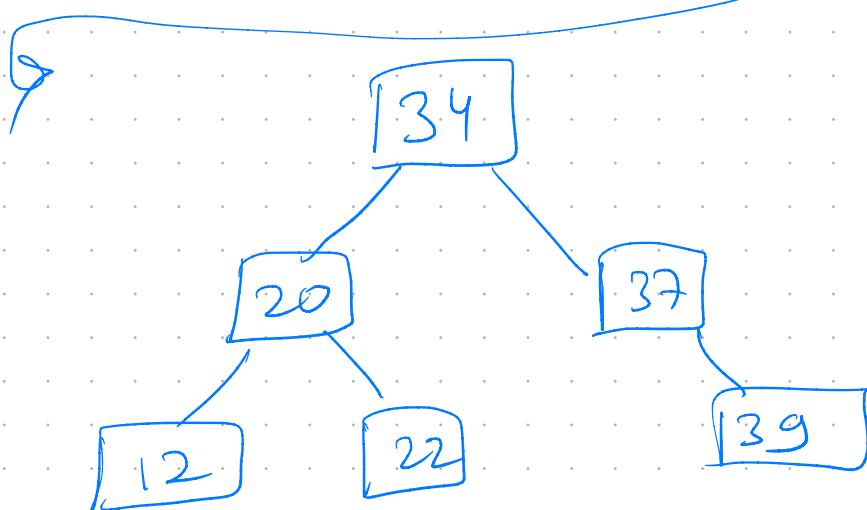
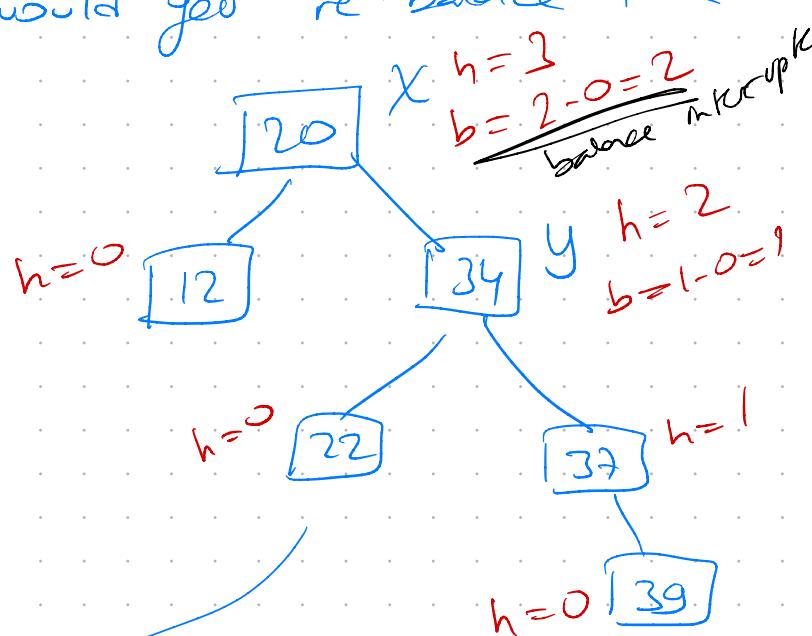
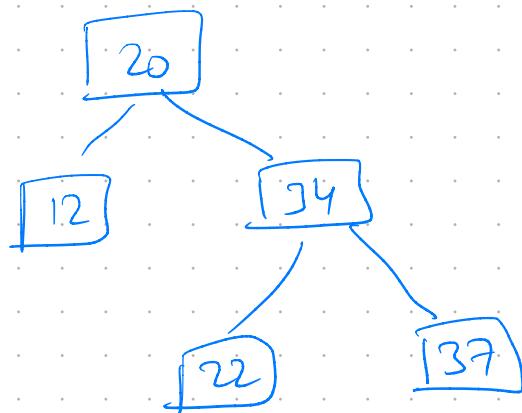


## Self balancing trees

1) Why do we use self-balancing trees? Briefly explain the most important reason.

