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How to Model the Impact of an Interest Rate Rise On Bonds, Equities and Other Assets Using Axioma's Multi-Asset Class Tools

# Philip Jacob, Melissa Brown & Diana Rudean

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The question is, how will the increase—or similar increases by other central banks—affect portfolio holdings?

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"History doesn't repeat itself but it often rhymes." - Attributed to Mark Twain

"Prediction is very difficult, especially if it's about the future." - Niels Bohr

# **Executive Summary**

An interest-rate increase is imminent, according to U.S. Federal Reserve Chair Janet Yellen.

The question is, how will the increase—or similar increases by other central banks--affect portfolio holdings?

Can the potential impact be modelled?

Axioma believes it can, and in this paper we provide a framework for those seeking to do so.

We begin by examining how short rates drive long rates, as their impact works its way through the real economy, shaping the yield curve. We then explore the linkage between bonds at the longer end of the curve, and stocks, as well as other assets.

We note that the historical linkage between equity and bond markets has shifted dramatically over the past 10 to 15 years, leading to results that run counter to common expectations, and we discuss some theories as to what might have caused this. Correlations between certain other asset classes and long rates have also shifted.

Key to this analysis is determining which correlations to use in stress tests. We attempt to find periods in history when changes in rates were similar to what we expect in the near future.

Taking all of this into account, considering both prevailing economic conditions and a data-driven analysis of what is happening in the real economy, we constructed stress tests that provided meaningful results of the returns to a multi-asset-class portfolio—results that can be used to inform investment and risk management decisions.

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#### 1. Introduction

Global interest rates have been trending down for the past 15 years, reaching record low levels, especially in developed markets (Figure 1). While international competition may drive down long-term global inflationary pressures, interest rates typically rise in the expansion phase of the business cycle. An interest rate increase by central banks will likely result in a turning point for interest rates overall, hence the growing anxiety among investors.

The U.S. Federal Reserve has been signaling the market for some time that an interest-rate increase is imminent. Chair Janet Yellen recently indicated that the Fed is on course to increase rates at its December 15 - 16, 2015 policy meeting, depending on the performance of the U.S. economy as of the time of this paper.

This study investigates the potential outcomes of a central bank's policy action on a broad-based portfolio consisting of assets with the following allocations:

- 10% commodities
- 20% US fixed-income
- 20% Global fixed-income
- 25% US equities
- 25% Global equities

Index constituents from major indices were used to implement the portfolio. Due to the assets in the portfolio, exposures to the following risk factors are present:

- US and global sovereign curves
- Credit spreads for corporate bonds
- Equity fundamental factors<sup>1</sup>
- Commodity future curves<sup>2</sup>
- Secondary risk factors, such as interest rate implied volatilities, which impact callable bond pricing
- FX cross rates

Given the multi-asset class risk factor exposures of the portfolio, a major part of the analysis involves justifying approaches to imputing the moves to factors other than the short-end of the US yield curve. To address this issue, and design stress tests relevant to a rate-rise scenario, the analysis examines the economic models linking changes in the policy rate, which drives short-term interest rates, to the wider economy and ultimately to the long-end of the yield curve. From there we focused on determining the possible impact on equity and other asset holdings.

<sup>&</sup>lt;sup>1</sup> We represent such exposures using Axioma's Global Equity Fundamental Factor Model.

<sup>&</sup>lt;sup>2</sup> The analysis uses an approach known to modeling commodity future risk known as "Constant Maturity Futures."

The stress tests conducted for this paper were designed to model the effect of a 100-basis-points policy rate rise by the U.S. Federal Reserve on a balanced multi-asset class portfolio under different correlation assumptions. Several methods can be used to arrive at correlation assumptions:

- 1. Using the market's current correlation structure;
- 2. Using historical correlations;
- 3. Directly replaying historical scenarios; and,
- 4. Perturbing a correlation matrix to create a matrix that reflects beliefs<sup>3</sup> on what a forward-looking set of correlations should be under such a rate-rise scenario.

The difference between the second (2) and the third case (3) is subtle. In the second case, we rely on multivariate multiple regression. Let us assume that each of the core risk factors,  $F^{(i)}$ , is represented by a time series of daily historical observations,  $f_t^{(i)}$ , ...,  $f_{t-\tau}^{(i)}$ . The risk factor returns depend on the type of the factor (for example, whether it is a spread, equity fundamental factor or zero coupon yield from a sovereign yield curve). The factor returns would then be computed using an appropriate return mapping function,  $r_t^{(i)} = g(f_t^{(i)}, f_{t-1}^{(i)})$ . A vector of factor returns is then formed,  $F_t^{(i)} = [r_t^{(i)}, ..., r_{t-M}^{(i)}]$ .

If we form a matrix from the m risk factors,  $F_t = [F_t^{(1)}, F_t^{(2)}, ..., F_t^{(m)}]^T$  then, the vector of betas of the non-core risk factors,  $Y_t = [Y_t^{(1)}, Y_t^{(2)}, ..., Y_t^{(m)}]^T$ , against the core factors at time, t, is estimated as

$$B_t = (F_t' F_t)^{-1} F_t' Y_t$$

So, predicted values for the non-core risk factors in a scenario described by the core factors  $reve{F}_{t}$  are

$$\widecheck{Y} = \widecheck{F}_{\mathsf{t}} B_t$$

However in case (3), no explicit estimate of the covariance  $(F_t{}'F_t)$  is used. Rather, factor changes are estimated directly by identifying appropriate historical date ranges,  $(t_1,t_2)$  on which to compute,  $r_{tot}^{(i)} = \prod_{k=t_1}^{t_2} r_{t_k}^{(i)}$ .

Holdings are repriced in all cases by computing stressed factor levels using the risk neutral pricing models for the assets.

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<sup>&</sup>lt;sup>3</sup> These beliefs may be driven by an investment committee, a risk manager, an economist, etc.

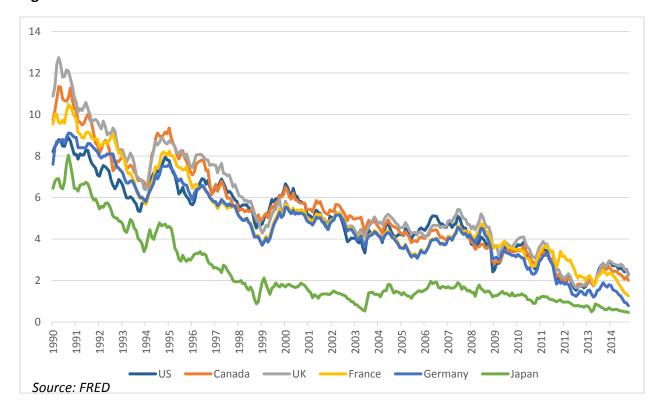


Figure 1. Historical Values of Government Bond Yields

# 2. Diverging Outcomes in Stress Tests

Identifying historical precedents to produce an accurate forecast of the future is difficult. Yet, while history may not repeat itself, it can still be a good guide for what to expect going forward.

