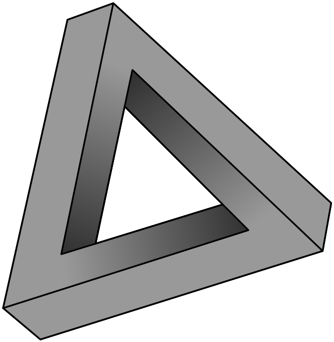
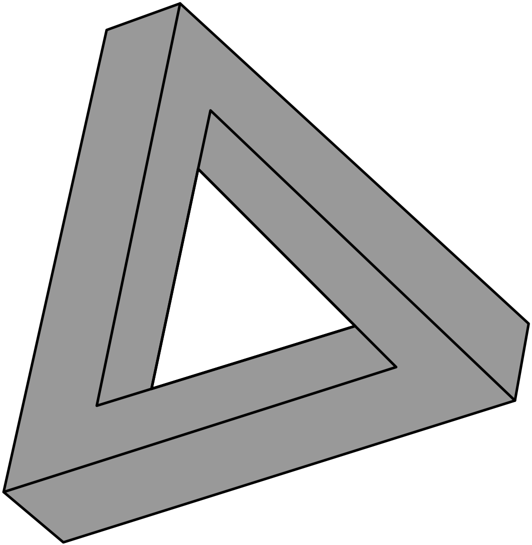
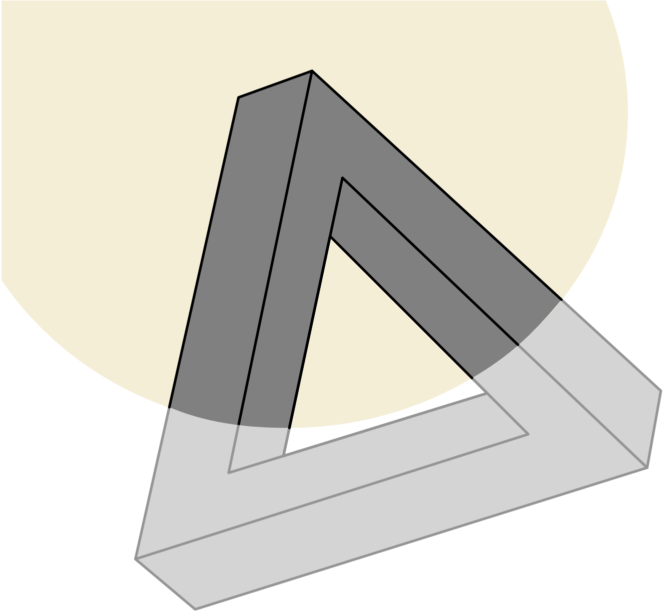
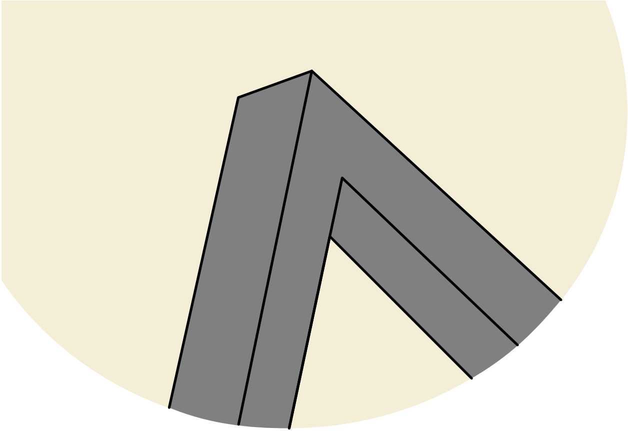
Cohomologie des figures impossibles

