The $S - shape$ Factor and Bond Risk Premia *

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Abstract

This paper examines the fourth principal component of the yields matrix, which is largely ignored in macro-finance forecasting applications, in the context of predicting excess bond returns. Using yields data from the Fama-Bliss and the Federal Reserve, we present the significant in-sample and out-of-sample predictive power of models including the fourth yield factor. Additionally, the "return-forecasting factor" in Cochrane and Piazzesi (2005) is shown to be a restricted linear combination of all yield factors and to be highly correlated with the second and fourth factors. We interpret the fourth yield factor as a factor representing "$S - shape$" (the shape of a sigmoid curve) and demonstrate the connection between the $S - shape$ factor and the yield curve.

Keywords: $S - shape$ factor, Yield curve, Bond risk premia, Principal component

JEL Classification: C1, E4, E5, E6, G1

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I. Introduction

There is a perception in the current literature that the first three yield factors, namely the *level*, *slope* and *curvature* factors, are enough for predictive exercises in macroeconomic and finance area. The *level* factor is approximated by the short-rate or the risk-free rate, the *slope* factor is approximated by the difference between long-term and short-term rates, and the *curvature* factor is often approximated by a butterfly spread (a mid-maturity rate minus a short- and long-rate average). The first factor, *level*, contributes most to the variation of yields and is thus non-negligible; the second factor, *slope* or *yield spread*, is proven highly significant in predictive regressions for multiple economic variables (e.g. growth rate of GDP, consumption, inflation, etc.); the third factor, *curvature* factor, has smaller variation compared to the first two factors and is insignificant in most predictive regressions. Thus, it seems natural to conclude that higher order yield factors, which have even smaller variations compared to the third factor, will not be useful for forecasting exercises.

However, this paper finds the fourth yield factor does have significant predictive power for excess bond returns. The fourth factor has a shape of S and represents how much S-shape like the yield curve is. If the shape of the yield curve becomes more S-shape like, the change would be reflected by the $S-shape$ factor. In another word, the $S-shape$ factor represents the interest change rate under median maturities is different from interest change rates under short and long maturities.

To test the predictive power of the $S-shape$ factor, we fit two nested models to data: the benchmark model includes the first four yield factors — *level*, *slope*, *curvature*, and $S-shape$ — while the baseline model only includes the first three yield factors. Comparing these two models, we can see how much more predictability the $S-shape$ factor brings for predicting excess bond return. To prove that the empirical evidence of predictability is reliable and robust, we use two data sets from the Fama-Bliss and the Federal Reserve, five sample periods which are 1964–2007, 1964–1984, 1985–2007, 1964–2002, and 1985–2012 to forecast bonds starting with four different maturates, $n = 5, 4, 3, 2$.

The in-sample and out-of-sample statistics present consistent results that the $S-shape$ factor is a significant predictor for excess bond returns: the coefficients before the $S-shape$ factors are significant under most cases evaluated by t-statistics; the predictive power stands robust for different data sets, different sample periods and bonds with different maturities; the increase of R^2 because of the additional $S-shape$ factor is notable. For example, for bonds that bought with 3 years maturates and sold with 2 years maturates, during sample period 1964-2007, the in-sample R^2 increase from 15% to 28%, which is a 87% increase; for forecasting period 1985-2007, the out-of-sample R^2 increase from 13% to 26%, which is a 100% increase.

To see how economic meaningful the $S-shape$ factor is, this paper makes use of analysis in Campbell and Thompson (2008). Campbell and Thompson (2008) calculate how much more expected return can be increased for a typical investor using forecasting variables. The typical investor is assumed to have a single-period investment horizon and mean-variance preference. Setting the risk-averse coefficient to 1, this paper finds that according to the Fama-Bliss data, the $S-shape$ factor can increase a typical investor's absolute return in the range of 5%–15%, which is 13%–48% proportionally; according to the Federal Reserve data, the $S-shape$ factor can increase the absolute return in the range of 9%–34%, which is 28%–131% proportionally.

As evidenced by empirical statistics, smaller variations of higher order yield factors do not necessarily mean that they are less useful for predictive purposes or less meaningful economically. Each yield factor contains different economic meaning, and the importance of economic implication that each factor conveys does not necessarily be proportional to its variance. The variance of the *slope* factor is close to 1% of that of the *level* factor but the *slope* factor or *yield spread* is found to be a much more significant predictor in macroeconomic forecasting applications than the *level* factor. The variation of the $S-shape$ factor is comparable to that of the *curvature* factor but this paper shows that the $S-shape$ factor has much stronger predictive power for bond risk premia than the *curvature* factor.

Yield factors are principal components of the yields matrix. Principal components of yields are derived from the variance-covariance matrix of yields through orthogonal transformation and are a set of linearly uncorrelated variables. The initial motivation of Principal Component Analysis (PCA) is to find the direction in the data with the most variation. Nowadays PCA has been applied in a variety of empirical settings. It has been developed as one popular latent variable approach or dynamic variable analysis. The benefits of PCA include that the common factors can be constructed without concerns on the structural instability of the data, and also that the extracted factors can be combined freely to construct other factors.

