Conditional Obligation Logics: Motivation, Varieties, and Future Work

Brandon Hopkins
23 February 2024

1 Prefatory

(1) Why doing deontic logic? Spring 2022, thinking about bridge principles, which strike me as very special kinds of conditional obligations/permissions, or at least would involve such.

(Φ) If $\gamma(\Gamma \Vdash C)$, then $\mathsf{N}^S(\alpha(\Gamma), \beta(C))$.

(2) Applicability? Well, AI... But also philosophically interesting. Questions: To what extent is normative language functional? To what extent is modal semantics limited, if at all?

(3) Bias? Particularism: Where do formal semantics and metaethics make contact, and in particular, where do opportunities for particularism appear in the semantics?

(4) Warning: Much to say, but no singular thesis yet. Per Rohan, I am here meditating in a garden of forking paths.

2 Preliminaries

2.1 Standard Stuff

Definition 2.1. A *basic deontic language* contains:

1. The propositional constants for falsity $\bot$ and truth $\top$.
2. A countable set of propositional variables $p_0, p_1, p_2, \ldots$.

   $\mathcal{P} = \{\bot, \top, p_0, p_1, p_2, \ldots\}$
3. The propositional connectives and punctuation: $\neg, \wedge, \vee, \rightarrow, \leftrightarrow,), ($.
4. The modal operators O and P.

Definition 2.2. The *formulas of deontic logic* are defined inductively as follows:

1. $\bot$, $\top$, and $p_0, p_1, p_2, \ldots$ are atomic formulas.
2. If A is a forumla, then $\neg A$, $\mathsf{O}A$, and $\mathsf{P}A$ are formulas.
3. If A and B are formulas, then $A \wedge B$, $A \vee B$, $A \rightarrow B$, and $A \leftrightarrow B$ are formulas.
4. Nothing else is a formula.

Definition 2.3. A *standard relational model* is a triple $\mathfrak{M} = \langle W, R, V \rangle$ such that

1. W is a set of points, or worlds;

   $W = \{w, v, u, \ldots\}$
2. R is a relation on W;

   $R\colon W \rightarrow \mathbb{P}(W \times W)$
3. V is a function assigning to each propositional variable p a set $V(p)$ of possible worlds.

   $V\colon \mathcal{P} \rightarrow \mathbb{P}(W)$

Definition 2.4. Let $\mathfrak{M} = \langle W, R, V \rangle$ be a standard relational model, then:

1. $\mathfrak{M}, w \Vdash \mathsf{O}A$ iff $\mathfrak{M}, v \Vdash A$ for all $v \in W$ where Rwv.
2. $\mathfrak{M}, w \Vdash \mathsf{P}A$ iff $\mathfrak{M}, v \Vdash A$ for some $v \in W$ where Rwv.

2.2 The System SDL and its Semantics

Standard Deontic Logic (SDL) comes in two variants. Its weaker variant is just the modal system KD, and its stronger variant is the modal system KU. Both systems have the same syntax, and the semantics are given in terms of standard relational models and truth conditions.

Disclaimer: Nobody believes SDL is any good, except as a jumping-off point.

Definition 2.5. The *system* SDL is given by rules R1-R3 and axioms A1-A3:

R1 All truth-functional tautologies are theorems.

R2 If $\vdash A$ and $\vdash A \to B$, then $\vdash B$ (MP)

R3 If $\vdash A$, then $\vdash \mathsf{O}A$ (NEC)

A1 $\vdash \mathsf{O}A \leftrightarrow \neg\mathsf{P}\neg A$ (DUAL)

A2 $\vdash \mathsf{O}(A \to B) \to (\mathsf{O}A \to \mathsf{O}B)$ (K)

A3 $\vdash \mathsf{O}A \to \mathsf{P}A$ (D)

In normal modal logics, $(\mathsf{O}A \to \mathsf{P}A) \leftrightarrow \neg(\mathsf{O}A \wedge \mathsf{O}\neg A) \leftrightarrow \mathsf{P}\top \leftrightarrow \neg\mathsf{O}\bot$.

Definition 2.6. The system SDL^+ is the result of extending SDL with A4:

A4 $\vdash \mathsf{O}(\mathsf{O}A \to A)$ (U)

Theorem 2.1. SDL is determined by the class of standard serial models, i.e., the class of models in which R satisfies the following condition:

"is determined by" = "is sound and complete with respect to"

(d) For all $w \in W$, there is some $v \in W$ such that Rwv.

