Discrete Problem: Incremental Pressure Correction scheme

Tentative velocity

$$\frac{u^* - u^n}{\delta t} + [u^* \cdot \nabla u^*]^{AB} - \frac{1}{2}\nu\Delta(u^* + u^n) + \nabla p^{n-1/2} = f^{n+1/2}$$

$$u^* = g(\cdot, t^{n+1}) \text{ on } \partial\Omega_D$$

$$\frac{1}{2}\nu\nabla(u^* + u^n) \cdot n = p^{n-1/2} n \quad \text{on } \partial\Omega_N$$

Pressure correction

$$-\Delta\phi = -\frac{1}{\delta t}\nabla \cdot u^* \qquad \nabla\phi \cdot n = 0 \quad \text{on } \partial\Omega_D$$

$$\phi = 0 \quad \text{on } \partial\Omega_N$$

Velocity update

$$u^{n+1} = u^* - \delta t \nabla\phi.$$

- Crank-Nicolson timestepping scheme
- Explicit Adam-Bashforth
- P2-P1 finite element pairs

u_{num}

d_n

Objective Function

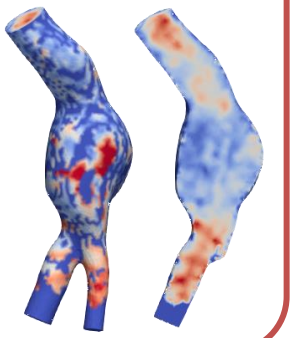
$$J(u) = \sum_{n=1}^{N} \int |T_n u - d_n^*|^2 dx$$

$$with\ T_n u(x) = u(x, t_n)\ \forall x\ in\ \Omega_{obs}$$

u_{obs}

$T_n u$

Experimental measurements



Tikhonov Regularisation Term

$$R(c) = ||c||^2_{\Gamma\times(0,T]}$$

$$||c||^2_{\Gamma\times(0,T]} = \left(\int_0^T \int_\Omega \frac{\alpha}{2}(|g_D|^2 + |\nabla g_D|^2) + \frac{\beta}{2}(|\dot{g}_D|^2 + |\dot{\nabla} g_D|^2) dx dt\right)^{1/2}$$

FENICS PROJECT

dolfin-adjoint

Adjoint Equations

L-BFGS

Optimization algorithm

Optimal Inlet BC

g_D

ll n° of minimization iteration < 10?

NO

YES

VarDA result