Recurrent Neural Network - Backpropagation Through Time

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Contents

Adopted from tutorial.

1 Download tokenized text from Brown Corpus and Import to CL

Download the corpus using nltk (Python Library)

```python
import nltk
from nltk.corpus import brown

nltk.download("brown")
data = brown.sents(categories=["science_fiction"])

with open("/tmp/sentences", mode='w') as file:
    file.write("( ")
    for sentence in data:
        file.write("( ")
        for word in sentence:
            file.write("\""+ word+"\" ")
        file.write(")\n")
    file.write(" )")
```

Load it into Common Lisp

```lisp
(defparameter *sentences* (uiop:read-file-form #p"/tmp/sentences"))
```

2 Create training data

The input to our Recurrent Neural Networks are vectors, not strings. So we create a mapping between words and indices, index-to-word, and word-to-index. For example, the word “friendly” may be at index 2001. A training example x may look like [0, 179, 341, 416], where 0 corresponds to SENTENCE-START. The corresponding label y would be [179, 341, 416, 1]. Remember that our goal is to predict the next word, so y is just the x vector shifted by one position with the last element being the SENTENCE-END token. In other words, the correct prediction for word 179 above would be 341, the actual next word.

2.1 Assign index to each word

```lisp
;; index-to-word & word-to-index
(defparameter *word-count* (make-hash-table :test #'equal))

(loop for s in *sentences* do
  (loop for w in s
        for word = (string-downcase w)
        for count = (gethash word *word-count* 0) do
          (setf (gethash word *word-count*) (1+ count))))
```

There are

```lisp
(hash-table-count *word-count*)
```

```
3032
```

unique words.

2.2 Create traning data by converting sentences to vector - also replace infrequent words with unknown-word

We remove less frequent words and take only 1000 words in our vocabulary. This limit is mainly because of the one-hot encoding we use for the words. So, the input vector size is 1000. The words that don't fall in our vocabulary are replaced by UKNONWN-WORD.

```
(defun hashtable-keys (hashtable)
  (let ((keys (make-array (hash-table-count hashtable) :fill-pointer 0)))
    (maphash (lambda (key value)
               (declare (ignore value))
               (vector-push key keys))
             hashtable)
    keys))

(defparameter *sorted-words* (sort (hashtable-keys *word-count*)
                                   #'> :key (lambda (word)
                                              (gethash word *word-count* 0))))

(defparameter *vocabulary_size* 1000)
(defparameter *index-to-word* (make-array *vocabulary_size*
                                          :element-type 'string
                                          :initial-element ""
                                          :adjustable t
                                          :fill-pointer 3))

(defparameter *word-to-index* (make-hash-table :test #'equal :size *vocabulary_size*))

;; start with index 3.
(defconstant +SENTENCE-START+ 0)
(defconstant +SENTENCE-END+ 1)
(defconstant +UNKNOWN-WORD+ 2)

(defparameter *data* (make-array (length *sentences*)
                                 :fill-pointer 0))

(let ((top-words (make-hash-table :test #'equal))
      (index-counter 2))
  (loop for w across *sorted-words*
        repeat (- *vocabulary_size* 3) do
          (setf (gethash w top-words) t))

  (loop for s in *sentences*
        for x = (make-array (+ (length s) 2) :fill-pointer 0) do
          (vector-push +sentence-start+ x)
          (loop for w in s
                for word = (string-downcase w)
                for index = (gethash word *word-to-index*) do

                  (if (gethash word top-words)
                      (progn
                        (unless index
                          (setf index (incf index-counter))
                          (setf (gethash word *word-to-index*) index)
                          (vector-push word *index-to-word*))
                        (vector-push index x))
                      (vector-push +unknown-word+ x)))
          (vector-push +sentence-end+ x)
          (print x)
          (setf (fill-pointer x) (+ (length s) 1))

          (vector-push-extend
           (list x
                 (make-array (+ (length s) 1)
                             :displaced-to x
                             :displaced-index-offset 1))
                *data*)))

(defun index-word (index)
  "Get word from index"
  (aref *index-to-word* index))

(defun word-index (word)
  "Get index from word"
  (gethash word *word-to-index*))
```

