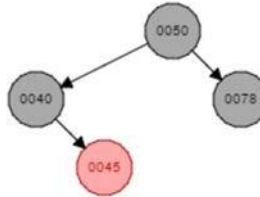Name: Breona Pizzuta   Date: 11/14/24

"I pledge my honor that I have abided by the Stevens Honor System"

Points earned: _____ / 74 = _____ %

1. Show the red-black tree after inserting a node with the key 0043. Use the document on Canvas that explains the insertion process succinctly. List the case you applied (i.e. 1, 2a, 3b), and write the steps you took to fix the tree (also listed in the document).

   a) Draw the tree after doing a regular binary search tree insertion of 0043. (3 points)
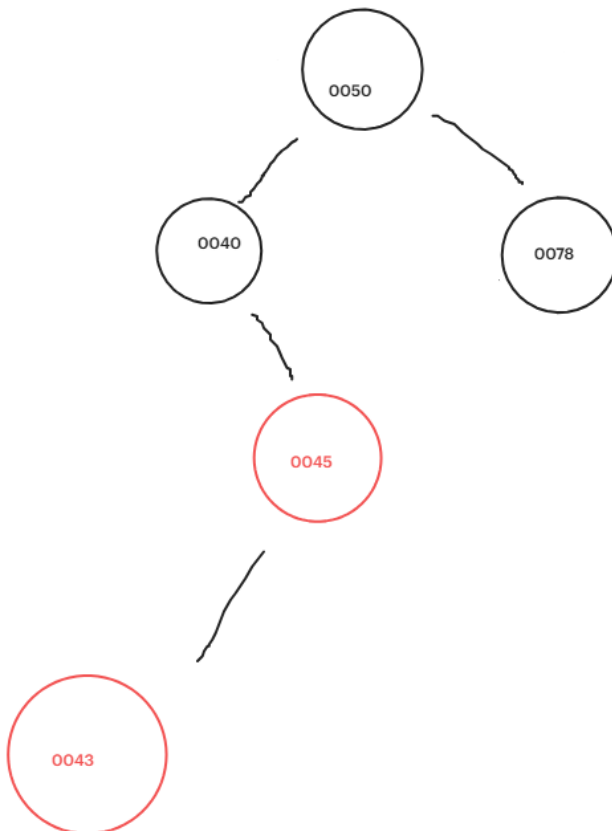
b) Which RBT property is violated? (3 points) <u>Property 4 (if a node is red, then both of its children are black) has been violated.</u>
Case seen after the regular binary search tree insertion: (3 points) <u>Case 2B</u>
Steps taken to fix the tree: (3 points) <u>Set z= the parent of z. After, right rotate on z.</u>

Draw the tree after taking the steps you just described. (3 points)
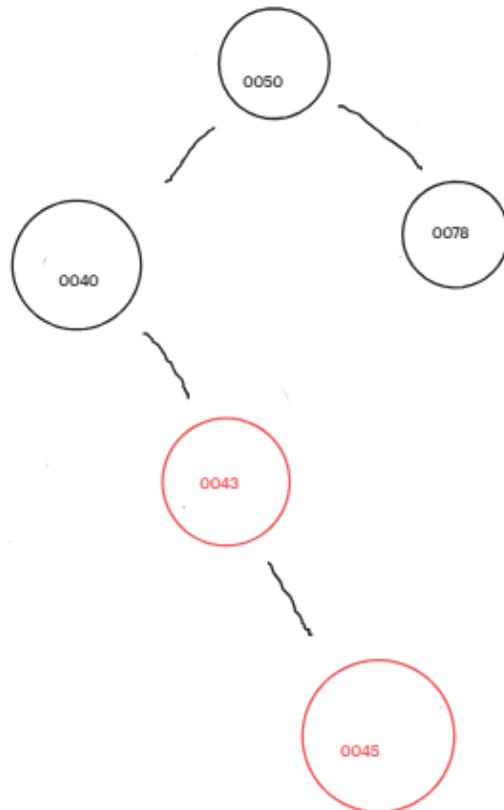
c) Which property is violated now? (3 points) <u>Property 4 (if a node is red, then both of its children are black) has been violated.</u>
Case seen after first fixup: (3 points) <u>Case 3b</u>
Steps taken to fix the tree: (3 points) <u>Change z's parent's color to black. Change z's grandparent's color to red. Left rotate z's grandparent.</u>
Draw the tree after taking the steps you just described. (3 points)

```
                    0050

      0043
                              0078


   0040           0045
```
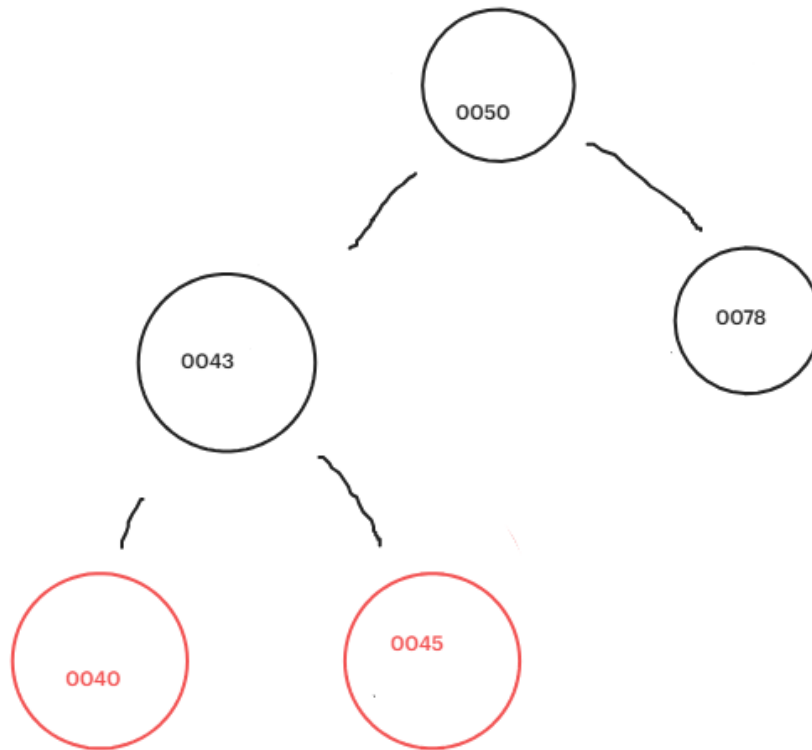
2. Draw the 2-3 tree after inserting each of the following keys. Redraw the whole tree for each part.
   a) 50 (1 point)

