Testing for a Difference in Population Means (for Independent Groups)

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2 Line Segment Title

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Parameter of Interest ($\mu_1 - \mu_2$): Body Mass Index or BMI (kg/m^2)

Research Question

Considering Mexican-American adults (ages 18 - 29) living in the United States, do males have a significantly higher mean Body Mass Index than females?

Task: Perform an independent samples t-test regarding the value for the difference in mean BMI between males and females.

2 Line Segment Title

Steps to Perform a Hypothesis Test

1. Define null and alternative hypotheses
2. Examine data, check assumptions, and calculate test statistic
3. Determine corresponding p-value
4. Make a decision about null hypothesis

Step 1: Define Hypotheses

Null: There is no difference in mean BMI

Alternative: There is a significant difference in mean BMI

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Significance Level = 5%

2 Line Segment Title

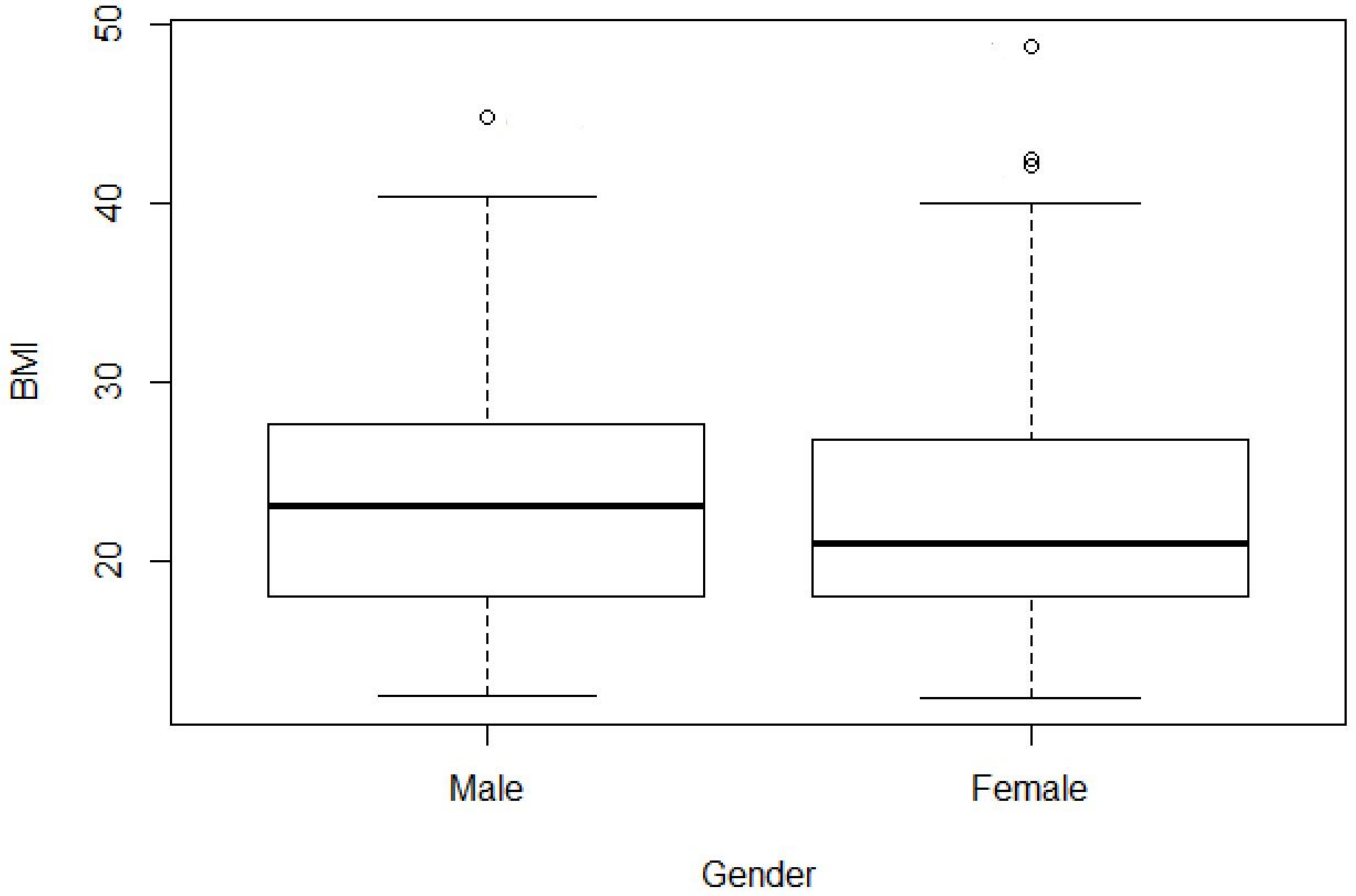
Step 2: Examine Data

Gender	BMI	Race	Age 18-29
1	19.9	1	1
2	17.0	1	1
2	26.7	1	1
1	25.6	1	1
…	…	…	…

The data was filtered to include only Mexican-American adults that were between the ages of 18 and 29.

Step 2: Examine Data

	Male	Female
Mean	23.57	22.83
St. Dev.	6.24	6.43
Min	12.5	12.4
Max	44.9	48.8
n	258	239



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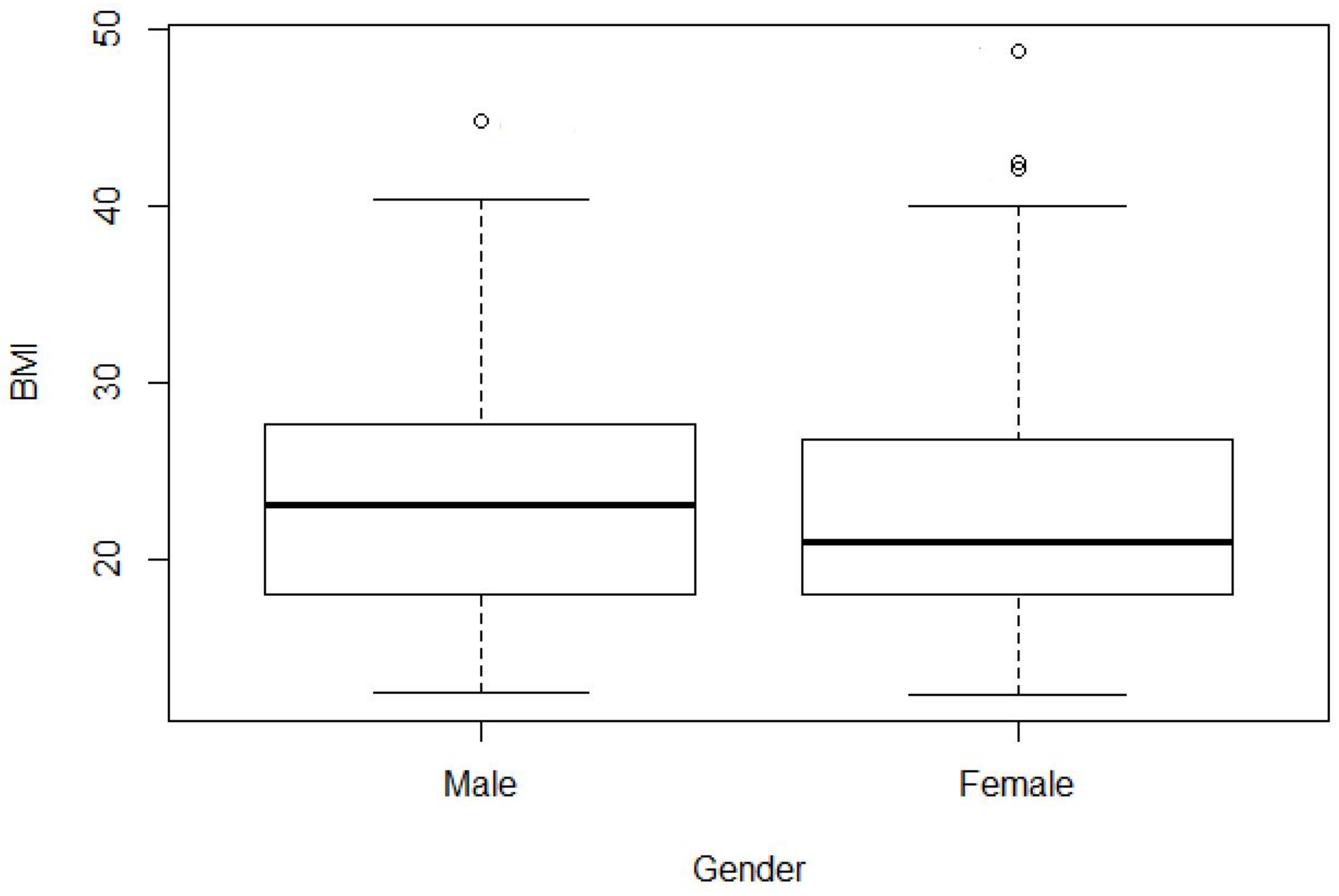

