PHYS405 - Possibly Useful Formulas

Fundamental Equations

$$
\begin{aligned}
i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t} &= \frac{-\hbar^2}{2m}\nabla^2\Psi(\vec{r},t) + V(\vec{r},t)\Psi(\vec{r},t) \\
\frac{-\hbar^2}{2m}\nabla^2\psi(\vec{r}) + V(\vec{r})\psi(\vec{r}) &= E\psi(\vec{r})
\end{aligned}
$$

Some Important Solutions

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) \qquad E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \qquad \xi = \sqrt{\frac{m\omega}{\hbar}}x$$

$$\psi_n(x) = \frac{\alpha}{\sqrt{2^n n!}}H_n(\xi)e^{-\xi^2/2} \qquad E_n = \left(n+\frac{1}{2}\right)\hbar\omega \qquad \alpha = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}$$

$$\psi_{N\ell m}(\vec{r}) = A_{N\ell}\, j_\ell\left(\frac{\beta_{N\ell}\, r}{a}\right)Y_\ell^m(\theta,\phi) \qquad E_{N\ell} = \frac{\beta_{N\ell}\pi^2\hbar^2}{2ma^2}$$

$$\psi_{n\ell m}(\vec{r}) = A_{n\ell}\, e^{-r/na}\left(\frac{2r}{na}\right)^\ell\left[L_{n-\ell-1}^{2\ell+1}\left(\frac{2r}{na}\right)\right]Y_\ell^m(\theta,\phi) \qquad E_n = \frac{E_1}{n^2}$$

Mathematical Relations

$$
\begin{aligned}
e^{i\theta} &= \cos\theta + i\sin\theta \\
\cos(\theta) &= \frac{1}{2}\left[e^{i\theta} + e^{-i\theta}\right] \\
\sin(\theta) &= \frac{1}{2i}\left[e^{i\theta} - e^{-i\theta}\right]
\end{aligned}
$$

$$
\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B
\end{aligned}
$$

$$
\begin{aligned}
\nabla^2 f &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \\
\nabla^2 f &= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial f}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial f}{\partial\theta}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}
\end{aligned}
$$

Fundamental Constants and Units

$$
\begin{aligned}
e &= 1.602\times10^{-19}\,\mathrm{C} \\
m_e &= 9.11\times10^{-31}\,\mathrm{kg} = 511\,\mathrm{keV/c^2} \\
m_p &= 1.67\times10^{-27}\,\mathrm{kg} = 938\,\mathrm{MeV/c^2} \\
\hbar &= 1.055\times10^{-34}\,\mathrm{J\cdot s} = 6.582\times10^{-16}\,\mathrm{eV\cdot s} \\
\hbar c &= 197.3\,\mathrm{eV\cdot nm}
\end{aligned}
$$

$$
\begin{aligned}
a &= 0.529\times10^{-10}\,\mathrm{m} \\
E_1 &= -13.6\,\mathrm{eV} \\
1\,\mathrm{J} &= 1\,\mathrm{N\cdot m} = 1\,\mathrm{kg}\cdot\frac{\mathrm{m}^2}{\mathrm{s}^2} \\
1\,\mathrm{eV} &= 1.602\times10^{-19}\,\mathrm{J}
\end{aligned}
$$

Special Functions

$$P_\ell^m(x) = \frac{(-1)^m}{2^\ell \ell!}(1-x^2)^{m/2}\left(\frac{d}{dx}\right)^{\ell+m}(x^2-1)^\ell$$

$$P_\ell^{-m}(x) = (-1)^m\frac{(\ell-m)!}{(\ell+m)!}P_\ell^m(x)$$

$$P_0^0(x) = 1$$

$$P_1^0(x) = x$$

$$P_1^1(x) = -\sqrt{1-x^2}$$

$$P_2^0(x) = \frac{1}{2}(3x^2-1)$$

$$P_2^1(x) = -3x\sqrt{1-x^2}$$

$$P_2^2(x) = 3(1-x^2)$$

$$H_n(\xi) = (-1)^n e^{\xi^2}\left(\frac{d}{d\xi}\right)^n e^{-\xi^2}$$

$$H_0(\xi) = 1$$

$$H_1(\xi) = 2\xi$$

$$H_2(\xi) = 4\xi^2 - 2$$

$$H_3(\xi) = 8\xi^3 - 12\xi$$

$$H_4(\xi) = 16\xi^4 - 48\xi^2 + 12$$

$$j_\ell(x) = (-x)^\ell\left(\frac{1}{x}\frac{d}{dx}\right)^\ell \frac{\sin x}{x}$$

$$j_0(x) = \frac{\sin x}{x}$$

$$j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x}$$

$$j_2(x) = \left(\frac{3}{x^3} - \frac{1}{x}\right)\sin x - \frac{3}{x^2}\cos x$$

$$L_q^p(x) = \frac{(-1)^p}{(p+q)!}\left(\frac{d}{dx}\right)^p\left[e^x\left(\frac{d}{dx}\right)^{p+q}\left(e^{-x}x^{p+q}\right)\right]$$

$$L_0^0(x) = 1$$

$$L_0^1(x) = 1$$

$$L_0^2(x) = 1$$

$$L_1^0(x) = 1 - x$$

$$L_1^1(x) = 2 - x$$

$$L_1^2(x) = 3 - x$$

$$L_2^0(x) = \frac{1}{2}x^2 - 2x + 1$$

$$L_2^1(x) = \frac{1}{2}x^2 - 3x + 3$$

$$L_2^2(x) = \frac{1}{2}x^2 - 4x + 6$$

Some Integrals

$$\int_0^\infty u^n e^{-u}du = n!$$

$$\int_{-\infty}^\infty e^{-u^2}du = \sqrt{\pi}$$

$$\int_{-\infty}^\infty u^2 e^{-u^2}du = \sqrt{\pi}/2$$

$$\int_{-\infty}^\infty u^4 e^{-u^2}du = \frac{3\sqrt{\pi}}{4}$$

$$\int_{-\infty}^\infty u^6 e^{-u^2}du = \frac{15\sqrt{\pi}}{8}$$

$$\int_0^\pi \sin(nu)\sin(mu)du = \frac{\pi}{2}\delta_{nm}$$

$$\int_0^\pi u\sin(nu)du = \frac{(-1)^{n+1}\pi}{n}$$

$$\int_0^\pi u\sin(nu)\sin(mu)du = \frac{2mn[(-1)^{m+n}-1]}{(m-n)^2(m+n)^2}$$

$$\int_0^\pi u\sin^2(nu)du = \frac{\pi^2}{4}$$

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$