Research on prediction of economics variables using yields factors has been fruitful, especially using the second yield factor or the *yield spread*. For example, using the *yield spread*, Campbell and Shiller (1991), Cochrane and Piazzesi (2005) and Fama and Bliss (1987) forecast future short yields; Ang, Piazzesi, and Wei (2006) and Estrella and Hardouvelis (1991) forecast real activity; Fama (1990) and Mishkin (1990) predict inflation; and Estrella and Mishkin (1998) predict future recessions. More recently, Diebold, Rudebusch, and Aruoba (2006) provide a macroeconomic interpretation of the yield factors by combining it with VAR dynamics for the macro-economy. There are also papers that investigate time-variation in the predictive power (see Benati and Goodhart (2008)), and for instability of it (see Stock and Watson (2003a)).

A few other papers run predictive regressions or construct affine term structure models including the fourth yield factor. Cochrane and Piazzesi (2008) regress average excess returns on all yield factors including the fourth factor. Le and Singleton (2013) point out that the fourth yield factor shows substantial correlation with the forward rate factor constructed by Cochrane and Piazzesi (2005) and also with real economic growth as demonstrated in Joslin, Priebsch, and Singleton (2013). Both of these variables are known to have strong predictive power for excess bond returns. However, to the author's best knowledge, this paper is the first to focus on exploring the economic information in the fourth yield factor or the $S-shape$ factor and its predictive power for excess bond returns.

Another contribution of this paper is to connect yield factors with the "return-forecasting factor" in Cochrane and Piazzesi (2005). Cochrane and Piazzesi (2005) regress excess bond returns on yield and forward rates and obtain a return-forecasting factor which is a linear combination of yields and forward rates. Their return-forecasting factor is influential in the bond return forecasting literature and serves as a benchmark factor in return forecasting exercises. Cochrane and Piazzesi (2008) conclude that the return-forecasting factor is not spanned by the first three yield factors.

This paper derives theoretically that the return-forecasting factor is a restricted linear combination of all yield factors in the data. The connection is verified using empirical data. All yield factors derived from the Fama-Bliss data can explain 100% of the variation of the return-forecasting factor. This paper also finds that the return-forecasting factor has a high correlation with the *slope* factor and the $S-shape$ factor.

To compare the predictability of yield factors and the return-forecasting factors, this paper builds a third model which only includes the return-forecasting factor as a single predictor. As expected, both yield factors and the return-forecasting factor present excellent in-sample statistics for predicting excess bond returns. There are some advantages of using yield factors to predict economic variables instead of using the return-forecasting factor though. The first advantage is that the estimation of loadings on yield factors is much less sensitive to the range of data used compared to the estimation of coefficients of the return-forecasting factor. The second is that the estimation of yield factors faces less econometric issues such as collinearity than the return-forecasting factor would face. The third is that it is straightforward to check which yield factor captures the most predictive information for excess bond returns and then research could focus more on these factors than on others.

The rest of the paper is organized as follows. Section 2 gives a brief analysis on yield factors. Section 3 demonstrates the theoretical connection between yield factors and the return-forecasting factor and also supplies the empirical evidence on their correlation. Sections 4 and 5 present the in-sample and out-of-sample statistics for predictive models, aiming to show the significant predictive power of the $S-shape$ factor. Section 6 applies expected returns analysis used in Campbell and Thompson (2008) and calculates how much expected return increases for a typical investor because of the additional $S-shape$ factor. Section 7 concludes.

II. Functions of Yield Factors

As pointed out in the introduction, yield factors are principal components derived from the variance-covariance matrix of yields. One projects the data onto the directions of eigenvectors of the variance-covariance matrix to get principal components. The first eigenvector of the variance-covariance matrix of the data corresponds to the largest eigenvalue of it and the first principal component explains most variation of the data. The number of directions used depends on the specific situation. This section follows the notation in Piazzesi (2010). Let $\boldsymbol{Y}$ represent the $m*n$ matrix of yields with different maturities. The variance-covariance matrix of $\boldsymbol{Y}$ can be written as

$$var(\boldsymbol{Y}) = \Omega\Lambda\Omega^T ,$$

where Λ is a diagonal matrix of eigenvalues of the matrix $var(\boldsymbol{Y})$ and Ω is an orthogonal matrix whose columns are standardized eigenvectors. Principal components (PCs) can be defined by

$$PC = \Omega^T \boldsymbol{Y} \ , \tag{II.1}$$

Details on this method can be found in Kent, Bibby, and Mardia (1979).

We use two data sets on yields to verify the robustness of our analysis. The first is the Fama-Bliss monthly data consisting of 1 through 5 year zero-coupon governemtn bond prices from CRSP and the second is monthly observations of market yields on U.S. Treasury securities at 3-month, 6-month, 1-year, 3-year, 5-year and 10-year from the Federal Reserve. In this section, the only sample data range considered is 1964-2007. We will consider more subsamples in the section of forecasting excess bond returns.

A. Level, Slope, Curvature

The first three yield factors are analyzed intensively in literature. Litterman and Scheinkman (1991) are among the first to interpret the first three latent factors. They named the first three factors as "*level*", "*steepness*" and "*curvature*". These names deliver much of the intuition for what drive yields as shown in figure 1.

INSERT FIG. 1 NEAR HERE

Figure 1 plots the first four yield factors' loadings on yields since each yield factor can be denoted as a linear combination of yields. These loadings are just the eigenvectors of the variance-covariance matrix of yields— columns in Ω in equation II.1. Each yield factor's loading is a vector of coefficients of yields of different maturities. We connect these points of loadings for each factor to make a line. Figure 1 shows us the shape of the fourth yield factor is stable S-shape like using different data sets.

