Homework assignment 6:
Suggested due date: Friday, December 1 2017 at 03:30pm

1. Make a binary search tree using these numbers

   5, 7, 1, 4, 3, 80, 54, 26, 30, 28, 8, 34, 37, 42, 0, 6, 52, 83, 81

2. Delete the below nodes in your binary search tree
   a. 52
   b. 34
   c. 37
   d. 6

3. Find the successor of the below numbers in the BST in question 1.
   a. 7
   b. 52
   c. 34
   d. 30
   e. 28
   f. 83

4. Suppose a binary search tree is to hold keys 1,4,5,10,16,17,21. Draw possible binary search trees for these keys, and having heights 2,3,4,5, and 6. (Hint: the height-2 tree should be a perfect tree.)

5. Suppose we have a BST that stores keys between 1 and 1000. If we perform a find on key 363, then which of the following sequences could not be the sequence of nodes examined when performing using the find() method.

   a) 2,252,401,398,330,344,397
   b) 924,220,911,244,898,258,362,363
   c) 925,202,911,240,912,245,363
   d) 2,399,387,219,266,382,381,278,363
   e) 935,278,347,621,299,392,358,363

6. Suppose a BST is constructed by repeatedly inserting distinct keys into the tree. If the number of nodes examined when *searching* for a key is **K**, what will be the number of nodes examined when *inserting* that key.

7. Prove that the maximum number of nodes in a binary tree with height h is 2^{h+1} -1.

8. Prove that it takes Ω(n log n) steps in the best case to build a binary search tree having n keys. Show work and explain.

9. Prove that, when a binary tree with n nodes is implemented by using links to the left and right child, then there will be a total of n+1 null links. (Hint: use induction)

10. A full node for a binary tree is one that has two children. Prove that the number of full nodes plus one equals the number of leaves of a binary tree. (Hint: use induction on the number of full nodes)

11. Prove or disprove: deleting keys x and y from a BST is commutative. In other words, it does not matter which order the keys are deleted. The final trees will be identical. If true, provide a proof. If false, provide a counterexample. (Hint: consider the binary search tree with nodes inserted in the following order: 1,10,5,2,8,7,9)

12. What is the minimum number of nodes that a balanced tree of height 5, 10, 15 can have?

13. Insert the below numbers into an initially empty AVL tree. Redraw the tree each time a rotation is required.
    a. 2, 1, 4, 5, 9, 3, 6, 7
    b. 9, 3, 6, 10, 8, 7, 1, 4, 5

14. Delete 9 and then 6 from the AVL tree in question 14 part b.

15. **Extra credit:** Prove that if keys 1, 2,…, 2^{k+1} are inserted in order into an initially empty AVL tree, then the resulting tree is perfect. (Hint: use induction.)