Runtime, Unbalanced worst-case can be  $O(n)$  while balanced search...  $O(\log n)$  time.

2) After inserting 29 into AVL tree, we obtain BST on right. How would you re-balance the right tree?



## Hashing:

1) Insert the US Presidents listed below into a hash table w/ 13 cells using linear probing and the simple hash function  $h(key) = key$ .

Key	Name	Index
44	Obama	5
40	Reagan	1
39	Carter	0
31	Hoover	5
28	Wilson	2
26	Roosevelt	0
25	McKinley	12
24	Cleveland	11
17	Johnson	4
7	Jackson	7

$$44 \% 13 = 5$$

$$40 \% 13 = 1$$

$$39 \% 13$$

$$31 \% 13$$

$$28 \% 13$$

$$26 \% 13$$

$$25 \% 13$$

$$17 \% 13$$

2) What is the expected number of comparisons to find an item in this table?

$$\text{Load factor} = \frac{10}{13} \approx 0.77$$

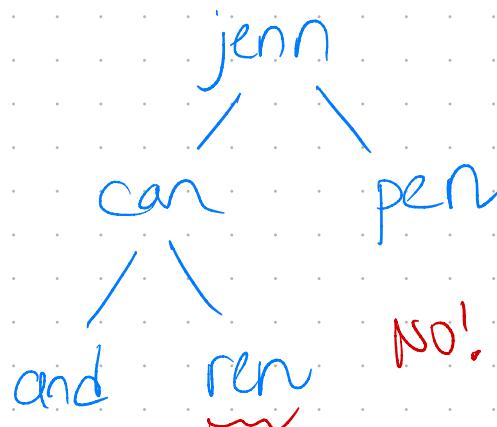
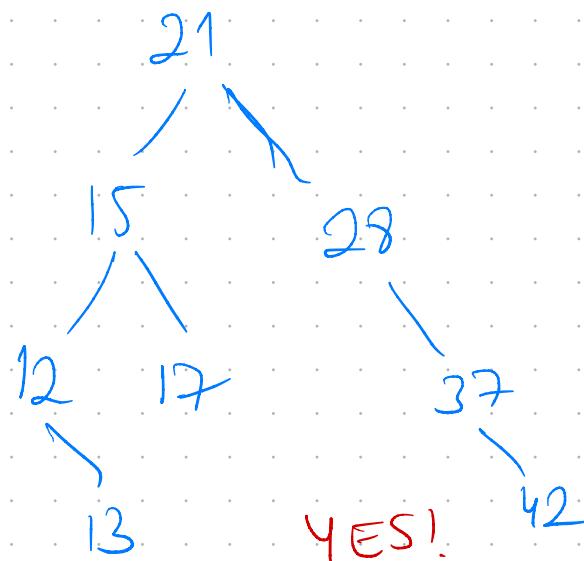
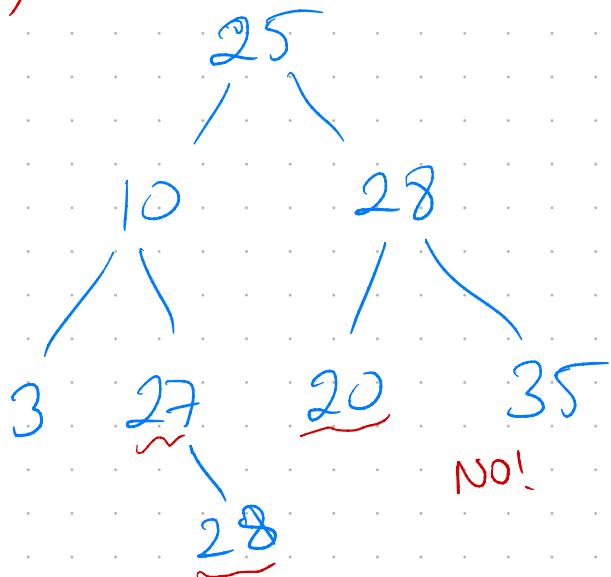
$$C = \frac{1}{2} \left( 1 + \frac{1}{1-L} \right)$$

$$C = \frac{1}{2} \left( 1 + \frac{1}{\frac{3}{13}} \right) = \frac{1}{2} \left( 1 + \frac{13}{3} \right) = \frac{1}{2} \cdot \frac{16}{3} \approx 2.66$$

