Exercise Week 08

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Time Schedule

- 10' Rekursion - Konzept mit Beispiel
- 3' Rekursion vs Iteration
- 5' Rekursion - Optimierung
- 10' Rekursionsbäume
- 20' Beispielaufgaben zu Rekursion
- 15' Pause
- 45' Q&A

Learning Objectives

- Verständnis: Rekursion

Rekursion - Konzept

```
1 int function(arg1, arg2){
2   //Call function recursively
3   function(arg1, arg2);
4 }
```

Rekursion - Konzept

```
1  int function(arg1, arg2){
2    // Termination condition
3    if(terminate){
4      return result;
5    }
6    // do stuff
7
8    //Call function recursively
9    function(arg1, arg2);
10 }
```

Rekursion - Beispiel I

```
1 double power(double x, int n){
2   if (n == 1){
3     return x;
4   }
5
6   return x*power(x, n-1);
7 }
```

Rekursion - Beispiel I

```
1  double power(double x, int n){
2    if (n == 1){
3      return x;
4    }
5
6    return x*power(x, n-1);
7  }
8
9  double itpower(double x, int n){
10   double result = 1;
11   for(int i = 0; i<n; i++){
12     result *=x;
13   }
14   return result;
15 }
```

Rekursion vs Iteration

Vorteile und Nachteile der Rekursion

- Vorteile:
  - Komplexe Probleme können relativ einfach gelöst werden.
  - Der Code ist übersichtlicher.
- Nachteile:
  - Möglicher Stack Overflow!
  - Langsamer
  - Debugging ist schwieriger.

Rekursion - Beispiel I - Optimierung

$$x^2 = x \cdot x$$
$$x^4 = x^2 \cdot x^2$$
$$x^8 = x^4 \cdot x^4$$
$$x^{16} = x^8 \cdot x^8$$
$$x^{20} = x^{16} \cdot x^4$$

Rekursion - Beispiel I - Optimierung

```
1 double power (double x, int n){
2   if (n == 1){
3     return x;
4   }
5
6 double temp = power(x, n/2);
7 return temp*temp;
8 }
```

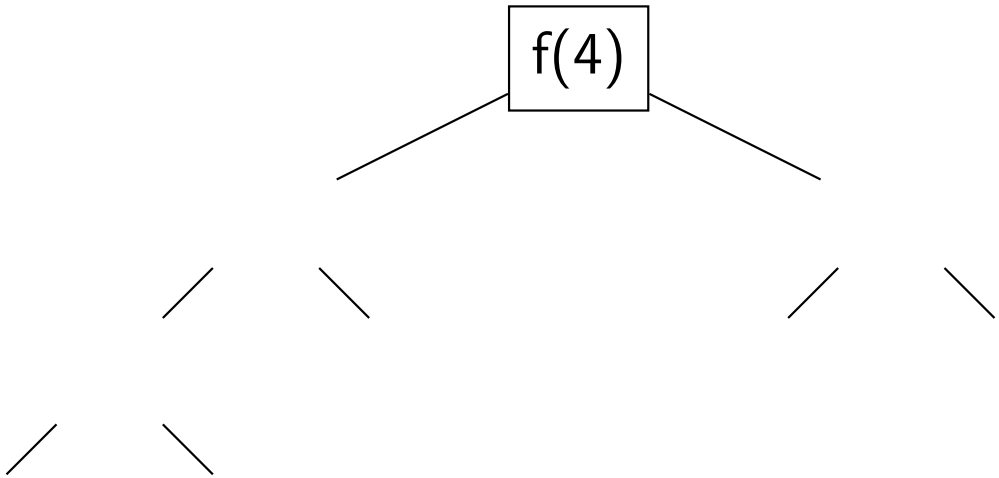
Rekursion - Beispiel I - Optimierung

```
1  double power (double x, int n){
2    if (n == 1){
3      return x;
4    }
5    else if (n%2 == 0) {
6      double temp = power(x,n/2);
7      return temp*temp;
8    }
9    else return x*power(x,n-1);
10 }
```