Theorem 2.2. SDL^+ is determined by the class of standard serial and shift-reflexive models, i.e., the class of models in which R satisfies (d) plus the following condition:

(u) For all $w, v \in W$, if Rwv then Rvv.

2.3 Big Problems for SDL

Conditionals and standard deontic operators don't play nice. This was first exposed by Chisholm [2]. How should we interpret the following set of statements? They seem to represent an imaginable state of affairs, and each strikes us as independent of the others.

This presentation is lifted from [4].

(1) It ought to be that Jones goes to assist his neighbors.
(2) It ought to be that, if Jones goes, then he tells them he is coming.
(3) If Jones doesn't go, then he ought not tell them he is coming.
(4) Jones doesn't go.

The three plausible transcriptions have issues:

	Option 1	**Option 2**	**Option 3**
(1)	$\mathsf{O}g$	$\mathsf{O}g$	$\mathsf{O}g$
(2)	$\mathsf{O}(g \to t)$	$\mathsf{O}(g \to t)$	$g \to \mathsf{O}t$
(3)	$\neg g \to \mathsf{O}\neg t$	$\mathsf{O}(\neg g \to \neg t)$	$\neg g \to \mathsf{O}\neg t$
(4)	$\neg g$	$\neg g$	$\neg g$

Option 1 is inconsistent.
In Option 2, (1) implies (3).
In Option 3, (4) implies (2).

Conflicting obligations. In SDL – and indeed, in all standard serial frames – $\neg\mathsf{O}\bot$ implies that $\neg\mathsf{O}(A \wedge \neg A)$, which implies that it can never be the case that $\mathsf{O}A \wedge \mathsf{O}\neg A$. But is the world not rife with conflicting obligations? Perhaps we can say that, ideally, any one *code* ought to be consistent, but not necessarily all of them together. But even if this is ideal, this is not how the world presents itself. We are often torn between conflicting obligations, and anyway, the existence of conflicting obligations does not actually entail that either everything is obligatory or everything is permissible. Shouldn't our logic tolerate conflicting obligations, even if not outright contradictions?

Interactions with the propositional constants $\top$ and $\bot$. Maybe $\neg\mathsf{O}\bot$ strikes us as plausible (it strikes me as plausible, even if not the inference shown above) – and perhaps even so does $\mathsf{P}\top$ – but what about $\mathsf{O}\top$, or $\neg\mathsf{P}\bot$? The standard reason to not want to validate $\mathsf{O}\top$ is that $\top$ – the proposition expressed by all tautologies – is not ordinarily the kind of thing that we are obliged to bring about. There is something off in saying, "It ought to be the case that all tautologies obtain," whereas there is no obvious problem in saying, "It ought not be the case that any contradiction obtains."

Much more broadly, my own interest is in understanding where metaethics and deontic logic make contact, and in particular in being able to provide a logic for particularism.

3 Standard Conditional Obligation

In the decade following Chisholm's puzzle, the semantics for dyadic deontic operators was developed by a number of people, and David Lewis [3] describes four such semantic analyses from a very high level.

Remark. In the early days of the semantics for dyadic modal operators, it appears that any such operator that had any analogy with *conditionals* would be symbolized as conditional probability had been symbolized for some decades. There are reasons to avoid this. For example, whereas $\mathbf{P}(A \mid A) = 1$ by definition, it is not clear that we should want to validate $\neg\mathsf{O}(\neg A/A)$. More below. To break free of the analogy, I take the liberty of everywhere symbolizing conditional obligation as "$A \mapsto\!\circ\, B$", with the dual, conditional permission, $\neg(A \mapsto\!\circ\, \neg B)$, being symbolized as "$A \mapsto\!\mathsf{P}\, B$".

Below, we assume that $\mathfrak{M} = \langle W, \mathscr{F}, \|\cdot\|^{\mathfrak{M}}\rangle$, where the value structures slot into the $\mathscr{F}$-slot.

3.1 Value Structures, Frames, and Our Choices

Definition 3.1. A *choice function* f is a function that assigns to each $X \subseteq W$ some $f(X) \subseteq X$, subject to two conditions:

(1) If $X \subseteq Y$ and $f(X) \neq \emptyset$, then $f(Y) \neq \emptyset$.