Examining the effects of changes in interest rates on a portfolio can be done in several ways. One could use a macroeconomic model. This presents the limitation of only using the factors included in that respective model. Also, the model would not be helpful if it did not include the interest rate as set by the central bank.

Alternatively, one could use the *current* correlation structure across markets as a predictor of how the portfolio will be affected by a move in interest rates. Correlations, however, can change, especially if there are substantial moves in key economic variables. Therefore, this might not be an optimal way to find an answer to this question.

Another option would be to search for a historical period similar to the current environment—though finding a time period with all of the same market components as the period of interest is difficult at best.

Finally, one could make assumptions about future correlations by combining knowledge of history with views on the current and future environment, while also applying economic theory to calibrate inputs to the stress test—the approach used in this study.

The complexity of stress testing the impact of an interest rise on a multi-asset class portfolio comes from having to make reasonable economic assumptions, from assessing the impact across an asset book, and from choosing appropriate stress test settings.

Some of the economic considerations to be assessed are listed below:

- Determining the anticipated size of a move in a stressed variable and the stress test horizon;
- Choosing historical scenarios that are the most similar to the current environment;
- Identifying the stage of the current economic cycle;
- Analyzing current, past, and future correlations (analyze the correlation matrix);
- Differentiating between today's conditions from best-guess historical scenarios (string together non-contiguous pieces of history, perturb correlation matrix); and,
- Weighting potential outcomes.

Stress testing could be performed as stepped changes in variables, replaying a historical scenario day-by-day or month-by-month, or by changing all variables in a single shock. As expected, testing difficulty rises with the complexity of instruments in the portfolio. Additionally, the choice of mechanisms to generate simulations could have a significant impact on the test results. Ideally, multiple simulations are used to combine insights from different periods in history.

All of these inputs could have a drastic impact on the results of the test. To exemplify, the analysis included testing the impact of a 100-basis-points increase in in the yield curve (across the yield curve) on a portfolio comprising linear instruments under two scenarios: using historical correlations and volatilities versus using the current market environment. They revealed divergent outcomes, as displayed in Figure 2.

Under the historical approach, a 100-basis-points increase in interest rates increased the portfolio returns by 3.5%. When replying on current market conditions for the stress test, portfolio returns increased by only 1.7%.

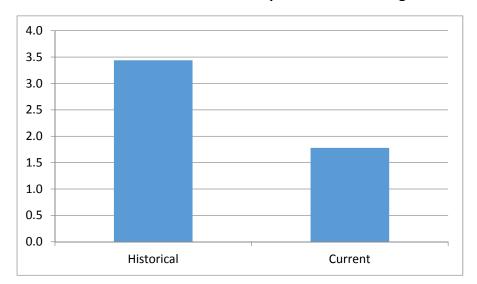


Figure 2. Different Choices for Stress Test Inputs Result in Divergent Outcomes

There is a series of steps required to ensure a reasonable and relevant outcome to a macro stress test:

Step 1. First, designing scenarios and generating views on the market requires an understanding of macroeconomic principles, particularly of the link between short- and long-term rates and how central bank actions impact the long end of the yield curve. One needs to start with a consistent and coherent theory on the shape of the yield curve and understand how monetary policy transmission operates.

Step 2. Once there is more clarity on long-term rates, one must establish the correlation between those longer-term rates and other asset classes and determine how these correlations will work their way into the portfolio.

Step 3. After establishing the scenarios one thinks will play out in case of policy rate rise, stress tests must be devised that reflect those scenarios. Then, one should examine the implications across a portfolio, evaluating the correlations across multi-asset class factors at different time horizons. One may also be interested in what underlies the changes in asset values (for example are there certain equity sectors that are likely to see a larger impact, which countries will have the biggest reaction, what are the currency impacts, etc.)

In the rest of this paper we will examine how higher short rates might shift the yield curve at the longer end, the theory behind the link between long rates and equities, how one might think about the current environment that will drive those links, how to design the appropriate stress test, and finally what some plausible outcomes might look like.

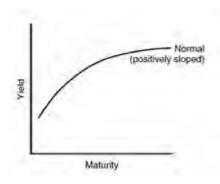
#### 3. The Yield Curve

The yield curve depicts the relationship between bond yields and maturities, and it has multiple roles in the financial industry. It acts as a reference for rate predictions uncovering investors' views on future rates and represents the most significant factor for investment grade bond pricing. The yield curve gives us a language for describing fixed-income strategies. It is typically estimated from bonds with similar risk drivers, covering many aspects of the market (e.g., sovereign yields, AA-corporates, municipal bonds).

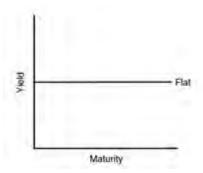
The yield curve is regarded as a leading indicator summarizing the market's expectation of the direction of interest rates and has a strong link to the wider economy. It may have various shapes (Figure 3).

Figure 3. Yield Curve Shapes: a, b, c, d.

a. Normal

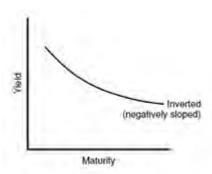


 A normal/upward sloping curve may reflect that the market expects the economy to continue growing, and the increase in interest rates that may accompany that growth.



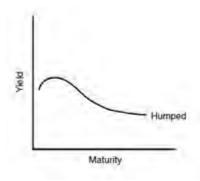
A flat curve is unusual and often can be a sign of turmoil in the market, as was the case with the US market in late 1989 and into the 1990s. One may see a flat yield curve when the central bank has raised short term rates to restrain the economy, but no parallel rise occurred at the long end of the curve, perhaps because the market was skeptical about future growth.

#### c. Inverted



The yield curve can be inverted (negatively sloped), which generally suggests the beginning of a recession. In such circumstances, the market expectation is for lower rates, in conjunction with slowing economic growth and lower inflation. This type of curve was observed in the U.S. in early 2000.