INSERT FIG. 2 NEAR HERE

Figure 2 plots the time series of the first four yield factors during 1964–2007 using the data from the Federal Reserve. The *level* factor is the most persistent series with an autocorrelation of 0.99. The *slope* factor is the second persistent series with an autocorrelation of 0.96. The autocorrelation for the *curvature* and $S - shape$ factors are 0.87 and 0.84, respectively. The Fama-Bliss data also reveals that the first yield factor is the most persistent among all yield factors. The autocorrelations of the first four yield factors using the Fama-Bliss data are 0.99, 0.94, 0.60 and 0.43, respectively.

INSERT FIG. 3 NEAR HERE

As shown in figure 1, the loadings of the *level* factor is flat and thus, the *level* factor roughly represents an average of yields of all maturities. The change of the *level* factor represents a parallel shift of the yield curve as indicated in figure 3(a). The loadings of the *level* factor using the Fama-Bliss data are (0.44, 0.45, 0.45, 0.45, 0.45). They are (0.41, 0.41, 0.41, 0.41, 0.41, 0.40) using the Federal Reserve data. Suppose yields of all maturities go up by 0.20%, then according to equation II.1, the value of the *level* factor would go up by (0.44+0.45+0.45+0.45+0.45)*0.20% or 0.45 according to the Fama-Bliss data and it would go up by (0.41+0.41+0.41+0.41+0.41+0.40)*0.20% or 0.49 according to the Federal Reserve data. The *level* factor goes up (down) if the overall level of yields goes up (down). Due to this fact, the *level* factor is often used as duration hedging in portfolio analysis as indicated in Litterman and Scheinkman (1991).

An important fact to note is that the parallel shifts in the yield curve do not cause other yield factors to change, which means this effect is uniquely captured by the *level* factor.

Under the same assumption that yields of all maturity go up by 0.20%, values of the *slope*, *curvature* and $S - shape$ factors would barely change. In another word, the change of the overall level of yields is uniquely represented by the *level* factor.

The *slope* factor is widely used in macroeconomic forecasting exercises. One reason for its popularity is that it can be approximated by the difference between a long rate and a short rate or *yield spread* and *yield spread* proves to be very informative about the future economy. A higher long rate than the short rate is likely to indicate a positive future economy while a higher short rate than the long rate often forecasts a recession or economic slowdown.

The *slope* factor's loadings on yields are (0.74, 0.23, −0.10, −0.35, −0.52) according to the Fama-Bliss data and are (0.48, 0.41, 0.25, −0.17, −0.38, −0.60) according to the Federal Reserve data. Its loadings are positive on yields of short maturities and are negative on yields of long maturities. Also, the absolute values of loadings are relatively big at the tail and relatively small in the middle. As shown in figure 3(b), the *slope* factor reflects difference of yields under short maturities and yields under long maturities. In another word, it reflects changes in the slope of the yield curve. If the short-term rate increases while the long-term rate does not change, the value of the *slope* factor would increase and the yield curve would appear more flat; whereas if the long term rate increase while the short term rate does not change, the value of the *slope* factor would decrease and the yield curve would appear steeper.

As mentioned in the introduction, empirical research approximates the *level* factor by the short rate or the risk free rate, the *slope* factor by the difference between the long rate and the short rate, the *curvature* factor by the butterfly spread or a mid-maturity rate minus a short- and long-rate average. The first two latent factors have high correlation with their proxies while the third latent factor has a relatively low correlation with its proxy. For example, Ang et al. (2006) find the *level* factor has a correlation of 0.96 with the short rate (three-month risk free rate) and we find that it is 0.98 using the Fama-Bliss data. The correlation between the *slope* factor and the *yield spread* is also close to 1. However, for the *curvature* factor the correlation between the latent factor and its empirical proxy is only around 0.5.

The *curvature* facto is also the least significant variable among the first three yield factors in predictive regressions. For example, Chen and Tsang (2013) show that the first three yield factors can help predict exchange rate movements, with the *slope* factor being the most robust one, but movements in the *curvature* factor have a much smaller effect on exchange rates. Litterman and Scheinkman (1991) interpret the *curvature* factor as representing how "hump-shaped" the yield curve is.

Figure 3(c) shows our interpretation on what the *curvature* factor represents. Its loadings on yields are (-0.48, 0.54, 0.46, -0.01, -0.52) using the Fama-Bliss data and are (-0.57, 0.02, 0.40, 0.50, 0.13, -0.50) using the Federal Reserve data. The factor has relatively large negative loadings at the tails and has relatively large positive loadings under median maturities. If yields at the short or long ends go up, the value of the *curvature* factor would decrease and the yield curve would become less hump-shaped. If yields under median maturities go up, the value of the *curvature* factor would increase and the yield curve would become more hump-shaped. Thus a higher value of the third principal component represents a more curvy yield curve as shown in figure 3(c) while other yield factors would not catch this effect as effectively. For example, under the same assumption, the value of the *slope* factor would not change much because the opposite signs of loadings at the short and long ends would offset the changes.

Current literature also builds connections between yield factors and macroeconomic variables. Since changes in the overall level of the yields or interest rates would change the value of the *level* factor, the *level* factor is often related to inflation or expected inflation. See

Diebold et al. (2006), Van Dijk, Koopman, Van der Wel, and Wright (2012). A higher level of inflation or expected inflation could be because the government is encouraging saving and investment and thus is increasing interest rates and a positive expected inflation rate or *level* factor is often a positive sign for future economy.