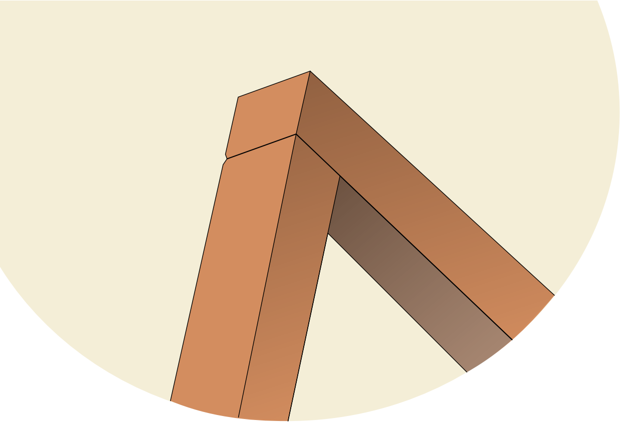
Basile Pillet

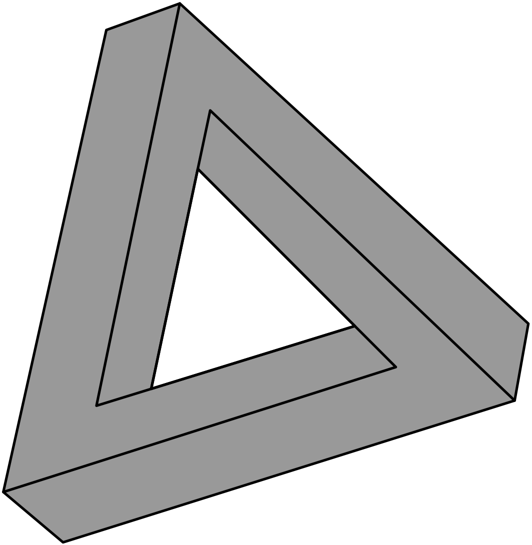


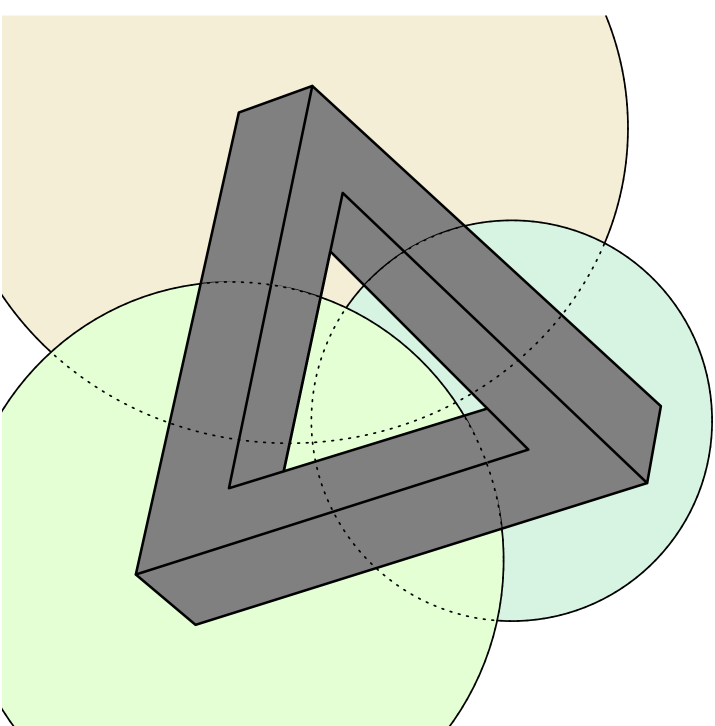


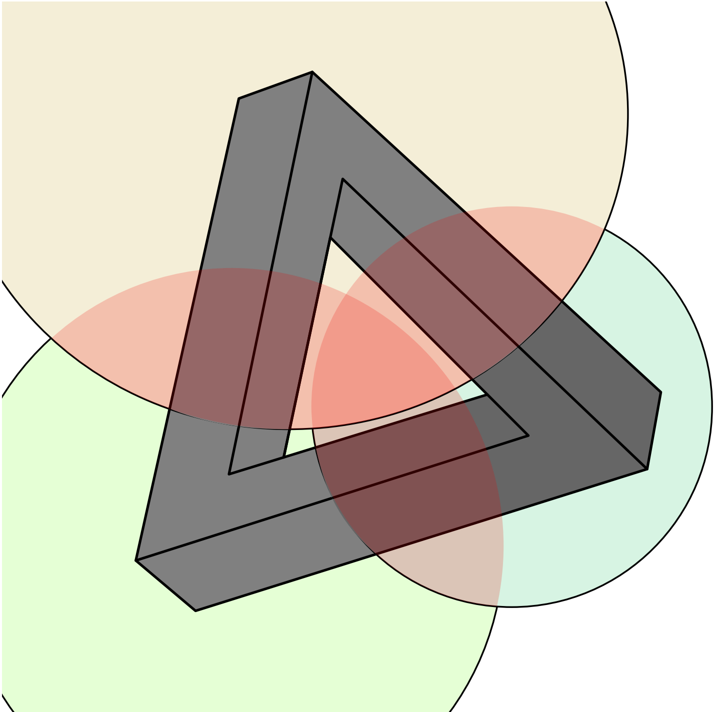


