Testing Hypotheses about a Population Mean

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Cartwheel Study

- 25 team members/colleagues (all adults) asked to perform a cartwheel
- **Variable**: Cartwheel Distance (in inches)

Research Question

Is the ***average*** cartwheel distance (in inches) for adults more than 80 inches?

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Is the ***average*** cartwheel distance for adults more than 80 inches?

Population: All adults

Parameter of Interest: population mean cartwheel distance μ

Perform a one-sample test regarding the value for the mean cartwheel distance for the population of all such adults.

Step 1: Define the Null and Alternative

- Null: Population mean CW distance (μ) is 80 inches
- Alternative: Population mean is _**greater than (>)**_ 80 inches

More compact notation:

- H_0: $\mu = 80$
- H_a: $\mu > 80$

where μ represents the population mean cartwheel distance (inches) for all adults

Significance Level = 5%

Step 2: Examine Results, Check Assumptions, Summarize Data via Test Statistic

```
df.describe()["CWDistance"]
```

```
count     25.000000
mean      82.480000
std       15.058552
min       63.000000
25%       70.000000
50%       81.000000
75%       92.000000
max      115.000000
Name: CWDistance, dtype: float64
```

n = 25 observations

Minimum = 63 inches
Maximum = 115 inches

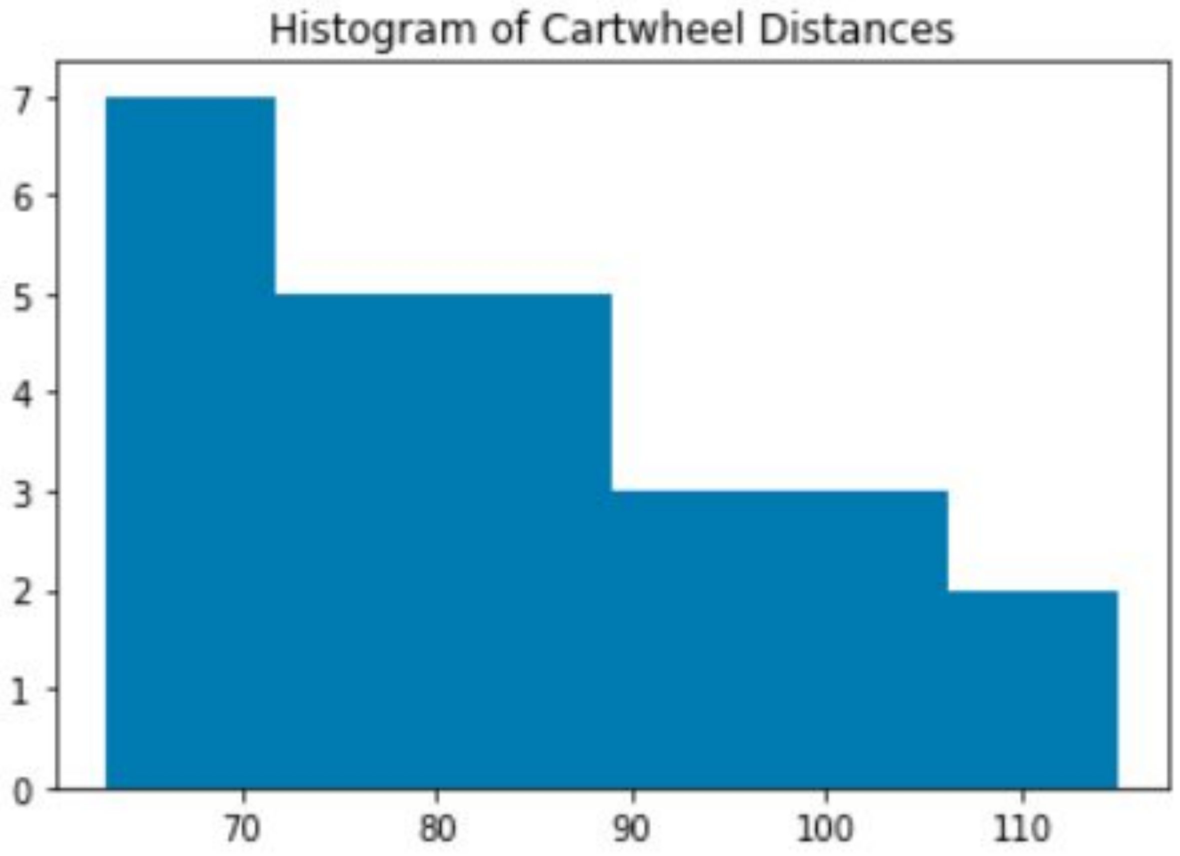
$\boxed{\text{Mean = 82.48 inches}}$
Standard Deviation = 15.06 inches