(2) If $X \subseteq Y$ and $X \cap f(Y) \neq \emptyset$, then $f(X) = X \cap f(Y)$.

An interpretation $\|\cdot\|^{\mathfrak{M}}$ is *based on* a choice function f iff:

$$\mathfrak{M}, w \Vdash A \mapsto\!\circ\, B \text{ iff } f(\|A\|^{\mathfrak{M}}) \neq \emptyset \text{ and } f(\|A\|^{\mathfrak{M}}) \subseteq \|B\|^{\mathfrak{M}}.$$

Definition 3.2. A *ranking* $\langle K, R\rangle$ is a pair such that

(1) $K \subseteq W$, and

(2) R satisfies the following three conditions:

 (a) $R \subseteq K \times K$,

 (b) R is transitive, and

 (c) for any $j, k \in K$, either Rjk or Rkj.

In other words, R is a *weak ordering* or *total preordering*.

An interpretation $\|\cdot\|^{\mathfrak{M}}$ is *based on* a ranking $\langle K, R\rangle$ iff:

Here, and below, we can read "Rkj" as "$k > j$".

$$\begin{aligned}\mathfrak{M}, w \Vdash A \mapsto\!\circ\, B &\text{ iff for some } j \in \|A \wedge B\|^{\mathfrak{M}} \cap K,\\ &\text{there is no } k \in \|A \wedge \neg B\|^{\mathfrak{M}} \cap K \text{ such that } Rkj.\end{aligned}$$

Definition 3.3. A *nesting* $ *over* W is a set of subsets of W – $\$ \subseteq \mathbb{P}(W)$ – such that, if $S, T \in \$$, then either $S \subseteq T$ or $T \subseteq S$.

An interpretation $\|\cdot\|^{\mathfrak{M}}$ is *based on* a nesting $ iff:

A nesting $ is *closed* iff, for any subset $\mathbf{S} \subseteq \$$, $\cup\mathbf{S} \in \$$. Per Lewis, closure has no semantic effect.

$$\mathfrak{M}, w \Vdash A \mapsto\!\circ\, B \text{ iff for some } S \in \$, S \cap \|A\|^{\mathfrak{M}} \neq \emptyset \text{ and } S \cap \|A\|^{\mathfrak{M}} \subseteq \|B\|^{\mathfrak{M}}.$$

Definition 3.4. An *indirect ranking* $\langle V, R, f\rangle$ *over* W is a triple such that:

(1) V is a set,

(2) R is a weak ordering of V, and

(3) $f\colon W \to \mathbb{P}(V)$.

An indirect ranking $\langle V, R, f\rangle$ is *linear* iff there are no two distinct members $v, u \in V$ such that both Rvu and Ruv. Again, per Lewis, linearity has no semantic effect.

An interpretation $\|\cdot\|^{\mathfrak{M}}$ is *based on* an indirect ranking $\langle V, R, f\rangle$ iff:

$$\begin{aligned}\mathfrak{M}, w \Vdash A \mapsto\!\circ\, B &\text{ iff for some } v \in f(j) \text{ such that } j \in \|A \wedge B\|^{\mathfrak{M}},\\ &\text{there is no } u \in f(k) \text{ for any } k \in \|A \wedge \neg B\|^{\mathfrak{M}} \text{ such that } Ruv.\end{aligned}$$

Properties of Value Structures *Trivial* value structures are those in which no world is evaluable. *Normal* value structures are those in which at least some world is evaluable. *Universal* value structures are those in which every world is evaluable. *Limited* value structures are those with no infinitely ascending sequences of better and better worlds. *Separative* value structures are those in which any world that

Below, it turns out that the only properties of value structures that make a difference to the logic are *normality* and *universality*.

surpasses various of its rivals taken separately also surpasses all of them taken together.

All limited value structures are separative, but not conversely.

(1) Nestings are equivalent to indirect rankings. (2) Rankings, separative nestings, and separative indirect rankings are equivalent and reducible to (1). Choice functions, limited rankings, limited nestings, and limited indirect rankings are equivalent and reducible to (1) and (2). All of this holds true whether we are considering trivial, normal, or universal value structures.

(?) Here Lewis claims that the real choice is not between different value structures, but between three levels of generality: limited, separative, and unrestricted. Yet, as we see below, these restrictions do not validate any new sentences; again, all that matters there is normality and universality. Is the point, then, that limitedness and separativeness make a difference to *falsification* and *invalidation*, if not to verification and validation?

