

Discrete Problem: Incremental Pressure Correction scheme

Tentative velocity $\frac{u^* - u^n}{\delta t} + [u^* \cdot \nabla u^*]^{AB} - \frac{1}{2} \nu \Delta(u^* + u^n) + \nabla p^{n-1/2} = f^{n+1/2}$

$$u^* = g(\cdot, t^{n+1}) \text{ on } \partial\Omega_D$$

$$\frac{1}{2} \nu \nabla(u^* + u^n) \cdot n = p^{n-1/2} n \text{ on } \partial\Omega_N$$

Pressure correction $-\Delta\phi = -\frac{1}{\delta t} \nabla \cdot u^* \quad \nabla\phi \cdot n = 0 \text{ on } \partial\Omega_D$
 $\phi = 0 \text{ on } \partial\Omega_N$

Velocity update $u^{n+1} = u^* - \delta t \nabla\phi.$

- Crank-Nicolson timestepping scheme
- Explicit Adam-Bashforth
- P2-P1 finite element pairs

Objective Function

$$J(u) = \sum_{n=1}^N \int |T_n u - d_n^*|^2 dx$$

with $T_n u(x) = u(x, t_n) \forall x \text{ in } \Omega_{obs}$

u_{num}

d_n

u_{obs}

$T_n u$

Tikhonov Regularisation Term

$$R(c) = \|c\|_{\Gamma \times (0,T]}^2$$

$$\|c\|_{\Gamma \times (0,T]}^2 = \left(\int_0^T \int_{\Omega} \frac{\alpha}{2} (|g_D|^2 + |\nabla g_D|^2) + \frac{\beta}{2} (|\dot{g}_D|^2 + |\nabla \dot{g}_D|^2) dx dt \right)^{1/2}$$

Experimental measurements



Adjoint
Equations
L-BFGS

Optimization
algorithm

Optimal Inlet BC

g_D

NO

If n° of
minimization
iteration < 10?

YES

VarDA result