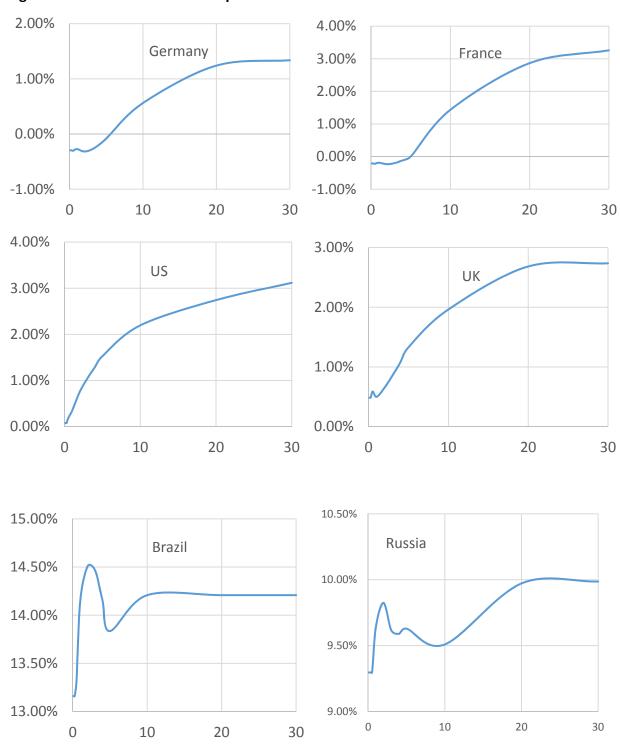
#### d. Humped



A humped shape is rare and may signal a future slowdown. The hump may represent market expectations of what will happen over the medium term (e.g., the country may default sometime in the next three to five years). This is the shape recently seen for Brazil and Russia.

Figure 4 below exhibits the various shapes of the curve observed in US, UK, Germany, France, Brazil, and Russia in September 2015.

**Figure 4. Current Yield Curve Shapes** 



The current US yield curve has a "normal" shape despite the extreme low levels of short-term interest rates. Negative rates at the short-end of the curve in the European market (see Figure 4 above) are very unusual and were only seen in deflationary Japan. The long-ends of the yield curves for France and Germany reflect the credit differences between them. Credit concerns also tend to produce mid-curve rises, as witnessed by the Russian yield curve.

Central banks have an important role in driving the shape of the yield curve. They all have similar mandates and tools to achieve their objectives. Central banks' mandates focus primarily on inflation, but they may also address concerns for stability, protection of the currency, or economic growth.

For example, central banks may target low and stable inflation or a certain level of economic growth (maximum output). Table 1 below shows the various mandates and tools used by the Fed, European Central Bank (ECB), Bank of England (BoE), and Bank of Japan (BoJ).

**Table 1. Central Banks' Mandates and Primary Tools** 

Institution	Mandate	Primary Tools	Meetings
ВоЕ	Price stability and confidence in currency; 2% inflation target	Repos, long-term refinancing operation	Monthly
ВоЈ	Price stability and stability of financial system; top focus on inflation	Open market operations (repurchase agreements)	Bi-weekly/ monthly
ECB	Price stability and growth, keeping consumer prices <2%; prevent over-appreciation of currency	Open market operations (repurchase agreements)	ECB
US Fed	Long-term price stability and sustainable growth	Open market operations (repurchase agreements)	8 meetings per year

The actions of central banks directly impact the short end of the curve. They may raise short-term interest rates if they expect increasing inflation or drive rates down if they expect deflationary pressure.

Figure 5 shows the European Central Bank policy rates between 1999 and 2015. The low levels of interest rates in the current European environment are striking.



Figure 5. European Central Bank Policy Rates (Refinance rate)

So far this paper has addressed how central banks directly impact the short-end of the curve, but the main interest is in what drives the shape of the entire curve. Known market-determining factors are already reflected in the shape of the curve. It is only when the market gets surprised that one sees a shift in the yield curve.

#### Long-term nominal rate = Long term real rates + Expected inflation

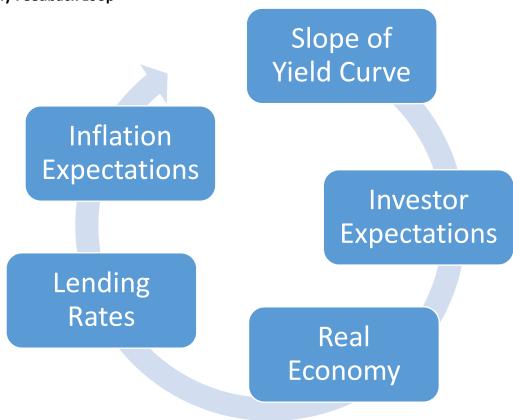
Short-term rates are indirectly linked to long-term rates. Long-term rates are influenced by the real economy, the performance of which is reflected in macroeconomic variables. The key variable is inflation expectations, which are indirectly impacted by the central bank policy action.

Several theories attempt to explain what drives the shape of the curve.

- The expectations hypothesis states that the slope of the curve reflects investor expectations for short-term rates, and long-term rates are determined by combining future expected short-term rates.
- The liquidity preference theory explains the shape of the curve by asserting that long term rates include a "term" or "liquidity" premium (i.e., investors need to be compensated for buying a longer term bond).
- The preferred habitat theory states that the shape of the curve reflects investors' distinct investment horizons. For example, the long end of the curve may be driven by pension funds that have a very long horizon, whereas the middle part of the curve may be driven by mortgage investors.

The slope of the curve affects the economy, and it is part of a feedback loop. Inflation expectations drive the slope of the curve, which in turn drives investors' expectations, leading to changes in lending rates. These changes ultimately impact the real economy, which affects inflation expectations—the variable we started with (see Figure 6). This is not to say that some of these variables will not move in other directions, but generally, this is how short-term rates get translated into longer-term rates.

Figure 6. Economy Feedback Loop



### 4. How are bonds and stocks linked?

Axioma surveyed historical correlations covering a period from the mid-1980s through to the present day. Using such correlations, the possible impact on a multi-asset class portfolio was imputed conditional on the change in interest rates, and, therefore, the study assessed what changes to the yield curve mean for other asset classes.

$$E(Y_i|X) = \mathbf{x}_i^T \hat{\beta}$$

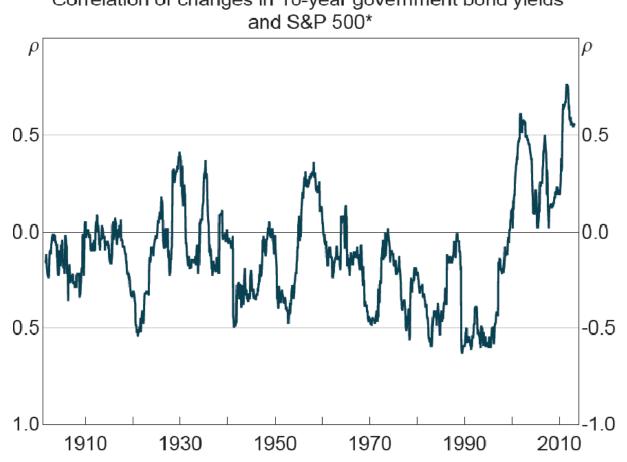
where 
$$\hat{\beta} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y}$$

History reveals that the correlation between stock returns and bond yields is not consistent over time; rather, it has varied significantly over time.