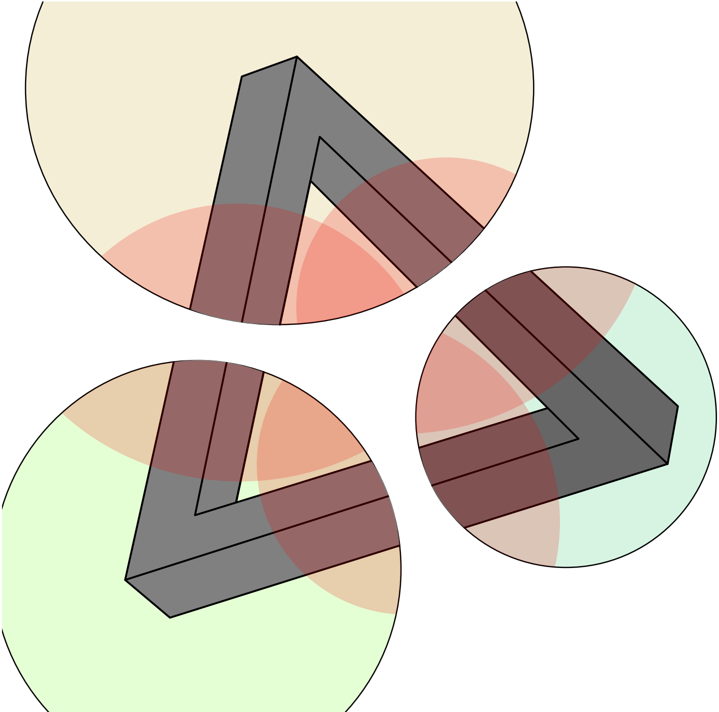


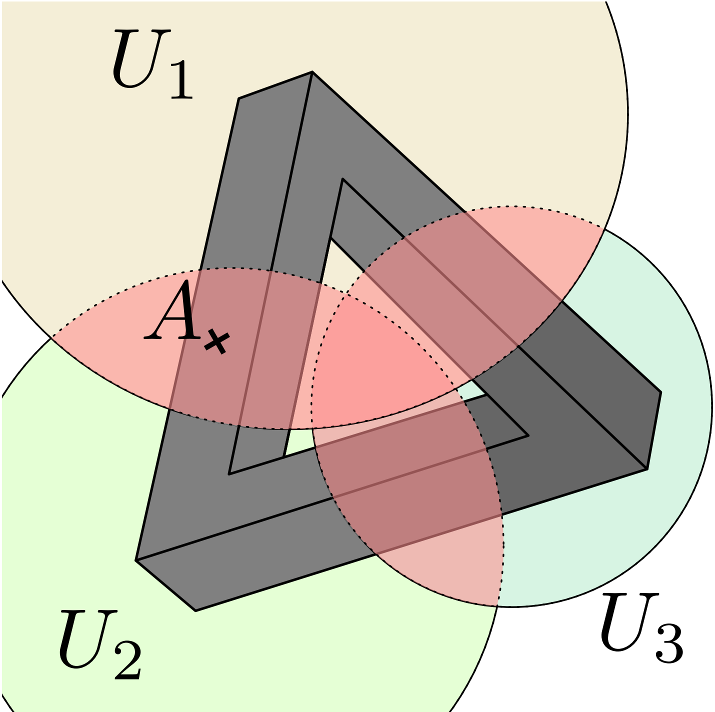


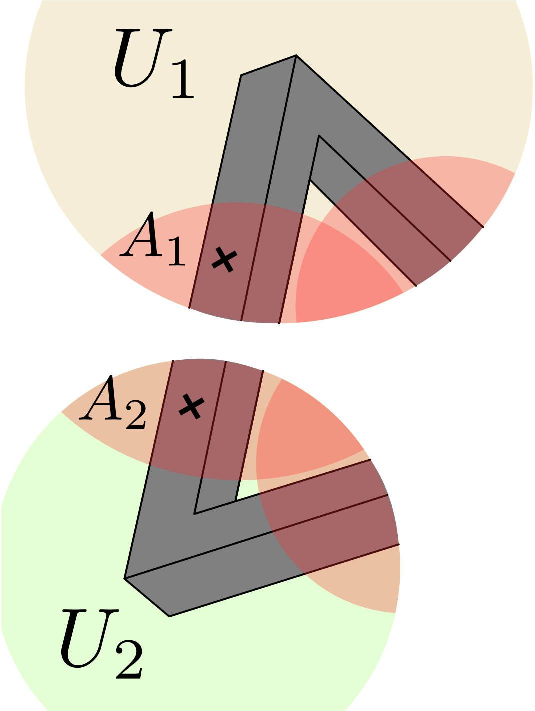