Step 2: Check Assumptions

Samples are considered simple random samples

Samples are independent from one another

Both populations of responses are approximately normal (or sample sizes are both 'large' enough)

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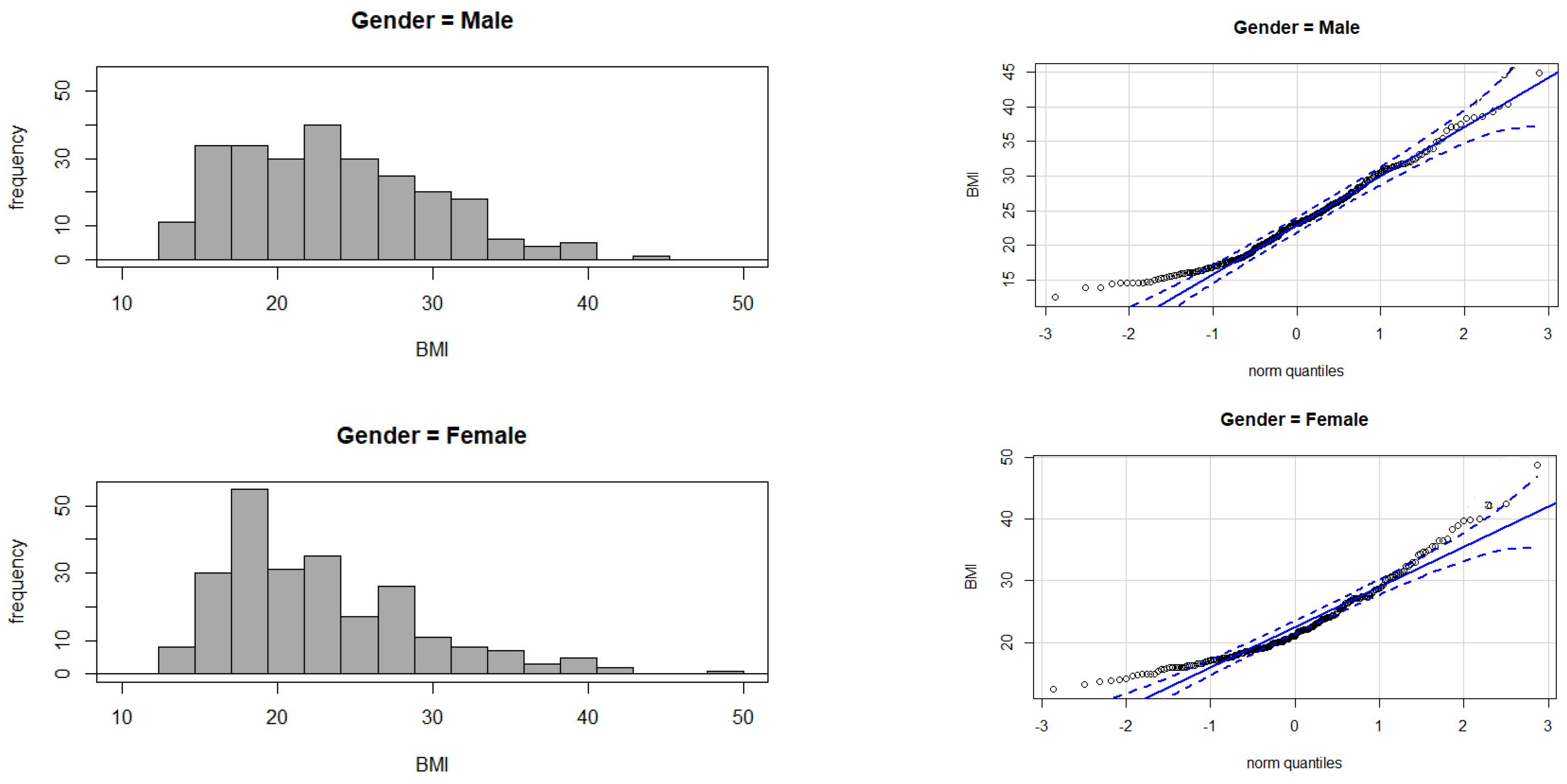