Figure 7 presents the correlations between S&P 500 returns and changes in US 10-year government bond over the past century. Throughout history, this correlation has mainly been negative—in other words, when interest rates increase, stocks go down. Since about 2000 the correlation has been positive, meaning when interest rates increased, stocks went up. Very recently correlations have been extremely high.

Figure 7. Correlation between S&P 500 and Long-Term Bond Yields

Correlation of changes in 10-year government bond yields



Source: S&P Inc.

The prior chart was based on monthly data. In Figure 8, below, we depict the relationship between equities and changes in the 10-year Constant Maturity Treasury (CMT) rate over a more recent period based on daily data. The "synchronous" line shows daily bond returns for the prior week correlated with stock returns for the same set of days. The 25-day lag shows correlations between bond returns and stock returns commencing 25 business days later. So, for example, daily bond returns in January would be correlated to stock returns in February. The "robust" line uses the same 25-day lag but adjusts for outliers, so that it dampens the

effect of correlations of unusually large market events. As the graph shows, the presence of outliers such as "Black Monday" considerably distorts correlations for a long period after the event. Use of an outlier detection algorithm as a component in a robust regression approach significantly improves the stability of the correlation estimates.

Many theories exist as to why the correlation has changed so significantly. In particular, it has been hypothesized that the aggressive actions of the Federal Reserve have played a major role in altering the relationship. We will discuss this further in the next section, after we review theory as to why stocks and bonds *should be* related.

8.0 0.6 0.4 0.2 0.0 -0.2 Synchronous — 25D lag —25D lag (robust) -0.4 Start and end of "window" -0.6 following Black Monday -0.8 1986 1988 1990 1992 1994 1996 1998 2000 2002 2004 2006 2008 2010 2012 2014

Figure 8. Correlation: Equity Index versus CMT 10Y

Source: Axioma, Inc.

An analysis of the correlation between stock returns, as represented by FTSE 100, and 10-year yields in UK shows a similar pattern. Similar to the US market, the correlation became positive after 2000 (see Figure 9).

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<sup>&</sup>lt;sup>4</sup> October 19<sup>th</sup>, 1987

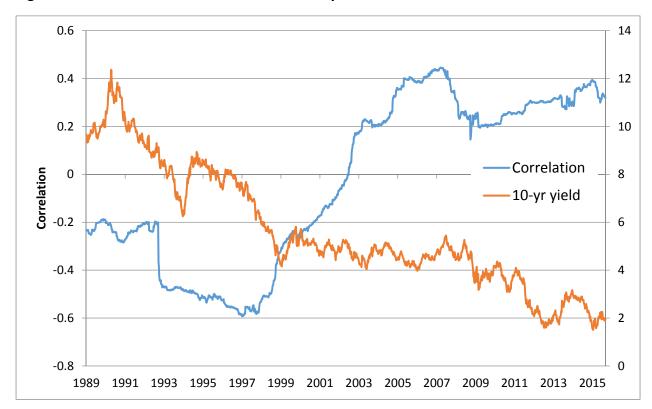


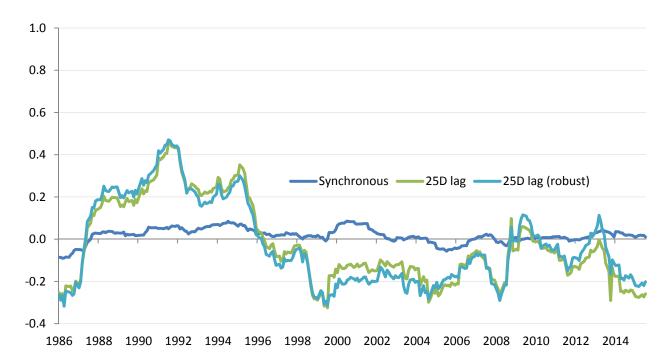
Figure 9. Correlation between FTSE 100 and 10-year CM Gilt Yields

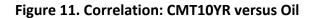
Axioma studied lagged correlations to assess whether the impact of a change in long-term rates is immediate on the equity market, or if it may take time for the impact to be seen in the system.

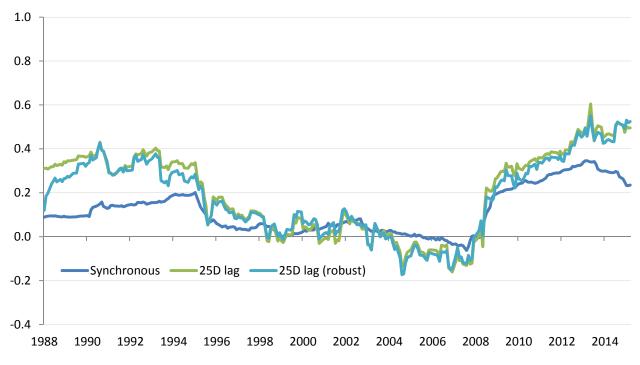
There is little correlation between changes in US 10-year government bond yields and changes in the price of gold that took place in the same period. However, the correlation of gold returns to changes in yields occurring in the prior month swung heavily from positive to negative through time (Figure 10).

Returns on oil and credit spreads are mostly positively correlated with bond yields throughout history. A huge increase in the level of correlations can be observed after the financial crisis of 2008 (Figure 11 and Figure 12).

Figure 10. Correlation: CMT10Y versus Gold







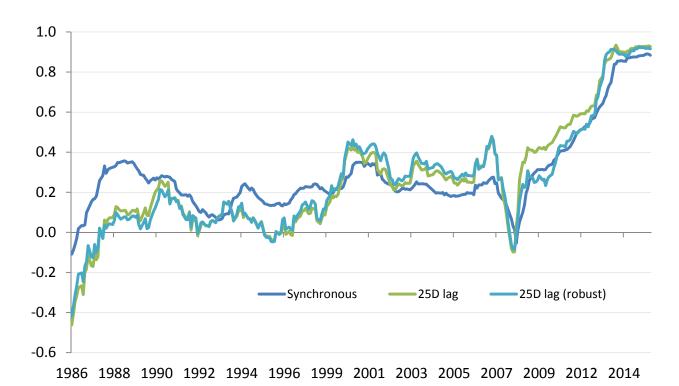


Figure 12. Correlation: CMT10Y versus Corporate Spreads

# 5. Bond Yields and Equity Market

Investors rely on a variety of models to explain equity returns. The "Fed Model" was widely used for many years and compares the earning yield on the stock market to current yields on long-term government bonds.

$$\frac{E_x}{P_x} = Y_{10}$$

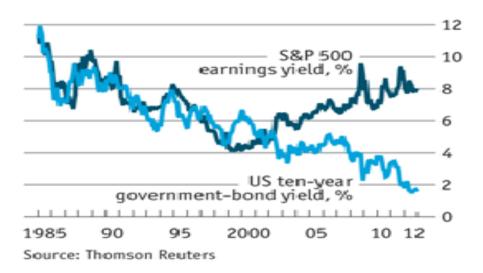
where  $E_x$  is earnings per share,  $P_x$  is share price,  $Y_{10}$  is nominal rate on 10-year Treasury bonds

This equation implies a negative correlation between treasury and equity yields, whereby higher stock prices are associated with lower yields. This relationship, however, has been broken since the Asian crisis in 2000.