Frames, and Six Logics The way we set value structures up above obscures another degree of freedom. Above, the *source* of value is held fixed over all worlds. But we might think that evaluations might depend on states of affairs, in which case *each world* would have its own value structure.

Definition 3.5. A *choice function frame* $\langle f_w \rangle_{w \in W}$ *over a set* W assigns a choice function f_w to each $w \in W$.

Definition 3.6. A *ranking frame* $\langle K_w, R_w \rangle_{w \in W}$ *over a set* W assigns a ranking $\langle K_w, R_w \rangle$ to each $w \in W$.

Definition 3.7. A *nesting frame* $\langle \$_w \rangle_{w \in W}$ *over a set* W assigns a nesting $\$_w$ to each $w \in W$.

Definition 3.8. An *indirect ranking frame* $\langle V_w, R_w, f_w \rangle_{w \in W}$ *over a set* W assigns a ranking $\langle V_w, R_w, f_w \rangle$ to each $w \in W$.

Like value structures, frames may assign the same value structure to every world – in which case he calls them *absolute*.

Lewis makes a number of arguments to the effect that the only properties of frames that make any difference to what sentences are valid are normality, universality, and absoluteness. Hence, we may distinguish *six* logics:

Definition 3.9. The following are sets of sentences valid in the indicated class of frames:

- **CO**: all frames
- **CD**: all normal frames
- **CU**: all universal frames
- **CA**: all absolute frames
- **CDA**: all absolute normal frames
- **CUA**: all absolute universal frames

Axiomatics

All six logics take R1-R4. **CO** takes A1-A8. **CD** takes A1-A9. **CU** takes A1-A8 and A10-A11. **CA** takes A1-A8 and A12-A13. **CDA** takes A1-A9 and A12-A13. **CUA** takes A1-A8, A10, and A12-A13.

Hence, normality corresponds to A9, universality corresponds to A10, and absoluteness corresponds to A12-A13—the oddball being A11, which is valid in non-absolute universal frames but not absolute ones. (?)

R1	All truth-functional tautologies are theorems.
R2	If $\vdash A$ and $\vdash A \rightarrow B$, then $\vdash B$ (MP)
R3	If $\vdash A \leftrightarrow B$, then $\vdash (C \boxright A) \leftrightarrow (C \boxright B)$. (RE$_R$)
R4	If $\vdash A \leftrightarrow B$, then $\vdash (A \boxright C) \leftrightarrow (B \boxright C)$. (RE$_L$)
A1	$(A \diamondright B) \leftrightarrow \neg(A \boxright \neg B)$ (DUAL)
A2	$(A \boxright (B \wedge C)) \leftrightarrow ((A \boxright B) \wedge (A \boxright C))$ (M), (C)
A3	$(A \boxright B) \rightarrow (A \diamondright B)$ (D)
A4	$(A \boxright \top) \rightarrow (A \boxright A)$
A5	$(A \boxright \top) \rightarrow ((A \vee B) \boxright \top)$
A6	$((A \boxright C) \wedge (B \boxright C)) \rightarrow ((A \vee B) \boxright C)$
A7	$((A \diamondright \bot) \wedge ((A \vee B) \boxright C)) \rightarrow (B \boxright C)$
A8	$(((A \vee B) \diamondright A) \wedge ((A \vee B) \boxright C)) \rightarrow (A \boxright C)$
A9	$\top \boxright \top$

A4-A13 are relatively easily interpreted, but here is where connections with standard monadic axioms start to blur.

A10 $A \to (A \mapsto_{\mathsf{O}} \top)$
A11 $(A \mapsto_{\mathsf{O}} \top) \to ((A \mapsto_{\mathsf{P}} \bot) \mapsto_{\mathsf{P}} \bot)$
A12 $(A \mapsto_{\mathsf{O}} B) \to (\neg(A \mapsto_{\mathsf{O}} B) \mapsto_{\mathsf{P}} \bot)$
A13 $(A \mapsto_{\mathsf{P}} B) \to (\neg(A \mapsto_{\mathsf{P}} B) \mapsto_{\mathsf{P}} \bot)$

4 Classical Systems and Minimal Models

This section draws heavily from Chellas [1].

4.1 For Monadic Operators

Classical systems are *minimal* modal logics in the sense that they are the weakest logics that are still recognizable as modal logics.

We here consider the case where our modal operators are O and P.

