Class 5: Design of Price and Advertising Elasticity Models

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1 Variable Transformations and Non-linear Effects

1.1 Previous Sales Model

$$Sales = \beta_0 + \beta_1 \times Price + \beta_2 \times Advertising + \beta_3 \times Display$$

e.g., $Sales = 100 - 100 \times Price + 50 \times Advertising + 1000 \times Display$

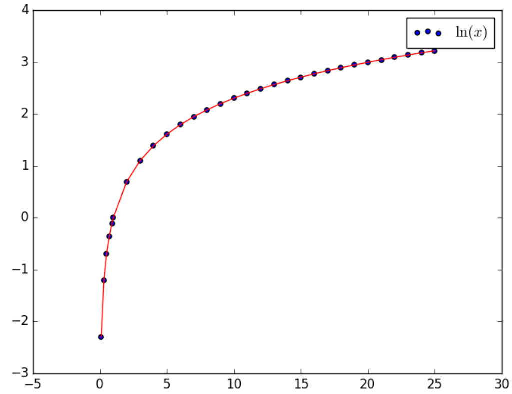
- Is this an adequate model of the sales marketing mix relationship?
- Problematic Implications:
  - Increasing advertising leads to consistent increase in sales
  - Increase in sales from a unit increase in advertising is same at all levels of advertising
  - A price decrease always lead to an increase in sales
    - Saturation point
- Solution: log transformation

1.2 Log Transformation

$$Sales = \beta_0 + \beta_1 \times \ln(Price) + \beta_2 \times \ln(Advertising) + \beta_3 \times Display$$

In general, two reasons to take a log:

1. A non-linear relationship (decreasing marginal return) exists between the independent and dependent variables.



- Examples: distance, income, etc

1.3 Non-Linear Effects: Inverted-U Relationship

- Likelihood of Purchasing Candy Bar = $1.1 + 3 \times$ Sweetness
  - So should we keep adding sugar?
- You can add a squared term (x^2 when you expect inverted-U relationship

For example, if our results are:

$$y = 1 + 3 \times x - 0.2 \times x^2$$

the predicted y values ($\hat{y}$) are:

x	1	2	3	4	5	6	7	8	9	10	11	12
$\hat{y}$	3.8	6.2	8.2	9.8	11	11.8	12.2	12.2	11.8	11.0	9.8	8.2

2 Elasticities

- The measurement of how responsive an variable is to a change in another
- The x (price, advertising, etc) elasticity of y (usually demand or supply) is:

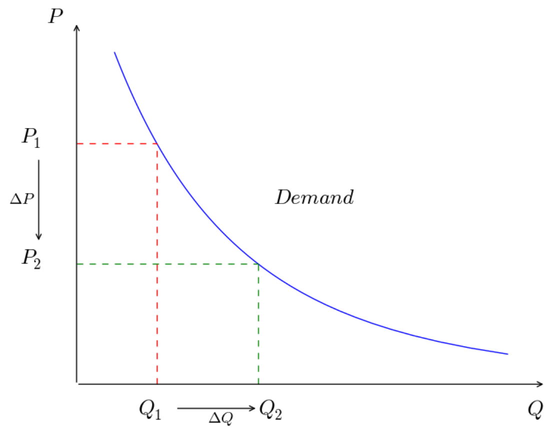
$$\frac{\text{Percentage change in } y}{\text{Percentage change in } x}$$

  - where

$$\text{Percentage change in } x = \frac{\Delta x}{x}$$

where Δ denotes the change

2.1 Price Elasticity of Demand



- Price elasticity of demand (PED):
  - Percentage change in quantity demanded in response to a **1% change** in price (holding constant all the other marketing mix variables)

$$\begin{aligned} PED &= \left| \frac{\text{\%Changes in Sales (Quantity Demanded)}}{\text{\%Changes in Own Price}} \right| \\ &= \left| \frac{\Delta Q}{Q} \Big/ \frac{\Delta P}{P} \right| \end{aligned}$$

Suppose Price increases 1%, and demand is (P)	Sales Decrease (Q)	Total Revenue $P \times Q$
Elastic $(PED > 1)$	More than 1 %	$\Downarrow$
Unit Elastic $(PED = 1)$	1 %	$\Longleftrightarrow$
Inelastic $(PED < 1)$	Less than 1 %	$\Uparrow$

Note

Note that PED only looks at the size of the change because the direction of the change is assumed to be negative.

2.1.1 Elasticities in Regression

- With a sample of historical data, you can measure the elasticities with the log-log model:

$$\ln(Q) = \beta_0 + \beta_1 \ln(P) + \varepsilon$$

- (The size of) β_1 represents the price elasticities of demand

$$\ln(Q) = 10 - 1.5\ln(P)$$

Why?