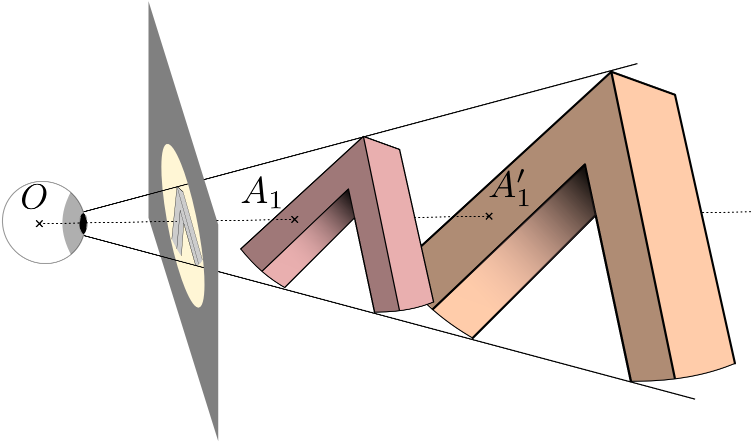




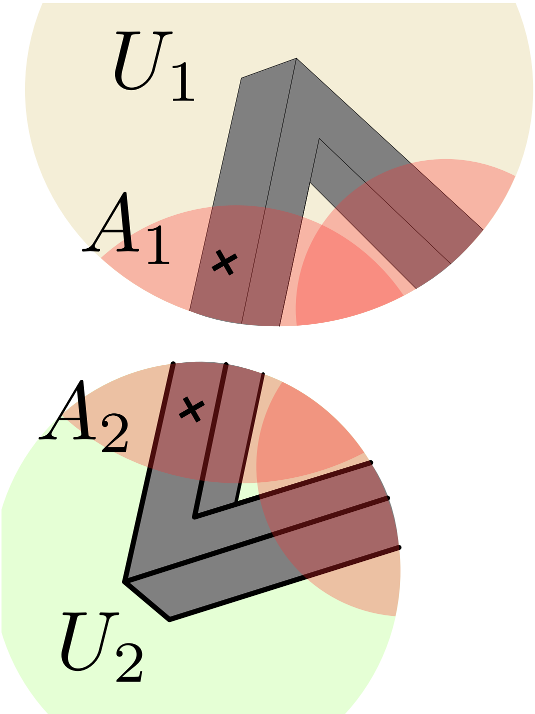

U_1
A
U_2
U_3



U_1
A_1
A_2
U_2

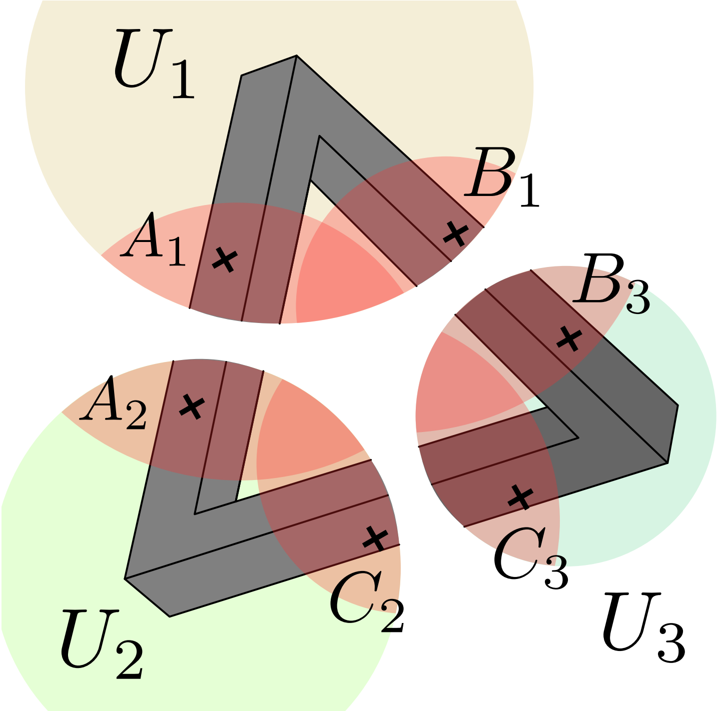


O
A_1
A_1'