Definition 4.1. A modal logic is *classical* iff it contains the following inference rules and axiom:

R1 All truth-functional tautologies are theorems.
R2 If $\vdash A \leftrightarrow B$, then $\vdash \mathsf{O}A \leftrightarrow \mathsf{O}B$. (RE)
A1 $\mathsf{P}A \leftrightarrow \neg\mathsf{O}\neg A$ (DUAL)

We call this weakest classical modal logic E.

There are in fact a number of ways to present classical systems, using different inference rules and axioms, but here I opt for the most modular way, which bears the most resemblance to the semantics.

Definition 4.2. A modal logic is *normal* iff it extends E with the following axioms:

A2 $\mathsf{O}(A \wedge B) \to (\mathsf{O}A \wedge \mathsf{O}B)$ (M)
A3 $(\mathsf{O}A \wedge \mathsf{O}B) \to \mathsf{O}(A \wedge B)$ (C)
A4 $\mathsf{O}\top$ (N)

We call this logic K.

Between E and K there are *six* logics, each of which combines A2-A4 with R1-R2 and A1 in different ways.

Minimal models allow us to semantically pull apart normal modal logic. We do this by trading in our relation R on W in relational models for a function f_N which maps worlds to sets of sets of worlds, otherwise called "neighborhoods". The neighborhood of w is the set of propositions which are obligated (or necessary) at w.

Definition 4.3. A *minimal model* is a triple $\mathfrak{M} = \langle W, f_N, P\rangle$ such that:

(1) W is a set,
(2) f_N is a function that maps each world in W to a set of sets of worlds, and
(3) P is a mapping from propositional constants and variables to sets of worlds,

$f_N : W \to \mathbb{P}(\mathbb{P}(W))$

$P : \mathcal{P} \to \mathbb{P}(W)$

where:

$$\mathfrak{M}, w \Vdash \mathsf{O}A \text{ iff } \|A\|^{\mathfrak{M}} \in f_N(w), \text{ and}$$
$$\mathfrak{M}, w \Vdash \mathsf{P}A \text{ iff } -\|A\|^{\mathfrak{M}} \notin f_N(w).$$

Theorem 4.1. E is determined by the class of minimal models.

Theorem 4.2. K is determined by the class of minimal models in which the following three conditions on N hold:

(m) If $X \cap Y \in f_N(w)$, then $X \in f_N(w)$ and $Y \in f_N(w)$.
(c) If $X \in f_N(w)$ and $Y \in f_N(w)$, then $X \cap Y \in f_N(w)$.
(n) $W \in f_N(w)$.

Closure under supersets. (A2)

Closure under intersections. (A3)

Contains the unit. (A4)

Once again, between minimal models with no conditions on f_N and those in which (m), (c), and (n) hold, there are six classes of models, which determine the six corresponding logics indicated above.

Remark. For any minimal model that's closed under supersets (satisfies condition (m)), the truth conditions for O given in Definition 4.3 are equivalent to the following:

$$\mathfrak{M}, w \Vdash \mathsf{O}A \text{ iff for some } X \in f_N(w), X \subseteq \|A\|^{\mathfrak{M}}.$$

This has a straightforward deontic reading: A is obligatory in w iff A follows from some part of the moral code in w.

Minimal Deontic Logic We are now in a position to look at the axiomatics and semantics of what Chellas calls *minimal deontic logic*.

Definition 4.4. *Minimal deontic logic* is the system given by the following inference rules and axioms:

R1	All truth-functional tautologies are theorems.	
R2	If $\vdash A \leftrightarrow B$, then $\vdash \mathsf{O}A \leftrightarrow \mathsf{O}B$.	(RE)
A1	$\mathsf{P}A \leftrightarrow \neg\mathsf{O}\neg A$	(DUAL)
A2	$\mathsf{O}(A \wedge B) \rightarrow (\mathsf{O}A \wedge \mathsf{O}B)$	(M)
A3	$\neg\mathsf{O}\bot$	(P_{O})

I do not have completeness results for this proposed axiomatization of minimal deontic logic. Chellas starts from the models and does not go into axiomatics.