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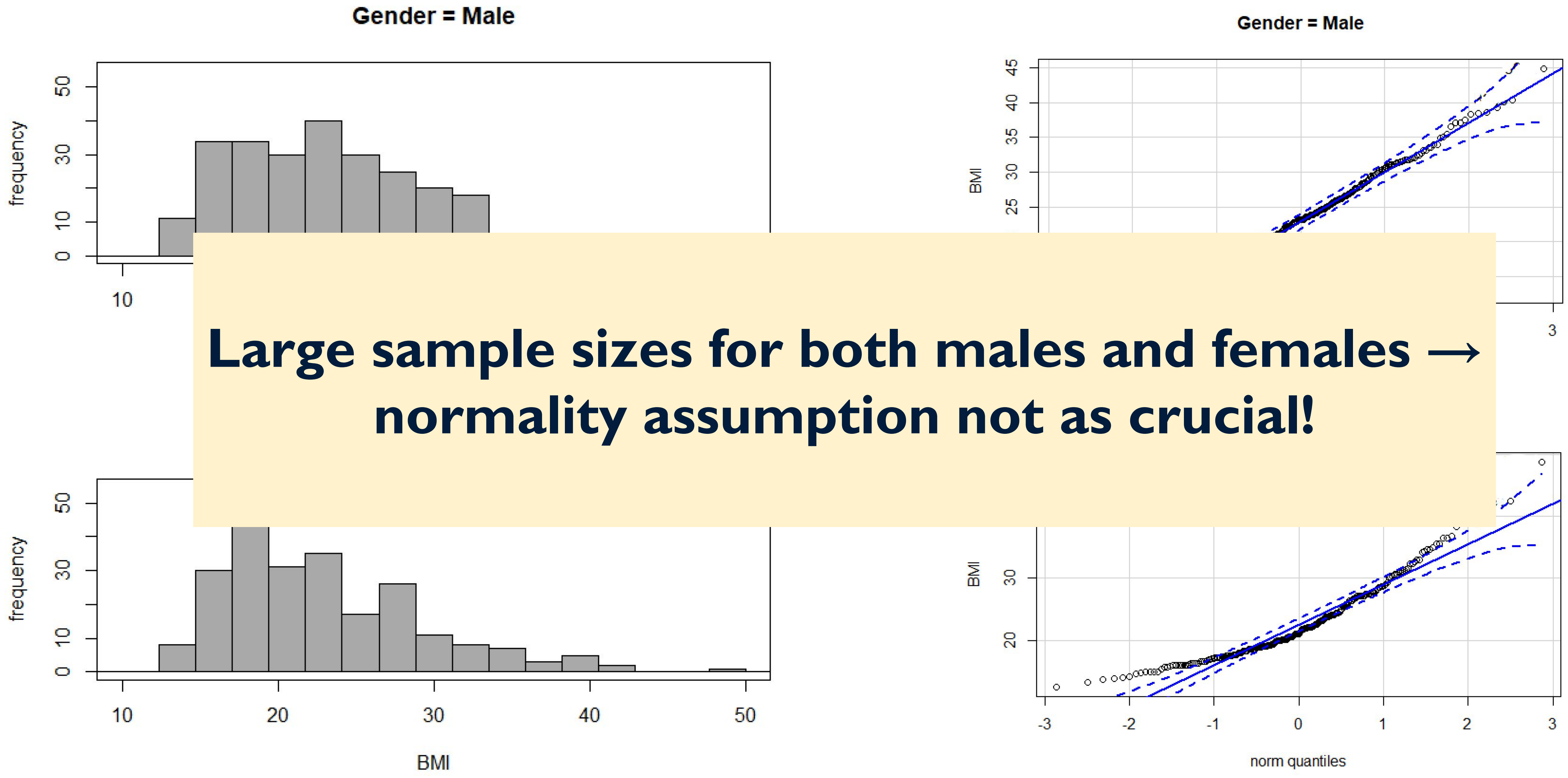

