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MSIM 580 Assignment One: Pegboard Game Analysis

Overview

While developing and testing the solutions to the pegboard game search functions, I discovered that the functions were taking a long time to produce results and many of the searches could have lasted hours or days. In order to compare results across functions and puzzles I added a constraint on the maximum amount of time the program spends searching and added a prompt for the user to specify the amount of time to search. The program then returns the best solution that was found given the time limit.

I also discovered that many initial pegboard layouts do not have a single peg solution; in this case the program outputs the best solution found. For this analysis I set the time limit to five minutes for each search before returning the best result found. By doing this I am still able to compare the results of searches that did not reach the single peg solution. The results of the program are included on the third page for the $n \times n$ puzzles and the fourth page for the bonus puzzles.

Heuristic Function

For the greedy best-first and A* searches I implemented a heuristic function that assigns a value based on the Manhattan distance (vertical + horizontal distances) from the center of the puzzle. After calculating the Manhattan distance the heuristic function also adds 1 to the value for each peg that is located on an edge of the puzzle. The results of the heuristic function are used to sort possible successor states in the greedy best-first and A* search functions.

Completeness

As I encountered long wait times while developing the program, it was clear that completeness would become an issue. By setting a maximum processing time I deliberately limited the search functions' abilities to generate complete solutions; however, by including the best solution found completeness can still be compared to an extent.

Of all of $n \times n$ the puzzles, only the 4x4, 5x5, and 6x6 pegboards were able to be completed by the depth-first, greedy best-first, and A* searches. The breadth-first search did not complete any of the $n \times n$ puzzles. While testing the program the breadth-first search did reach 2 remaining pegs in the 4x4 puzzle and would have eventually completed the puzzle.

According to the Peg Solitaire article at <http://recmath.org/pegsolitaire/> and based on experimentation with the program, not all starting pegboards have a single peg solution. I am unsure if the 7x7 pegboard I used as a starting point has a single peg solution. This could

explain the dramatic increase in processing time compared to the 4x4, 5x5, and 6x6 pegboards. For the 8x8 pegboard I did use a known-good 8x8 starting pegboard from the Peg Solitaire article and in a test run of 20 minutes the 8x8 did reach 3 pegs using the depth-first search.

The depth-first search generated the most complete solutions for the 7x7 and 9x9 pegboards with 3 and 7 pegs remaining respectively. The greedy best-first search and A* search generated more complete solutions for the 8x8 and 10x10 pegboards with 4 and 11 pegs remaining respectively for both searches.

Optimality

The A* search reached its results using the least number of nodes out of the four search functions and the depth-first search visited the most number of nodes. Although the A* search would eventually produce optimal results, the depth-first search was able to outperform the A* results for the 5x5, 7x7, and 9x9 puzzles.

Time Complexity

By setting a uniform execution time, the time complexity can be easily compared for the four functions by comparing the total number of nodes visited for the unsolved puzzles. The additional time needed to sort possible successor states using the heuristic function limited the total number of nodes the greedy best-first search and A* search could visit in the same amount of time as the depth-first search. This meant the depth-first search was able to outperform other searches by visiting more nodes.

Space Complexity

The depth-first search and the greedy best-first search use substantially less space than the breadth-first search and the A* search. This limits the overall performance which can be seen by comparing the results of the greedy best-first and the A* search. The greedy best-first search is able to visit substantially more nodes than the A* search in the same amount of time.

Pegboard Game NxN Search Results Summary

Breadth-First Search Results

	4x4	5x5	6x6	7x7	8x8	9x9	10x10
Solution Found	No	No	No	No	No	No	No
Pegs Left	4	16	27	41	56	73	92
Total Nodes	492,416	3,092,628	890,855	13,231,362	11,944,797	10,072,978	8,780,323
Comp. Time	5:00.003	5:00.009	5:00.011	5:00.013	5:00.010	5:00.20	5:00.015

Depth-First Search Results

	4x4	5x5	6x6	7x7	8x8	9x9	10x10
Solution Found	Yes	Yes	Yes	No	No	No	No
Pegs Left	1	1	1	3	5	7	15
Total Nodes	300	3,196,133	288,157	29,970,328	26,394,925	22,812,320	19,047,227
Comp. Time	0:00.005	0:01.667	0:05.319	5:00.003	5:00.002	5:00.003	5:00.003

Greedy Best-First Search Results

	4x4	5x5	6x6	7x7	8x8	9x9	10x10
Solution Found	Yes	Yes	Yes	No	No	No	No
Pegs Left	1	1	1	5	4	8	11
Total Nodes	509	5,909,128	600	25,325,467	22,405,775	18,744,248	16,062,363
Comp. Time	0:00.016	0:46.527	0:00.023	5:00.018	5:00.008	5:00.005	5:00.008

A* Search Results

	4x4	5x5	6x6	7x7	8x8	9x9	10x10
Solution Found	Yes	Yes	Yes	No	No	No	No
Pegs Left	1	1	1	5	4	8	11
Total Nodes	253	2,469,541	306	12,598,722	11,163,393	9,444,336	8,236,734
Comp. Time	0:00.026	0:41.268	0:00.076	5:00.005	5:00.001	5:00.001	5:00.001

Pegboard Game Bonus Search Results Summary

Bonus Breadth-First Search Results

	Bonus 1	Bonus 2	Bonus 3	Bonus 4	Bonus 5	Bonus 6
Solution Found	No	No	No	No	No	Yes
Pegs Left	29	37	30	24	33	1
Total Nodes	13,759,350	8,675,013	12,173,379	13,756,025	9,561,096	27,463
Comp. Time	5:00.014	5:00.017	5:00.021	5:00.014	5:00.027	0:02.544

Bonus Depth-First Search Results

	Bonus 1	Bonus 2	Bonus 3	Bonus 4	Bonus 5	Bonus 6
Solution Found	No	No	No	Yes	No	Yes
Pegs Left	3	5	4	1	5	1
Total Nodes	30,150,781	21,279,223	26,332,219	13,160,312	22,227,147	2,986
Comp. Time	5:00.001	5:00.002	5:00.001	0:00.366	5:00.002	0:00.004

Bonus Greedy Best-First Search Results

	Bonus 1	Bonus 2	Bonus 3	Bonus 4	Bonus 5	Bonus 6
Solution Found	No	No	No	No	No	Yes
Pegs Left	5	4	3	2	5	1
Total Nodes	25,467,759	17,412,443	22,608,051	25,649,191	18,115,547	5,849
Comp. Time	5:00.003	5:00.001	5:00.001	5:00.001	5:00.001	0:00.109

Bonus A* Search Results

	Bonus 1	Bonus 2	Bonus 3	Bonus 4	Bonus 5	Bonus 6
Solution Found	No	No	No	No	No	Yes
Pegs Left	5	4	3	2	5	1
Total Nodes	13,226,216	8,252,429	11,413,529	13,140,036	9,060,289	2,925
Comp. Time	5:00.001	5:00.001	5:00.001	5:00.001	5:00.001	0:00.196