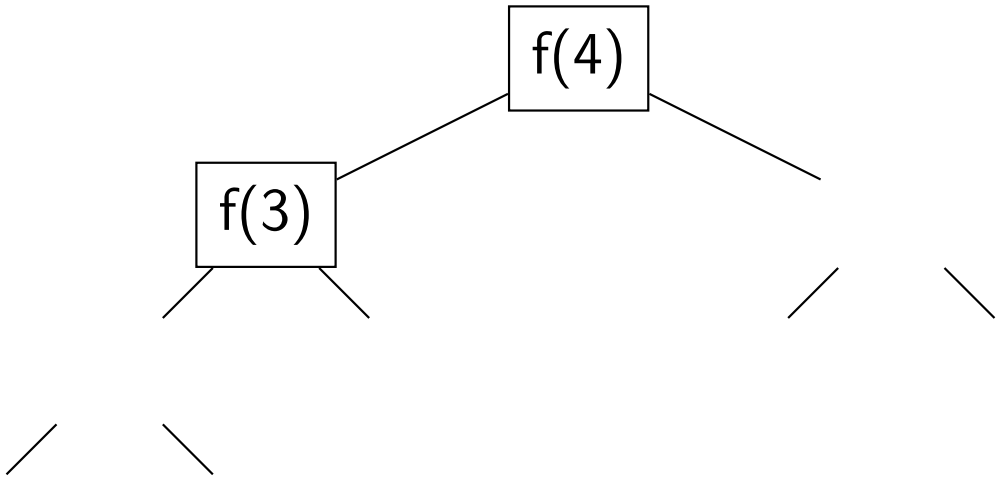
Rekursionsbäume

```
1 //POST: return value is the n-th
2 //Fibonacci number F(n)
3 unsigned int fib(const unsigned int n){
4   if (n == 0) return 0;
5   if (n == 1) return 1;
6   return fib(n-1) + fib(n-2); //n>1
7 }
```

