Counting Problems 45 Breeze

w/ MEETING

1) a) 1 2 3 4 5 6 7 — want 5 letter strings

e multiplicity of 2

		total
case 0 e's: $\binom{5}{5}$ MTING	$\frac{5!}{5!\,0!}$	1
case 1 e's: $\binom{5}{4}$ choose 1e and 4 remaining unique letters	$\frac{5!}{4!\,1!}$	5
case 2 e's: $\binom{5}{3}$ choose 2es and 3 remaining unique letters	$\frac{5!}{3!\,2!}$	10

= unique 16 subsets

b) subsets 0 e's: only 1 way to choose letters, all letters unique so 5! strings

1e: 5 subsets each with 5! arrangements

2e: 10 subsets each with $\frac{5!}{2!}$ ← identical e arrangements

$1 \cdot 5! + 5 \cdot 5! + 10 \cdot \frac{5!}{2!} =$

$120 + 600 + 10 \cdot 60 =$ 1320 dif strings

2) 52 cards, 5 card hand — pair, pair, single

13 kinds of card levels (2 → ace)

$C(13,2) = \frac{13!}{2!(11!)} = \frac{13 \cdot 12}{2} = 78$

4 kind of card suits $C(4,2) = \frac{4!}{2!(2!)} = \frac{24}{4} = 6$

1st pair → 2nd pair → $C(4,2) = 6$

remaining card: 11 other ranks, 4 suits so $11 \times 4 = 44$

total: $78 \times 6 \times 6 \times 44 =$ 123552 combinations

if same color: spades, clubs = black; diamond, hearts = red

↳ now $C(2,1) = 2$ ← choose 1 of same color; do twice, once for each pair

for 5th single remaining: $C(13,2) \cdot C(2,1)^2 \times 11 \text{ types} \times 4 \text{ suits}$

$78 \times 2 \times 2 \times 44 =$ 13728 combos

3) 13 students, 2 teams, 6p, 7p

How many dif ways teams pick if 2 best p cant be on same team

PA → blue or → red; PB → red → blue } 2 scenarios for best 2p

11 students left → 6, → 5

$C(11,5) = \frac{11!}{5!(6!)} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{5!} = 462$

red wins or blue wins → $2 \cdot 462 \cdot 2 =$ 1848 combos

4) 16 stars, 7 bars — 16 songs, 7 couples, 1 minivan, 1 couple only 1 song

couple w/ fight: 0 or 1 songs

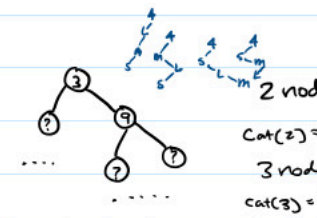
C(n+k-1, k-1)

0: 16 among 6 $C(21,5) = \frac{21!}{5!(16!)} = 20{,}349$

1: 15 among 6 $C(20,5) = \frac{20!}{5!(15!)} = 15{,}504$ +

35,853 combos

5) BST 12 nodes root v:3 Rchild(Root) v: 9



2 nodes: 2 ways → 9 longer than 3 tho so only 1 way: root + R or L kid

$Cat(2) = 4!/3!2! = \frac{4}{2} = 2$

3 nodes: 5 ways

$cat(3) = 6!/4!3! = \frac{6 \cdot 5}{3 \cdot 2} = \frac{30}{6} = 5$

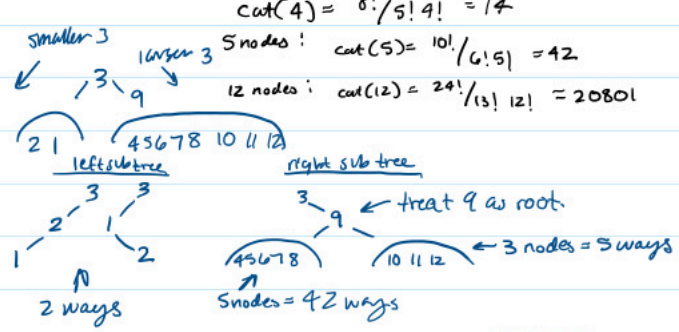
Catalan(n) = (2n)!/(n+1)!n!

4 nodes: 14 ways

$cat(4) = 8!/5!4! = 14$

5 nodes: $cat(5) = 10!/6!5! = 42$

12 nodes: $cat(12) = 24!/13!12! = 20801$



BST(2) * BST(5) * BST(3) = 420 ways

2 * 42 * 5 =

6) indist obj (friends) into indist boxes (nurses)

each nurse min. 1 friend.

4 nurses working: 9 ways

1117	1144	1333
1126	1225	2224
1135	1234	2233

3 nurses working: 8 ways

118	226
127	235
136	244
145	334

total: 17 ways

PROBABILITY PROBLEMS #5 Breeze

1) 21 students uniformly at random chosen to answer Q. Asks 13 Q's. Prob no student answers more than 1 Q?

all ways! permutation w/ repetion: 21^{13}

only 1 q each! ways to do Q w/o repetion: $P(21,13) = \frac{21!}{(21-13)!} = \frac{21!}{8!}$ ← ways to arrange 13 students from group of 21 so each only answers 1 question. (21 options first Q, 20 options second, etc)

task: we need to find the # of permutations w/o repetion. order matters w/ questions bc dif Qs

$$P(n,k) = \frac{n!}{(n-k)!}$$

Probability no student answer more than 1 Q!

