

**Title :** Strong piecewise linearity regularization based on the  $\ell_0$ -norm.  
Application to the estimation of the Covid19 reproduction number

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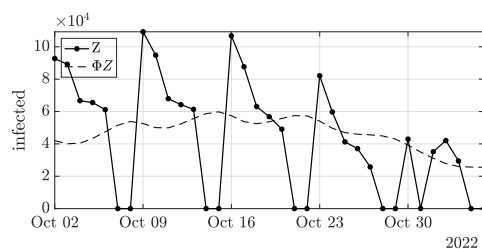
**Location :** Laboratoire des Sciences du Numérique de Nantes, Centrale Nantes

**Date/duration :** Spring 2023 ; 4 to 6 months

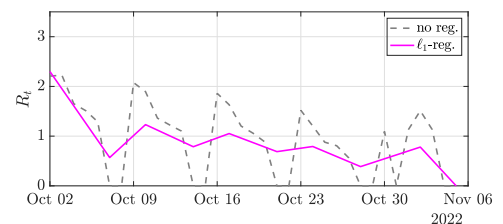
**Salary :** ~ 600 euros per month (standard internship compensation)

**Abstract :** Since the beginning of the Covid-19 pandemic, daily new infections (solid line, left plot) have been monitored. Yet, it is not informative enough about the dynamics of the pandemic and decision makers rapidly got interested in the *reproduction number*  $R_t$  at day  $t$ , quantifying the number of people infected by one sick individual :  $R_t > 1$ , corresponds to exponential spread of the virus ; while for  $R_t < 1$ , the epidemic is vanishing.  $R_t$  is linked to daily new infections by Cori's model [1] stating that the number of new cases follows a nonstationary Poisson distribution whose instantaneous parameter at day  $t$  is proportional to  $R_t$ . Direct estimation of  $R_t$  provides very irregular estimates (dashed gray line, right plot). To obtain realistic estimates, it has been proposed to enforce temporal regularity using a  $\ell_1$  penalization on the second order derivative of  $R_t$  [2]. Yet, the  $\ell_1$ -norm only enables to obtain *approximate* sparsity : the second-order derivative of the obtained estimates are not sparse (see the magenta curve on right plot). Recent approaches [3] have proposed numerical schemes enforcing sparsity by using a  $\ell_0$ -norm based regularization term. The corresponding solution is then likely to provide more accurate estimates, despite their much higher numerical complexity. In the considered case, however, the relatively small dimension of the problems should make it possible to use such exact approaches. For the moment, these techniques have mostly be used coupled to quadratic data-fidelity terms, which does not cover the estimation of  $R_t$  proposed in [2].

The purpose of this project are : 1) to get familiar with variational tools for the estimation of its reproduction number ; 2) to understand the numerical challenges raised by the  $\ell_0$ -norm ; 3) to identify the major deadlocks in using an  $\ell_0$ -norm penalization in the estimation of the reproduction number  $R_t$  ; 4) to develop a propotype algorithm in MATLAB that implements the branch-and-bound optimization of the  $\ell_0$  norm solution.



Daily new infections in France



Estimated reproduction number

**Bibliography :**

- [1] Cori, A., Ferguson, N. M., Fraser, C., & Cauchemez, S. (2013). A new framework and software to estimate time-varying reproduction numbers during epidemics. *American journal of epidemiology*, vol. 178, no 9, p. 1505-1512.
- [2] Abry, P., Pustelnik, N., Roux, S., Jensen, P., Flandrin, P., Gribonval, R., ... & Garnier, N. (2020). Spatial and temporal regularization to estimate COVID-19 reproduction number  $R(t)$  : Promoting piecewise smoothness via convex optimization. *Plos One*, vol. 15, no 8.
- [3] Ben Mhenni, R., Bourguignon, S., & Ninin, J. (2021). Global optimization for sparse solution of least squares problems. *Optimization Methods and Software*, 1-30.

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