MAPPING IN THE COMPLEX PLANE

James J. Hennessey, S.J.

In dealing with complex numbers and variables there frequently arises the relation such that the dependent complex variable w is a function of the independent complex variable z. The complex variables, w and z, just as the complex numbers, have the real variable as a special case. In algebraic symbols the complex variables may be written:

$$(1) \qquad z = x + i\,y$$
$$(2) \qquad w = f(z).$$

Since w is complex and a function of z, we have

$$(3) \qquad w = u + i\,v.$$

that is, u and v must be functions of x and y.

The question arises how can we represent this relationship geometrically. Since we have

$$w = u + i\,v$$
$$z = x + i\,y$$

we have four real variables, u, v, x, and y, to deal with. This suggests a figure in four dimensions. Quite conveniently the z points are represented in one plane called the Z-plane and the w-points in another plane called

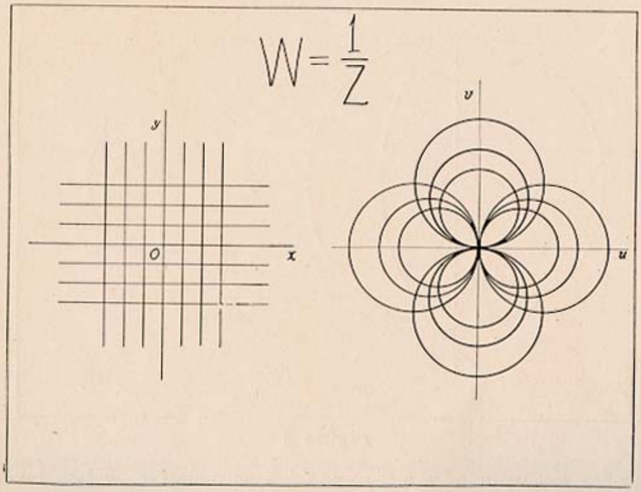

Figure 1

the W-plane. As in considerations of functions of the real variable when x varies, its function also takes on various values; so with these complex variables there is a relationship between the two planes. As any curve is traced in the Z-plane by the point P, a corresponding curve will be traced, or, technically, mapped, in the W-plane by the point Q. Whether the W-plane will map upon the entire Z-plane or only upon a part of it depends upon the character of the functions.