In normal modal logics, $(\mathsf{O}A \rightarrow \mathsf{P}A) \leftrightarrow \neg(\mathsf{O}A \wedge \mathsf{O}\neg A) \leftrightarrow \mathsf{P}\top \leftrightarrow \neg\mathsf{O}\bot$. Whereas, absent closure under intersections, $\neg\mathsf{O}\bot$ and $\mathsf{P}\top$ come apart from $(\mathsf{O}A \rightarrow \mathsf{P}A)$ and $\neg(\mathsf{O}A \wedge \mathsf{O}\neg A)$. That is, whereas the former are expressed in an axiom, the latter cannot be derived in this system. This affords us the ability to express conflicting obligations without being able to derive any contradictions. Where (P) and (D) come apart – i.e., where $\neg\mathsf{O}\bot \not\leftrightarrow (\mathsf{O}A \rightarrow \mathsf{P}A)$ – (P) is the more plausible, and so is chosen as the deontic basis.

Theorem 4.3. Minimal deontic logic is determined by the class of minimal models in which the following conditions on f_N hold:

(m) If $X \cap Y \in f_N(w)$, then $X \in f_N(w)$ and $Y \in f_N(w)$.

(p) $\emptyset \notin f_N(w)$.

4.2 Minimal Models for Dyadic Operators

Definition 4.5. A *minimal conditional model* is a triple $\mathfrak{M} = \langle W, f_S, P\rangle$ such that:

(1) W is a set,

(2) f_S is a function that maps propositions at worlds to sets of sets of worlds, and

(3) P is a mapping from propositional constants and variables to sets of worlds,

where:

$$\mathfrak{M}, w \Vdash A \Rightarrow B \text{ iff } \|B\|^{\mathfrak{M}} \in f_S(w, \|A\|^{\mathfrak{M}})$$

$f_S \colon W \times \mathbb{P}(W) \rightarrow \mathbb{P}(\mathbb{P}(W))$

$P \colon \mathcal{P} \rightarrow \mathbb{P}(W)$

Obviously, here our language includes a dyadic operator $\Rightarrow$.

Definition 4.6. A *standard conditional model* is a minimal conditional model in which f_S satisfies the following conditions:

(cm) If $Y \cap Y' \in f_S(w, X)$, then $Y \in f_S(w, X)$ and $Y' \in f_S(w, X)$;

(cc) If $Y \in f_S(w, X)$ and $Y' \in f_S(w, X)$, then $Y \cap Y' \in f_S(w, X)$;

(cn) $W \in f_S(w, X)$.

As before, between minimal and normal there are six other ways to combine these conditions, yielding six other logics.

Minimal Conditional Obligation

Definition 4.7. The logic of *minimal conditional obligation* is determined by the class of minimal conditional models in which the following conditions hold:

(cm) If $Y \cap Y' \in f_S(w, X)$, then $Y \in f_S(w, X)$ and $Y' \in f_S(w, X)$;

(cp) If $X \neq \emptyset$, then $\emptyset \notin f_S(w, X)$.

Definition 4.8. The logic of minimal conditional obligation is given by the following inference rules and axioms:

R1	All truth-functional tautologies are theorems.	
R2	If $\vdash A \leftrightarrow B$, then $\vdash (C \vDash\!\mathsf{O}\, A) \leftrightarrow (C \vDash\!\mathsf{O}\, B)$.	(RE_{R})
R3	If $\vdash A \leftrightarrow B$, then $\vdash (A \vDash\!\mathsf{O}\, C) \leftrightarrow (B \vDash\!\mathsf{O}\, C)$.	(RE_{L})
A1	$(A \vDash\!\mathsf{P}\, B) \leftrightarrow \neg(A \vDash\!\mathsf{O}\, \neg B)$	(DUAL)
A2	$(A \vDash\!\mathsf{O}\, (B \wedge C) \rightarrow ((A \vDash\!\mathsf{O}\, B) \wedge (A \vDash\!\mathsf{O}\, C)))$	(M)
A3	$\Diamond A \rightarrow \neg(A \vDash\!\mathsf{O}\, \bot)$	($\mathrm{P}^{*}_{\mathsf{O}}$)

Here, like in Definition 4.4, I do not have completeness results, so it is not totally clear whether this is the full logic.

Chellas offers the alethic A3 instead of the non-alethic $\neg(A \vDash\!\mathsf{O}\, \bot)$ in order to avoid the consequence that $\neg(\bot \vDash\!\mathsf{O}\, \bot)$. But why not want to treat conditioning on $\bot$ identically to conditioning on $\top$, i.e., as a typical monadic obligation operator? It's not clear to me that one of these is better than the other, if the trade-off is that we have to start minding alethic context.

