Data 609 Module 2 HW

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Ex.1

Show $x^2 + exp(x) + 2x^4 + 1$ is convex.

Solution:

$f(\alpha x + \beta y) \leq \alpha f(x) + \beta f(y)$

$(\alpha x + \beta y)^2 + exp(\alpha x + \beta y) + 2(\alpha x + \beta y)^4 + 1 \leq \alpha(x^2 + exp(x) + 2x^4 + 1) + \beta(y^2 + exp(y) + 2y^4 + 1)$

Using $\alpha + \beta = 1$ we can simplify the inequality and rewrite it as

$2\alpha x^4 + \alpha x^2 + \alpha exp(x) + 2\beta y^4 + \beta y^2 + \beta exp(y) + 1 - ((\alpha x + \beta y)^2 + exp(\alpha x + \beta y) + 2(\alpha x + \beta y)^4 + 1) \geq 0$

$2\alpha x^4 + \alpha x^2 + \alpha exp(x) + 2\beta y^4 + \beta y^2 + \beta exp(y) - (\alpha x + \beta y)^2 - exp(\alpha x - \beta y) - 2(\alpha x + \beta y)^4 \geq 0$

The inequality is always true which shows that $x^2 + exp(x) + 2x^4 + 1$ is convex.

Ex.2

Show that the mean of the exponential distribution $p(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 (\lambda > 0) \\ 0 & x < 0 \end{cases}$ is $\mu = \frac{1}{\lambda}$ and its variance is $\sigma^2 = \frac{1}{\lambda^2}$

Solution:

To find the mean of the exponential distribution solve $\mu = E[X] = \int_0^\infty x\lambda e^{-\lambda x}$

Solve using integration by parts $\int uvdx = u\int vdx - \int u'(\int vdx)dx$

$[-xe^{-\lambda x}]_0^\infty + \int_0^\infty e^{-\lambda x}dx =$

$[-xe^{-\lambda x}]_0^\infty + [-\frac{1}{\lambda}e^{-\lambda x}]_0^\infty =$

$(0 - 0) + (0 + \frac{1}{\lambda}) = \frac{1}{\lambda}$

To find the variance of the exponential distribution solve $\sigma^2 = Var[X] = E[X^2] - E[X]^2$

$E[X^2] = \int_0^\infty x^2\lambda e^{-\lambda x}$ which can be solved by again using integration by parts

$[-x^2e^{-\lambda x}]_0^\infty + \int_0^\infty 2xe^{-\lambda x}dx =$

$[-x^2e^{-\lambda x}]_0^\infty + [-\frac{2}{\lambda}xe^{-\lambda x}dx]_0^\infty + \frac{2}{\lambda}\int_0^\infty e^{-\lambda x}dx$ Again use integration by parts =

$[-x^2e^{-\lambda x}]_0^\infty + [-\frac{2}{\lambda}xe^{-\lambda x}dx]_0^\infty + \frac{2}{\lambda}[-\frac{1}{\lambda}xe^{-\lambda x}dx]_0^\infty =$

$(\frac{2}{\lambda})(\frac{1}{\lambda}) = \frac{2}{\lambda^2}$

$E[X^2] - E[X]^2 = \frac{2}{\lambda^2} - (\frac{1}{\lambda})^2 = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$

Ex.3

It is estimated that there is a typo in every 250 data entries in a database, assuming the number of typos can obey the Poisson distribution. For a given 1000 data entries, what is the probability of exactly 4 typos? What is the probability of no typo at all? Use R to draw 1000 samples with $\lambda = 4$ and show their histogram.

Solution:

```
lambda <- 4
x1 <- 4
four_typos <- (lambda ^ x1 * exp(-lambda)) / factorial(x1)
four_typos
```

```
## [1] 0.1953668
```

The probability of exactly 4 typos is 19.54%.

```
lambda <- 4
x2 <- 0
zero_typos <- (lambda ^ x2 * exp(-lambda)) / factorial(x2)
zero_typos
```

```
## [1] 0.01831564
```

The probability of no typos at all is 1.83%.

```
# Built in r function to check answers
q1 <- dpois(4, lambda = 4)
q1
```

```
## [1] 0.1953668
```

```
q2 <- dpois(0, lambda = 4)
q2
```

```
## [1] 0.01831564
```

```
samples <- 1:1000
values <- rpois(1000, lambda = 4)
hist(values)
```


Histogram of values
Frequency
0
50
100
150
200
0
2
4
6
8
10
values