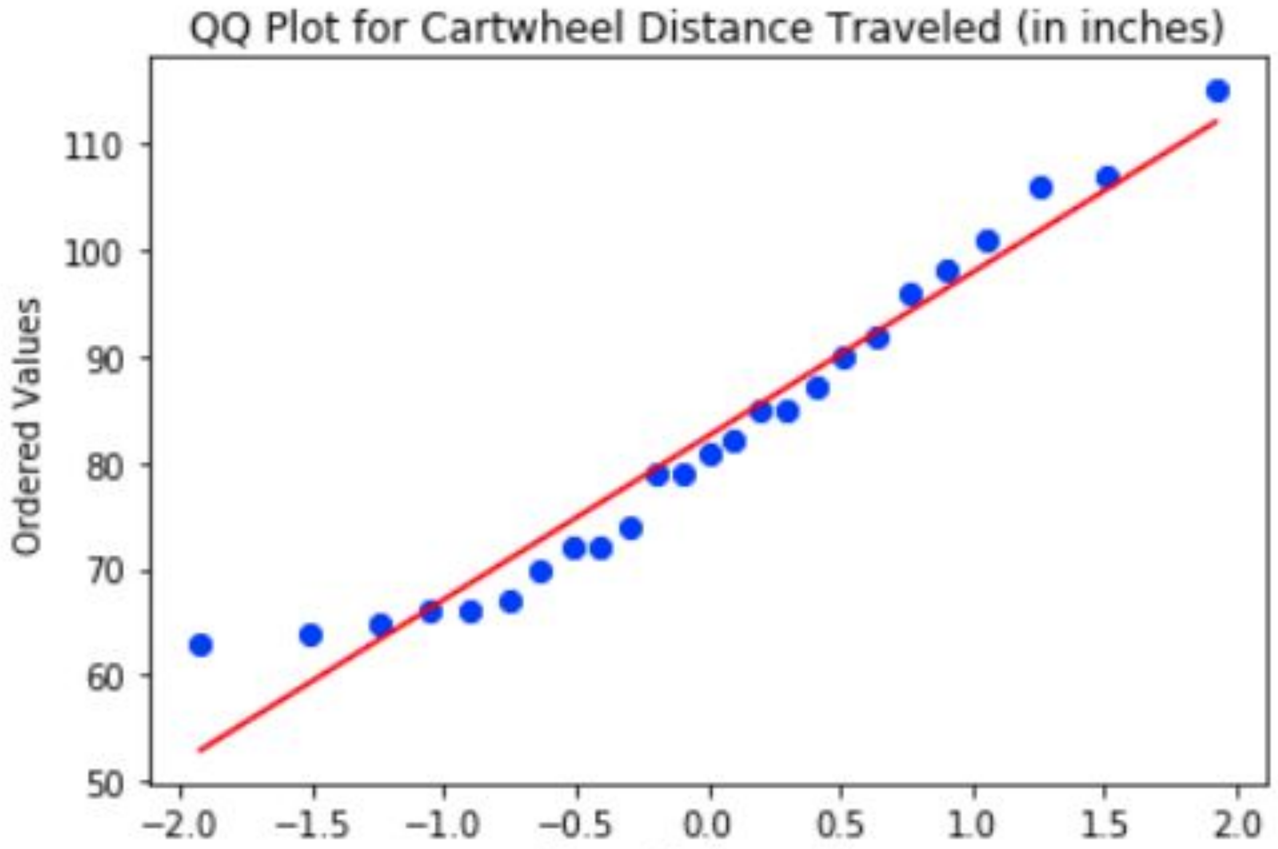
Step 2: Examine Results, Check Assumptions, Summarize Data via Test Statistic

Assumptions:

- Sample of **CW Distance measurements** considered a **simple random sample**
- **Normal** distribution for CW Distances in population (not as critical given large sample size, but still graph our data!)