Patterns exhibited by the market (see Figure 13) support the thesis adopted in this analysis, whereby the correlation between bond yields and equity returns depends on the level of the yield curve, and stage in the economic cycle.

Figure 13. Equity and Bond Valuations

#### Equity and bond valuations



The "Fed Model" has been supplanted by the Capital Structure Substitution Theory, which accounts for taxation effects. This theory states that the supply side (i.e., company management) drives the relationship between the stock market's earnings yield and the yield curve.

$$\frac{E_x}{P_x} = Y_{10}[1-T]$$

where T is the after-tax interest rate on corporate bonds.

When company management makes decisions to issue stocks or bonds, it takes into account tax effects. The stock versus bond yield relationship is no longer driven by market forces but driven internally by management. While the relationship held over the period 2000-2005, this theory also failed during the 2008 financial crisis.

So, theory says that equities should be negatively correlated with changes in interest rates. This theoretical relationship can be explained by the Fed Model, the Capital Structure Substitution Model, or a dividend discount model. However, as we have shown, correlations between US and UK stock returns and bond yields have been positive for about 15 years. The same is likely true for many other equity markets. Since the choice of correlation is a key component driving the results of our stress test, we need to understand it better.

Some observers have suggested that the level of interest rates is a key driver of this relationship. When interest rates are low, as they have been for some time now, investors are more worried about deflation than inflation. A move by the Fed to increase rates is therefore a positive signal, since the implied concern of the Fed is that inflation is heating up. If rates are high, however, the opposite is true. The market worries about inflation, and that concern is

exacerbated by the Fed's move. Others believe the shift from market to management influence, as described by the Capital Structure Substitution Theory, may also play a role in the current positive level of correlation.

Since current levels of interest rates are at or near historical low levels (even negative at the short end in some cases) we believe conclusions are speculative at best. The answer to looking for negative or positive correlations may lie in the horizon of the stress test and the expected magnitude of the shift. For short-term stress tests that look for small shifts in the yield curve using a more recent (positive) correlation may be more appropriate, but for a longer-term test and/or a large increase in rates older (negative) correlations may be a better choice.

# 6. Monetary Policy Transmission

The transmission mechanism of monetary policy is the process through which central bank decisions affect the economy. This mechanism reveals how policy-induced changes in short-term interest rates filter through the system, change the shape of the yield curve, and impact asset price levels. It is difficult to predict the precise effect of monetary policy on the market, but having a view on it is essential in designing a stress test.

#### From Policy Rates to the Yield Curve

As described earlier and shown in Figure 6, central bank interest-rate decisions directly affect short-term interest rates (e.g., mortgage rates, bank deposit rates). Simultaneously, interest-rate policy announcements drive expectations in the market regarding the future course of the economy. The central bank's actions reflect its view on the economy and drive how "popular" estimates are shaped, in turn affecting asset prices and exchange rates.

Central bank interest-rate policy has a material impact on both individual and business decision-making, changing spending, savings, and investment patterns. Therefore, changes in interest rates affect the demand in the market. High interest rates drive higher savings and strengthen the domestic currency as the economy pulls in foreign investments. Lower rates provide incentives for increased spending and investments.

As demand in the market adjusts relative to domestic supply, it influences inflation levels.

#### **How the Transmission Mechanism Operates**

The central bank determines rates in the wholesale money market. Its role is to be the supplier of "base money" (M0) (i.e., notes and coins in circulation and bank deposits). Effectively, it defines the rate for lending M0 to the private sector.

As discussed, central banks have different channels for implementing their policies. For example, the Bank of England uses the repo rates on short-dated securities, while the Fed uses the lending window.

The short end of the curve follows the policy rate closely, but the implications of the policy decisions on long-term rates are ambiguous. According to the Expectation Theory, the long end of the curve is driven primarily by the average of current and future expected short-rates.

A short-term rate rise could cause an expectation of lower future rates, which leads to a reduction in inflation expectations. Alternatively, a short-term rate rise could be an indicator that future short-term rates will continue to rise due to a growing economy.

#### Asset Pricing Response to an Increase in Interest Rates

Investment grade bonds are inversely rated to interest rates, and therefore rising policy rates lead to lower bond prices, particularly for shorter-dated bonds.

On the equities side, a simple model like the dividend discount model implies that equity prices are expected to fall as rates rise due to heavier discounting. The picture becomes more blurry, however, when one considers the effects of a policy change on market participants' confidence, the feedback loop in the economy, and interactions with other physical assets (such as house prices).

In summary, the market perception of policy rate moves influences its expectations on activity levels in the real economy. This perception changes the beliefs of financial market participants and affects their decision making.

This makes it hard to predict the impact of a change in interest rates. Let us assume that the central bank makes a policy decision to increase interest rates. Under one scenario, this rise this might indicate that the central bank believes the economy is growing faster, boosting growth expectations and investments. Under a different scenario, the rates rise might indicate that the central bank thinks the economy is running up against capacity constraints, leading to inflation. The same action of the central bank could be driven by various motivations, leading to separate market reactions and very different correlation patterns across asset classes.

The key is understanding the policy motivations and, in particular, establishing where we are in the economic cycle when the central bank actions are taking place.

#### Scale and Horizons

The timeframe over which policy effects are felt and the scale of these effects on the shape of the yield curve is a large area of economic research. The Bank of England documented their internal models used to assess the impact on the real economy of policy rate changes. Their viewpoint is that from the moment they are making a policy rate decision, acting on the short end of the curve, the flow through the broader economy takes about one year to propagate and to influence the long end of the curve.

The timeframe of propagation is important as one sets up a stress test. One can capture either the asset repricing occurring at the short end of the curve as the central bank changes rates, or the total influence of central bank's actions, which will promulgate beyond the initial change.

This paper investigates stress testing over the longer horizon in order to examine asset repricing beyond the initial move of the central bank. Another important input into the stress test is the scale of the change at the long end of the curve. The methodology employed in this study captures both short-rate and eventual long-rate changes to the curve.

The Bank of England estimates that a 1% interest-rate rise lowers gross domestic product ("GDP") output by 0.2% - 0.35% and reduces inflation expectations, which drives the long end of the curve down by 0.2% - 0.4%. While these estimates are specific to the UK economy, it is anticipated that these horizon and scale estimates are applicable internationally and are used in the stress test assumptions.

