We can formulate the general expressions for the Bürmann series using functions $\phi(z)$ with vanishing derivatives $\phi^{(1,2,3,\dots,v)}(z_0)$. For instance, series of powers of functions of this type can give convergent expansions for functions that are defined by integrals.

Defining the integrand as the basis function of a Bürmann series as explained in (10) we will find a rapidly converging series ~~for~~ ~~the~~ representation of $\operatorname{erf}(z)$.
If we choose the basis function $\phi(z)$ to be equal to the first derivative $f'(z)$ of $f(z)$, we find, by using

$$C_n(f;\phi,z_0) = \frac{1}{n}\frac{1}{\phi'(z_0)}\frac{dC_{n-1}(f;\phi,z_0)}{dz_0}. \qquad (5)$$

The recursive expression and the first 3 coefficients.

$$C_n(f;f',z_0) = \frac{1}{n}\frac{1}{f''(z_0)}\frac{d}{dz_0}C_{n-1}(f;f',z_0).$$

$$B_1 = f'(z_0) \qquad (10).$$

$$B_2 = \frac{1}{2}\left(f''(z_0)^2 - f'(z_0)f'''(z_0)\right)/f''(z_0).$$

The idea of expanding an analytic function using its derivative as a basis function is fruitful for cases where the function is ~~def~~ defined by an integral. (It will be shown that solutions to linear and non-linear problems of diffusion or heat transfer can be expressed as integrals.) We get.

$$f(z) = \int_{z_0}^{z} f'(\zeta)\, d\zeta \rightarrow f(z) = \sum_{n=0}^{\infty} B_n(f; f', z_0)\left(\frac{f'(z)-f'(z_0)}{f''(z_0)}\right)^n$$

We will use the error function to demonstrate the efficiency of the Bürmann series using the first derivative as a basis function. We define the function $f(z)$ and the basis function $\phi(z)$ by

$$f(z) = \frac{\sqrt{\pi}}{2}\,\mathrm{erf}(z) = \int_0^z e^{-\zeta^2}\, d\zeta \rightarrow \phi(z) = f'(z) = e^{-z^2}$$

$$\phi'(z) = f''(z) = -2z\,e^{-z^2}$$

The error function will be expanded around the origin $z_0 = 0$, where we will find that $\phi'(z_0) = 0$. This expansion thus calls for the application of the generalized form of the Burmann series

$$f(z) = f(z_0) + \sum_{n=1}^{\infty}\sum_{r=0}^{n-1} \frac{f^{(r+1)}(z_0)}{r!}\,\frac{1}{n}\,R_{n-r-1}\left(\phi^*, \frac{n}{r+1}\right)\left(\sqrt[r+1]{\frac{(r+1)!\,(\phi(z)-\phi(z_0))}{\phi^{(r+1)}(z_0)}}\right)^n \qquad (14).$$

Hence we have to set, according to

$$\Theta(\phi, z_0, z) = \sqrt[\nu+1]{\phi(z) - \phi(z_0)}, \qquad (28).$$

$$\Theta(\phi, 0, z) = \sqrt{1-e^{-z^2}}$$

To evaluate (14) we use the following relations for the derivatives of the integrand:

$$\phi^{(2n)}(0) = 2(-1)^n \frac{(2n-1)!}{(n-1)!}$$

$$\phi^{2n+1}(0) = 0.$$

(32) different. The result of this calculation performed up to order

(33). $$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \operatorname{sgn} x \cdot \sqrt{1-e^{-x^2}} \left(1 - \frac{1}{12}\left(1-e^{-x^2}\right) - \frac{7}{480}\left(1-e^{-x^2}\right)^2 - \frac{5}{896}\left(1-e^{-x^2}\right)^3 - \frac{787}{276480}\left(1-e^{-x^2}\right)^4 \ldots\right.$$

~~wenfunlauted~~ To show that this approach is superior to a common Taylor expansion in a plot, we calculate the power series in z up to order 5. graph + description.

Due to the uniform convergence of (32) we can write:

$$\operatorname{erf}(z) = \frac{2 \operatorname{sgn} x}{\sqrt{\pi}} \sqrt{1-e^{-x^2}} \left(c_0 + c_1 e^{-x^2} + c_2 e^{-2x^2} + \ldots\right.$$

Using $\lim_{z\to\infty} \operatorname{erf}(z) = 1$ and $\lim_{z\to\infty} e^{-kz^2} = 0$ in (33)

we find $c_0 = \frac{\sqrt{\pi}}{2}$