Step 2: Examine Results, Check Assumptions, Summarize Data via Test Statistic





Histogram and Normal Q-Q Plot suggest modest deviations from **normality**

Note: reasonable sample size + CLT … normality assumption not so crucial

Step 2: Examine Results, Check Assumptions, Summarize Data via Test Statistic

H_0: $\mu = 80$

H_a: $\mu > 80$

- Is sample mean of 82.48 inches *significantly greater than* hypothesized mean of 80 inches?

standard error of the sample mean $= \dfrac{\sigma}{\sqrt{n}}$

***estimated* standard error of the sample mean** $= \dfrac{s}{\sqrt{n}}$

Step 2: Examine Results, Check Assumptions, Summarize Data via Test Statistic

Test Statistic: Assuming sampling distribution of sample mean is normal,

$$t = \frac{best\ estimate - null\ value}{estimated\ std\ error} = \frac{\bar{x} - 80}{\frac{s}{\sqrt{n}}}$$

$$= \frac{82.48 - 80}{\frac{15.06}{\sqrt{25}}} = \frac{2.48}{3.012} = \mathbf{0.82}$$

Test Statistic Interpretation

- $$t = \frac{best\ estimate - null\ value}{estimated\ std\ error} = \frac{\bar{x} - 80}{\frac{s}{\sqrt{n}}} = \frac{82.48 - 80}{\frac{15.06}{\sqrt{25}}} = 0.82$$

Our **sample** mean is **only 0.82** (*estimated*) **standard errors above null value of 80 inches**

Step 3: Determine P-Value

- If null hypothesis was true, would a test statistic value of only $t = 0.82$ be unusual enough to reject the null?

- **P-value** = Probability of seeing test statistic of 0.82 or *more extreme* assuming the null hypothesis is true.

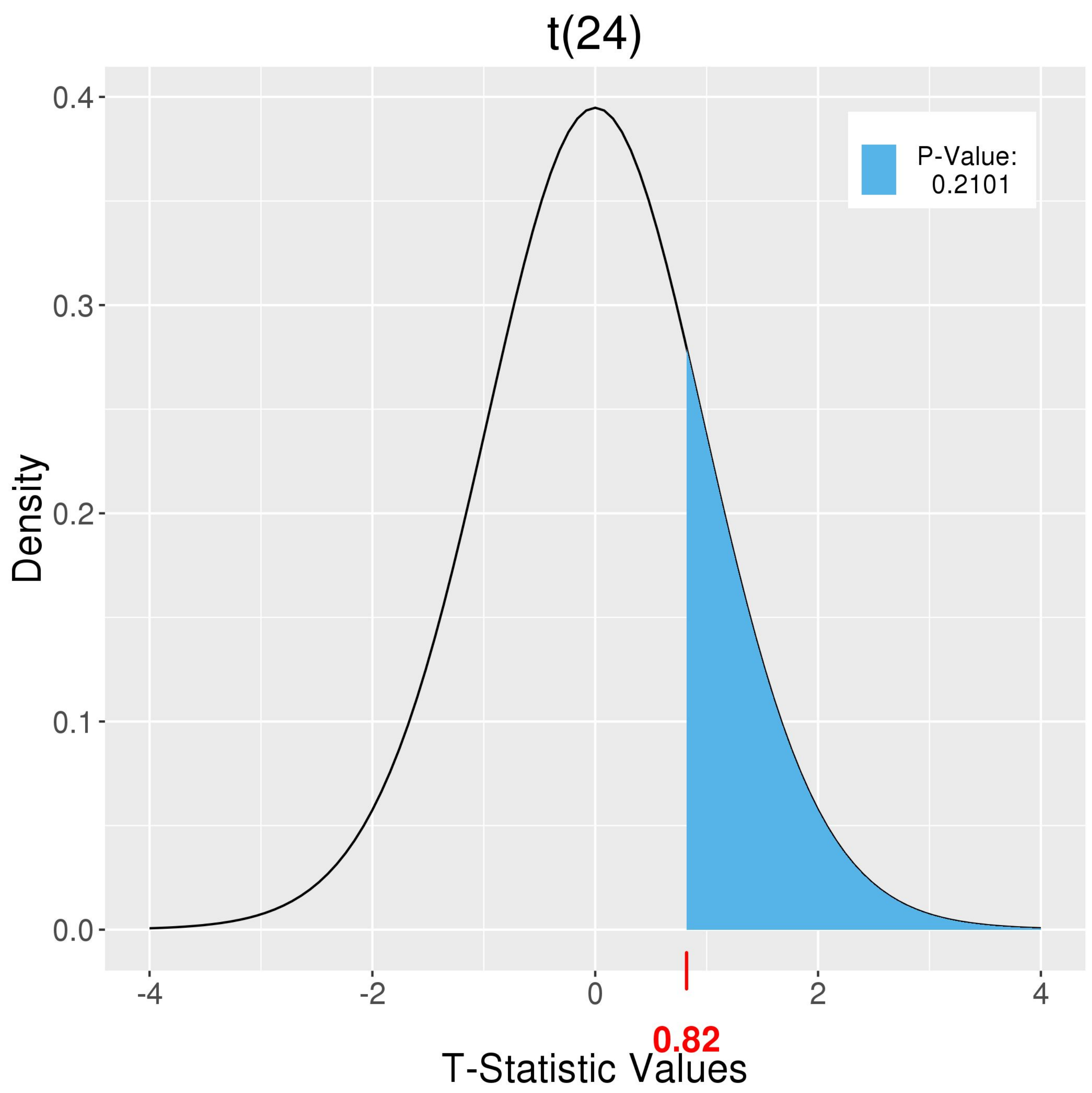
- If null hypothesis was true, t test statistic follows a Student t distribution with degrees of freedom $n - 1 = 25 - 1 = 24$.

