Data 609 Module 1 HW

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2/14/2021

Ex.1

Find the minimum of $f(x, y) = x^2 + xy + y^2 in (x, y) \in R^2$

Solution:

The stationary conditions are $\frac{\partial f}{\partial x} = 2x + y = 0$ and $\frac{\partial f}{\partial y} = x + 2y = 0$

From the second condition we get either x = -2y or y = $\frac{-x}{2}$

Substituting x = -2y into the first condition gives us

2(-2y) + y = 0

-4y + y = 0

-3y = 0

y = 0

Substituting y = $\frac{-x}{2}$ into the first condition gives us

2x + $\frac{-x}{2}$ = 0

$\frac{3x}{2}$ = 0

x = 0

So x = 0 and y = 0. These also hold true for the second condition.

Hessian Matrix $H = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

fxx = 2x + y = 2

fxy = 2x + y = 1

fyx = x + 2y = 1

fyy = x + 2y = 2

H is positive definite which means that the point (0,0) is a minimum.

Ex.2

For $f(x) = x^4 in R$, it has a global minimum at x = 0. Find its new minimum if a constraint $x^2 \geq 1$ is added.

Solution:

$f(x) = x^4$ and $g(x) = x^2$

$\text{II} = f(x) + \mu[g(x)]^2 = x^4 + \mu[x^2]^2 = x^4 + \mu[x^4]$

$\text{II}'(x) = 4x^3 + 4\mu x^3 = 0 = \text{II}'(x) = 4x^3(1+\mu) = 0$

X can be either 1 or -1. The new minimum is x = 1.

Ex.3

Use a Lagrange multiplier to solve the optimization problem $minf(x,y) = x^2+2xy+y^2$, subject to $y = x^2-2$

Solution:

$L = f(x,y) + \lambda h(x,y)$ $h(x,y) = x^2 - 2 - y$

$L = x^2 + 2xy + y^2 + \lambda(x^2 - 2 - y)$

$\frac{\partial L}{\partial x} = 2x + 2y + 2\lambda x = 0$, $\frac{\partial L}{\partial y} = 2x + 2y - \lambda = 0$, $\frac{\partial L}{\partial \lambda} = x^2 - 2 - y = 0$

For the second condition, $2x + 2y - \lambda = 0$ gives us $\lambda = 2(x+y)$

Plugging $\lambda = 2(x+y)$ into the first condition gives us

$2x + 2y + 2(2(x+y))x = 0$

$2x + 2y + 4x^2 + 4xy = 0$

$2(x + y + 2x^2 + 2xy) = 0$ which gives us $y = \frac{-2x^2-x}{2x+1}$

Substitute $y = \frac{-2x^2-x}{2x+1}$ into the third condition gives us

$x^2 - 2 - \frac{-2x^2-x}{2x+1} = 0$ when simplified gives x = 1 or x = - 2.

Plugging x = 1 into the third condition gives us y = -1 and plugging in x = -2 gives us y = 2. We get two points (1, -1) and (-2, 2).

The optimality for $minf(x,y) = x^2 + 2xy + y^2$ is (1, -1) with $f_{min} = 0$ and (-2, 2) with $f_{min} = 0$