Large sample sizes for both males and females → normality assumption not as crucial!

Step 2: Calculate Test Statistic

- H$_0$: $\mu_1 - \mu_2 = 0$ vs. H$_a$: $\mu_1 - \mu_2 \neq 0$

Best Estimate: $\bar{x}_1 - \bar{x}_2 = 23.57 - 22.83 = 0.74$

Is our sample mean difference of 0.74 kg/m^2 significantly different than 0?

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Best Estimate: $\bar{x}_1 - \bar{x}_2 = 23.57 - 22.83 = 0.74$

Is our sample mean difference of 0.74 kg/m^2 significantly different than 0?

We'll use a test statistic to find out!

Test Statistic

A measure of how far our sample statistic is from our hypothesized population parameter, in terms of <u>estimated</u> standard errors

The further away our sample statistic is, the less confident we'll be in our null hypothesized value

Step 2: Calculate Test Statistic

- $$t = \frac{best\ estimate\ - null\ value}{estimated\ standard\ error}$$

Step 2: Calculate Test Statistic

Pooled Approach

The variance of the two populations are assumed to be equal $(\sigma_1^2 = \sigma_2^2)$

Unpooled Approach

The assumption of equal variances is dropped

Step 2: Calculate Test Statistic

Pooled Approach:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) \; - 0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Step 2: Calculate Test Statistic

Pooled Approach:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

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- **Unpooled Approach:**

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Pooled or Unpooled?

