

1) Comment trouver p et q tels que $p, q \in \mathbb{Z}$, $q \neq 0$, $\left| \frac{p}{q} - \sqrt{2} \right| < \varepsilon$

Poser $\begin{cases} p_0 = 1 \\ q_0 = 1 \end{cases}$

$$\begin{cases} p_{n+1} = p_n^2 + 2q_n^2 \\ q_{n+1} = 2p_n q_n \end{cases}$$

$$\frac{p_{n+1}}{q_{n+1}} = \frac{p_n}{2q_n} + \frac{q_n}{p_n}$$

Alors $\lim_{n \rightarrow +\infty} \left(\frac{p_n}{q_n} \right) = \sqrt{2}$

(ADMIS)

$$\begin{cases} p_1 = 3 \\ q_1 = 2 \end{cases}$$

$$\begin{cases} p_2 = 17 \\ q_2 = 12 \end{cases}$$

$$\frac{17}{12} - \sqrt{2} \approx 0,002...$$

2) $(x \in \phi) \Rightarrow$ "je suis le pape"

$$X = \{\text{moi}\} \cap \{\text{pape}\}$$

2 cas

- $X \neq \phi$ donc moi = pape de!
- $X = \phi$ ou $x \in \phi$ donc $x \in \{\text{moi}\} \cap \{\text{pape}\}$ et moi = x = pape oh!

3) Dessin

$$f(x, y) = \frac{xy}{x^2 + y^2}$$

