Soit $M^{\mathbb{R}}$ HK $\quad M^{\mathbb{R}} = (M^{\mathbb{R}}_{top}, g, I, J, K)$ $\quad 2n/\mathbb{C}$

$Z = Z(M^{\mathbb{R}}) \xrightarrow{f} \mathbb{P}^1$ son espace des twisteurs $\quad 2n+1/\mathbb{C}$

Soit $M^{\mathbb{C}}$ voisinage de $M^{\mathbb{R}}$ dans $\mathrm{Sec}(f) = \{\sigma : \mathbb{P}^1 \to Z \mid \text{section de } f\}$

$\to$ voisinage de $M^{\mathbb{R}}$ dans l'espace des déformations de droites twistorielles.

On sera amené à le rechercher. $\quad \dim M^{\mathbb{C}} = 4n \ /\mathbb{C}$

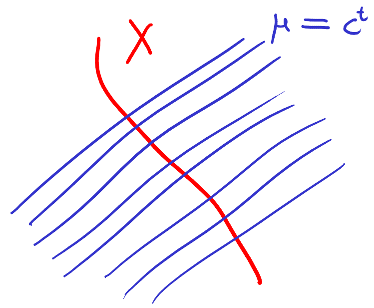
$\to$ On notera $M = M^{\mathbb{C}}$

Soit $W = \{(\sigma, p) \in M \times Z \mid \sigma(f(p)) = p\}$ $\quad \dim = 4n+1 \ /\mathbb{C}$

$$\begin{tikzcd} & & W^{4n+1} \arrow[dl, "\mu"'] \arrow[dr, "\nu"] & \\ \mathbb{P}^1 & Z^{2n+1} \arrow[l, "f"'] & & M^{4n} \end{tikzcd}$$

$X = \nu^{-1}(\sigma) = \{ p \in Z \mid \sigma(f(p)) = p\}$
$\simeq \mathbb{P}^1$

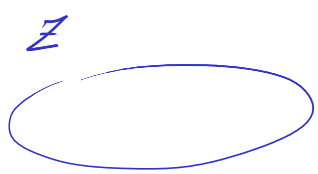
$\mu^{-1}(p) = \{\sigma \in M \mid \sigma(f(p)) = p\}$
$= \left\{ \begin{array}{l} \text{déformations passant par } p \\ \text{de la droite twist. passant par } p \end{array} \right\}$

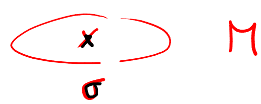


$0 \to T_{F_\mu} \to TW \to \mu^* TZ \to 0$

$0 \to \mu^*\Omega^1_Z \to \Omega^1_W \to \Omega^1_\mu \to 0$

$\boxed{\Omega^1_\mu = T^*_{F_\mu}}$





On a : $\quad 0 \to \mu^{-1}\mathcal{O}_Z \to \mathcal{O}_W \xrightarrow{d_\mu} \Omega^1_\mu \xrightarrow{d_\mu} \Omega^2_\mu \to \dots$

?

d'où en $\otimes\, \mu^{-1}\mathcal{O}_Z(TZ)$ $\qquad \mu^*\mathcal{O}_Z = \mu^{-1}\mathcal{O}_Z \otimes_{\underline{\mathbb{C}}} \mathcal{O}_W$

... ?

On a :

$$0 \longrightarrow \mu^{-1}\mathcal{O}_Z(TZ) \longrightarrow \mathcal{O}_W(\mu^*TZ) \xrightarrow{d_\mu} \Omega^1_\mu(\mu^*TZ) \xrightarrow{d_\mu} \Omega^2_\mu(\mu^*TZ) \cdots$$

résolut° acyclique !

donc
$$H^1(W, \mu^{-1}\mathcal{O}_Z(TZ)) = \frac{\ker\ \Gamma(\Omega^1_\mu(\mu^*TZ)) \xrightarrow{\Gamma d_\mu} \Gamma(\Omega^2_\mu(\mu^*TZ))}{\operatorname{Im}\ \Gamma(\mathcal{O}_W(\mu^*TZ)) \xrightarrow[\Gamma d_\mu]{} \Gamma(\Omega^1_\mu(\mu^*TZ))}$$

$$\left.\begin{array}{l} \text{or [Buchdahl]} \\ \forall p\ \ \mu^{-1}(p) \text{ simplement connexe} \end{array}\right\} \Rightarrow \quad H^1(Z, TZ) \overset{\mu^*}{\simeq} H^1(W, \mu^{-1}\mathcal{O}_Z(TZ))$$