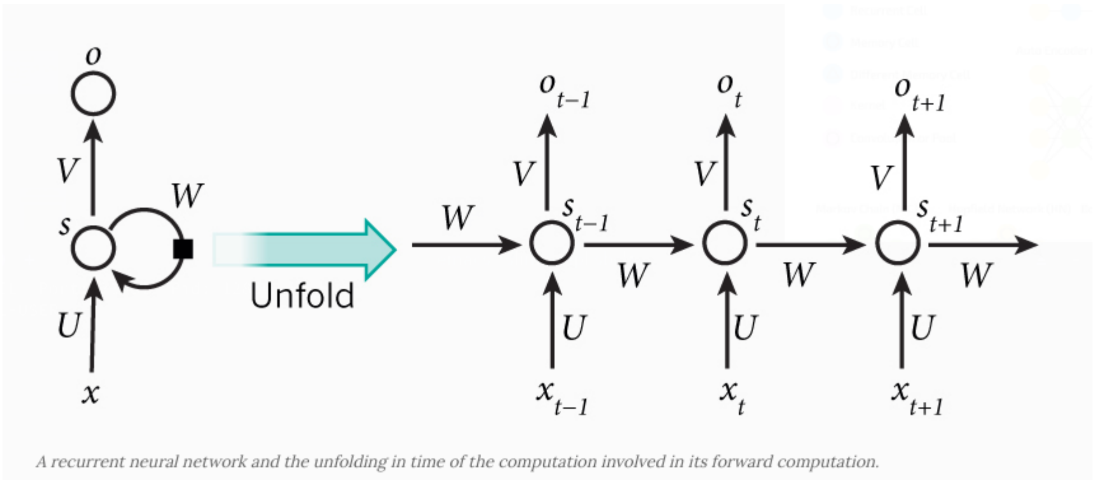
Here's an actual training example from the text (with corresponding index of the words):

```
(print (first *sentences*))
(print (aref *data* 0))
```

```
("Now" "that" "he" "knew" "himself" "to" "be" "self" "he" "was" "free" "to"
 "grok" "ever" "closer" "to" "his" "brothers" "," "merge" "without" "let" ".")
(#(0 3 4 5 6 7 8 9 10 5 11 12 8 13 14 15 8 16 17 18 2 19 20 21)
 #(3 4 5 6 7 8 9 10 5 11 12 8 13 14 15 8 16 17 18 2 19 20 21 1))
```

3 RNN Structure

Let's get concrete and see what the RNN for our language model looks like. The input x will be a sequence of words (just like the example printed above) and each $\mathrm{x_t}$ is a single word. But there's one more thing: Because of how matrix multiplication works we can't simply use a word index (like 36) as an input. Instead, we represent each word as a one-hot vector of size vocabulary-size. For example, the word with index 36 would be the vector of all 0's and a 1 at position 36. So, each $\mathrm{x_t}$ will become a vector, and x will be a matrix, with each row representing a word. We'll perform this transformation in our Neural Network code instead of doing it in the pre-processing. The output/predictions of our network p has a similar format. Each $\mathrm{p_t}$ is a vector of vocabulary-size elements, and each element represents the probability of that word being the next word in the sentence.



o
V
W
s
U
x
Unfold
W
o_{t-1}
o_t
o_{t+1}
V
s_{t-1}
s_t
s_{t+1}
U
x_{t-1}
x_t
x_{t+1}
A recurrent neural network and the unfolding in time of the computation involved in its forward computation.

Figure 1: RNN (copied from the tutorial; (In this document $\mathrm{o_t}$ is replaced with $\mathrm{p_t}$))

$s_t = \tanh(Ux_t + Ws_{t-1})$ $p_t = \mathrm{softmax}(Vs_t)$

Let our vocabulary size be C, and hidden layer size (i.e. number of neurons in the hidden layer be H) then

- $x_t \in R^C$
- $p_t \in R^C$

- $s_t \in R^H$
- $U \in R^{H \times C}$
- $V \in R^{C \times H}$
- $W \in R^{H \times H}$

This is valuable information. Remember that U,V and W are the parameters of our network we want to learn from data. Thus, we need to learn a total of 2HC + H^2 parameters. In the case of C=1000 and H=100 that's 210,000. The dimensions also tell us the bottleneck of our model. Note that because x_t is a one-hot vector, multiplying it with U is essentially the same as selecting a column of U, so we don't need to perform the full multiplication. Then, the biggest matrix multiplication in our network is Vs_t. That's why we want to keep our vocabulary size small if possible.