4.3 Chellas' Fusion

Chellas redefines conditional obligation in terms of a model with both a monadic deontic operator O and a dyadic conditional operator $\Rightarrow$. He calls the former D and the latter CKD. He argues that the logic of conditional obligation given here is identical to the one above.

Definition 4.9. D is determined by the class of minimal models $\mathfrak{M} = \langle W, f_N, P \rangle$ in which the following conditions are satisfied:

(m) If $Y \cap Y' \in f_N(w)$, then $Y \in f_N(w)$ and $Y' \in f_N(w)$;

(p) $\emptyset \notin f_N(w)$.

Hence, conflicting obligations do not lead to contradiction, and the only restriction is that contradictions are not obligated.

Definition 4.10. CKD is determined by the class of minimal conditional models $\mathfrak{M} = \langle W, f_S, P \rangle$ in which the following conditions are satisfied:

(cm) If $Y \cap Y' \in f_S(w, X)$, then $Y \in f_S(w, X)$ and $Y' \in f_S(w, X)$;

(cc) If $Y \in f_S(w, X)$ and $Y' \in f_S(w, X)$, then $Y \cap Y' \in f_S(w, X)$;

(cn) $W \in f_S(w, X)$;

(cd) If $X \neq \emptyset$, then $\emptyset \notin f_S(w, X)$.

Definition 4.11. D + CKD ($\mathrm{CO_C}$ for short) is determined by the class of models $\mathfrak{M} = \langle W, f_N, f_S, P \rangle$ in which f_N behaves as it does in D and f_S behaves as it does in CKD.

Chellas' motivation for redefining conditional obligation this way appears to be that he want to be able to *prove*, in the logic, that monadic obligation is equivalent to conditional obligation where we condition on $\top$. Hence, if we add

(def) $f_S(w, W) = w$

to the conditions on f_S, then $\vdash A \leftrightarrow (\top \Rightarrow A)$, in which case $\vdash \mathsf{O}A \leftrightarrow (\top \mathrel{\vdash\!\circ} A)$.

Definition 4.12. In $\mathrm{CO_C}$, $A \mathrel{\vdash\!\circ} B \; =_{\text{def}} \; A \Rightarrow \mathsf{O}B$, so that

$$\mathfrak{M}, w \Vdash A \mathrel{\vdash\!\circ} B \text{ iff } \{w \in W \colon \|B\|^{\mathfrak{M}} \in f_N(w)\} \in f_S(w, \|A\|^{\mathfrak{M}})$$

i.e., iff there is some $X \in f_S(w, \|A\|^{\mathfrak{M}})$ such that $X \subseteq \{w \in W \colon \|B\|^{\mathfrak{M}} \in f_N(w)\}$.

Axiomatics for $\mathrm{CO_C}$? Chellas claims $\mathrm{CO_C}$ yields the same logic as minimal conditional obligation. Well, I'm not so sure. Some proofs will be in order.

Intuitions about how neighborhoods muddy Lewis' story? In the inference rules and axioms of these minimal logics, we can see Lewis' list much reduced, as well as some new players. Proofs about what is equivalent to what, what follows from what, etc., are in order. One ambition is to be able to describe in terms of minimal models and conditional models the logics Lewis describes above. The idea would be to rebuild all of the axioms of all of the logics Lewis describes in terms of minimal models and conditional models.

The models of most, if not all, of the logics Lewis describes involve closure under intersections (c) and, it would seem, some variation of containing the unit (n). But there are forks, too. Lots of questions to ask about relationships between these logics.

References

[1] Brian F. Chellas. *Modal Logic: An Introduction*. New York: Cambridge University Press, 1980.

[2] Roderick M. Chisholm. "Contrary-to-Duty Imperatives and Deontic Logic". In: *Analysis* 24.2 (1963), pp. 33–36. DOI: 10.1093/analys/24.2.33.

[3] David K. Lewis. "Semantic analyses for dyadic deontic logic". In: *Logical Theory and Semantic Analysis: Essays Dedicated to Stig Kanger on His Fiftieth Birthday*. Ed. by Sören Stenlund. D. Reidel, 1974.

[4] Paul McNamara and Frederik Van De Putte. "Deontic Logic". In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta and Uri Nodelman. Fall 2022. Metaphysics Research Lab, Stanford University, 2022.