$$\frac{\text{favorable events}}{\text{total events}} = \frac{21!}{8!(21^{13})} = 0.0082\ldots \quad \boxed{P = 0.82\%}$$

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2) fundamental principle of counting. all digits must be unique, must be odd

total possibilities: 00000 → 99999 = 100,000

Binomial Prob formula: $P(X=k) = \binom{n}{k} \times p^k \times (1-p)^{n-k}$

n = total tries, p = 1 num meet criteria, k = successful tries

first digit any even except 0 (2,4,6,8) → 4 choices

2nd: any unchosen even → 3 choices

3: 7 options (3,5,6,7,8,9,0) → any unchosen by 1st, 2nd or 5th

4: 6 options

5th: (1,3,5,7,9) → any odd

$4 \times 3 \times 7 \times 6 \times 5$ ← order of selection matters so 7*6 not 7 choose 2

$$P(\text{one \# meeting criteria}) = \frac{4\times3\times7\times6\times5}{100\,000} \quad \frac{\text{favorable}}{\text{total}} = \frac{2520}{100{,}000} = 0.0252 = 2.52\%$$

$P(\text{7/10 \#s meet criteria})$

$$C\binom{10}{7} \times \left(\frac{4\times3\times7\times6\times5}{100{,}000}\right)^7 \times \left(1 - \left(\frac{4\times3\times7\times6\times5}{100{,}000}\right)\right)^{10-7}$$

$120 \times (0.0252)^7 \times (0.9748)^3 = \boxed{7.17 \times 10^{-10}}$ probability of 7/10 matching criteria

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3) roll 3 six sided dice. Event A: 2 dice show 4+. Event B: all 3 dice show same val

are A & B independant?

independant if: $P(A \cap B) = P(A) \times P(B)$ (both occur)

exactly 2 dice show above 4 → $C(3,2)$

ways to show above 4 for each dice → $V\binom{i}{ii}$ → 4,5,6; $V(iii)$ → 1,2,3,4,5,6

3rd dice option shows below 4

$P(A) = C(3,2) \times \left(\frac{3}{6}\right) \times \left(\frac{3}{6}\right) \times \left(\frac{3}{6}\right) = 0.375$

$+= \left(\frac{3}{6}\right)\times\left(\frac{3}{6}\right)\times\left(\frac{3}{6}\right) = 0.125$ ← all 3 show 4+

$0.125 + 0.375 = 0.5 = 50\%$

$P(B) = \frac{\text{favored } 6}{\text{total } 6\times6\times6 = 6^3} = \frac{1}{36} = 0.0\overline{27} = 2.7\%$

6 favorable outcomes (all 1s, all 2s, etc)

3 dice

both A & B only happen if all 3 dice show all 4, or all 5, or all 6.

3 favored outcomes, of 6^3 total. $P(A \cap B) = \frac{3}{6^3} = \frac{1}{72} = 0.013\overline{8} \approx 1.38\%$

$P(A \cap B) = P(A) \times P(B)$

$\frac{1}{72} = \frac{1}{2} \times \frac{1}{36}$

$\frac{1}{72} = \frac{1}{72}$ A & B are INDEPENDANT

PROBABILITY continued

4) POKER STRAIGHTS

↳ 5 cards in sequential order
ace higher or low okay, but can't wrap around ace

- 10 possible sequences: A-2-3-4-5, ... 10, J, Q, K, A
- 4 suits for each card, so each sequence has 4^5 possibilities
- exclude straight flushes (all same suit) → 4 of for each sequence (1 per suit)

Tot St. Hands = num sequences × (combinations per seq − straight flushes)

↓

Total straight hands = $10(4^5 - 4)$

$$P = \frac{\text{tot Straight hands}}{\text{Total any hands}} = \frac{10 \times (4^5 - 4)}{C(52,5)} = \frac{10200}{2598960} = 0.0039246... = \boxed{0.392\%}$$

prob of straight hand in 1 hand draw

expected # of hands: Bernoulli (success or fail) $= \frac{1}{P} = \frac{1}{0.0039246...} = 254.803... =$ **254 expected hands to be dealt a straight**

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5) Basketball team superstar wins 75% of time when plays. When doesn't play win 40%.
5 game stretch: chance play next 5 games was 65%. Team won 3 of 5 games.
What is prob superstar played these 5 games. (plays all or none)

Bayes Theorem:

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

binomial distribution formula: $P(x=k) = \binom{n}{k} \times p^k \times (1-p)^{n-k}$

A = played = 0.65
B = team won 3 of 5

$$P(A|B) = \frac{P(B|A) \times 65\%}{P(B)}$$

$$P(B) = [P(B|A) \times P(A)] + [P(B|A') + P(A')]$$

↓ ↓

35%

$P(B|A) = C(5,3) \times 0.75^3 \times (1-0.75)^{5-3} = 0.2637 = 26.37\%$

$P(B|'A) = C(5,3) \times 0.4^3 \times (1-0.4)^{5-3} = 0.2304 = 23.04\%$

$P(B) = (0.2637 \times 0.65) + (0.2304 \times 0.35) = 0.252073 \approx 25.2\%$

$$P(A|B) = \frac{0.2637 \times 0.65}{0.252073} = 0.679981593 \approx \boxed{68\%}$$

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6) 37 red 43 gold remove all 80 (one at a time), whats P last ball was red?

prob last ball red $= \frac{37}{80} = 0.4625 = \boxed{46.25\%}$

equally likely for any ball to be last regardless of color.

so P(last red) just = P(red)