Remember the interpretation of β, the coefficient of in linear regression is:

$$\frac{dQ}{dP} = \frac{d(\beta_0 + \beta_1 P + \varepsilon)}{dP} = \beta_1$$

That is, the increase in Q when P increases by 1 unit
When you have a log-log model:

$$\ln(Q) = \beta_0 + \beta_1 \ln(P) + \varepsilon$$

$$\Rightarrow Q = \exp(\beta_0 + \beta_1 \ln(P) + \varepsilon)$$

$$\frac{dQ}{dP} = \underbrace{\exp(\beta_0 + \beta_1 \ln(P) + \varepsilon)}_{Q} \cdot \underbrace{\beta_1 \cdot \frac{1}{P}}_{\text{Chain Rule}} = Q \cdot \beta_1 \cdot \frac{1}{P}$$

Hence,

$$\beta_1 = \frac{dQ}{Q} \cdot \frac{P}{dP} = \frac{dQ}{Q} \Bigg/ \frac{dP}{P}$$

NOTE: This derivation will not be on the exam

2.2 Cross Price Elasticities

- Elasticity between variables of two different products
- Important because many products are **complements** or **substitutes** to each other:
  - Cannibalization
- Product 1's cross-price elasticity of product 2 would be:
  - The impact of product 2's percentage change in prices on the percentage change in product 1's sales

$$\frac{\text{Changes in Sales (Quantity Demanded)}}{\text{Changes in Price of Another Good}} = \frac{\Delta Q}{Q} \Bigg/ \frac{\Delta P_{other}}{P_{other}} = \frac{d\ln(Q)}{d\ln(P_{other})}$$

2.2.1 Cross Price Elasticities in Regression

- Hence, if you have the following model:

$$\ln(Q) = \beta_0 + \beta_1 \ln(P) + \beta_2 \ln(P_{other}) + \varepsilon$$

  β_2 reflects the cross price elasticity

- If β_2 is positive ...
  - Other good price increases, your sales increase $P_{other} \uparrow (Q_{other} \downarrow) \Rightarrow Q \uparrow$
  - e.g. Raise price of salsa and sales of cheese dip increase
  - Two products are *substitute*
- If β_2 is negative ...
  - Other good price increases, your sales decrease $P_{other} \uparrow (Q_{other} \downarrow) \Rightarrow Q \downarrow$
  - e.g. Raise price of salsa and sales of chips decrease
  - Two products are *complements*
- If β_2 is zero (insignificant) ...
  - Other good price increases, your sales don't change
  - e.g. Raise price of salsa and sales of pasta sauce not affected
  - Two products are *independent*

2.3 The log-log sales response model

$$\ln(\text{sales in period } t) = \beta_0 + \beta_1 \times \ln(\text{own price in period} t) + \beta_2 \times \ln(\text{competitor price in period } t) + \varepsilon_t$$

- This model typically fits the data much better than the linear model
- Coefficients to log(prices) may be interpreted as price elasticities

2.4 Advertising Elasticity of Demand (AED)

- Just like the price elasticities of demand,
- A measure to show the responsiveness of the quantity demanded of a good (or service) to a change in the level of advertising:

$$
\begin{aligned}
AED &= \frac{\text{\%Changes in Sales}}{\text{\%Changes in Advertising}} \\
&= \frac{\Delta Q}{Q} \Bigg/ \frac{\Delta A}{A}
\end{aligned}
$$

2.4.1 AED Regression

Similarly, β_1 in the following regression equation represents the advertising elasticities of demand:

$$
\ln(Q) = \beta_0 + \beta_1 \cdot \ln(A) + \varepsilon
$$

2.5 Price and Advertising Elasticity Model

You can estimate all the elasticities with the following model:

$$
\begin{aligned}
\ln(\text{Sales in period } t) = \beta_0 + \beta_{own} \times \ln(\text{Own Price in period } t) \\
+\beta_{cross} \times \ln(\text{Other Good Price in period } t) \\
+\beta_{ad} \times \ln(\text{Advertising}_t) \\
+\beta_{display} \times \text{Display}_t
\end{aligned}
$$

- Interpretation of Coefficients:
  - β_{own}: own price elasticity
  - β_{cross}: cross price elasticity
  - β_{ad}: advertising elasticity
  - $\beta_{display}$: impact of display on sales

Interpretation of $\beta_{display}$

The Display dummy equals to 1 when the product is on display. Hence, $\beta_{display}$ represents the increase in ln(Sales) when the product is on display comparing to the baseline case where the product is not on display. That is,

$$
\begin{aligned}
\beta_{display} &= \ln(\text{Sales}_{Display}) - \ln(\text{Sales}_{NoDisplay}) \\
\Rightarrow \beta_{display} &= \ln\left(\frac{\text{Sales}_{Display}}{\text{Sales}_{NoDisplay}}\right)
\end{aligned}
$$

$$\Rightarrow \exp(\beta_{display}) = \frac{\text{Sales}_{Display}}{\text{Sales}_{NoDisplay}}$$

Hence, $\exp(\beta_{display})$ is the ratio between the two sales. For example, if $\beta_{display}$ were 0.3, $\exp(0.3) = 1.35$, which means a product's sales increase by 35% when it is on display compared to when it is not.