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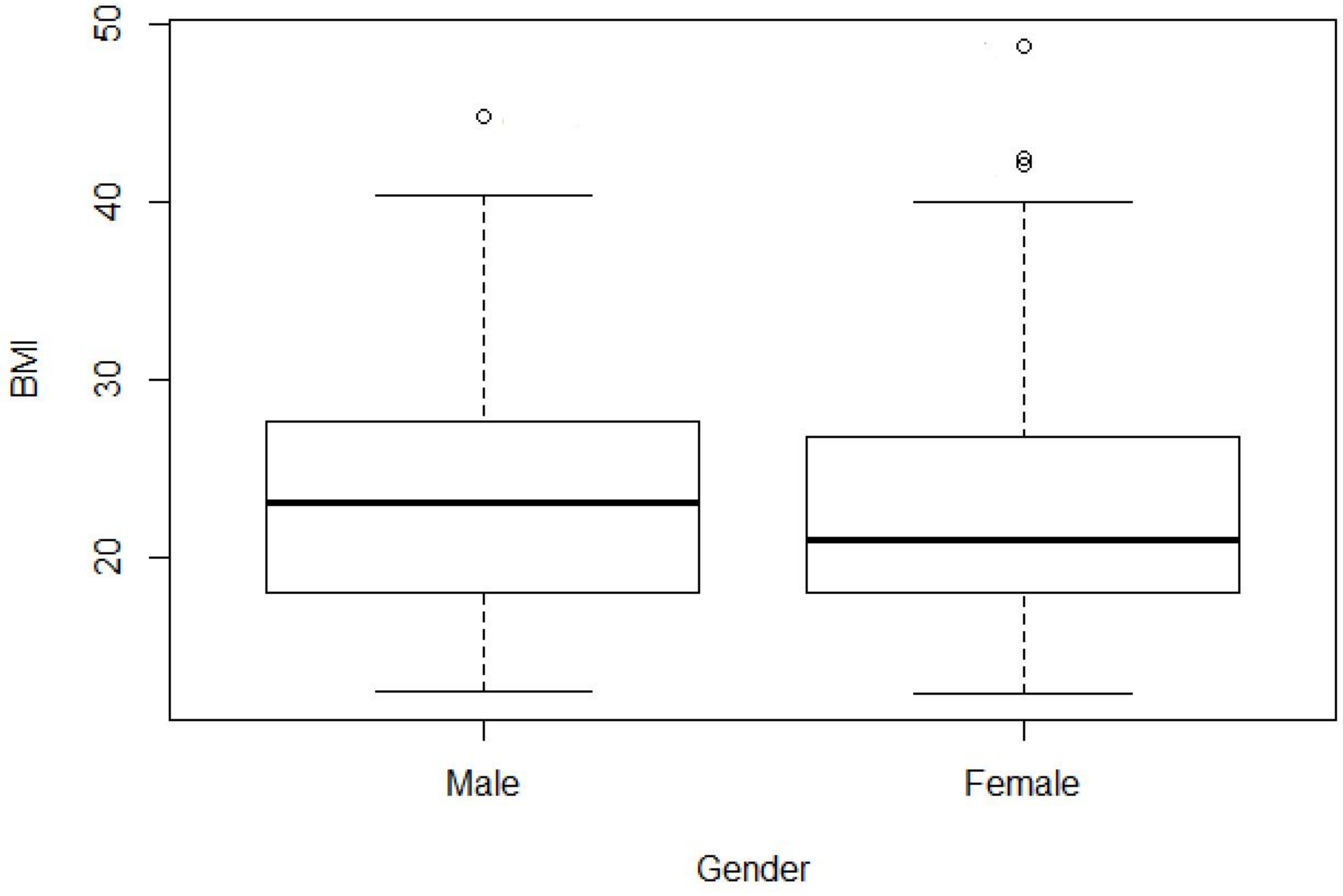

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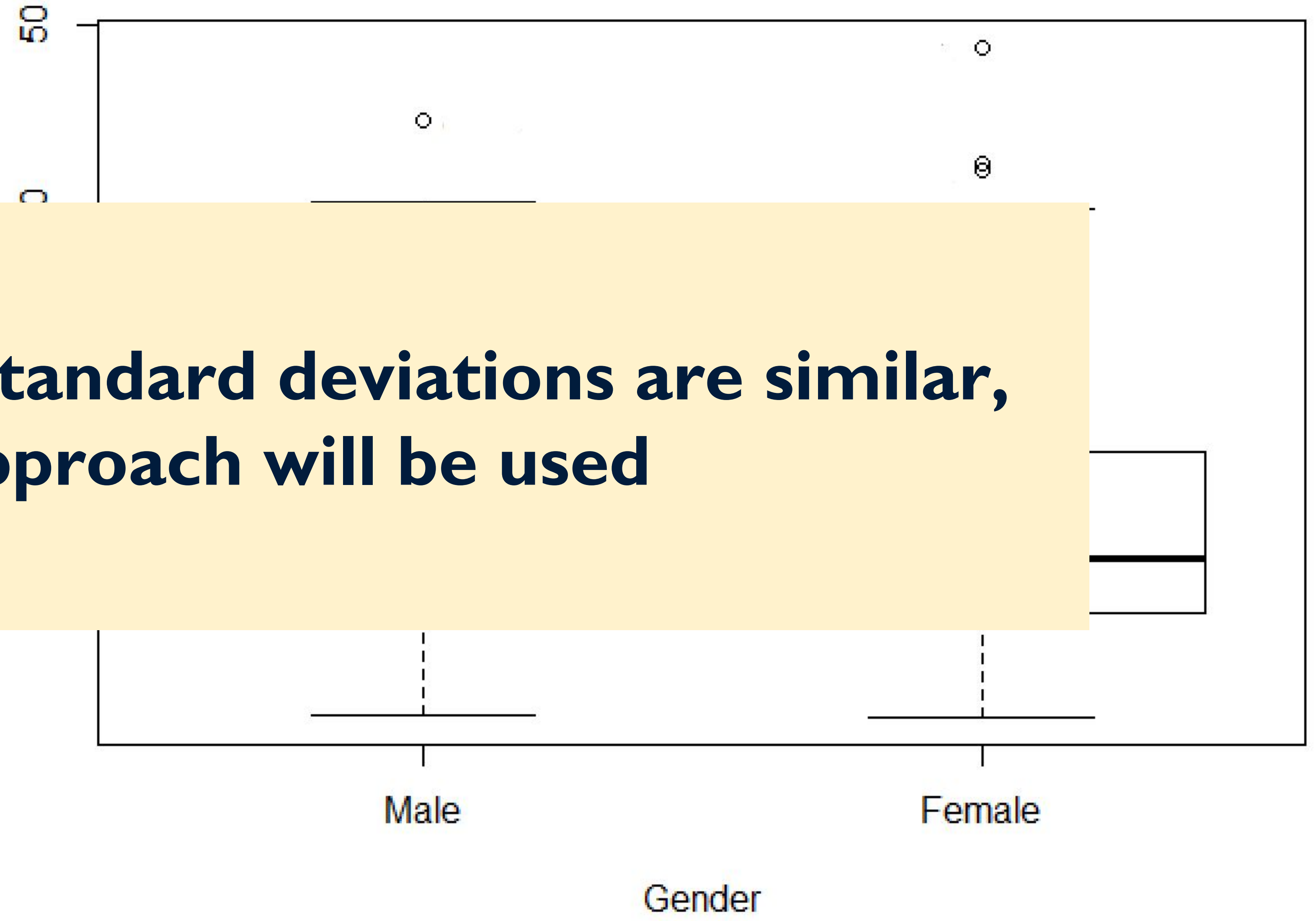