Meanwhile, literature connects the *slope* factor with real activity as its macroeconomic representation. See Estrella and Mishkin (1998), Stock and Watson (2003b) and Kurmann and Otrok (2012). One interpretation is that when central banks tighten monetary policy, the short rate increases and a recession often follows. Another interpretation is that lower long rate than short rate signals that peoples expectation on future short rates is low and the economy is likely to slow down. Hence, a flat or inverted yield curve is typically considered as a signal for an economic slowdown or a recession.

However, it is not as clear yet to see what it means for the economy when the yield curve becomes more hump-shaped. Current literature also finds it difficult to connect it with a specific macroeconomic variable. Litterman and Scheinkman (1991) find that changes in the *curvature* of the yield curve are associated with changes in yields volatility. However, as pointed out by Piazzesi (2010), stochastic volatility behaves like a *curvature* factor in some estimated models but it turns out to be so persistent that it becomes the *level* factor in others. Also, results in Litterman and Scheinkman (1991) are difficult to replicate on more recent and non-U.S. data as pointed out by Diebold and Rudebusch (2012).

B. The $S-shape$ Factor

As seen in figure 1, the fourth yield factor has a shape of S which is the reason why we name it the $S-shape$ factor. Its loadings on yields are (0.15, −0.53, 0.23, 0.64, −0.49) according to the Fama-Bliss data and are (0.40, −0.20, −0.58, 0.45, 0.32, −0.39) according to the Federal Reserve data. If we separate different maturities of yields into four different regions: short, median short, median long and long, the value of the $S - shape$ factor would go up if yields under short or median long maturities increase, or if yields under median short maturities or long maturities decrease. Also notice that the sign of loadings in neighbor regions are opposite to each other. The loadings of short maturities are positive but the loadings of median short maturities are negative; the loadings of long maturities are negative but the loadings of median long maturities are positive. If the yield curve become more S-shape like, the change would be reflected in the $S - shape$ factor.

INSERT FIG. 4 NEAR HERE

Figure 3(d) shows our understanding of what $S-shape$ factor represents. The $S-shape$ factor measures yields change rate under "median short — median long" maturities are different from yields change rates under "short — median short" and "median long — long" maturities. If yield curve changes in the direction of figure 3(d), the curve would appear more S-shape like, also the change rate under median maturities would be much higher than that under both ends. For example, as show in historical yield curve of Nov. 2007 in figure 4(b), the yield curve is flat under short and long ends but has a clear upward trend under median maturities. The S-shape character of the yield curve is also documented in previous literatures including Nelson and Siegel (1987). Meanwhile, it is important to notice that the S-shape of the yield curve can take another form as shown in figure 4(a).

The shape of the yield curve is often upward but takes more abnormal forms during recessions. It can be reverted, S-shaped, flat or displaying a mixed form. On the whole, the shape of the yield curve changes more frequently and more dramatically than what people would normally expect. As discussed in previous subsection, macroeconomic research ties the change of expected inflation to the *level* factor, the real output with business cycle to the *slope* factor, and nothing in particular to the *curvature* factor, it is still left as a question what macro variable should be tied to the $S - shape$ factor. Monetary policy seems a good candidate. However, monetary policy has no reason to just impact the $S-shape$ factor but not other yield factors. We leave this interesting question for further research.

III. Yield Factors and the "Return-Forecasting Factor"

This section demonstrates the theoretical connection between yield factors and the "return-forecasting factor" in Cochrane and Piazzesi (2005) and verifies the correlation with empirical data. We build the theoretical connection using the Fama-Bliss data since that was the data used to create the return-forecasting factor in Cochrane and Piazzesi (2005). However, the theoretical relationship derived is in fact data-free. We use both the Fama-Bliss data and the Federal Reserve data to provide empirical evidence on the correlation.

A. Notation

The analysis here uses the same notation as Cochrane and Piazzesi (2005). The notation for log bond price is

$$p_t^{(n)} = \text{log price of } n\text{-year discount bond at time } t \text{ with face value 1.}$$

The parentheses are used to distinguish maturity from exponentiation in the superscript. The log yield is

$$y_t^{(n)} \equiv -\frac{1}{n} p_t^{(n)},$$

and the log forward rate at time t for loans between time $t+n-1$ and $t+n$ is

$$f_t^{(n)} \equiv p_t^{(n-1)} - p_t^{(n)},$$

and the log holding period return from buying an n-year bond at time t and selling it as an $(n-1)$-year bond at time $t+1$ is

$$r_{t+1}^{(n)} \equiv p_{t+1}^{(n-1)} - p_t^{(n)}.$$

The excess log return is denoted by

$$rx_{t+1}^{(n)} \equiv r_{t+1}^{(n)} - y_t^{(1)}.$$

The same letters without n index are used to denote vectors across maturity, e.g.,

$$\mathbf{rx}_{t+1} \equiv \left[rx_{t+1}^{(2)} \;\; rx_{t+1}^{(3)} \;\; rx_{t+1}^{(4)} \;\; rx_{t+1}^{(5)} \right]^T.$$

When used as right hand variables, these vectors include an intercept, e.g.,

$$\mathbf{y}_t \equiv \left[1 \;\; y_t^{(1)} \;\; y_t^{(2)} \;\; y_t^{(3)} \;\; y_t^{(4)} \;\; y_t^{(5)} \right]^T,$$

$$\mathbf{f}_t \equiv \left[1 \;\; y_t^{(1)} \;\; f_t^{(2)} \;\; f_t^{(3)} \;\; f_t^{(4)} \;\; f_t^{(5)} \right]^T.$$