U_1
A_1
A_2
U_2

$$d_{12} = \frac{\text{distance du point représenté par } A_1 \text{ à l'observateur}}{\text{distance du point représenté par } A_2 \text{ à l'observateur}}$$

$$d_{12} = \frac{OA_1}{OA_2}$$



U_1
B_1
A_1
B_3
A_2
C_3
C_2
U_3
U_2

$$d_{12} = \frac{OA_1}{OA_2}$$

$$d_{12} = \frac{OA_1}{OA_2}$$

$$d_{31} = \frac{OB_3}{OB_1}$$

$$d_{12} = \frac{OA_1}{OA_2}$$

$$d_{31} = \frac{OB_3}{OB_1}$$

$$d_{23} = \frac{OC_2}{OC_3}$$

Recollement

Pour se recoller

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il faut

- que A_1 et A_2 se superposent

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- que C_2 et C_3 se superposent

Recollement

Pour se recoller

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- que A_1 et A_2 se superposent : $d_{12} = 1$
- que B_1 et B_3 se superposent
- que C_2 et C_3 se superposent

Recollement

Pour se recoller

il faut

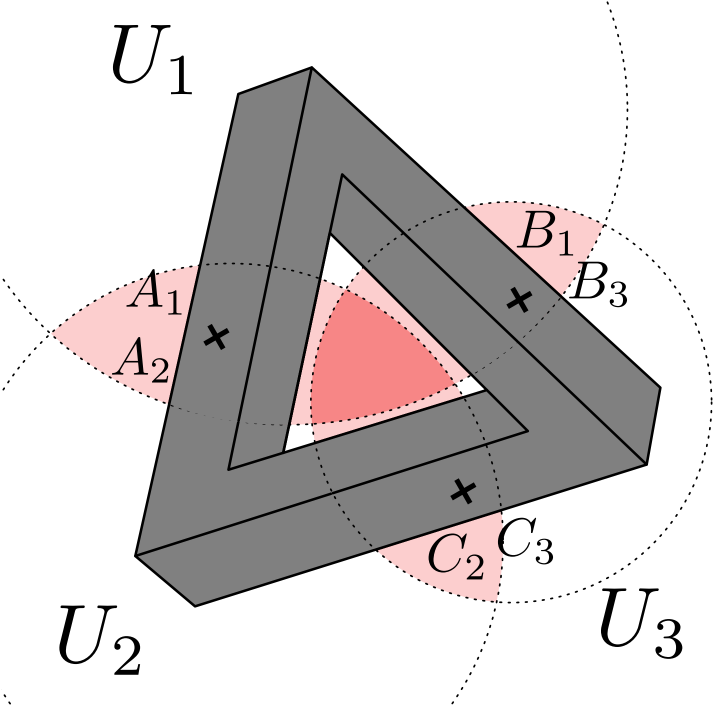
- que A_1 et A_2 se superposent : $d_{12} = 1$
- que B_1 et B_3 se superposent : $d_{31} = 1$
- que C_2 et C_3 se superposent