                    50

   b) 76 (1 point)

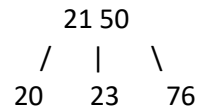   50 76

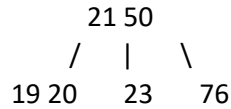   c) 23 (3 points)

```
              50
             / \
          23    76
```
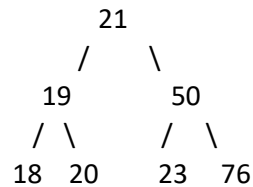
   d) 21 (3 points)

```
              50
             / \
       21 23    76
```

e) 20 (3 points)

```
        21 50
        /  |   \
      20   23   76
```

f) 19 (3 points)

```
        21 50
        /  |   \
    19 20  23   76
```

g) 18 (3 points)

```
          21
          /   \
        19     50
       / \    / \
      18 20  23  76
```

3. Read pages 241-242 in the textbook. Using that information, write pseudocode for computing the LCM of an array A[1..n] of integers. You may assume there is a gcd() function available. (6 points)
   ALGORITHM *LCM*(A[1..n]):
       // Computes the least common multiple of all the integer in array A
       //input an array of integers; output the LCM
   len ← n
   if len = 0 return 0
   answer ← A[0]
   for i ← 1 to len do
           answer ← A[i] * answer / gcd(A[i], answer)
   return answer

4. Horner's method: $p(x) = 4x^4 + 5x^3 - 2x^2 - 4x + 7$
   a) Repeatedly factor out x in the polynomial above so that you can apply Horner's method. Write your final expression for $p(x)$. (5 points)
   $$p(x) = 4x^4 + 5x^3 - 2x^2 - 4x + 7$$
   $$p(x) = x(4x^3 + 5x^2 - 2x - 4) + 7$$
   $$p(x) = x(x(4x^2 + 5x - 2) - 4) + 7$$
   $$p(x) = x(x(x(4x + 5) - 2) - 4) + 7$$

   b) Show values of the array P[0..n] as needed to apply Horner's method. (3 points)
   P[0...n] = [7, -4, -2, 5, 4]

c) Apply Horner's method to evaluate the polynomial at x 2. Make a table as we did in class showing the values x, p, n, and i, and then state your final answer for $p(2)$. (5 points)

x	p	n	i	
2	4	4		(2*4) + 5 =13
	13		3	(2*13) -2 = 24
	24		2	(2*24)-4= 44
	44		1	(2*44)+7 = 95
	95		0	

$p(2) = 95$

d) Use **synthetic** (not long) **division** to divide $p(x)$ by x - 2 to check your work. Be sure to show your work. (5 points)

x_0	x^4	x^3	x^2	x^1	x^0
2	4	5	-2	-4	7
	0	(2*4)=8	(2*13)=26	(2*24)=48	(2*44)=88
0	4	13	24	44	95

$$p(x) = 4x^4 + 5x^3 - 2x^2 - 4x + 7 = (4x^3 + 13x^2 + 24x + 44)(x - 2) + 95$$

5. Rewrite the *LeftRightBinaryExponentiation* algorithm on page 237 in the textbook to work for $n = 0$ as well as any positive integer. *No credit will be given for answers that simply start with an if statement for n = 0.* (6 points)

ALGORITHM *LeftRightBinaryExponentiation*(a, b(n)):
    // Computes a^n

product ←1
for i ←I downto 0 do
        product ←product *product
        if bi = 1 product ←product *a
return product