Over bars are used to denote averages across maturity, e.g.,

$$\overline{rx}_{t+1} \equiv \frac{1}{4} \sum_{n=2}^{5} rx_{t+1}^{(n)}.$$

With this notation, Cochrane and Piazzesi (2005) build the return-forecasting factor by running the regression

$$\frac{1}{4}\sum_{n=2}^{5} rx_{t+1}^{(n)} = \hat{\gamma}_0 + \hat{\gamma}_1 y_t^{(1)} + \hat{\gamma}_2 f_t^{(2)} + \cdots + \hat{\gamma}_5 f_t^{(5)} + \hat{\varepsilon}_{t+1},$$

or

$$\overline{\mathbf{rx}}_{t+1} = \hat{\boldsymbol{\gamma}}^T \mathbf{f}_t + \hat{\varepsilon}_{t+1}.$$

Where $\boldsymbol{\gamma}^T$ denotes a vector consisting of a constant and coefficients, $[\gamma_0\ \ \gamma_1\ \ \gamma_2\ \ \gamma_3\ \ \gamma_4\ \ \gamma_5]$, and $\hat{\varepsilon}$ is the estimated regression residuals. The estimated return-forecasting factor is $\hat{\boldsymbol{\gamma}}^T \mathbf{f}_t$.

B. Theoretical Correlation Between PCs and CP

For the rest of the paper, we denote the return-forecasting factor as CP and the yield factors or principal components of the yields matrix as PCs. This section demonstrates the theoretical correlation between CP and PCs.

Since by definition,

$$CP_t \equiv \boldsymbol{\gamma}^T \boldsymbol{f}_t = \gamma_0 + \gamma_1 y_t^{(1)} + \gamma_2 f_t^{(2)} + \cdots + \gamma_5 f_t^{(5)},$$

and $y_t^{(1)} = -p_t^{(1)}$, $f_t^{(2)} = p_t^{(1)} - p_t^{(2)}$, $f_t^{(3)} = p_t^{(2)} - p_t^{(3)}$, $f_t^{(4)} = p_t^{(3)} - p_t^{(4)}$, $f_t^{(5)} = p_t^{(4)} - p_t^{(5)}$, one can show with substitution that CP can be denoted as a linear combination of log bond prices:

$$CP_t \equiv \gamma_0 + \gamma_1(-p_t^{(1)}) + \gamma_2(p_t^{(1)} - p_t^{(2)}) + \cdots + \gamma_5(p_t^{(4)} - p_t^{(5)}).$$

After collecting terms, it becomes:

$$CP_t \equiv \gamma_0 + (-\gamma_1 + \gamma_2)p_t^{(1)} + (-\gamma_2 + \gamma_3)p_t^{(2)} + \cdots + (-\gamma_4 + \gamma_5)p_t^{(4)} + (\gamma_5)p_t^{(5)}. \qquad \text{(III.2)}$$

One can write the yield factors into the following equations according to equation II.1:

$$\begin{aligned}
PC1_t &= \alpha_{1,0} + \alpha_{1,1}y_t^{(1)} + \alpha_{1,2}y_t^{(2)} + \alpha_{1,3}y_t^{(3)} + \alpha_{1,4}y_t^{(4)} + \alpha_{1,5}y_t^{(5)},\\
PC2_t &= \alpha_{2,0} + \alpha_{2,1}y_t^{(1)} + \alpha_{2,2}y_t^{(2)} + \alpha_{2,3}y_t^{(3)} + \alpha_{2,4}y_t^{(4)} + \alpha_{2,5}y_t^{(5)},\\
PC3_t &= \alpha_{3,0} + \alpha_{3,1}y_t^{(1)} + \alpha_{3,2}y_t^{(2)} + \alpha_{3,3}y_t^{(3)} + \alpha_{3,4}y_t^{(4)} + \alpha_{3,5}y_t^{(5)},\\
PC4_t &= \alpha_{4,0} + \alpha_{4,1}y_t^{(1)} + \alpha_{4,2}y_t^{(2)} + \alpha_{4,3}y_t^{(3)} + \alpha_{4,4}y_t^{(4)} + \alpha_{4,5}y_t^{(5)},\\
PC5_t &= \alpha_{5,0} + \alpha_{5,1}y_t^{(1)} + \alpha_{5,2}y_t^{(2)} + \alpha_{5,3}y_t^{(3)} + \alpha_{5,4}y_t^{(4)} + \alpha_{5,5}y_t^{(5)},
\end{aligned}$$

where the first, second, third, fourth and fifth yield factors are denoted as $PC1$, $PC2$, $PC3$, $PC4$ and $PC5$ respectively; α represents the loadings of yield factors on yields. Since $y_t^{(1)} = -p_t^{(1)}$, $y_t^{(2)} = -\frac{1}{2}p_t^{(2)}$, $y_t^{(3)} = -\frac{1}{3}p_t^{(3)}$, $y_t^{(4)} = -\frac{1}{4}p_t^{(4)}$ and $y_t^{(5)} = -\frac{1}{5}p_t^{(5)}$, yield factors can be rewritten into following equations:

$$\begin{aligned}
PC1_t &= \alpha_{1,0} - \alpha_{1,1}p_t^{(1)} - \frac{1}{2}\alpha_{1,2}p_t^{(2)} - \frac{1}{3}\alpha_{1,3}p_t^{(3)} - \frac{1}{4}\alpha_{1,4}p_t^{(4)} - \frac{1}{5}\alpha_{1,5}p_t^{(5)}, && \text{(III.3)}\\
PC2_t &= \alpha_{2,0} - \alpha_{2,1}p_t^{(1)} - \frac{1}{2}\alpha_{2,2}p_t^{(2)} - \frac{1}{3}\alpha_{2,3}p_t^{(3)} - \frac{1}{4}\alpha_{2,4}p_t^{(4)} - \frac{1}{5}\alpha_{2,5}p_t^{(5)}, && \text{(III.4)}\\
PC3_t &= \alpha_{3,0} - \alpha_{3,1}p_t^{(1)} - \frac{1}{2}\alpha_{3,2}p_t^{(2)} - \frac{1}{3}\alpha_{3,3}p_t^{(3)} - \frac{1}{4}\alpha_{3,4}p_t^{(4)} - \frac{1}{5}\alpha_{3,5}p_t^{(5)}, && \text{(III.5)}\\
PC4_t &= \alpha_{4,0} - \alpha_{4,1}p_t^{(1)} - \frac{1}{2}\alpha_{4,2}p_t^{(2)} - \frac{1}{3}\alpha_{4,3}p_t^{(3)} - \frac{1}{4}\alpha_{4,4}p_t^{(4)} - \frac{1}{5}\alpha_{4,5}p_t^{(5)}, && \text{(III.6)}\\
PC5_t &= \alpha_{5,0} - \alpha_{5,1}p_t^{(1)} - \frac{1}{2}\alpha_{5,2}p_t^{(2)} - \frac{1}{3}\alpha_{5,3}p_t^{(3)} - \frac{1}{4}\alpha_{5,4}p_t^{(4)} - \frac{1}{5}\alpha_{5,5}p_t^{(5)}, && \text{(III.7)}
\end{aligned}$$

Equations III.2 — III.7 clearly indicate that both CP and PCs can be expressed as linear combinations of log bond prices of different maturities. Therefore, the log bond prices serve as the foundation of the theoretical connection between the return-forecasting factor and principal components.

As shown in equations III.3 — III.7, we have five left-hand side variables and five right-hand side variables. Solving them as multiple equations, we can in fact denote log bond prices p_t as linear combinations of principal components PCs. Then back to equation III.2, we can rewrite the return-forecasting factor as a linear combination of principal components:

$$CP_t = \beta_0 + \beta_1 PC1_t + \beta_2 PC2_t + \beta_3 PC3_t + \beta_4 PC4_t + \beta_5 PC5_t. \tag{III.8}$$

Thus, we have shown that the return-forecasting factor is a restricted linear combination of all yield factors in the data. Note that this analysis works with arbitrary data sets with any numbers of yield factors.

C. Empirical Evidence

This subsection supplies empirical evidence on the correlation between the return-forecasting factor and yield factors. To do this, we continue to use both the Fama-Bliss and the Federal Reserve data. The sample period in this subsection is still set to 1964-2007.

Cochrane and Piazzesi (2008) conclude that the return-forecasting factor is not spanned by the standard *level*, *slope* and *curvature* factors. Their conclusion is reasonable since the return-predicting factor is not a linear conbination of those three yield factors and those three factors do not explain all the variation of the return-forecasting factor.

However, as shown in the previous subsection, the return-forecasting factor is in fact a linear combination of all yield factors in the data if the return-forecasting factor and yield factors are derived from the same data set. Moreover, the return-forecasting factor is shown to be highly correlated with the *slope* factor and the $S-shape$ factor. According to the Fama-Bliss data, the return-forecasting factor has a correlation of -0.76 with the *slope* factor and a correlation of 0.52 with the $S - shape$ factor. Meanwhile, the correlation with the *level* factor ranks the third — 0.30, and the correlations with the *curvature* factor and the fifth factor are 0.25 and -0.05, respectively. According to the Federal Reserve data, the return-forecasting factor has a correlation of -0.74 with the *slope* factor and a correlation of 0.28 with the $S - shape$ factor. The correlation with the *level* factor is 0.24 and the correlations with the *curvature* factor, the fifth factor and the sixth factor are 0.06, 0.02 and -0.03, respectively.

INSERT TABLE. I NEAR HERE

Table I presents statistics of five univariate regressions and two multivariate regressions using the Fama-Bliss data. We regress the return-forecasting factor onto yield factors to verify the derived correlation. The regressed coefficients of yield factors are stable: the coefficients of the *level* factor are around 0.003, the coefficients of the *slope* factor are around -0.06, the coefficients of the *curvature* factor are around 0.13, and the coefficients of the $S - shape$ factor are around 0.39. $\overline{R}^2$ of the second multivariate regression which includes the first four yield factors as right-hand variables can be rounded to 100%, which means the first four yield factors explain almost all the variation of the return-forecasting factor. Univariate regressions also show that the variation of the return-forecasting factor can be explained mostly by the *slope* factor and the $S-factor$ with $\overline{R}^2$ of 57% and 27%, respectively. The *level* factor and and the *curvature* factor can explain 9% and 6% of the variation of the yield curve, respectively. The fifth principal components is insignificant in the univariate regression. Comparing $\overline{R}^2$ of two multivariate regressions in table I , we see that the $S-shape$ factor plays an important role in explaining the variation of the return-forecasting factor by adding almost 30% $\overline{R}^2$.