4 Initialization

We start by declaring a RNN network class and initializing our parameters.Initializing the parameters U,V and W is a bit tricky. We can't just initialize them to 0's because that would result in symmetric calculations in all our layers. We must initialize them randomly. Because proper initialization seems to have an impact on training results there has been lot of research in this area. It turns out that the best initialization depends on the activation function (tanh in our case) and one recommended approach is to initialize the weights randomly in the interval from $\left[-\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right]$ where n is the number of incoming connections from the previous layer. This may sound overly complicated, but don't worry too much it. As long as you initialize your parameters to small random values it typically works out fine.

```
(defun make-random-matrix (m n)
  (let ((matrix (make-array (list m n) :element-type 'double-float
                             :initial-element 0d0))
        (1/sqrtn (coerce (/ (sqrt n)) 'double-float)))
    (loop for i from 0 below m do
      (loop for j from 0 below n
            for random = (random (* 2 1/sqrtn)) do
            (setf (aref matrix i j)
                  (- random 1/sqrtn))))
    matrix))

(defclass network ()
  ((H :accessor H :initarg :H)
   (C :accessor C :initarg :C)
   (U :accessor U )
   (V :accessor V )
   (W :accessor W )))

(defmethod initialize-instance :after ((n network) &key)
  (with-slots (H C) n
    (setf (slot-value n 'U) (make-random-matrix H C))
    (setf (slot-value n 'V) (make-random-matrix C H))
    (setf (slot-value n 'W) (make-random-matrix H H))))
```

5 Forward Propagation

```
(defun matrix-dot-vector (matrix vector)
  "pointwise operate f on matrix . vector"
  (let* ((m (array-dimension matrix 0))
         (n (array-dimension matrix 1))
         (result (make-array m :element-type 'double-float :initial-element 0d0
                                 :fill-pointer 0)))
    (loop for i from 0 below m do
      (vector-push (loop for j from 0 below n
                         summing (* (aref matrix i j)
                                    (aref vector j)))
                   result))
    result))

(defun matrix-dot-index (matrix index)
  "matrix . vector; where vector is one shot representation of index"
  (let* ((m (array-dimension matrix 0))
         (n (array-dimension matrix 1))
         (result (make-array m :element-type 'double-float :initial-element 0d0
                                 :fill-pointer 0)))
    (loop for i from 0 below m do
      (vector-push (aref matrix i index)
                   result))
    result))

(defun softmax% (vector)
  "Destructively calculates softmax"
  (map-into vector (lambda (x) (exp x)) vector)
  (let ((sum (reduce #'+ vector)))
    (map-into vector (lambda (x) (/ x sum)) vector)
    vector))

(defun map-into2 (function &rest sequences)
  "map `function' into the first of the `sequences'"
  (apply #'map-into (first sequences) function sequences))

(defmethod forward-propagate ((n network) x)
  (with-slots (U V W) n
    (let ((s (make-array (length x))) ;; hidden state at each timestep
          (p (make-array (length x)))) ;; output at each timestep
      (loop for time from 0 below (length x)
            for xt = (aref x time) do
        (setf (aref s time)
              (if (= time 0)
                  (matrix-dot-index U xt)
                  (map-into2 (lambda (x y)
                                (tanh (+ x y)))
                             (matrix-dot-index U xt)
                             (matrix-dot-vector W (aref s (1- time))))))
        (setf (aref p time)
              (softmax% (matrix-dot-vector V (aref s time)))))
      (values p s))))
```

6 Loss Function

To train our network we need a way to measure the errors it makes. We call this the loss function L, and our goal is find the parameters U,V and W that minimize the loss function for our training data. A common choice for the loss function is the cross-entropy loss. If we have N training examples (words in our text) and C classes (the size of our vocabulary) then the loss with respect to our predictions p and the true labels y is given by:

$$L(y,p) = -\frac{1}{N}\sum_{n\in N}\log(\vec{y}_n \cdot \vec{p}_n) \tag{1}$$

```
(defun loss (output y-indices)
  "Loss for a single sentence;
output `output' from the network, the actual target `y' "
  (/ (loop for p_t across output
           for y across y-indices
           summing (log (aref p_t y)))
      -1))

(defmethod calculate-total-loss ((n network) inputs targets)
  "Loss for all sentences 'inputs' and 'outputs'"
  (/ (loop for input in inputs
           for target in targets
           summing (loss (forward-propagate n input) target))
     (loop for i in inputs summing (length i))))
```

