THE HARMONIC FORMULA OF FOURIER AND BESSEL.

The harmonic formula of Fourier and Bessel is usually presented in one or other of the two following forms:

(I) $y = M + p_1 \cos x + q_1 \sin x + p_2 \cos 2x + q_2 \sin 2x + p_3 \cos 3x + q_3 \sin 3x + \ldots$

(II) $y = M + u_1 \sin(U_1 + x) + u_2 \sin(U_2 + 2x) + u_3 \sin(U_3 + 3x) + \ldots$

It is evident that these two formulæ are one and the same under different form, for developing (II) we have:

$$y = M + u_1 \sin U_1 \cos x + u_1 \cos U_1 \sin x + u_2 \sin U_2 \cos 2x + u_2 \cos U_2 \sin 2x + \ldots \text{etc.}$$

and making in the expression—

$$\begin{array}{cc} u_1 \sin U_1 = p_1 & u_1 \cos U_1 = q_1 \\ u_2 \sin U_2 = p_2 & u_2 \cos U_2 = q_2 \\ \text{etc.} & \text{etc.} \end{array}$$

we find that (II) is reduced to (I).

The practical applications of this formula are many and important.

Fourier's theorem (say Thomson and Jait) is not only one of the most beautiful results of modern analysis, but may be said to furnish an indispensable instrument in the treatment of nearly every recondite question in modern physics. To mention only sonorous vibrations, the propagation of electric signals along the telegraph wire, and the conduction of heat by the earth's crust, as subjects in their generality intractable without it, is to give but a feeble idea of its importance.

In the present study we limit ourselves to the application which is made of the formula in meteorology; and we will endeavor to show what each one of its elements represents, and point out its utility and practical application.[1]

The greater number of the phenomena studied in meteorology are periodic, and the curves which represent them are consequently periodic curves—that is to say, curves which represent the movement of a point which passes an indefinite number of times over the same path in the same time.

Fourier proved that any periodic curve could be resolved into a series of harmonic curves of periods 1, $\frac{1}{2}$, $\frac{1}{3}$, etc., of the given curve, and that *only one combination* of these elementary curves was possible to reproduce a specified curve. This corresponds with the fact observed by Helmholtz that the same composite sound is always resolved into the same elementary sounds.

Prescinding for the moment from the practical utility which this resolution of curves may have in meteorology, let us examine how the formula which we are studying is nothing more than the analytical expression of periodic curves in terms of their harmonic components.

A harmonic curve is one which represents graphically the harmonic motion of a point.

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[1] One of the best practical treatises which we know on the employment of Fourier's series in mathematical physics is "An Elementary Treatise on Fourier's Series and Spherical Harmonics" by W. E. Byerly, published by Ginn & Co., New York.

The original work of Fourier himself "Théorie Analytique de la Chaleur" is perhaps as up to date as any other modern publication on this subject. Mellor's "Higher Mathematics for Students of Chemistry and Physics," Longmans, London and New York, may also be consulted.