_mono90.fits. So, in the end of one run of Jedisim we will get four important output files, two for non-rotated galaxies and two for rotated galaxies.

5 STAR CREATION

When we run jedisim, we get chromatic and monochromatic versions of simulated galaxies. These galaxies images (lsst.fits, lsst90.fits, lsst_mono.fits and lsst_mono90.fits) takes into consideration of PSF convolution with galaxy images and we also add fixed value of sky noise 10 as galaxy simulation configuration parameter. However, we do not inject the stellar contamination and WCS addition to the simulated images. In this section we will discuss the process of creating simulated stars and combining them with the galaxy images. The DMSTACK and OBS_FILE pipeline demands that the input images should have stars associated with them and have World Coordinate System (WCS) values attached to the input fits files.

5.0.1 Create text file for star positions

Here, from section 3.1.4 we have 21 PSF files for 21 narrow bands of whole flux region so as to study the chromatic effects of PSF. We also have weighted PSF files for bulge, disk and, monochromatic cases (psfb.fits, psfd.fits and, pdfm.fits). The jedisim simulation output files have a shape of NAXIS1 = 3398 and NAXIS2 = 3398 but these images are created after re-scaling HST images with LSST pixscale. The HST simulated images have the size of NAXIS1 = 12288 and NAXIS2 = 12288. So we need to create star.fits files with initial shape of 12288 and later we will convolve it with the respective PSF to get the desired shape. While creating the star files, we want to exclude the stars at the border of the images, so we choose some offset to deal with this. In this simulation we chose an offset of 10%, this means we have steller objects only in the region between *offset* to *0.1 * 12288* and there are no stars between 0 to offset and so on. To use the same stars for all the simulation for the verification, we keep the fixed seed of random number generator and randomly generate *n_star* number of stars. Here we choose to create 100 stars. The star.txt files will have n_star (eg. 100) rows with two columns x and y as the position of stars to be created with. We will later choose the pixel value of stars.

5.0.2 Create star.fits file from text file

Now we have set up the framework to create the star fits files. We have the positions of stars to be created we need to choose the pixel value of stars and randomly create the stars. Here, we choose fixed pixel value *star_val* of 10,000. Then we create *n_star* = 100 random numbers and multiply them with fixed value of *star_val* = 10,000. We make sure that result is reproducible by keeping the random number generator seed constant. Here, this star.fits file does not depend on redshift. We can use the same star.fits file for different redshift simulations.

5.0.3 Convolve and re-scale star.fits with psf bdm files

We have single star.fits file that does not depend on the redshift. We also have three weighted PSF files for bulge, disk and, monochromatic cases, viz. (psfb.fits, psfd.fits and, psfm.fits) which depend on the choice of redshift. We convolve the star.fits files with these three different PSF files. The resultant files have the shape of 12288 but the final output of jedisim has the shape of 3398, so we re-scale the convolved image with LSST pixscale of 0.2 and get the shape 3398 so that we can add these files to jedisim outputs.

5.0.4 Add stars to jedisim outputs

The jedisim simulation gives the simulation images such as:
jout_z1.5_ngals10k_000_099/lsst_000.fits, *jout_z1.5_ngals10k_000_099/lsst90_000.fits*,
jout_z1.5_ngals10k_000_099/lsst_mono_000.fits,
jout_z1.5_ngals10k_000_099/lsst_mono90_000.fits and we also have star files such as:
stars_z1.5_100_100000/starb_z1.5_100_100000.fits,
stars_z1.5_100_100000/stard_z1.5_100_100000.fits,
stars_z1.5_100_100000/starm_z1.5_100_100000.fits. Here in star files 100 means there are

100 stars and 100,000 means a single star has pixel value of 100,000 multiplied by some random number between 0 to 1.

To run the DMSTACK pipeline, we need to add stars to chromatic and monochromatic files. For the 0 degree and 90 degree rotated versions of monochromatic files (lsst_mono.fits, lsst_mono90.fits), we combine the starm file to these files.

For chromatic cases (lsst.fits and lsst90.fits) we add both starb and stard files to them. In the end, we also need to add the fake world co-ordincates system (WCS) to these files. In summary the input file for DMSTACK looks like this:

$$wcs\_star\_100\_100000\_z1.5\_lsst\_000.fits =$$

jout_z1.5_ngals10k_000_099/lsst_000.fits +

stars_z1.5_100_100000/starb_z1.5_100_100000.fits +

stars_z1.5_100_100000/stard_z1.5_100_100000.fits +

WCS (5.1)

$$wcs\_star\_100\_100000\_z1.5\_lsst90\_000.fits =$$

jout_z1.5_ngals10k_000_099/lsst90_000.fits +

stars_z1.5_100_100000/starb_z1.5_100_100000.fits +

stars_z1.5_100_100000/stard_z1.5_100_100000.fits +

WCS (5.2)

$$wcs\_star\_100\_100000\_z1.5\_lss_mono\_000.fits =$$

jout_z1.5_ngals10k_000_099/lsst_mono_000.fits +

stars_z1.5_100_100000/starm_z1.5_100_100000.fits +

WCS (5.3)

$$wcs\_star\_100\_100000\_z1.5\_lss_mono90\_000.fits =$$

jout_z1.5_ngals10k_000_099/lsst_mono90_000.fits +

stars_z1.5_100_100000/starm_z1.5_100_100000.fits +

WCS (5.4)

6 DMSTACK ANALYSIS

7   DMSTACK ANALYSIS

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APPENDIX: AN APPENDIX

A.1 A Section in the Appendix