

Problem Set 8

Bhavna Phogaat
ECON833: Computational Methods for Economists
Fall 2021

Economic Model that involves dynamic optimization

Let us assume that a firm wants to maximize profits from new customer acquisition by spending on marketing.

Here firm is the agent.

Let existing customer at time t : C_t

Let new customer acquisition at time t : $N(a_t)$

where a_t is the amount of money spent on the marketing channels at time t .

It is an infinite time horizon problem.

Objective: Maximize profits by getting the optimal advertisement spending

$$\max_{a_t} \sum_{t=0}^{\infty} \beta^t \pi(C_t, a_t) \quad (1)$$

Bellman Equation:

$$V(C_1) = \max_a \pi(C, a) + \beta V(C') \quad (2)$$

Transition:

$$\begin{aligned} C' &= \rho C + N(a) \\ N' &> 0, N'' < 0 \end{aligned} \quad (3)$$

Per period payoff:

$$\pi(C, a) = \rho C - a \quad (4)$$

Control variable = a

State variable = C

If $a > \rho C$, then negative cash flow. This represents borrowing.

FOC:

$$\begin{aligned} -1 + \beta \rho N'(a) &= 0 \\ \rho N'(a) &= 1/\beta, a > 0 \end{aligned} \quad (5)$$

Marginal revenue and the product of advertisement equals the time cost of money.

Simplifying the problem:

Let us take

$$\begin{aligned} N(a) &= a^\alpha, \alpha \leq 1 \\ N(a) &= \ln(a) \end{aligned} \quad (6)$$

Consider the transition equation:

$$\begin{aligned} C' &= \rho C + N(a) \\ N^{-1}(C' - \rho C) &= a \end{aligned} \quad (7)$$

The per period payoff is:

$$\pi(C, a) = \rho C - a \quad (8)$$

Writing this in terms of the state variable C:

$$\pi(C, C') = \rho C - N^{-1}(C' - \rho C) \quad (9)$$

Since,

$$N(a) = \ln(a) \quad (10)$$

and

$$\begin{aligned}C' &= \rho C + N(a) \\C' &= \rho C + \ln(a) \\C' - \rho C &= \ln(a) \\e^{C' - \rho C} &= a\end{aligned}\tag{11}$$

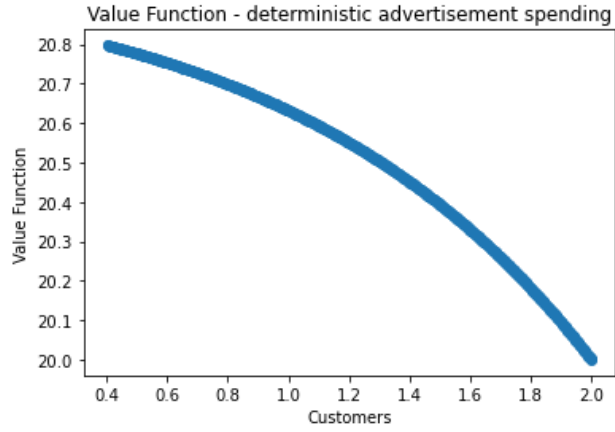
Value Function:

$$\begin{aligned}V(C_1) &= \max_{0 \leq C' \leq C} \pi(C, C') + \beta V(C') \\V(C_1) &= \rho C - N^{-1}(C' - \rho C) + \beta V(C')\end{aligned}\tag{12}$$

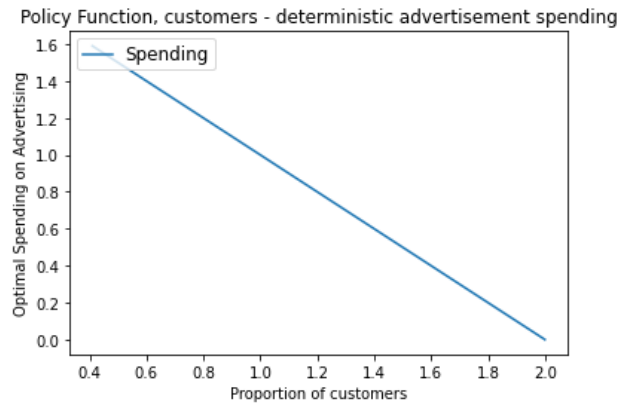
Policy Function:

$$a = N^{-1}(C' - \rho C)\tag{13}$$

Graphs:



(a) Value Function Plot



(b) Policy Function Plot

Figure 1: Graphical Representation

The graphs show how the spending on advertisement is reduced as new customers are acquired. If you look at it from a business point of view, some of the spending is being balanced from the profits that are earned from the total customers that the firm has at any time. Any credit on spending is also adjusted from the profits earned from the existing customers.