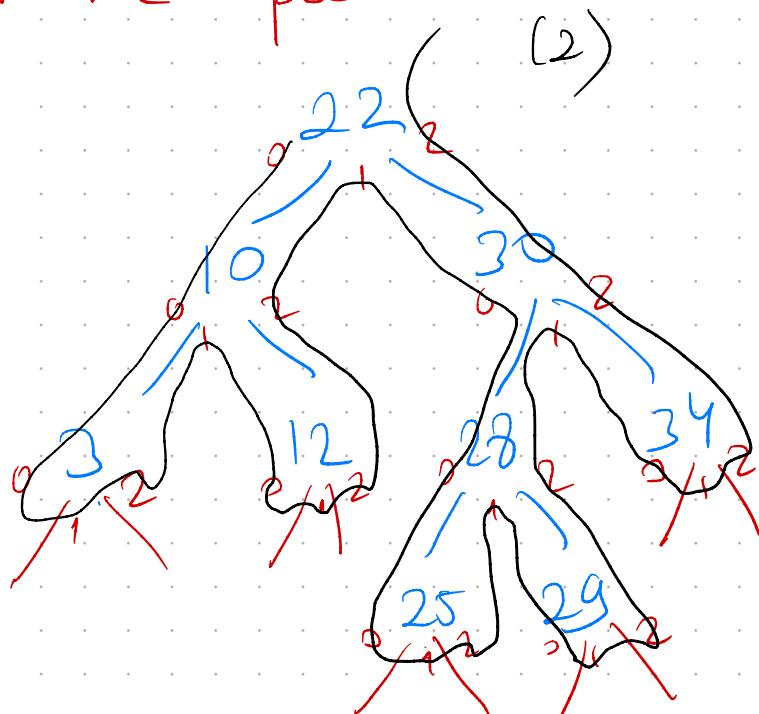
0	Carter
1	Reagan
2	Wilson
3	Roosevelt
4	Johnson
5	Obama
6	Hoover
7	Jackson
8	
9	
10	
11	Cleveland
12	McKinley

# Binary Search Trees:

1) BST or not?



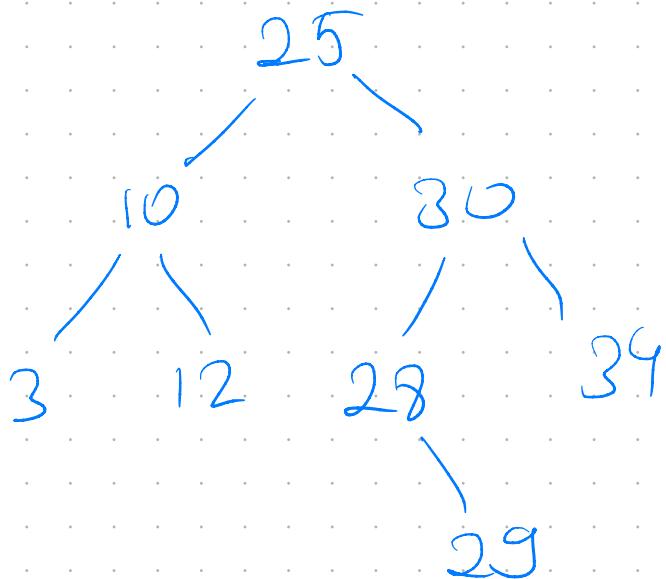
2) Find the post-order in the following BST.



3, 12, 10, 25, 29, 28, 34, 30, 22

3) Delete 22 from the previous BST.

You will change it w/ the successor of the node that you want to delete.  
(min in the right subtree.)