Recollement

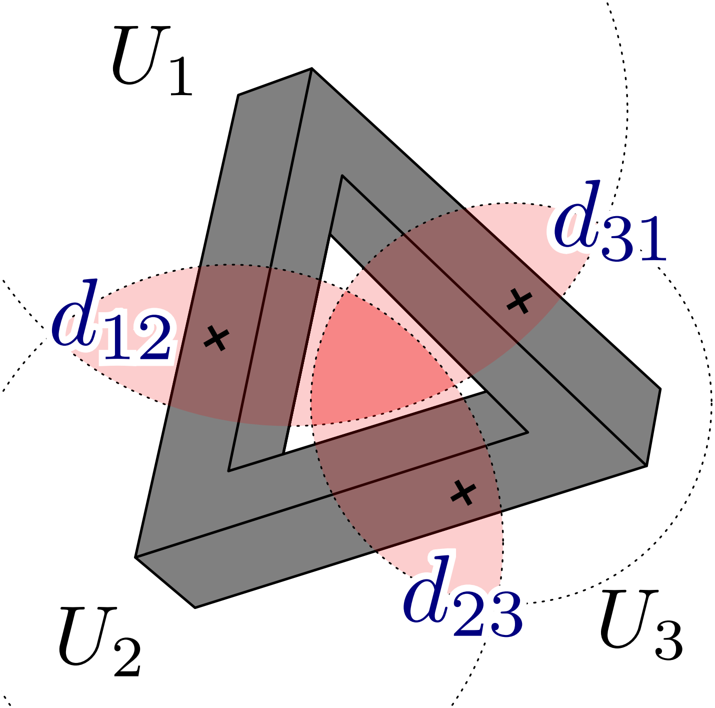
Pour se recoller

il faut

- que A_1 et A_2 se superposent : $d_{12} = 1$
- que B_1 et B_3 se superposent : $d_{31} = 1$
- que C_2 et C_3 se superposent : $d_{23} = 1$



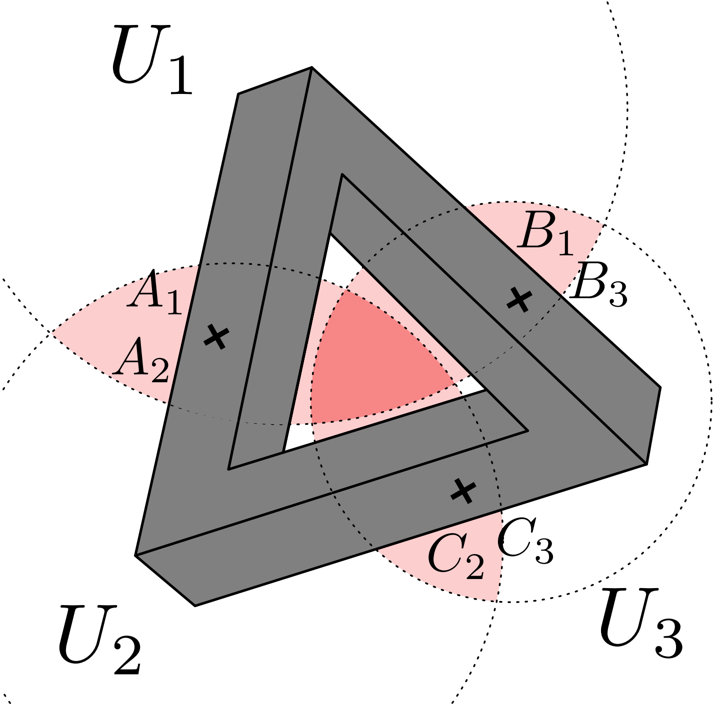
U_1
B_1
B_3
A_1
A_2
C_3
C_2
U_2
U_3



U_1
d_{31}
d_{12}
d_{23}
U_2
U_3

Les d_{ij} forment un **cocycle**.

Que ce passe-t-il si on multiplie toutes les dimensions de l'objet 1 par $\lambda_1 \in \mathbb{R}^{+*}$ ainsi que sa distance à l'observateur ?


U_1
A_1
A_2
B_1
B_3
C_2
C_3
U_2
U_3

$$d_{12} \mapsto$$

$$d_{31} \mapsto$$

$$d_{23} \mapsto$$

$$d_{12} \mapsto \lambda_1 d_{12}$$

$$d_{31} \mapsto$$

$$d_{23} \mapsto$$

$$d_{12} \mapsto \lambda_1 d_{12}$$

$$d_{31} \mapsto \frac{d_{31}}{\lambda_1}$$

$$d_{23} \mapsto$$

$$d_{12} \mapsto \lambda_1 d_{12}$$

$$d_{31} \mapsto \frac{d_{31}}{\lambda_1}$$

$$d_{23} \mapsto d_{23}$$

Recollement 2

Il existe une manière de redimensionner les trois objets telle que

$$d_{12} = d_{23} = d_{31} = 1$$

si et seulement si

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$$d_{12} = \frac{\lambda_1}{\lambda_2} \quad , \qquad d_{31} = \frac{\lambda_3}{\lambda_1} \quad , \qquad d_{23} = \frac{\lambda_2}{\lambda_3}$$

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si et seulement si

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on dit alors que les d_{ij} forment un **cobord**.

