Fattening

Y complexe manifold. $\qquad X \subseteq Y$ closed submanifold

$\mathcal{J} \subseteq \mathcal{O}_Y$ ideal of functᵒ vanishing on X

$$X^{(0)} = \left(X, (\mathcal{O}_Y/\mathcal{J})_{|X}\right) \simeq (X, \mathcal{O}_X) \qquad\qquad \mathcal{O}(TY) = \mathrm{Der}(\mathcal{O}_Y, \mathcal{O}_Y)$$

$$\mathcal{J}/\mathcal{J}^2 \simeq N^*_X$$

$\uparrow$ function vanishing at order 1 on X but not more.

...

$$X^{(1)} = \left(X, (\mathcal{O}_Y/\mathcal{J}^2)_{|X}\right)$$

$$\simeq \left(X, (\mathcal{O}_Y/\mathcal{J}) \otimes_{\mathcal{O}_Y} \mathcal{O}_Y(N^*_X)_{|X}\right)$$

$$\simeq \left(X, \mathcal{O}_X(N^*X)\right)$$

$$\left(\begin{array}{ccc} \mathcal{J}/\mathcal{J}^2 \otimes N_X & \longrightarrow & \mathcal{O}_X \\ f\ ,\ v & \longmapsto & v(f) \end{array}\right) \text{ non deg.}$$

...

$$X^{(m)} \simeq \left(X, (\mathcal{O}_Y/\mathcal{J}^{m+1})_{|X}\right)$$

Cas intrinsèque

$(X, \mathcal{O})$ var. complexe.

GROSSISSEMENT D'ORDRE m DE COD. k

$$X^{(m)} = (X, \mathcal{O}_{(m)}) \quad \text{tel que} \quad \mathcal{O}_{(m)} \underset{loc}{\simeq} \frac{\mathcal{O}[\xi^1, \xi^2, \dots, \xi^k]}{(\xi^1, \xi^2, \dots, \xi^k)^{m+1}} \quad \left(=: \mathcal{O}_{m,k}\right)$$

Si $X^{(m)}$ GROSSISSEMENT $\qquad \mathcal{J}_{(m)} \subseteq \mathcal{O}_{(m)}$ idéal des nilpotents

$$\mathcal{O}_{(\ell)} = \mathcal{O}_{(m)}/\mathcal{J}_{(m)}^{\ell+1} \qquad\qquad X^{(\ell)} = (X, \mathcal{O}_{(\ell)})$$

donc $\mathcal{O}_{(m)} \twoheadrightarrow \mathcal{O}_{(\ell)} \quad \rightsquigarrow \quad X^{(\ell)} \longrightarrow X^{(m)}$

$$X^{(1)} = (X, \mathcal{O}_{(1)}) \qquad \mathcal{O}_{(1)} = \left(\mathcal{O}_{(m)}/\mathcal{J}^2_{(m)}\right) \qquad \mathcal{J}_{(1)} = \mathcal{J}_{(m)}/\mathcal{J}^2_{(m)} \subseteq \mathcal{O}_{(1)}$$

$$\mathcal{J}_{(1)} \simeq \mathcal{O}(N^*_X)$$

Car $X = X^{(0)} \subseteq X^{(1)}$ a un fibré normal