1) Write the following method public boolean isBalanced();

A binary tree is balanced if both of its subtrees are balanced & the height of its left subtree differs from the height of its right subtree by at most 1.

```
public boolean isBalanced() {
```

```
    if (root == null)
```

```
        return true;
```

```
    return balanceOfNode(root);
```

```
}
```

```
public boolean balanceOfNode(Node n) {
```

```
    int leftHeight, rightHeight;
```

```
    if (n == null)
```

```
        return true;
```

```
    leftHeight = height(n.left);
```

```
    rightHeight = height(n.right);
```

```
    if (Math.abs(leftHeight - rightHeight) <= 1 &&  
        isBalanced(n.left) && isBalanced(n.right))
```

```
        return true;
```

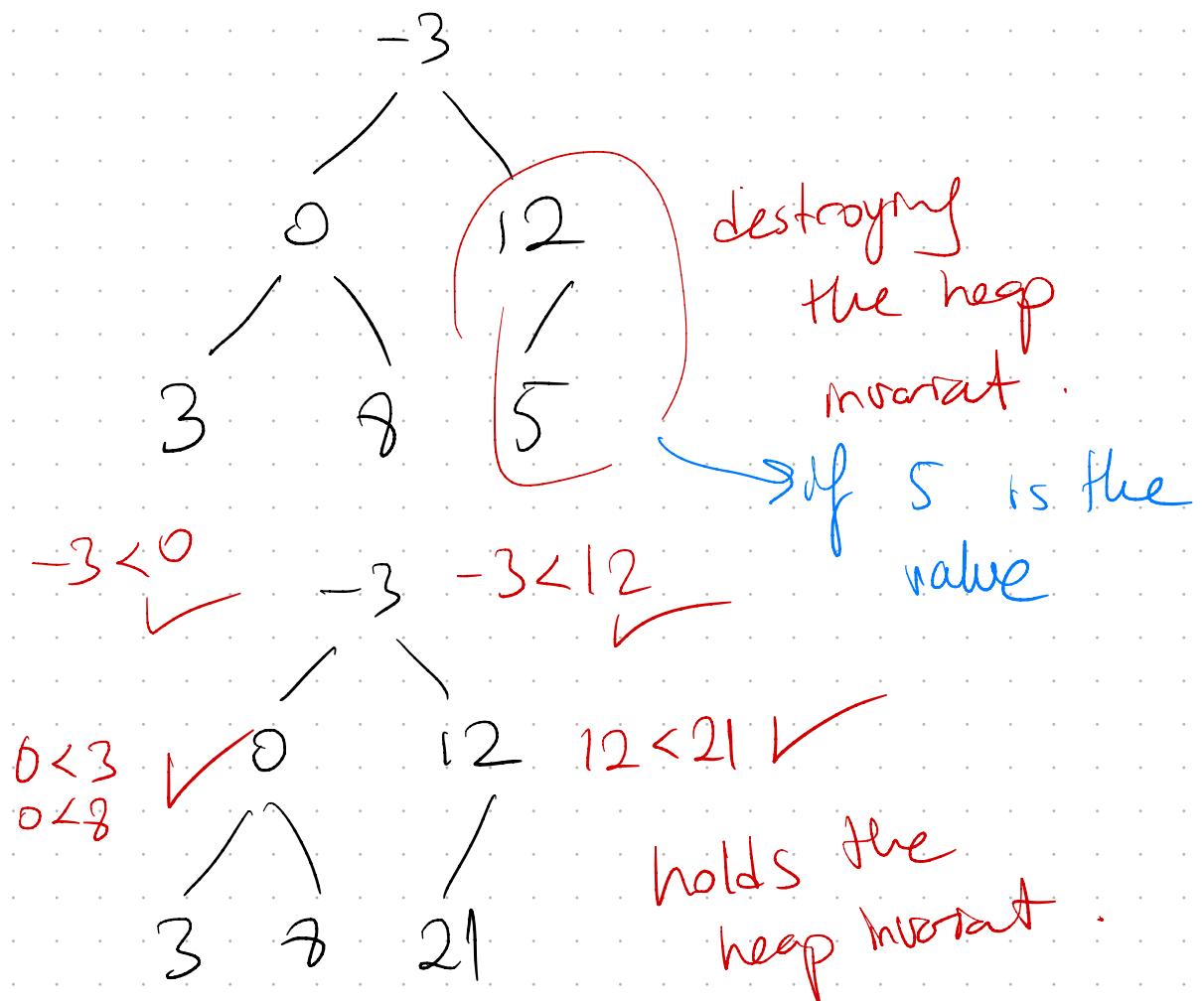
```
    return false;
```

```
}
```

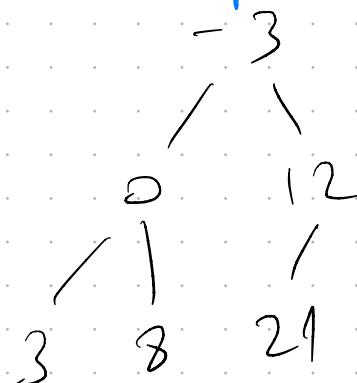
## Heaps

1) Show whether the following array is min heap or not. Justify your answer by drawing the tree representation and verifying the heap invariant is upheld.

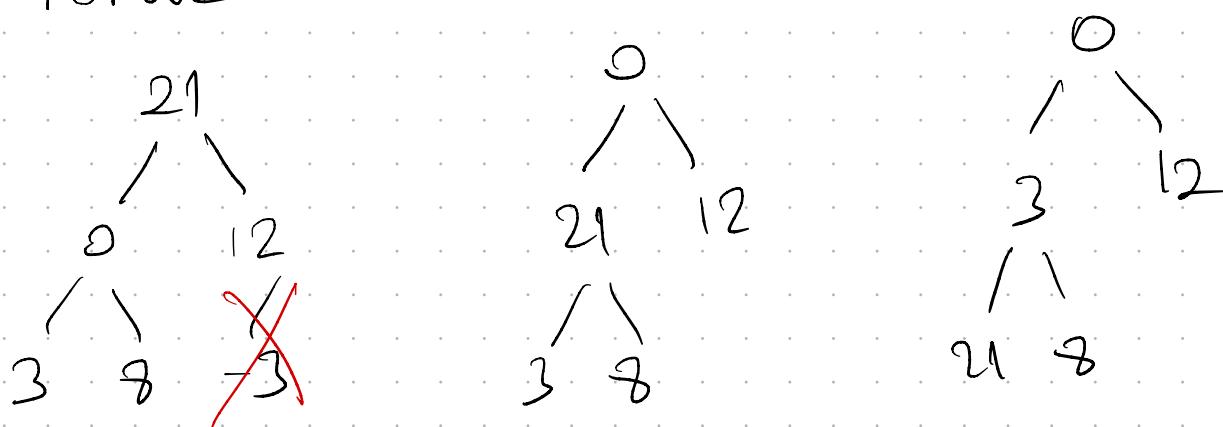
-3	0	12	3	8	21
P	L	r			



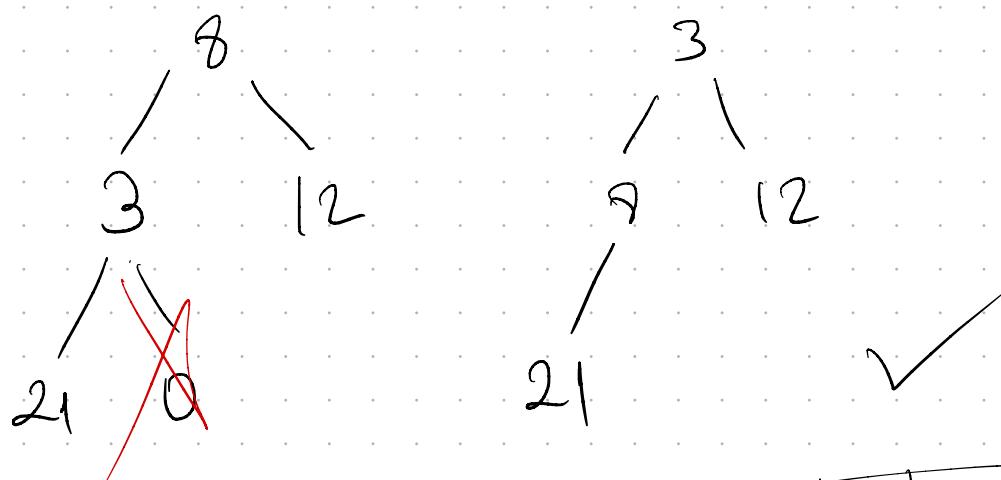
2) Apply heap removal operation twice & show the resulting heap after first & second removals, both in tree format and its array implementation.  
Show all node swaps.