6.1 Testing forward-propagation and loss calculation

```
(let ((network (make-instance 'network :c 1000 :h 100))
      (*data* (make-array 100 :displaced-to *data*)))
  (calculate-total-loss network
                        (map 'list #'first *data*)
                        (map 'list #'second *data*)))
```

```
6\.907459817685422d0
```

We have C words in our vocabulary, so each word should be (on average) predicted with probability 1/C, which would yield a loss of logC = log1000 = 6.9077554.

Our value is also quite close, so we are in the right track.

7 Training the RNN with SGD and Backpropagation Throught Time (BPTT)

7.1 Derivative of loss function wrt output

$\vec{o}$ is the output $V.\vec{s}$ and the softmax function turns that into probabilities $\vec{p}$

$$p_j = \frac{e^{o_j}}{\sum_k e^{o_k}}$$

and the loss for this output is

$$L = -\sum_j y_j \log p_j$$

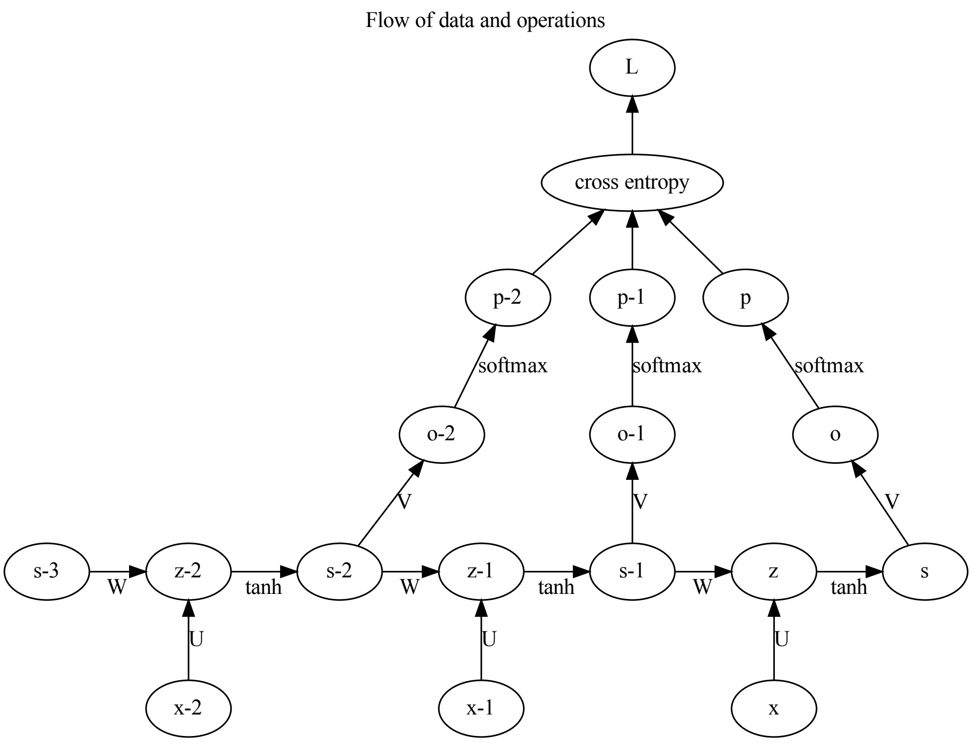
where $\vec{y}$ is one-shot representation of desired output implying $\sum_j y_j = 1$

Thus, following the derivation here you get the derivative to be:

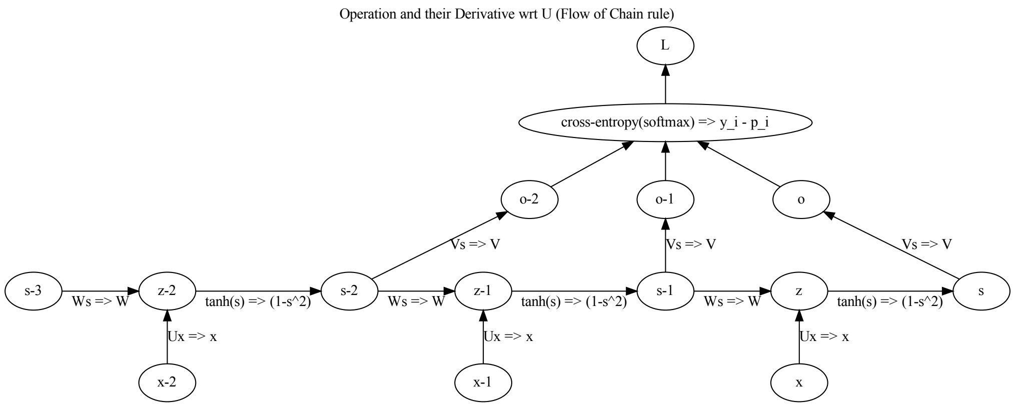
$$\frac{\partial L}{\partial o_i} = p_i - y_i$$

7.2 Diagramatic representation

```
digraph G {
        rankdir = "BT"
        "s-1" -> z [label=W];
        z -> s [label=tanh];
        s -> o [label=V]
        x -> z [label=U]
        o -> p [label=softmax]
        {p;"p-1"; "p-2"} -> "cross entropy"
      "cross entropy" -> L


        "s-2" -> "z-1" [label=W];
        "z-1" -> "s-1"  [label=tanh];
        "s-1" -> "o-1"  [label=V]
        "x-1" -> "z-1"  [label=U]
        "o-1" -> "p-1"  [label=softmax]

        "s-3" -> "z-2"  [label=W];
        "z-2" -> "s-2"  [label=tanh];
        "s-2" -> "o-2"  [label=V]
        "x-2" -> "z-2"  [label=U]
        "o-2" -> "p-2"  [label=softmax]

        {rank = same; "x-2"; "x-1"; x}
        {rank = same; "s-3"; "z-2"; "s-2"; "s-1"; s; "z-1"; z}
        {rank = same; "o-2"; "o-1"; o}

        labelloc = t
        label = "Flow of data and operations"

}
```



```
digraph G {
        rankdir = "BT"
        "s-1" -> z [label="Ws => W"];
        z -> s [label="tanh(s) => (1-s^2)"];
        s -> o [label="Vs => V"]
        x -> z [label="Ux => x"]
        {o;"o-1"; "o-2"} -> "cross-entropy(softmax) => y_i - p_i" -> L


        "s-2" -> "z-1"  [label="Ws => W"];
        "z-1" -> "s-1"  [label="tanh(s) => (1-s^2)"];
        "s-1" -> "o-1"  [label="Vs => V"]
        "x-1" -> "z-1"  [label="Ux => x"]

        "s-3" -> "z-2" [label="Ws => W"];
        "z-2" -> "s-2" [label="tanh(s) => (1-s^2)"];
        "s-2" -> "o-2" [label="Vs => V"]
        "x-2" -> "z-2" [label="Ux => x"]

        {rank = same; "x-2"; "x-1"; x}
        {rank = same; "s-3"; "z-2"; "s-2"; "s-1"; s; "z-1"; z;}
        {rank = same; "o-2"; "o-1"; o}
        labelloc="t"
        label = "Operation and their Derivative wrt U (Flow of Chain rule)"
}
```



