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Note on the Velocity of Light

RICHARD A. MILLER AND ADOLFO LOPEZ
Manila Observatory, Mirador, Baguio City, Philippines
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IN a survey article on determinations of the velocity of light,[1] it was noted that measurements at optical frequencies of the velocity of electromagnetic waves appear to give lower values than measurement at microwave frequencies. The difference between the two types of measurements is beyond the assigned limits of error. We wish to propose a possible source of systematic error for optical determinations, namely, the time taken for reflection from a mirror.

Bergstrand,[2] by an optical determination, using a single light beam with a modified Kerr cell apparatus obtained, however, a result very close to the results at microwave frequencies. He obtained 299 792±0.25 km/sec. The time taken for reflection from a mirror did not enter into his results insofar as for his distance D he used

$$D=K+[(2N-1)/8]\lambda, \tag{1}$$

where N is an integer, λ is the wavelength of the signal modulation, and K is entered as a constant dependent on the apparatus. This K absorbs the error which the one mirror used in the experiment could have introduced.

Work with standing microwaves in resonant cavities as far as possible eliminates end effects or reflection effects and thus Essen[3] obtained a value 299 792.5±3 km/sec for the velocity of electromagnetic waves, practically the equal of Bergstrand's value. It would seem the 14 km/sec or greater discrepancy between the value accepted from optical measurements from 1950 to 1955 and the newer values from microwave measurements is possibly due to a systematic error in optical measurements.

In many of the earlier optical determinations of the velocity of light, there is evidence that the assigned limits of error have been too optimistic. Hence, we will restrict ourselves to an analysis of a series of experiments which have been most extensively and painstakingly carried out in recent years. The rotating mirror apparatus used by Michelson and by Pease and Pearson up to the year 1935 has yielded results to which Birge in 1941 assigned the smallest probable error of all optical measurements, ±4 km/sec.

Pease and Pearson's apparatus, between the time of departure from and return to the rotating mirrors for the light beam, interposes thirteen reflecting surfaces into the path of the beam. Their value for light velocity comes 19 km short of the presently accepted approximate value of 299 793 km/sec. Their light path was close to 1.5958×10^{6} cm long, or somewhat over ten miles long. At 2.99793×10^{10} cm/sec this distance would be traversed in $5\,323\,006\times10^{-11}$ sec, whereas the time for their derived velocity would be $5\,323\,343\times10^{-11}$ sec; or a delay has been introduced of 2.58×10^{-10} sec/reflection.

Reflection from mirror surfaces cannot be instantaneous, such as from mathematical surfaces. Reflection involves the free and bound electrons of the reflecting surface. Free electrons play a greater role than bound electrons in reflection of visible light from metallic surfaces.[4] An individual electron is forced into oscillation by the incoming light wave and reemits the energy after an interval of time T which is the reciprocal of the damping constant of the electron, existing under the conditions at, and close to the reflecting surface. An exact reckoning of the damping constant or reciprocally, the relaxation time, is beyond the scope of this short letter which strives merely to point out the problem of a delay, however small, introduced by reflections from mirror surfaces.

Simple radiation damping of a free electron in a very thin gas would, by classical calculations[5] take place in 1.58×10^{-8} sec. This is too great a lag by a factor of 163 to explain Michelson's results given in the foregoing. However, free electrons in a mirror are disturbed by collisions and the relaxation time will be of the order of magnitude of the reciprocal of twice the collisional frequency of the electrons.[5] What will be the collisional frequency where the electrons turn back the light wave? Lorentz theory indicates that perpendicular reflection for light at 6000 A will occur when electron concentration is $2.9\times10^{21}/\text{cm}^3$. This is 1/20 the accepted number of free electrons/cm³ deep in the metal, silver.[6] Therefore reflection will occur well before the light waves reach the depth of maximum collisional frequency inside the metal. The collisional frequency of the electrons at the point where the light waves will be turned back, at 1/20th of the concentration deep within the metal, will be much less than the value deep within the metal.

If we consider that the electrons in question will be only 1/20 as dense, then their mean free path because of this will be greater by a factor of 20. Also, at this density of 1/20, close to the metal boundary, most of the collisions of electrons will be with fellow free electrons, rather than with the silver atoms which are held deeper. Electrons have at most one quarter[7] the collisional cross section of atoms. Crudely, this brings a factor of 4 into the mean free path. Now Kittel[6] gives 5×10^{-6} cm as the mean free path of free electrons deep within the metal silver. Hence 5×10^{-4} cm is taken for the mean free path, where electrons are 1/20 as dense.

Furthermore, the velocity of electrons will be less where their density is less because, near the surface of the metal, electrons will have partially overcome the work function. Kittel again gives 1.4×10^{8} cm/sec as the velocity of free electrons deep within silver. Somewhat arbitrarily, we take 1/100 of this value for our velocity at 1/20 the electron density. Mean free path divided into velocity gives a collisional frequency of 3×10^{9} sec^{-1}. The reciprocal of twice this collisional frequency gives 1.7×10^{-10} sec an order of magnitude agreement with the delay perhaps introduced by a mirror reflection.

Arguments based on experimentally determined phase changes upon reflection cannot give the time occupied in reemitting the light. An electron absorbs and reemits light so that the phase of the wave is changed, but ϕ the phase change may or may not be ωt, where ω is the known angular phase velocity of the light wave and t is the time taken to re-emit the energy. ϕ does not necessarily give t. ϕ is measured. ωt may be $2\pi n+\phi$ where n is an integer.

Because of the uncertainty in the time required to re-emit the electromagnetic energy impinging on reflecting surfaces, it would appear desirable that optical determinations of the velocity of electromagnetic waves eliminate this factor. Null methods by which possible reflection time is eliminated are safest. Michelson's method would involve the task of determining the apparatus constant. Optimistically, it might possibly offer a method of determining the time taken for each reflection from a mirror.

The authors are the first to admit that the present treatment is far from adaquate—yet they feel justified in presenting ideas, which may provoke much more competent development.

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