Donc $H^1(Z,TZ)$ est la coh du complexe

$$\cdots\ \Gamma(\mathcal{O}_W(\mu^*TZ)) \longrightarrow \Gamma(\Omega^1_\mu(\mu^*TZ)) \longrightarrow \Gamma(\Omega^2_\mu(\mu^*TZ))\ \cdots$$

sur W

et donc

$$\boxed{\begin{array}{l} H^1(Z,TZ) \text{ est la coh. du complexe} \\ \cdots\ \Gamma(M, \mathcal{E}^0) \longrightarrow \Gamma(M, \mathcal{E}^1) \longrightarrow \Gamma(M, \mathcal{E}^2)\ \cdots \\ \text{sur } M \end{array}}$$

où

$$\boxed{\mathcal{E}^p = \nu_* \Omega^p_\mu(\mu^*TZ)}$$

$\sigma \in M$,

$X = \nu^{-1}(\sigma)$

$$\mathcal{E}^p_\sigma = H^0(X, \Omega^p_\mu(\mu^*TZ))$$

$$\begin{tikzcd} X \arrow[r, "j"'] \arrow[d] & W \arrow[d, "\nu"] \\ \{\sigma\} \arrow[r, hook, "i"'] & M \end{tikzcd}$$

$$\begin{aligned} \mathcal{E}^p_\sigma &= i^{-1}\nu_*\Omega^p_\mu(\mu^*TZ) \\ &= (\nu_{|X})_* j^{-1}\Omega^p_\mu(\mu^*TZ) \\ &= \Gamma(X, j^{-1}\Omega^p_\mu(\mu^*TZ)) \\ &= \Gamma(X, \Omega^p_\mu(\mu^*TZ)_{|X}) \end{aligned}$$

$$\begin{tikzcd} A \arrow[r, "j"] \arrow[d, "g"'] & B \arrow[d, "f"] \\ A' \arrow[r, "i"'] & B' \end{tikzcd} \quad \swarrow F$$

$$i^{-1}f_*F = g_*j^{-1}F \ ?$$

$$\operatorname{Hom}(i^{-1}f_*F, g_*j^{-1}G) = \operatorname{Hom}(f_*F, i_*g_*j^{-1}G)$$

$$(ig = fj) \qquad = \operatorname{Hom}(f_*F, f_*j_*j^{-1}G)$$

$$\uparrow f_*$$

$$\eta : 1 \overset{\bullet}{\longrightarrow} j_*j^{-1} \qquad \eta_*1 \in \operatorname{Hom}(F, j_*j^{-1}F)$$

En savoir plus sur d_μ et D_μ

$$\Omega^p_W = \Lambda^p \Omega^1_W \quad \text{or}$$

$$0 \to T_{F_\mu} \to TW \to \mu^* TZ \to 0$$

$$0 \to \mu^*\Omega^1_Z \to \Omega^1_W \xrightarrow{\pi} \Omega^1_\mu \to 0$$

donc

$$\begin{tikzcd} \Lambda^p\Omega^1_W \arrow[r] \arrow[d, phantom, "\shortparallel"] & \Lambda^p\Omega^1_\mu \arrow[r] \arrow[d, phantom, "!!"] & 0 \\ \Omega^p_W & \Omega^p_\mu & \end{tikzcd}$$

ou alors

$$\Omega^p_\mu = \Omega^1_\mu \wedge \Omega^{p-1}_W$$

$$\begin{tikzcd} \Omega^p_W \arrow[r, "d"] \arrow[d] & \Omega^{p+1}_W \arrow[d, "\pi"] \\ \Omega^p_\mu \arrow[r, dashed, "d_\mu"] & \Omega^{p+1}_\mu \end{tikzcd}$$

$$\pi\circ d : \Omega^p_W \longrightarrow \Omega^{p+1}_\mu \qquad \ker(\pi\circ d) = \{\gamma \in \Omega^p_W \mid d\gamma_{|F} = 0 \ \forall F \text{ fibre}\} \ ?$$