INSERT TABLE. II NEAR HERE

Table II presents statistics with similar implications with table I by using yield factors derived from the Federal Reserve treasury yields. The return-forecasting factor is still derived from the Fama-Bliss data. Since the return-forecasting factor and yield factors are now derived from different data sets and contain different sets of information, the $\overline{R}^2$ in the multivariate regressions can not reach 100%. However, table II verifies that the *slope* factor has the highest correlaton with the return-forecasting factor, -0.74, the $S-shape$ factor ranks the second with a correlation of 0.28, and the *level* factor ranks the third with a correlation of 0.24. Other factors do not have notable correlations with the return-forecasting factor. It is 0.06, 0.02 and -0.03 for the third, fifth and sixth factors respectively.

Given the high correlation between the return-forecasting factor and yield factors, if the return-forecasting factor has predictive power for excess bond returns, it is reasonable to infer that yield factors have predictive power too. In fact, the inference is correct and yield factors do have significant predictive power for excess bond returns. The difference lies in that the estimation of the return-forecasting factor is much less stable than the estimation of yield factors and is very data dependent. Recall that the definition of the return-forecasting factor is a linear combination of yield and forward rates. The coefficients of these rates are estimated by running a linear regression of average excess bond returns on these rates. For different sample periods, the estimated coefficients may change substantially and in fact they do. Yield factors, on the other hand, are estimated through decomposing the variance-covariance matrix of the yields and can be taken as non-stochastic. The estimated loadings of yield factors stay the same during different sample periods.

The difficulty of estimating the return-forecasting factor becomes more serious when working with large datasets that have ten or more yields with different maturities. The econometric problem of multicollinearity will show up when we include a large number of yields and forward rates on the right hand side. The estimation of coefficients would be very unstable because the right hand side variables are highly correlated. Yield factors, on the other hand, do not have to face this problem. We will also show in the following section that yield factors are robust predictors for excess bond returns across different datasets and the $S-shape$ factor adds on to the predictability significantly.

IV. Bond Returns Forecast

This section presents the in-sample and out-of-sample statistics of predictive regressions for excess bond returns. The main purpose of this section is to verify the predictive power of the $S-shape$ factor and also to compare the predictive ability between the return-forecasting factor and yield factors.

A. In-Sample Forecast

In order to compare their predictabilities, we make use of three regression models. $Model\,1$ is a baseline model that include the first three yield factors as predictors. $Model\,2$ is the benchmark model which includes predictive factors in $Model\,1$ plus the $S-shape$ factor. We can check the marginal contribution for prediction of the $S-shape$ factor by comparing these two models. $Model\,3$ just contains a single predictor — the return-forecasting factor. Section 3 has shown that using the Fama-Bliss data, the first four yield factors can explain almost all the variation in the return-forecasting factor, and by comparing $Model\,2$ and $Model\,3$, we can see how yield factors and the return-forecasting factor perform relatively in predictive regressions.

The mathematic expressions for the three predictive regressions are:

$$
\begin{aligned}
Model\,1: \; rx_{t+1}^{(n)} =& \delta_{1,0} + \delta_{1,1} PC1_t + \delta_{1,2} PC2_t + \delta_{1,3} PC3_t + \varepsilon_{t+1}^{(n)}, \\
Model\,2: \; rx_{t+1}^{(n)} =& \delta_{2,0} + \delta_{2,1} PC1_t + \delta_{2,2} PC2_t + \delta_{2,3} PC3_t + \delta_{2,4} PC4_t + \varepsilon_{t+1}^{(n)}, \\
Model\,3: \; rx_{t+1}^{(n)} =& \delta_{3,0} + \delta_{3,1} CP_t + \varepsilon_{t+1}^{(n)},
\end{aligned}
$$

Following Cochrane and Piazzesi (2005), this section presents regression results of excess log bond returns with four different maturities: $n = 2, 3, 4, 5$. $N = 2$ represents the case of buying a 2-year bond and selling it as a 1-year bond, and $n = 3$ represents the case of buying a 3-year bond and selling it as a 2-year bond. Similar meanings apply to the cases for $n = 4$ and $n = 5$. Figure 5 plots average excess bond returns $\overline{rx}$ and the lagged $S - shape$ factor derived from the Fama-Bliss data. These two time series have a correlation of 0.28. Figure 6 plots average excess bond returns $\overline{rx}$ and the lagged $S - shape$ factor derived from the Federal Reserve data. These two time series have a correlation of 0.34. Interestingly, although average excess bond returns $\overline{rx}$ is derived from the Fama-Bliss data, it has a higher correlation with the fourth yield factor derived from the Federal Reserve data than with the fourth yield factor derived from the Fama-Bliss data.

INSERT FIG. 5 NEAR HERE

INSERT FIG. 6 NEAR HERE

It is customary to check for the stability of regressors, so we include different sample periods into regressions. Cochrane and Piazzesi (2005) use the sample data of 1964–2002. We consider this sample period for comparison and also divide the whole sample period from 1964 into some other subsample periods for different considerations. The great recession post–2007 may have changed many economic variables' usual meanings. Many financial ratios such as dividend price ratio lose their usual predictive power during this financial crisis period. The benchmark sample period is set to 1964–2007 to avoid the abnormal impact of financial crisis. Another potential interesting subsample is 1985–2007 since the period of 1985–2007 is called "Great Moderation". The subsample period pre–1985 is also considered. In addition, we present statistics during the period of 1985–2012 to show the impact of the financial crisis. We follow previous research's conventions to assume that the return-forecasting factor, excess returns and yield factors are stationary.