Le *triangle de Penrose* existe

ssi

les d_{ij} forment un cobord

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Si c'est le cas alors

$$d_{12} \times d_{23} \times d_{31} =$$

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Si c'est le cas alors

$$d_{12} \times d_{23} \times d_{31} = \frac{\lambda_1}{\lambda_2} \times \frac{\lambda_2}{\lambda_3} \times \frac{\lambda_3}{\lambda_1} =$$

Le *triangle de Penrose* existe

ssi

les d_{ij} forment un cobord

Si c'est le cas alors

$$d_{12} \times d_{23} \times d_{31} = \frac{\lambda_1}{\lambda_2} \times \frac{\lambda_2}{\lambda_3} \times \frac{\lambda_3}{\lambda_1} = 1$$

Contradiction

$$1 = d_{12} \times d_{23} \times d_{31}$$

Contradiction

$$\begin{aligned} 1 &= d_{12} \times d_{23} \times d_{31} \\ &= \frac{OA_1}{OA_2} \times \frac{OC_2}{OC_3} \times \frac{OB_3}{OB_1} \end{aligned}$$

Contradiction

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1 &= d_{12} \times d_{23} \times d_{31} \\
&= \frac{OA_1}{OA_2} \times \frac{OC_2}{OC_3} \times \frac{OB_3}{OB_1} \\
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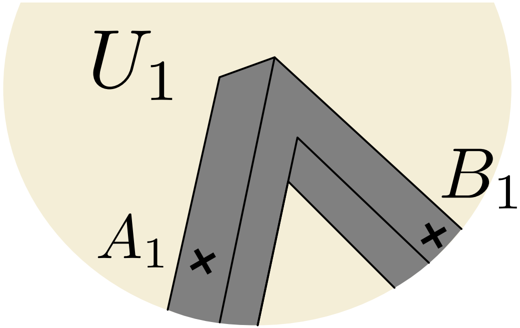
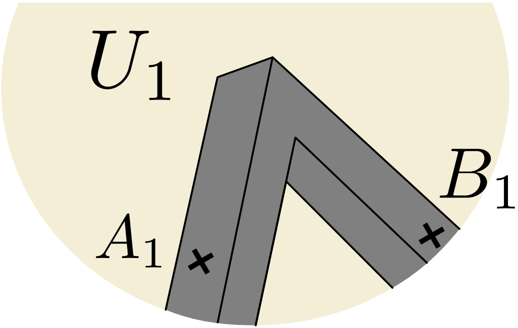
\end{aligned}$$

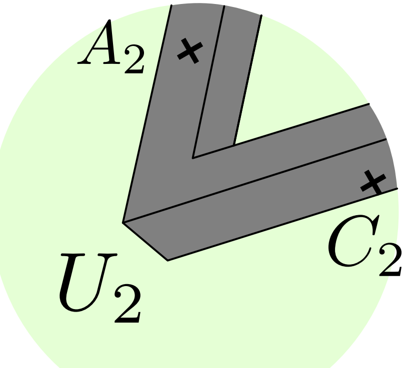

U_1
B_1
A_1



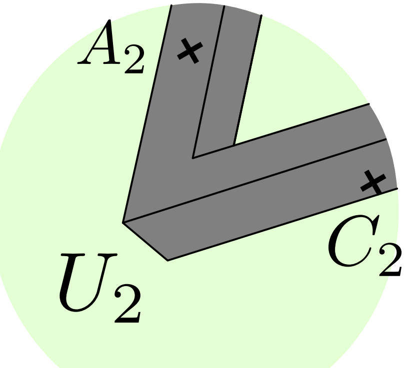
$OA_1 < OB_1$

Contradiction

$$
\begin{aligned}
1 &= d_{12} \times d_{23} \times d_{31} \\
&= \frac{OA_1}{OA_2} \times \frac{OC_2}{OC_3} \times \frac{OB_3}{OB_1} \\
&= \underbrace{\frac{OA_1}{OB_1}}_{<1} \times \frac{OC_2}{OA_2} \times \frac{OB_3}{OC_3}
\end{aligned}
$$

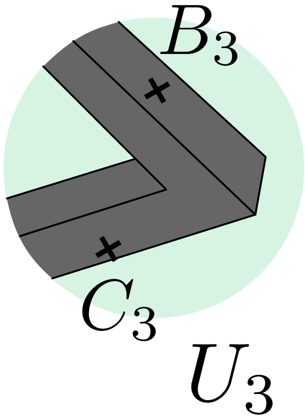

A_2
C_2
U_2

$OC_2 < OA_2$



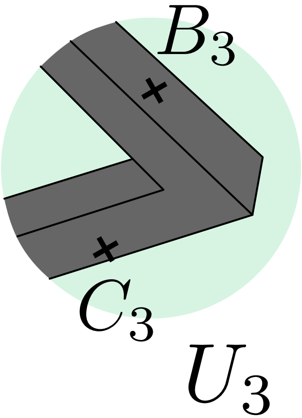
Contradiction

$$
\begin{aligned}
1 &= d_{12} \times d_{23} \times d_{31} \\
&= \frac{OA_1}{OA_2} \times \frac{OC_2}{OC_3} \times \frac{OB_3}{OB_1} \\
&= \underbrace{\frac{OA_1}{OB_1}}_{<1} \times \underbrace{\frac{OC_2}{OA_2}}_{<1} \times \frac{OB_3}{OC_3}
\end{aligned}
$$