Because the IQR's and standard deviations are similar, the pooled approach will be used

Step 2: Calculate Test Statistic

- $$t = \frac{(\bar{x}_1 - \bar{x}_2) \ - 0}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{(23.57 - 22.83)}{\sqrt{\frac{(257)6.24^2 + (238)6.43^2}{495}} \sqrt{\frac{1}{258} + \frac{1}{239}}}$$

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$$t = \frac{0.74}{0.0898 * 6.332} = \mathbf{1.30}$$

Step 2: Calculate Test Statistic

- $t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{(n_1 \cdots}{}}}$ … $\frac{(23.57 - 22.83)}{\cdots \frac{1}{258} + \frac{1}{239}}$

Our difference in sample means is only 1.30 (*estimated*) standard errors above the null difference of 0 kg/m²

$$t = \frac{0.74}{0.0898 * 6.332} = \mathbf{1.30}$$

Step 3: Determine p-value

- $$t = 1.30$$

If the null hypothesis $(\mu_1 - \mu_2 = 0)$ were true, would a test statistic value of 1.30 be unusual enough to reject the null?

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If the null hypothesis $(\mu_1 - \mu_2 = 0)$ were true, would a test statistic value of 1.30 be unusual enough to reject the null?

p-value: assuming the null hypothesis is true, it is the probability of observing a test statistic of 1.30 or more extreme

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Our alternative hypothesis is two-sided $(\mu_1 - \mu_2 \neq 0)$ so we will check both the upper and lower tail