#### Where Are We in the Cycle?

Capacity indicators (e.g., employment rate, productivity) are a crucial ingredient when trying to interpret the current stage of the economy in the business cycle and, therefore, to estimate the likely size and direction of cross-asset correlations.

The Keynesian model (or Aggregate Supply-Demand model) is based on John M. Keynes' work *General Theory of Employment, Interest & Money*. It is a macroeconomic model that evaluates the factors that affect real GDP level and price level through the movements of aggregate demand and aggregate supply curves. The model explains business cycles.

The model shows that as economic "booms" progress, the marginal cost of input variables (such as labor, commodities, etc.) start rising. While the economy approaches its maximum capacity, the prices of input variables increase, and that is what generates inflation. In turn, central banks step in to increase rates and decrease inflation. This sequence of actions generates economic cycles.

One can see these cycles reflected in historical data. Figure 14 below presents Capacity Utilization, as defined by the Fed, and short-term and long-term interest rates. It is clear that economic cycles are picked up by the capacity utilization measure and by the short-term interest rate cycle. Long-term rates, however, respond far less to economic cycles, especially during recessions.

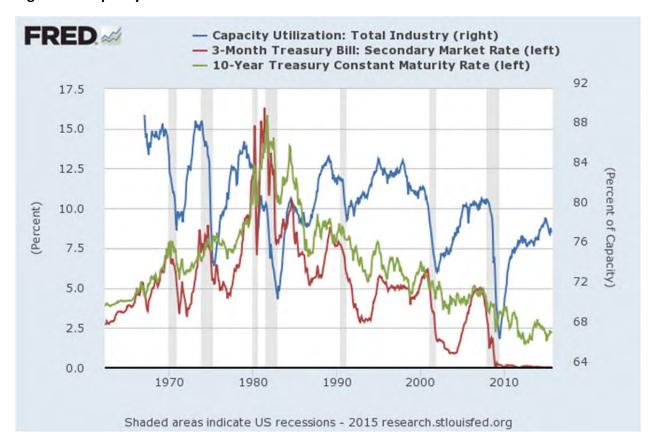


Figure 14. Capacity Utilization vs. Interest Rates

In conclusion, the link between policy actions at the short end of the curve and changes in the curve at the long end take place through the real economy.

Inflation expectations are the primary driver of long-term rates and are set by capacity constraints in the economy. Productivity and employment levels determine when capacity is reached.

The output gap is the difference between actual GDP and potential GDP. A positive output gap results in domestic inflationary pressure (unit costs growth). Economic "booms" lead to inflation, and this drives the long end of the curve.

All of this helps us estimate where we may be today with respect to the economic cycle, and from there to predict the direction in which correlations may move, and which historical periods are most relevant to use as an input to the stress test.

# 7. Implementation of Stress Tests

This section illustrates how to translate macroeconomic stresses into the "language" of Axioma's stress testing tools.

History is the first point of call. It helps define relationships between market risk factors and macroeconomic factors and provides context for the observed correlations. One must differentiate, however, between the past and the present situation.

One cannot look at history directly to determine the correlations between factors, because there is significant heterogeneity in regimes. This heterogeneity is caused by changes in the structure of the economy between crises, changes in patterns of trade, and financial innovation (and innovation in general) that lead to new factors and risks.

#### **Axioma's Tools**

Axioma's products provide support for designing stress tests to model the effect of a 100-basis-points policy-rate rise by the central bank on a balanced multi-asset class portfolio.

- The Macroeconomic Models provide an understanding of equity portfolio sensitivities to key economic variables.
- The Fixed-Income models include factors related to the yield curve and inflation and can be used for estimating risk in a fixed income portfolio.
- Axioma Risk includes a flexible and sophisticated stress-testing framework, which was
  used for this study. It supports historical mapping and asset repricing, and also handles
  aging effects and multiple other assumptions for stress testing.

Various mechanisms exist to propagate "core" factor changes to other financial factors that govern asset repricing. One could look at historical relationships or use regression techniques, which impose a particular parametric form for the relationships. These require choosing a certain period of history.

A more complex method is one where correlations are customized. Axioma tools can be used to specify stresses on core factors and adjust the correlation matrix.

#### **Customizing the Correlation Matrix**

One may take a historical view on pair-wise correlations and perturb it to incorporate beliefs of how future correlations might differ from historical ones.

The analysis begins with choosing a prior for the correlation matrix ( $\mathcal{C}_p$ ), possibly based on different frequencies for different pairs of factors. The prior may not be positive semidefinite. Axioma tools find a "nearby" matrix,  $\hat{\mathcal{C}}$ , that maintains required mathematical properties for the matrix.

As mentioned before, the user can specify particular views: the expectation of correlation between given portfolios or the expectation of correlation between factors.

Additionally, the user can specify the confidence of views:

$$\min \|\hat{\mathcal{C}} - \mathcal{C}_p\| + s^T \Sigma^{-1} s$$

$$diag(\hat{C}) = I$$

# 8. Design and Results of Stress Tests

#### **Central Scenario**

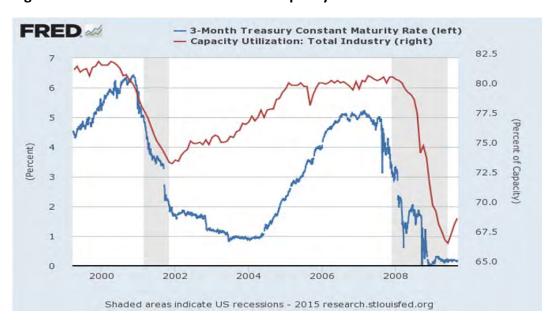
The central scenario in this study represents the scenario believed to have the highest probability and it is described below.

First, Axioma analyzed economic cycles and observed that capacity is still well below historical turning points. Therefore, the economy is in a mid-cycle rate-rise period where one expects continuing positive correlations between yields and risk-bearing assets.

One of the major challenges for stress testing an increase in interest rates is the historically abnormal low levels of current interest rates across most western economies. Looking at past history for insights might not be very useful. Markets are experiencing a globalization of low-rate regimes (exporting deflation). Additionally, increasing productivity renders capacity measurements invalid.

For the central scenario Axioma chose the period of rate rises that occurred over 2005–2006. Specifically, a window from 7 February 2005 to 8 August 2005 was selected, which exhibited an approximately 1% rise in yields (from 2.51% to 3.54%). As seen in Figure 15 below, this window covers a mid-cycle with recession starting in 2008.





The model portfolio used for stress testing is a broad multi-asset portfolio containing 10% commodities, 25% global equity, 20% global fixed income, 25% US equity, and 20% US fixed income. Axioma examined the change in the portfolio value triggered by a 1% rise in interest rates under various historical periods.