B_3
C_3
U_3

$OB_3 < OC_3$



Contradiction

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&= \underbrace{\frac{OA_1}{OB_1}}_{<1} \times \underbrace{\frac{OC_2}{OA_2}}_{<1} \times \underbrace{\frac{OB_3}{OC_3}}_{<1}
\end{aligned}
$$

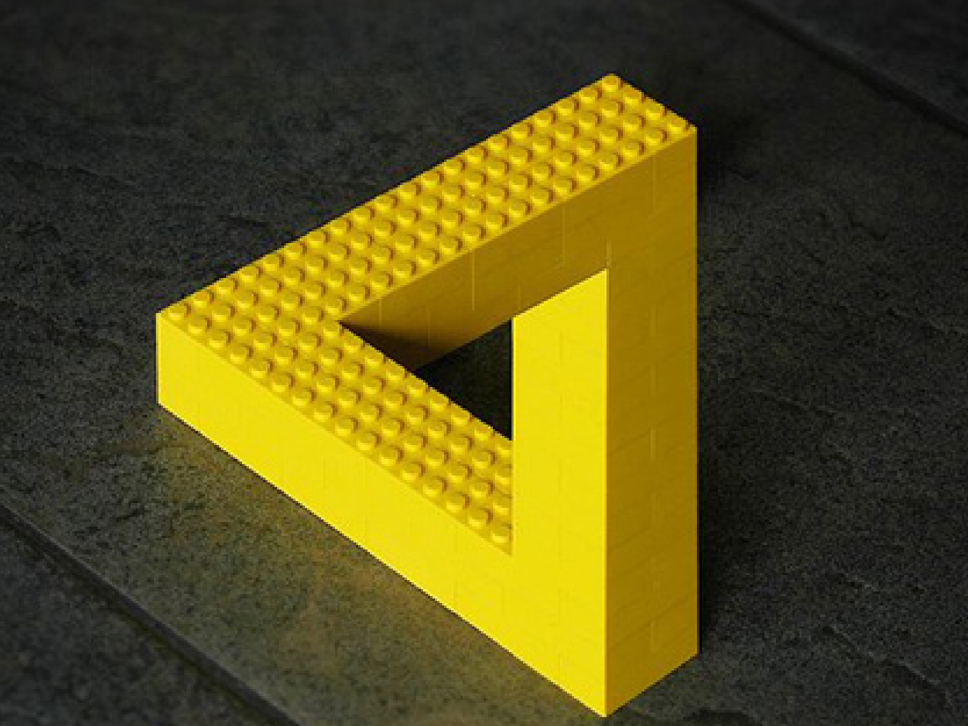
Contradiction

$$\begin{aligned}
1 &= d_{12} \times d_{23} \times d_{31} \\
&= \frac{OA_1}{OA_2} \times \frac{OC_2}{OC_3} \times \frac{OB_3}{OB_1} \\
&= \underbrace{\frac{OA_1}{OB_1}}_{<1} \times \underbrace{\frac{OC_2}{OA_2}}_{<1} \times \underbrace{\frac{OB_3}{OC_3}}_{<1} \\
&< 1
\end{aligned}$$

Contradiction

$$\begin{aligned}1 &= d_{12} \times d_{23} \times d_{31} \\ &= \frac{OA_1}{OA_2} \times \frac{OC_2}{OC_3} \times \frac{OB_3}{OB_1} \\ &= \underbrace{\frac{OA_1}{OB_1}}_{<1} \times \underbrace{\frac{OC_2}{OA_2}}_{<1} \times \underbrace{\frac{OB_3}{OC_3}}_{<1} \\ &< 1\end{aligned}$$

Le *triangle de Penrose* n'existe pas.



- Roger Penrose, *On the Cohomology of Impossible Figures.* Leonardo 25, no. 3/4 Visual Mathematics : Special Double Issue (**1992**), pp. 245-247