COMP 330 Winter 2021
Midterm

Belle Pan 260839939

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Solution 1.
In this question the alphabet is fixed as $\{a, b\}$.

- **Write a regular expression for the language of strings containing a's only when they occur as part of a block of consecutive a's of even length. Thus the legal strings cannot contain an a by itself or a sub-string of 3 or 5 or 7 consecutive a's. Thus $baabbb$ is accepted, so is $aabaabaaaabbaabbaa$ and so is $bbbbbbb$ which has no consecutive pair of a's. However $baaab$ is not allowed as this has three consecutive a's nor is $bababaab$ or $baaaaab$.**

- **Design a DFA (not an NFA) for this language. A picture is preferred. You must show the dead state if there is one. For full credit your machine must have no more than 3 states including the dead state (if there is one).**

Below is the DFA with the regular expression for the language:

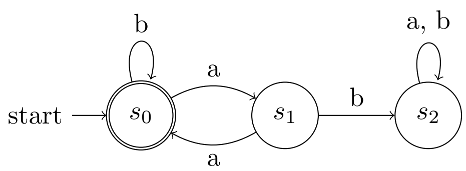

Figure 1: DFA accepting words in the language $(b^* + (aa)^*)^*$.

Solution 2.
Show, using the pumping lemma, that the following language is not regular. The alphabet is $\Sigma = \{a, b\}$. I prefer answers formatted as a game against the demon.

$$L = \{a^i b^j | i - j = 2, i, j > 0\}$$

Below is a proof using the demon - angel strategy of showing the pumping lemma:

1. The demon picks a number $p > 0$.
2. The angel picks the word $a^{p+2}b^p$.
3. The demon is forced to pick a y value that is made of exclusively a's due to the constraints $|xy| \leq p$ and $|y| > 0$. Suppose that $y = a^k$ such that $0 < k \leq p$.
4. The angel chooses $i = 2$ such that the new word becomes $a^{p+2+k}b^p$. This word is not in the language as $(p + 2 + k) - (p) = 2 + k \neq 2$.

Thus, by the pumping lemma, $L = \{a^i b^j | i - j = 2, i, j > 0\}$ is indeed not a regular language.

Solution 3.
Are the following statements true or false? No explanations are required. We have some fixed alphabet that we are working with.

1. **If L is a non-regular language and R is a regular language then $L \cap R$ must be regular.**
2. **If L is a non-regular language and R is a regular language then $L \cup R$ cannot be regular.**
3. **For every regular language there is a unique minimal NFA.**
4. **When we run the minimization algorithm on a DFA we cannot be sure that it will always terminate.**
5. **If L_1 is an infinite regular language and L_2 is a finite language then the DFA to recognize L_1 must have more states than the DFA to recognize L_2.**

Below are the corresponding true/false responses:

1. False.
2. False.
3. False.
4. False.
5. False.