For a comparison data point, a stress test for the Lehman Brothers default period was performed in order to investigate the scale of the impact over a well-known event.

Second, the central scenario was run for the period 7 February to 8 August 2005.

As mentioned before, one needs to consider not only the market changes that occurred concurrently with the central bank actions but also the later changes following the time period necessary for the market to react to the initial central bank actions. It was assumed that the impact of a policy rate change to take 12 months and the third stress test was run covering the period between 8 August 2005 and 2 July 2006.

Stress test results were very diverse depending on the period covered. The portfolio would lose about 2.20% over the Lehman Brothers default period. This is not entirely surprising, given the relatively heavy weight in fixed income assets in the portfolio. The central scenario reflects a gain of 3.45%.

When an extra six months beyond the initial change in policy rate were added, a 4.32% increase in portfolio value was observed. Allowing for the long-term propagation effect gives a much deeper change coming from the equity portion of the portfolio, than running the stress test over the horizon of the policy actions themselves (2.8% rise in Global equities, 1.9% rise in US Equities).

The final test, which uses recent correlation data, shows that the portfolio would rise about 1.75%.

Each stress test results are presented in the figures below. Mode details can be found in Appendix A.

Figure 16. Stress Test Results

#### a. Contributional View

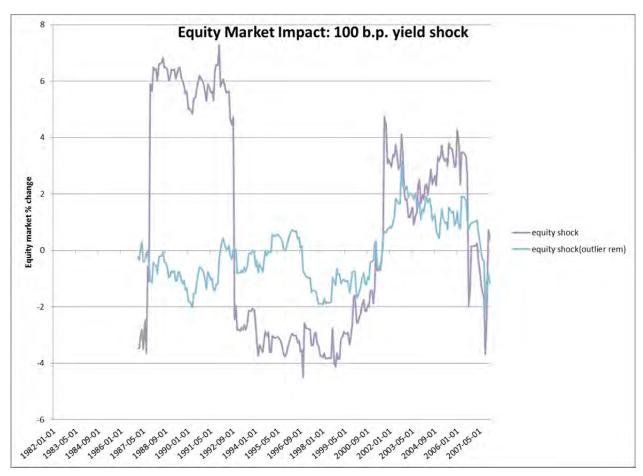
Reporting Levels	In Present Value	dil Lehman B	others Default Period	ad	In Hist 2/7/05-8/8/05	in	In Hist 8/8/05-2/7/06	and	IR 100b.p. Trans5YWeekly
	USD		USD		USD		USD		USD
▲ Fund of Fund Example Feb 13 2015	100,000,000		-2.200		3.455	i.	4.321		1.748
► Commodity Portfolio (1)	10,000,000		-0.176		1.594		0.802		1.261
▶ Global Equity Portfolio (22)	25,000,000		-1.255		1.693		2.827		0.963
▶ Global Fixed Income Portfolio (19)	20,000,000		0.026		-0.594		-1.012		-0.265
▶ US Equity Portfolio (1)	25,000,000		-0.973		1.202	1	1.910		0.467
▶ US Fixed Income Portfolio (1)	20,000,000		0.178		-0,439		-0.205		-0.678

#### b. Standalone View

Reporting Levels	III Portfolio Wgt	In Hist 2/7/05-8/8/05	In Hist 8/8/05-2/7/06	IR 100b.p. Trans5YWeekly
	USD	USD	USD	USD
▲ Fund of Fund Example Feb 13 2015	100	3.455	4.321	1.748
Commodity Portfolio (1)	10	15.936	8.019	12.611
▶ Global Equity Portfolio (22)	25	6.770	11.307	3.854
▶ Global Fixed Income Portfolio (19)	20	-2.970	-5.059	-1.327
▶ US Equity Portfolio (1)	25	4.809	7.639	1.867
▶ US Fixed Income Portfolio (1)	20	-2.195	-1.026	-3.389

Axioma also analyzed the equity market impact of a 100-basis-points yield shock, when using correlations from 1982 to 2008. The graph shows the impact on a broad-based US equity book of a shock of 100 basis points to the US yield curve. This graph highlights the criticality of choosing a representative historical time-period and of the importance of using a robust technique in computing correlations if the effect of outliers, such as "Black Monday," is not to be over emphasized (see Figure 17 below).

Figure 17. Modeling Using 1988/9 Losses



#### 9. Conclusions

Elementary finance suggests that there may often be a negative relationship between interest rate changes and stock market indices. That is, as interest rates fall (and bond prices rise), the stock market rises; and as interest rates rise (and bond prices fall), the stock market falls. This relationship has held through most of history, but over the past 15 years we have seen this relationship reverse from negative to positive.

This negative correlation could have multiple explanations. For instance, as interest rates fall, stocks become more attractive than bonds. Also, a decrease in rates has implications on the broader economy. Companies and consumers can borrow more cheaply, thus stimulating the economy, which in turn drives the stock market up.

There could also be a cut-off point for the interest rate where the bond-yield/equity-yield relationship changes. For example, if rates are lower than 5%, rising rates may increase stock prices. In this situation, rising interest rates may have a signaling effect, indicating the economy is moving out of a negative period.

To put together an effective stress test, one must have a good sense of where one is in the economic cycle. As mentioned, capacity indicators can help in making this determination. Of major importance is the current level of interest rates.

This analysis is based on the following two hypotheses:

- 1. The economy may be in a later stage of the economic cycle. The market may view an increase in rates by the central bank as a signal that the economy will approach capacity in the foreseeable future.
- 2. The economy could also be mid-cycle, where an increase in rates is perceived by the market as the central bank believing there is more growth ahead. This will eventually drive inflation, which the central bank would want to stall.

Capacity utilization is now well below the 85% levels seen in 2001. This suggests the market might be in the middle-stage of the economic cycle, due to the extended recession following the financial crisis of 2008.

Rates are currently low, indicating that we are somewhere early in the economic cycle. The middling capacity utilization values and weighting of probability lead us to the assumption that we are in the middle of the economic cycle and that the central bank believes there is more growth ahead.

This implies that a rate rise will be accompanied by further equity price rises and a continued positive correlation between rates and stocks.

The analysis presented in this paper is based on a methodology known as "historical stress testing," where market conditions, which reflect the macro environment to be reproduced, are played against a current multi-asset class portfolio.

Eventually, correlations between bond yields and equities will turn negative, and the model will need to be re-estimated using a different historical period or an alternative stress-testing methodology.