7.3 Code

Lets implement Backpropagation through time

```
(defun incf-outer-product (place vec-a vec-b)
  "Add the outer product of ‘vec-a’ and ‘vec-b’ into ‘place’"
  (let ((n (array-dimension place 0))
        (m (array-dimension place 1)))
    (assert (= n (length vec-a)))
    (assert (= m (length vec-b)))
    (loop for i from 0 below n do
      (loop for j from 0 below m do
        (incf (aref place i j)
              (* (aref vec-a i)
                 (aref vec-b j)))))
    place))

(defun incf-outer-product-with-index (place vec-a vec-b-index)
  (let ((n (array-dimension place 0))
        (m (array-dimension place 1)))
    (assert (= n (length vec-a)))
    (assert (< vec-b-index m))
    (loop for i from 0 below n do
      (incf (aref place i vec-b-index)
            (aref vec-a i)))
    place))

(defun matrix-T-dot-vector (matrix vector)
  "Multiply transpose of ‘matrix’ with ‘vector’"
  (destructuring-bind (m n) (array-dimensions matrix)
    (assert (= m (length vector)))
    (let ((result (make-array n)))
      (loop for j from 0 below n do
        (setf (aref result j)
              (loop for i from 0 below m
                      summing (* (aref matrix i j)
                                 (aref vector i)))))
      result)))

(defun bptt (n x y &key (bptt-truncate 4))
  (declare (optimize (debug 3)))
  (check-type n network)
  (with-slots (U V W) n
    (multiple-value-bind (p s) (forward-propagate n x)
      ;; p[t] = softmax(o[t] = V s[t]), s[t] = tanh(Ux + W s[t-1])
      (let ((dL/dU (make-array (array-dimensions (U n))))
            (dL/dV (make-array (array-dimensions (V n))))
            (dL/dW (make-array (array-dimensions (W n))))
            dL/do
            (di (make-array (array-dimension W 0))))

        ;; replace p with dL/do = p - y
        (loop for i from 0 below (length p)
              for index across y do
                (decf (aref (aref p i) index) 1))
        (setf dL/do p)

        ;; for each output backwards
        (loop for time from (1- (length y)) downto 0 do
          ;; dL/dV += L_i s[t]^j
          (incf-outer-product dL/dV
                              (aref dL/do time) (aref s time))

          ;; di = L_k V^k_i [ 1 - (s^i)^2]
          (loop for i from 0 below (length di) do
            (setf (aref di i)
                  (* (- 1 (expt (aref (aref s time) i) 2))
                     (loop for k from 0 below (array-dimension V 1)
                           summing (* (aref (aref dL/do time) k)
                                      (aref V k i))))))

          ;; accumulate error for bptt-truncate steps back in time
          (loop for time2 from time downto (max 0 (- time bptt-truncate)) do
            ;; dL/dW += d_i s^j_,-1 + d_i,-1 s^j_,-2 + ...
            (unless (= 0 time2)
              (incf-outer-product dL/dW
                                  di (aref s (- time2 1))))
            ;; dL/dU += d_i x^j + d_i,-1 x^j_,-1 + ...
            (incf-outer-product-with-index dL/dU
                                           di (aref x time))

            ;; d_i,-n = d_m,-n+1 W^m_i [ (1 - (s^i_, -n)^2)]
            (unless (= 0 time2)
              (map-into di
                        (lambda (dW_i s^i)
                          (* dW_i (- 1 (expt s^i 2))))
                        (matrix-t-dot-vector W di)
                        (aref s time2)))))
        (values dL/dU dL/dV dL/dW)))))
```

