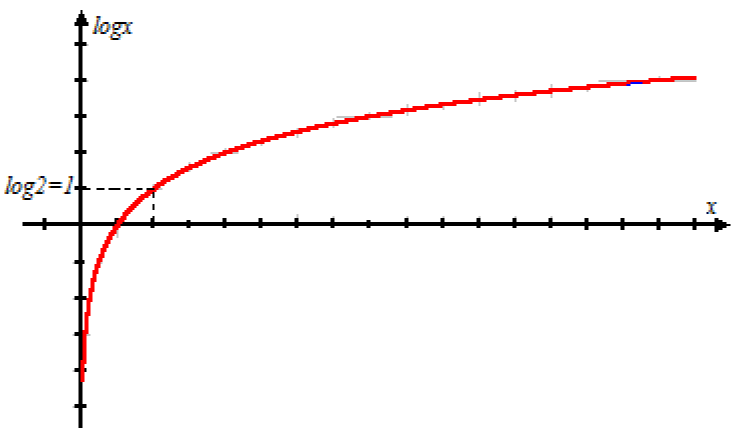
Series

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = (\frac{n(n+1)}{2})^2 \qquad \sum_{i=0}^{n} ar^i = \frac{a(r^{n+1}-1)}{r-1}$$

Logarithms



$$\log_b x = n \Rightarrow b^n = x$$

$$\log_b(x^a) = a\log_b(x)$$

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

$$\log_b(\frac{x}{y}) = \log_b(x) - \log_b(y)$$

$$b^{\log_b x} = x$$

$$y^{\log_b x} = x^{\log_b y}$$

$$\log_b(x) = \frac{1}{\log_x(b)}$$

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

Derivatives

$$(\ln(u))' = \frac{u'}{u}$$

$$(\log_b(x))' = \left(\frac{\ln x}{\ln b}\right)' = \left(\frac{1}{\ln b}\right)(\ln x)' = \frac{1}{x \ln b}$$

$$(e^u)' = u'e^u \Rightarrow (b^x)' = b^x \ln b \Rightarrow because: (b^x)' = (e^{\ln b^x})' = (\ln b^x)' e^{\ln b^x} = (x \ln b)' e^{\ln b^x} = (\ln b) e^{\ln b^x}$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \frac{1}{\cos^2 x} = \sec^2 x$$

$$(\cot x)' = -\frac{1}{\sin^2 x} = -\csc^2 x$$

Growth of functions

$\exists$ means "**there exist**", $\forall$ means "**for all**"

C and k are constants

f(n) is a polynomial of degree a

g(n) is a polynomial of degree b

Big-Oh

$f(n) = O(g(n)) \leftrightarrow \exists C > 0, \exists k \geq 0$ such that $|f(n)| \leq C|g(n)| \;\; \forall n \geq k$

Or

$f(n) = O(g(n)) \leftrightarrow a \leq b$

Big-Omega

$f(n) = \Omega(g(n)) \leftrightarrow \exists C > 0, \exists k \geq 0$ such that $|f(n)| \geq C|g(n)| \;\; \forall n \geq k$

Or

$f(n) = \Omega(g(n)) \leftrightarrow a \geq b$

Big-theta

$f(n)=\theta(g(n)) \leftrightarrow f(n)=O(g(n))$ and $f(n)=\Omega(g(n))$

Or

$f(n)=\theta(g(n)) \leftrightarrow a=b$

Little-Oh

$f(n)=o(g(n)) \leftrightarrow \lim_{n\to\infty} \frac{f(n)}{g(n)}=0$

Or

$f(n)=o(g(n)) \leftrightarrow a<b$

Little-Omega

$f(n)=\omega(g(n)) \leftrightarrow \lim_{n\to\infty} \frac{f(n)}{g(n)}=\infty$

Or

$f(n)=\omega(g(n)) \leftrightarrow a>b$

Theorem

1- $f(n)+g(n)=\theta(\max(f(n),g(n)))$

2- $f(n)=\theta(h(n)), g(n)=\theta(k(n)) \Rightarrow f(n)g(n)=\theta(h(n)k(n))$

3- $f(n)=O(h(n)), h(n)=O(g(n)) \Rightarrow f(n)=O(g(n))$ *it holds for **Ω,Θ,o,ω** too.*

4- $f(n)$ is decreasing $\rightarrow \sum_{i=1}^{n} f(i)=\theta(\int_1^n f(x)dx)$

5- $f(n)$ is increasing, and $f(n)=O(\int_1^n f(x)dx)$ $\rightarrow \sum_{i=1}^{n} f(i)=\theta(\int_1^n f(x)dx)$

6- $\forall k>0, \varepsilon>0 \Rightarrow \log^k n=o(n^{\varepsilon})$

Growth	**Terminology**
$O(1)$	constant growth
$O(\log n)$	logarithmic growth
$O(n)$	linear growth
$O(n \log n)$	log-linear growth
$O(n^b)$	polynomial growth
$\Omega(n^k)$ for every k	superpolynomial growth
$\Omega(b^n)$ for some $b > 1$	exponential growth