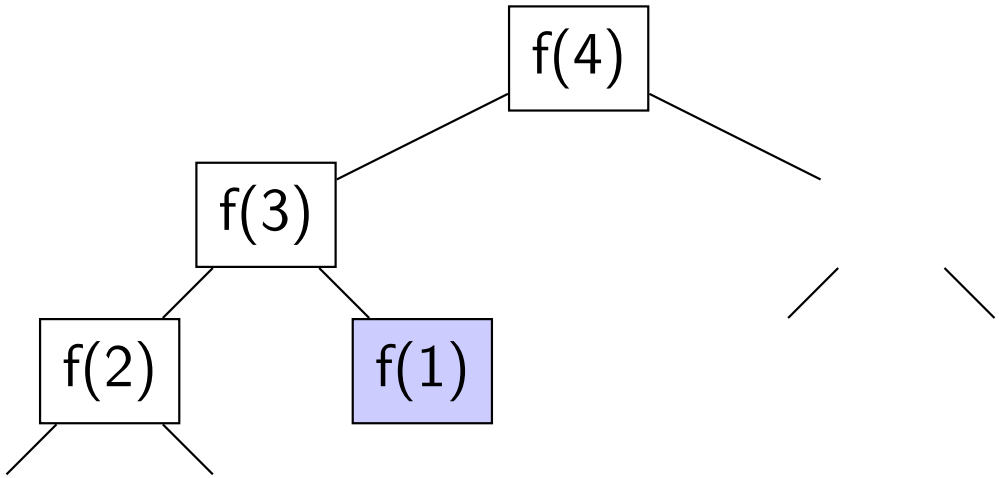
Rekursionsbäume



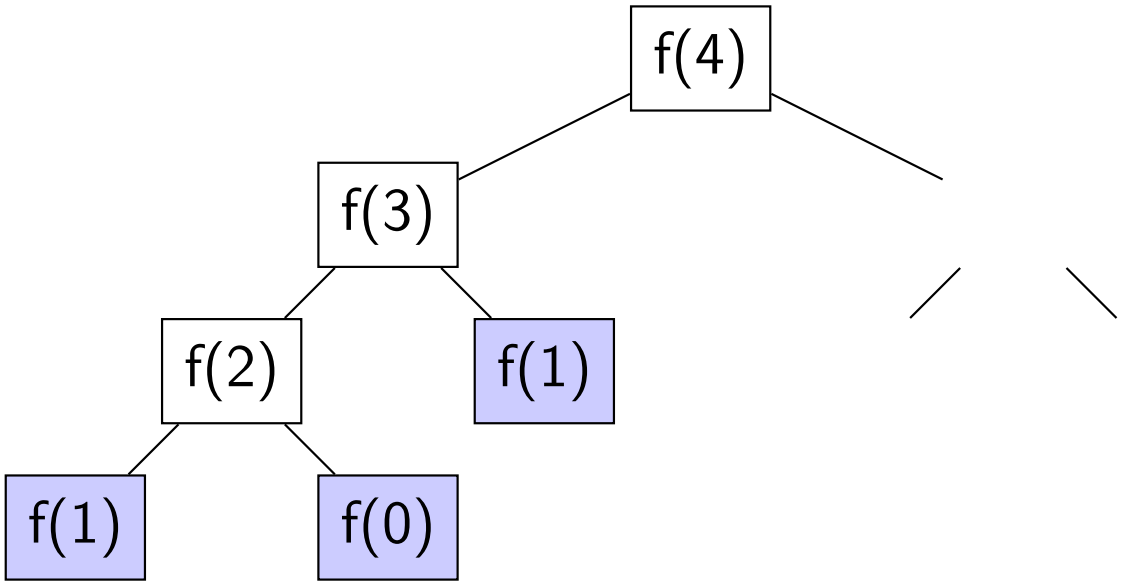
Rekursionsbäume



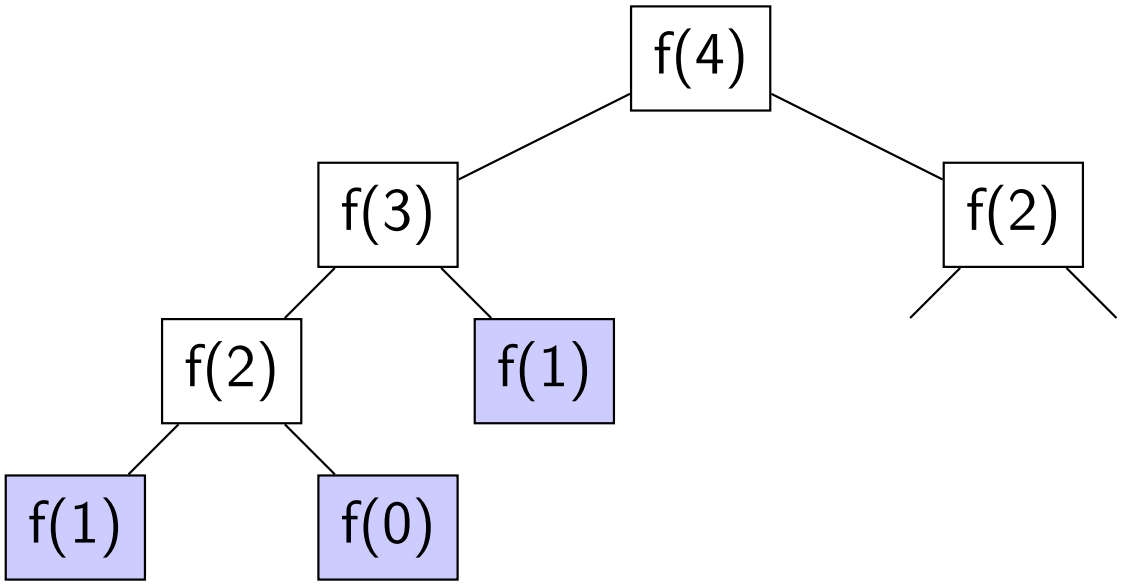
Rekursionsbäume



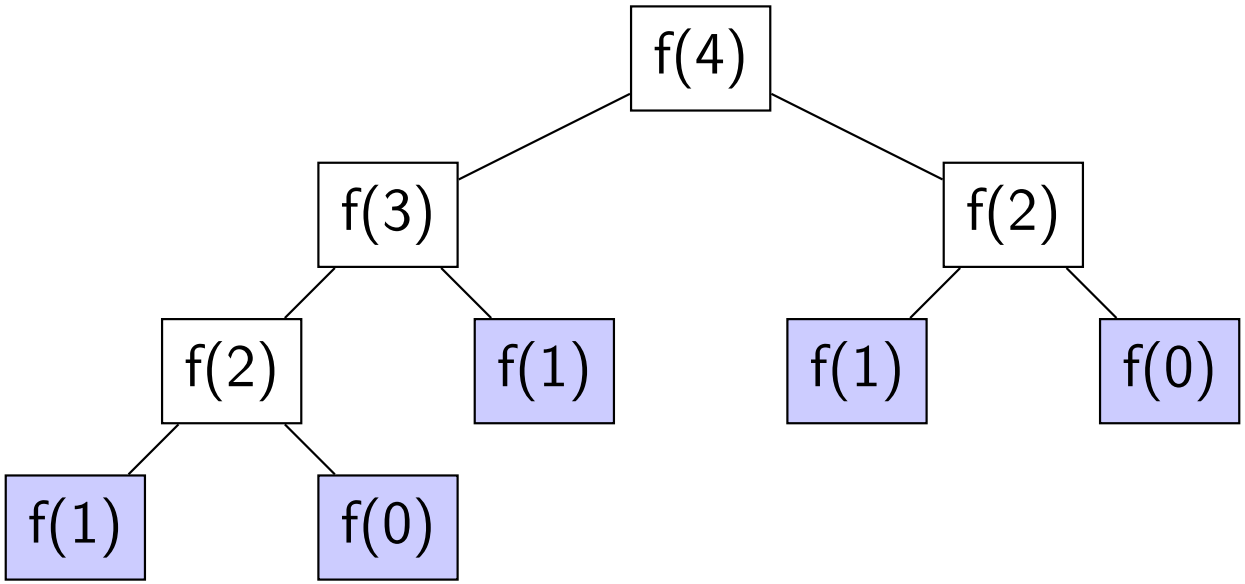
Rekursionsbäume



Rekursionsbäume



Rekursionsbäume



Fibonacci Iterativ

```
1  // POST: return value is the n-th
       Fibonacci number F(n)
2  unsigned int fib2 (const unsigned int n) {
3  if (n == 0) return 0;
4  if (n <= 2) return 1;
5  unsigned int a = 1; // F_1
6  unsigned int b = 1; // F_2
7  for (unsigned int i = 3; i <= n; ++i) {
8  unsigned int a_prev = a; // F_i-2
9  a = b; // F_i-1
10 b += a_prev; // F_i-1 += F_i-2 -> F_i
11 }
12 return b;
13 }
```

Rekursion - Aufgabe I $\sim$ 5'

Aufgabe

- Schreiben Sie die folgende Funktion in iterativer Form.

```
1 unsigned int f (const unsigned int n)
2 {
3   if (n<= 2) return 1;
4   return f(n-1) + 2*f(n-3);
5 }
```

Rekursion - Lösung I

```
1  unsigned int f_it (const unsigned int n) {
2    if(n<=2) return 1;
3    unsigned int a = 1; // f(0)
4    unsigned int b = 1; // f(1)
5    unsigned int c = 1; // f(2)