7.4 Test of BPTT

We can always check backpropagation with an acutal gradient computed using

$$\frac{\partial f(x;\theta)}{\partial\theta} = \lim_{h\to 0}\frac{f(x;\theta+h)-f(x;\theta)}{h}$$

```
(defun gradient-check (n x y)
  (declare (optimize (debug 3)))
  (with-slots (U V W) n
```

```lisp
  (multiple-value-bind (dL/dU dL/dV dL/dW) (bptt n x y :bptt-truncate 40)
    ;; for each variable take a random parameters and check it
    (let ((loss (loss (forward-propagate n x) y))
          (new-loss nil)
          (diff 0.0001)
          (actual-derivative nil))
      (loop repeat 5 do
        (loop for derivative-var in (list dL/dU dL/dV dL/dW)
              for var in (list U V W)
              for name in '(U V W)
              for i = (random (array-dimension var 0))
              for j = (random (array-dimension var 1))
              for original-value = (aref var i j) do
                (format t "~& Checking at ~d,~d of ~s" i j name)
                (incf (aref var i j) diff)
                (setf new-loss (loss (forward-propagate n x) y))
                (setf (aref var i j) original-value
                      actual-derivative (/ (- new-loss loss) diff))
                (format t "~&   Actual Derivative: ~f" actual-derivative)
                (format t "~&   Computed Derivative: ~f" (aref derivative-var i j))
                (unless (= actual-derivative 0.0d0)
                  (when (< (abs (/ (aref derivative-var i j) actual-derivative)) .9)
                    (format t "~&[WARNING] Relative Error > 90% !!!"))))))))


(let ((n (make-instance 'network :c 1000 :h 10)))
  (destructuring-bind (x y) (aref *data* 0)
    (gradient-check n x y)))
```

```
 Checking at 1,933 of U
   Actual Derivative: 0.0
   Computed Derivative: 0.0
 Checking at 866,6 of V
   Actual Derivative: -0.000019541062595428733
   Computed Derivative: -0.000019541967203488295
 Checking at 0,9 of W
   Actual Derivative: -0.013063465519599794
   Computed Derivative: 0.02276368401665077
 Checking at 2,122 of U
   Actual Derivative: 0.0
   Computed Derivative: 0.0
 Checking at 839,1 of V
   Actual Derivative: 0.0001507862672844925
   Computed Derivative: 0.00015078568191896963
 Checking at 1,0 of W
   Actual Derivative: 0.0007341455629026561
   Computed Derivative: -0.015454469780238055
 Checking at 0,159 of U
   Actual Derivative: 0.0
   Computed Derivative: 0.0
 Checking at 330,9 of V
   Actual Derivative: -0.00010548660878157174
   Computed Derivative: -0.0001054866726425041
 Checking at 7,1 of W
   Actual Derivative: -0.011276178498253225
   Computed Derivative: -0.0033533307913708663
[WARNING] Relative Error > 90% !!!
 Checking at 2,118 of U
   Actual Derivative: 0.0
```

```
    Computed Derivative: 0.0
  Checking at 558,3 of V
    Actual Derivative: -0.00008898126904709327
    Computed Derivative: -0.00008898159965005954
  Checking at 6,5 of W
    Actual Derivative: 0.0005569091995027611
    Computed Derivative: -0.015009585574136189
  Checking at 3,381 of U
    Actual Derivative: 0.0
    Computed Derivative: 0.0
  Checking at 922,4 of V
    Actual Derivative: 0.000035769006438858855
    Computed Derivative: 0.000035768414341222246
  Checking at 4,3 of W
    Actual Derivative: 0.014327403128352624
    Computed Derivative: 0.029498057807640476
```

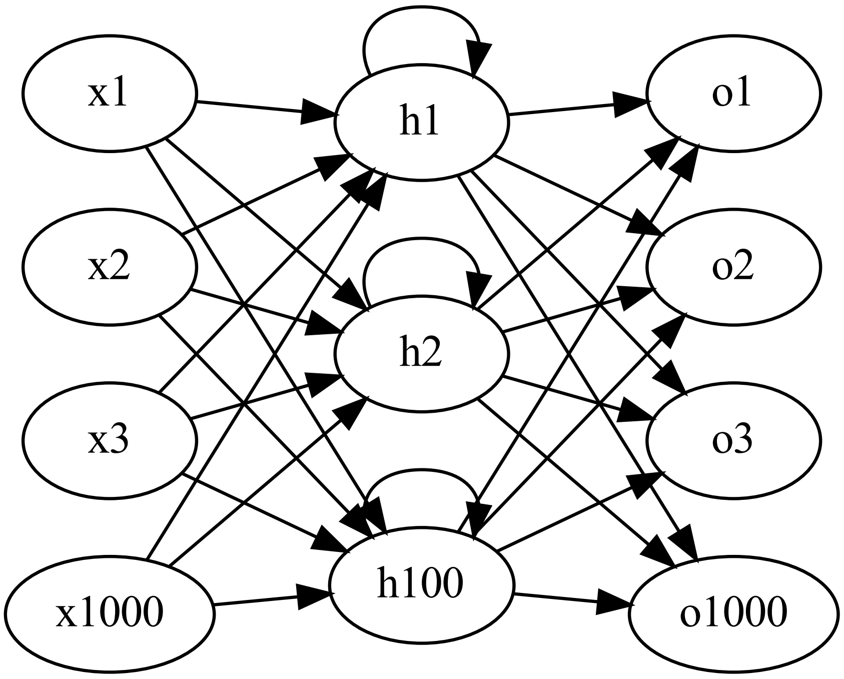
This shows that our BPTT algorithm has higher probability of being correct. only few check are giving warning. and that can be because of numerical errors/rounding.