remove -3



remove 0



0	3	12	21	8
---	---	----	----	---

3	8	12	21
---	---	----	----

## Sorting

1) Use selection sort to sort the following values.

0	1	2	3	4	5	6	n-2
17	42	4	12	25	48	7	9 to 5
i	j	m				n-1	6 steps total

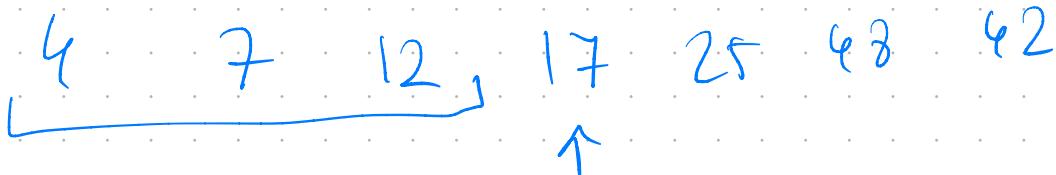
1<sup>st</sup>  
Iteration



2<sup>nd</sup>  
Iteration



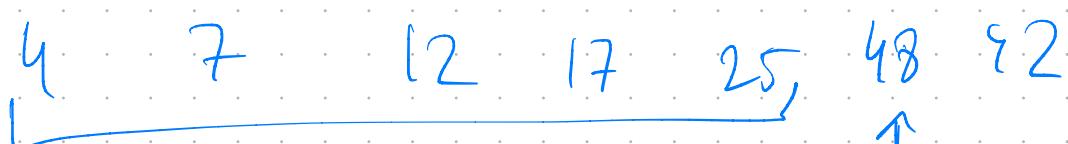
3<sup>rd</sup>  
Iteration



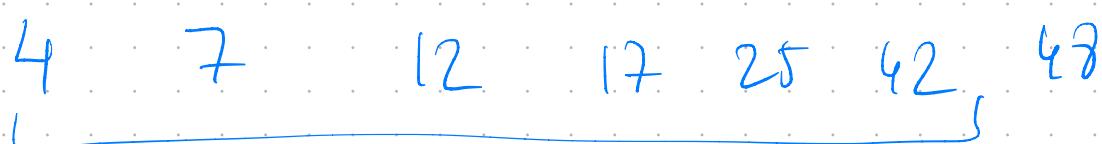
4<sup>th</sup>  
Iteration



5<sup>th</sup>  
Iteration



6<sup>th</sup>  
Iteration



1) Use MergeSort to sort the following values

72 35 18 22 43 12 52 21

[18, 22, 35, 72]

72 35 18 22

72 35

72 35

1<sup>st</sup>

8<sup>th</sup>

[12, 21, 43, 52]

43 12 52 21

72 35 18 22

2<sup>nd</sup>

5<sup>th</sup>

(12, 43)

12<sup>th</sup>

[21, 52]

(35, 72)

18 22 (18, 22)

43 12

52 21

6<sup>th</sup> / 7<sup>th</sup>

18 22

10<sup>th</sup> / 11<sup>th</sup>

43 12

52 21

3<sup>rd</sup> / 4<sup>th</sup>

72 35

13<sup>th</sup> / 14<sup>th</sup>