- Since we have a one tailed test to the right
  - More extreme measured to the right (upper tail).

Step 3: Determine P-Value

P-value = 0.21

If population mean CW distance was really was 80 inches, then observing a sample mean of 82.48 inches (i.e. a t statistic of 0.82) or larger is **quite likely**.



Step 4: Make a Decision about the Null

Since our P-value is much bigger than 0.05 significance level,
weak evidence against the null
→ we **fail to reject the null!**

Based on estimated mean (82.48 inches),
we ***cannot support***
the population mean CW distance
is greater than 80 inches

90% Confidence Interval Estimate

Mean = 82.48 inches

Standard Deviation = 15.08 inches

n = 25 observations → t^* = 1.711

$$\bar{x} \quad \pm \quad t^* \left(\frac{s}{\sqrt{n}}\right)$$

$$82.48 \pm 1.711\left(\frac{15.06}{\sqrt{25}}\right)$$

$$82.48 \pm 1.711(3.012)$$

$$82.48 \pm 5.15$$

$$\boxed{(77.33 \text{ inches}, 87.63 \text{ inches})}$$

Note: 80 inches is <u>IN</u> confidence interval of reasonable values for population mean CW distance

What if Normality Doesn't Hold?

- Not convinced that CW Distance follows a normal distribution in the population?

  □ **non-parametric test** that does not assume normality

- Non-parametric analog of the one sample t-test

  = **Wilcoxon Signed Rank Test**

  ~ uses median to examine location of distribution of measurements

What if Normality Doesn't Hold?

Wilcoxon Signed Rank Test Result: p-value >> 0.05

Fail to reject the null that population median CW distance 80 inches

Conclusion is robust to potential violations of normality!

For the population of interest (all adults)
~ regardless of assumptions made and inference approach used
 There is **not** sufficient evidence to support that the population mean CW distance is more than 80 inches

Summary

- Hypothesis Tests are used to put theories about a parameter of interest to the test **~ parameter = population mean**
- Basic Steps:
  - **State hypotheses (and select significance level)**
  - **Examine results, check assumptions, summarize via test statistic**
  - **Convert test statistic to P-value**
  - **Compare P-value to significance level to make decision**
- Assumptions for One-sample (t) Test for Population mean
  - **Data considered a random sample**
  - **Population of responses is normal (else n large helps)**
- Know how to interpret the **p-value, decision, and conclusion**