7.5 Stochastic Gradient Descent

```lisp
(defun update-matrix (M diff rate)
  (loop for i from 0 below (array-dimension M 0) do
    (loop for j from 0 below (array-dimension M 1) do
          (setf (aref M i j)
                (- (aref M i j)
                   (* rate (aref diff i j)))))))

(defmethod sgd-step ((n network) x y learning-rate)
  (multiple-value-bind (dL/dU dL/dV dL/dW) (bptt n x y)
    (with-slots (U V W) n
      (update-matrix U dL/dU learning-rate)
      (update-matrix V dL/dV learning-rate)
      (update-matrix W dL/dW learning-rate))))

(defmethod train-with-sgd ((n network) xs ys
                           &key (learning-rate 0.005) (epoch 100)
                             (evaluate-loss-after 5))
  (loop for i from 0 below epoch
        with losses = nil
        for clock-time = (get-internal-real-time) do
    (when (and (not (= i 0))
               (= 0 (mod i evaluate-loss-after)))
      (push (calculate-total-loss n xs ys) losses)
      (format t "~& Loss = ~f" (first losses))
      (when (and (> (length losses) 1)
                 (> (first losses) (second losses)))
        ;; when loss increased
        (setf learning-rate (* 0.5 learning-rate))
        (format t "~&Loss increased; so learning-rate is decreased to ~f" learning-rate)))

    (loop for y in ys
          for x in xs
          for i from 0 do
          (sgd-step n x y learning-rate)
          (when (= 0 (mod i 50))
            (format t "~&     ~d examples learned" i)))

    (format t "~& Epoch ~d done in ~f seconds." i
            (/ (- (get-internal-real-time) clock-time)
                   internal-time-units-per-second))))
```

8 Train!!

Lets initialize a RNN with a single hidden layer. The input and output layers have 1000 neurons and the hidden layer has 100 neurons.



```
(defparameter *network* (make-instance 'network
                                       :c *vocabulary_size*
                                       :h 100))
(train-with-sgd *network*
                (subseq (map 'list #'first *data*) 100)
                (subseq (map 'list #'second *data*) 100))
```

```
    0 examples learned
    50 examples learned
Epoch 0 done in 17.47 seconds.
    0 examples learned
    50 examples learned
Epoch 1 done in 17.55 seconds.
    0 examples learned
    50 examples learned
Epoch 2 done in 17.46 seconds.
    0 examples learned
    50 examples learned
Epoch 3 done in 17.480001 seconds.
    0 examples learned
    50 examples learned
Epoch 4 done in 17.58 seconds.
Loss = 4.710042288388026
    0 examples learned
    50 examples learned
Epoch 5 done in 22.71 seconds.
    0 examples learned
    50 examples learned
Epoch 6 done in 17.32 seconds.
    0 examples learned
    50 examples learned
Epoch 7 done in 17.47 seconds.
    0 examples learned
    50 examples learned
Epoch 8 done in 17.68 seconds.
    0 examples learned
    50 examples learned
Epoch 9 done in 17.53 seconds.
Loss = 4.497564963056905
    0 examples learned
    50 examples learned
```

We can see that the loss is decreasing. However the time taken for each epoch (just 100 sentences) is ~ 17 seconds.

9 Lets check predictions

```lisp
(defun argmax (vector)
    (loop with h = (aref vector 0)
          with hi = 0
          for i from 1 below (length vector)
          for v = (aref vector i) do
            (when (> v h)
              (setf h v
                    hi i))
          finally (return hi)))

(defun prediction (n x)
  (let ((p (forward-propagate n x)))
    (loop for pword across p
          for i from 0
          with aword = nil do
            (if (< i (length x))
                (setf aword (aref x i))
                (setf aword nil))
            (format t "~& ~s ~t~t~t ~s" (index-word aword)
                    (index-word (argmax pword))))))

(prediction *network* (first (aref *data* 0)))
```

```
""      "‘‘"
"now"      ""
"that"      ""
"he"     "."
"knew"     "."
"himself"     "."
"to"     "."
```

```
"be"     "."
"self"     "."
"he"     "."
"was"     "."
"free"     "."
"to"     "."
"grok"     "."
"ever"     "."
"closer"     "."
"to"     "."
"his"     "."
"brothers"     "."
","     "."
""     "."
"without"     "."
"let"     "."
"."     ""
```

Unfortunately with just 9 epochs of learning, the RNN hasn't learnt anything yet.