6    for (int i = 3; i<n; ++i){
7      int a_prev = a      // f(i-3)
8      a = b;              // f(i-2)
9      b = c;              // f(i-1)
10     c = b+ 2*a_prev;    // f(i)
11   }
12   return c + 2*a;
13 }
```

Rekursion - Aufgabe II $\sim$ 5'

Aufgabe

- Schreiben Sie die folgende Funktion in iterativer Form.

```
1 unsigned int f (const unsigned int n)
2 {
3   if (n==0) return 1;
4   return f(n-1) + 2*f(n/2);
5 }
```

Rekursion - Lösung II

```
1  unsigned int f_it (const unsigned int n)
2  {
3    if (n==0) return 1;
4
5    std::vector<unsigned int> f_val(n+1,0);
6    f_val[0] = 1;
7    for (int i = 1; i<=n; ++i)
8      f_val[i] = f_val[i-1] + 2*f_val[i/2];
9
10   return f_val[n];
11 }
```

Beispiel: Self assessment

```
1  int f(const int* begin, const int* end){
2    int val1 = *begin;
3    if(++begin != end){
4      const int val2 = f(begin,end);
5      if(val2>val1) { val1 = val2;}
6    }
7    return val1}
8
9  int g(const int *begin, const int *end){
10   int val = *begin;
11   for (const int * it=++begin; it != end;
         ++it){
12     if (*it > val){ val = *it}
13   }
14   return val
15 }
```