Il suffit de définir

$$d_\mu : \mathcal{O}_W \longrightarrow \Omega^1_\mu$$
$$\Omega^1_\mu \longrightarrow \Omega^2_\mu$$

$$\begin{tikzcd} \mathcal{O}_W \arrow[r, "d"] \arrow[dr, dashed, "d_\mu"'] & \Omega^1_W \arrow[d, "\pi"] \\ & \Omega^1_\mu \end{tikzcd}$$

or $0 \to \mu^{-1}\mathcal{O}_Z \to \mathcal{O}_W$

$$\begin{tikzcd} \theta_1, \theta_2 \in \Omega^1_W \arrow[r, "d"] \arrow[d] & \Lambda^2\Omega^1_W \arrow[d, "\pi"] \\ \gamma \in \Omega^1_\mu \arrow[r, dashed, "?"] & \Omega^2_\mu \end{tikzcd}$$

$$\theta_1 - \theta_2 \in \mu^*\Omega^1_Z$$

donc $d(\theta_1-\theta_2) \in \mu^*\Lambda^2\Omega^1_Z \subseteq \Lambda^2\Omega^1_W$

$$= \sum_i \mu^*\varphi_i \wedge \mu^*\psi_i \qquad \varphi_i, \psi_i \in \Omega^1_Z$$

$$\pi\left(\sum_i \mu^*\varphi_i \wedge \mu^*\psi_i\right) = 0$$

wikipedia semble être pour

$$\Omega^p_\mu = \Lambda^p \Omega^1_\mu$$

$$\Lambda^p\pi : \sum \varphi_{i_1}\wedge\cdots\wedge\varphi_{i_p} \mapsto \sum \pi\varphi_{i_1}\wedge\cdots\wedge\pi\varphi_{i_p}$$

$$\ker(\Lambda^p\pi) \subseteq \ker\pi \wedge \Omega^{p-1}_W$$

Concrètement

$\gamma \in \Omega^p_\mu$

p-forme le long des fibres de μ

→ On relève γ en une p-forme $\theta \in \Omega^p_W$

→ on différentie $d\theta \in \Omega^{p+1}_W$

→ on projette (restreint aux feuilles) $\pi d\theta \in \Omega^{p+1}_\mu$

Q : la surjectivité $\Omega^p_W \to \Omega^p_\mu$ est-elle vraie ?!

$$\Omega^p_\mu(\mu^*TZ) \xrightarrow{d_\mu} \Omega^{p+1}_\mu(\mu^*TZ) = \Omega^{p+1}_\mu \otimes_{\mathcal{O}_W} \mathcal{O}_W(\mu^*TZ)$$

$$\nu_*\Omega^p_\mu(\mu^*TZ) \xrightarrow{D_\mu} \nu_*\Omega^{p+1}_\mu(\mu^*TZ)$$

$$\begin{tikzcd} \Gamma(V,\mathcal{E}^p) \arrow[d, "D_\mu"] & = & \Gamma(\nu^{-1}(V), \Omega^p_\mu(\mu^*TZ)) \arrow[d, "d_\mu"] \\ \Gamma(V,\mathcal{E}^{p+1}) & = & \Gamma(\nu^{-1}(V), \Omega^{p+1}_\mu(\mu^*TZ)) \end{tikzcd}$$

$f \in \mathcal{O}_H(V)$, $\theta \in \mathcal{E}^p(V)$

$$\begin{aligned} D_\mu(f\theta)\circ\nu &= d_\mu((f\circ\nu)(\theta\circ\nu)) \\ &= \pi(d((f\circ\nu)(\varphi\circ\nu))) \\ &= \pi(d(f\circ\nu)\wedge\varphi\circ\nu + (f\circ\nu)d(\varphi\circ\nu)) \\ &= (f\circ\nu)\pi d(\varphi\circ\nu) + d_\mu(f\circ\nu)\wedge\pi(\varphi\circ\nu) \end{aligned}$$

$$\nabla : E \longrightarrow E\otimes\Omega^1_W$$

$$D_\mu(f\theta) = f D_\mu(\theta) + D_\mu(f)\wedge\theta \qquad \text{où } \varphi \text{ relevé de } \theta$$

$D_\mu(\theta) = 0$ ssi $d_\mu(\theta\circ\nu) = 0$

ssi $d(\varphi\circ\nu) \in \ker(\pi)$ pour φ relevé de θ

ssi $d\varphi\circ\nu\cdot d\nu \in \mu^*\Omega^2_Z$? what is going on.

ssi $(d\varphi\circ\nu\circ\mu)(d\nu\circ\mu) \in \Omega^2_Z$