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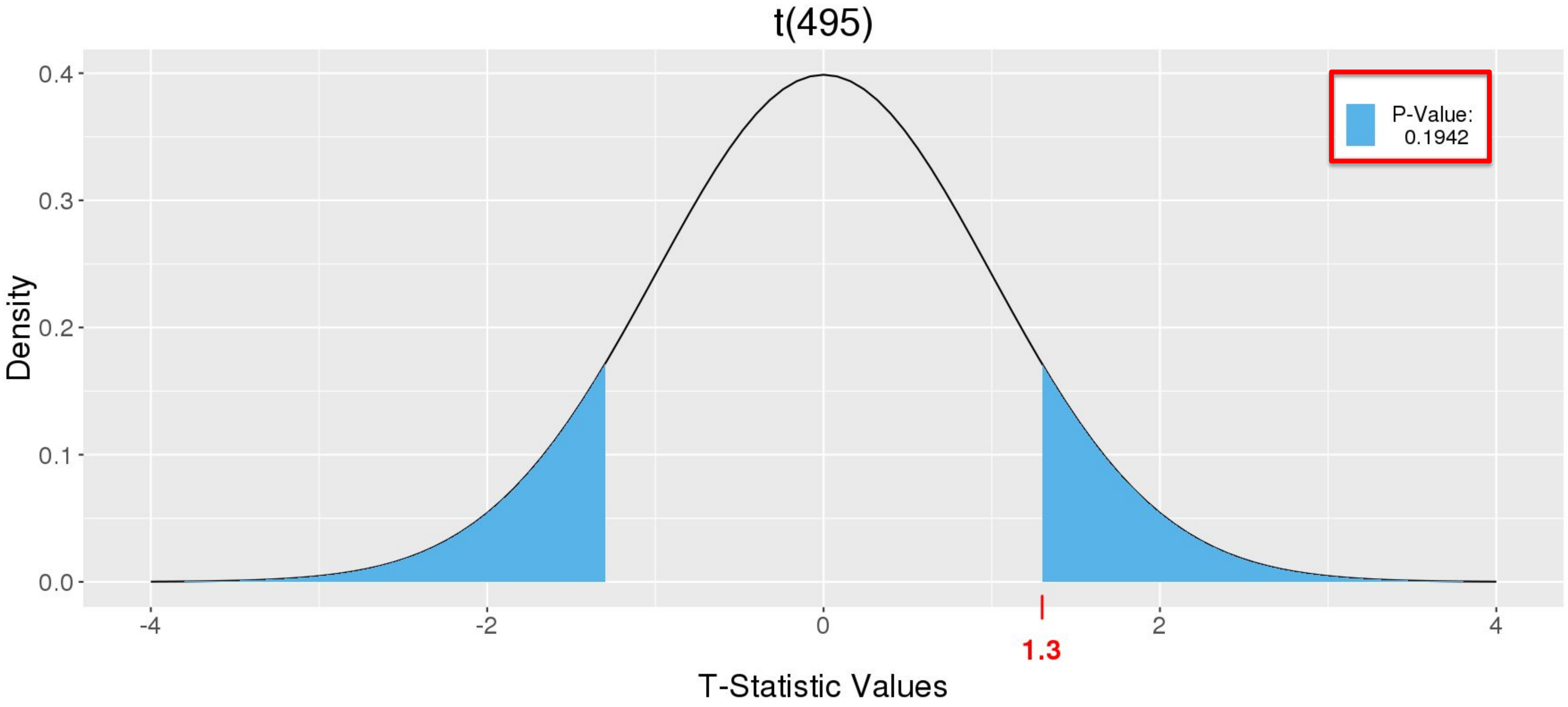

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p-value = 0.19

If the difference in population mean BMI between males and females was really 0 kg/m^2,
then observing a difference in sample means of 0.74 kg/m^2 (i.e. a t-statistic of 1.30) or more extreme is **fairly likely**.

Step 4: Make a Decision

Our p-value is larger than the 0.05 significance level, which means there is weak evidence against the null.

Thus, we **fail to reject the null!**

Step 4: Make a Decision

Based on our estimated difference in sample means, we cannot support that there is a significant difference between the population mean BMI for males and the population mean BMI for females for the population of all Mexican-Americans adults (ages 18 - 29) living in the U.S.

95% Confidence Interval Results

- In a previous lecture, we calculated the 95% CI for the difference in mean BMI between males and females

$$\left(-0.385\ {}^{kg}/_{m^2}\ ,\ 1.865\ {}^{kg}/_{m^2}\right)$$

Our test value of 0 kg/m^2 falls within our interval. This is a reasonable value for the difference in mean BMI.

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- Assumptions for Two-Sample (t) Test for Population Means
  ~ data are two simple random sample, independent
  ~ both populations of responses are normal (else n large helps)
- Know how to interpret the **p-value, decision, and conclusion**