# Appendix 1

Figure A. Effect on Global Equities

Reporting Levels	III Present Value	Lehman Brothers Default Period	In Hist 2/7/05-8/8/05	In Hist 8/8/05-2/7/06	IR 100b.p. Trans3YWeekly
	USD	USD	USD	USD	USD
▲ Global Equity Portfolio (22)	25,000,000	-1.380	1.693	2.827	-3.505
AU	833,160	-0.109	0.057	0.098	-0.025
BE	120,256	-0.007	0.003	0.025	-0.023
BR	108,874	-0.009	0.030	0.056	-0.010
CA	576,491	-0.014	0.128	0.140	-0.074
CH	1,334,283	-0.066	0.201	0.187	-0.201
CN	404,804	-0.077	0.094	0.155	0.009
DE	985,712	-0.067	0.054	0.110	-0.145
DK	102,128	-0.002	0.005	0.003	-0.016
ES	318,768	-0.028	0.002	0.018	-0.053
FR	923,605	-0.037	0.119	0.081	-0.136
GB	2,834,032	-0.158	0.144	0.104	-0.357
HK	86,094	0.000	0.013	0.007	-0.002
IT	89,221	-0.010	0.016	-0.001	-0.016
JР	993,854	-0.089	-0.022	0.594	-0.153
KR	264,754	-0.031	0.033	0.083	-0.023
MX	56,523	-0.004	0.011	0.027	-0.006
NL	163,401	-0.013	0.007	0.012	-0.027
RU	81,510	-0.022	0.023	0.101	-0.019
SE	121,930	-0.005	0.014	-0.001	-0.019
TW	131,957	-0.013	0.005	0.027	-0.01
US	14,412,146	-0.610	0.738	0.978	-2.184
ZA	56,497	-0.011	0.016	0.021	-0.01

Figure B. Global Fixed-Income: Country Decomposition

Reporting Levels	III Portfolio Wgt	In Hist 2/7/05-8/8/05	In Hist 8/8/05-2/7/06	IR 100b.p. Trans5YWeekly
	USD	USD	USD	USD
▲ Fund of Fund Example Feb 13 2015	100.00	3.455	4.321	1.748
▶ Commodity Portfolio (1)	10.00	15.936	8.019	12.611
▶ Global Equity Portfolio (22)	25.00	6.770	11.307	3.854
■ Global Fixed Income Portfolio (19)	20.00	-2.970	-5.059	-1.327
AT	1.41	-3.009	-4.937	-1.531
AU	0.59	-2.699	-5.123	-3.502
BE	0.20	-3.408	-4.696	0.281
DE	1.98	-3.083	-5.353	-1.035
DK	0.80	-2.948	-5.027	-0.561
ES	0.40	-3.319	-5.719	-2.926
FI	0.40	-2.666	-5.110	-1.379
FR	4.80	-2.908	-5.140	-1.066
GB	1.01	-3.168	-4.761	0.225
IE	0.20	-3.648	-5.101	1.962
IT	0.40	-2.787	-5.273	-3.086
KR	0.20	-3.105	-5.099	-0.735
KY	0.20	-3.503	-4.391	0.580
LU	1.56	-2.858	-4.826	-1.392
NL	2.22	-3.258	-4.887	-2.615
NO	0.80	-3.256	-4.865	-0.271
SE	1.41	-2.610	-5.202	-1.312
TR	0.40	-2.585	-5.138	-0.760
US	1.00	-2.633	-5.032	-2.021
▶ US Equity Portfolio (1)	25.00	4.809	7.639	1.867
▶ US Fixed Income Portfolio (1)	20.00	-2.195	-1.026	-3.389

Figure C. Global Equity: Country Decomposition

Reporting Levels	III Portfolio Wgt	and I	n Hist 2/7/05-8/8/05	.iil	In Hist 8/8/05-2/7/06	.11	IR 100b.p. Trans5YWeekly
	USD		USD		USD		USD
▲ Fund of Fund Example Feb 13 2015	100.00		3.455		4.321		1.748
▶ Commodity Portfolio (1)	10.00		15.936		8.019		12.611
▲ Global Equity Portfolio (22)	25.00		6.770		11.307		3.854
AU	0.83		6.782		11.769		2.257
BE	0.12		2.682		20.779		5.821
BR	0.11		27.229		51.425		16.694
CA	0.58		22.232		24.340		4.919
CH	1.33		15.097		14.042		1.195
CN	0.40		23.228		38.364		5.730
DE	0.99		5.516		11.188		7.531
DK	0.10		4.525		3.297		7.701
ES	0.32		0.775		5.495		5.561
FR	0.92		12.912		8.788		10.717
GB	2.83		5.076		3.657		9.310
HK	0.09		15.184		8.300		-3.098
IT	0.09		18.138		-1.133		15.187
JP	0.99		-2.217		59.792		4.597
KR	0.26		12.571		31.279		-1.571
MX	0.06		18.778		48.032		12.509
NL	0.16		4.553		7.575		8.404
RU	0.08		28.096		124.279		13.647
SE	0.12		11.433		-0.815		6.108
TW	0.13		3.556		20.622		-0.667
US	14.41		5.124		6.783		2.080
ZA	0.06		29.088		37.049		-2.383

Figure D. Global Equity: Sector Decomposition

Reporting Levels	all Portfolio Wgt	all I	n Hist 2/7/05-8/8/05	In Hist 8/8/05-2/7/06	IR 100b.p. Trans5YWeekly
	USD		USD	USD	USD
▲ Fund of Fund Example Feb 13 2015	100.00		3.455	4.321	1.74
▶ Commodity Portfolio (2)	10.00		15.936	8.019	12.61
▲ Global Equity Portfolio (10)	25.00		6.770	11.307	3.85
▲ Consumer Discretionary (5)	2.39		2.470	6.294	1.85
Automobiles & Components	0.68		-0.533	17.788	7.85
Consumer Durables & Apparel	0.23		11.603	9.263	1.77
Consumer Services	0.23		-1.287	20.669	-1.64
Media	0.58		-2.639	0.855	1.48
Retailing	0.67		8.125	-6.692	-2.69
■ Consumer Staples (3)	3.24		4.122	3.259	1.34
Food & Staples Retailing	0.64		1.750	-3.965	0.34
Food, Beverage & Tobacco	2.00		5.689	4.865	2.02
Household & Personal Products	0.60		1.441	5.643	0.15
▶ Energy (1)	2.72		19.240	10.697	11.21
▲ Financials (4)	5.27		1.492	17.242	4.57
Banks	3.70		1.044	16.116	5.36
Diversified Financials	0.84		-1.569	19.139	2.49
Insurance	0.61		7.949	12.979	2.86
Real Estate	0.13		3.679	57.849	3.55
▶ Health Care (2)	3.35		11.980	6.256	2.79
▶ Industrials (2)	1.70		3.787	10.404	2.57
▶ Information Technology (3)	3.89		9.642	17.572	2.40
▶ Materials (1)	0.99		5.827	14.529	6.87
▶ Telecommunication Services (1)	1.20		0.230	12.788	2.23
▶ Utilities (1)	0.25		-1.603	1.406	1.07