For regressions including yield factors as regressors, we apply the Ordinary Least Squares (OLS) method to estimate the coefficients and the method in Newey and West (1994) to fix the standard error estimation problem caused by heteroskedasticity and autocorrelation among errors. Newey and West (1994) use bartlett kernel and automaticly choose the bandwidth considering correlations among errors. For regressions including the return-forecasting factor as a singular variable, we use the two-step OLS to estimate the coefficients and the method in Hansen (1982) to fix the standard error estimation problem caused by generated regressors.

INSERT TABLE III NEAR HERE

The stability of estimated coefficients of predictors reflects predictors' predictive ability. The $S-shape$ factor and the return-forecasting factor have relatively more stable estimated coefficients than the *curvature* factor. From table III we can see that there is a sign change for the coefficients of the *curvature* factor from 1964–1984 to 1985–2007. For bonds with different maturities, the *curvature* and the $S-shape$ factor have more predictive power for excess bond returns during pre-"Great Moderation" period than during "Great Moderation" period. The return-forecasting factor appears to be a significant predictor during post-1985 periods for bond with different maturities. During pre–1985 period, the return-forecasting factor is a significant predictor for bonds with maturities $n = 4$ and $n = 5$, but not for bonds with maturities $n = 2$ and $n = 3$.

Across different data sets, the $S - shape$ factor proves to be a much stronger predictor than the *curvature* factor. Yield factors derived from the Fama-Bliss data appear more significant in predictive regressions than yield factors derived from the Federal Reserve data, which is within the expectation, since the dependent variables in the regressions — excess bond returns are derived from the Fama-Bliss data.

INSERT TABLE IV NEAR HERE

Table IV compares performances of models in term of the in-sample $\overline{R}^2$ and shows what is the marginal gain in the $\overline{R}^2$ with an additional predictor: the $S - shape$ factor. $Model\,2$ with the $S - shape$ factor performs better than $model\,1$ in all cases: the $S - shape$ factor significantly increases the $\overline{R}^2$. For example, for bonds with $n = 3$, during period 1964–2007, according to the Fama-Bliss data, the $S - shape$ factor increases the in-sample $\overline{R}^2$ from 18% to 27%, which is a 50% increase ; according to the Federal Reserve data, the increase is from 15% to 28%, which is a 87% increase. For bonds with all maturities, comparing to the Fama-Bliss data, the Federal Reserve data indicates a larger amount of increase in the $\overline{R}^2$ because of the $S - shape$ factor.

$Model\,2$ using the Fama-Bliss data is not supposed to outperform $model\,3$ since we have shown in previous section that the return-forecasting factor contains the information of all yield factors in the Fama-Bliss data. However, as we can see from table IV, the third column and the six column have similar values, which means, $model\,2$ performs as good as $model\,3$ without considering the fifth factor.

Comparing $model\,2$'s performances under two datasets, surprisingly we find that the in-sample $\overline{R}^2$ of $model\,2$ using the Federal Reserve data is even bigger than the in-sample $\overline{R}^2$ using the Fama-Bliss data, even given that the Fama-Bliss data is where excess bond returns are derived from. Also, $Model\,2$ using the Federal Reserve data outperforms $model\,3$ during almost all subsamples except the subsample period covering the financial crisis.

Statistics of sample period 1964–2002 give similar implications with those of period 1964–2007. Cochrane and Piazzesi (2005) use data of 1964–2002 and forecast average excess bond returns with an R^2 around 35%. We are able to verify their finding in table IV. Also, the in-sample $\overline{R}^2$ of each model during 1964–2002 is bigger than the in-sample $\overline{R}^2$ during 1964–2007. This is consistent with the fact that all three models perform better during the earlier period 1964-1984 than during the later period 1985–2007. There is a significant decrease in the $\overline{R}^2$ comparing pre- and post-1985. For example, for bonds with $n = 2$, comparing sample period 1964–1984 with 1985–2007, according to the Federal Reserve data, the in-sample R^2 drops from 30% to 17% for *model* 1, it drops from 52% to 29% for *model* 2; for *model* 3, the in-sample R^2 of drops from 36% to 22%. On the whole, according to the Federal Reserve data, the in-sample R^2 during post–1985 are around one half of the R^2 during post–1985.

To summarize, bonds with different maturities give similar implications. First, both the $S - shape$ factor and the return-forecasting factor prove to be strong predictors for predicting bond risk premia. Second, predictors are more useful during the pre–1985 period than during the "Great Modification" or financial crisis period. Third, the $S - shape$ factor adds significant predictive power to the regressions and the first four yield factors together can outperform the return-forecasting factor.

B. Out-of-Sample Forecast

For out-of-sample performances, we focus on comparing *model* 1 with *model* 2 to emphasize the additional predictive power brought by the $S - shape$ factor. Among different methods to measure out-of-sample performances, we choose to calculate the Root Mean Squared Error (RMSE) and the out-of-sample R^2. RMSE is a standard method widely used in measuring the out-of-sample performances (see Bordo and Haubrich (2008)). Meanwhile, calculation of the out-of-sample R^2 makes it convenient to compare with the in-sample R^2.

Our way to calculate the out-of-sample RMSE and R^2 is standard: regress the true excess bond returns on fitted excess bond returns and extract the residuals and the R^2 of fitting. The fitted excess return at time t is estimated by multiplying coefficients estimated through time $t-1$ with predictors at time t. We use recursive regressions starting from 20 years. Since the sample starts from 1964, the forecast period starts from 1985.

INSERT TABLE V NEAR HERE

Table V presents the RMSE of *model* 1 and *model* 2. The RMSE of *model* 2 is smaller than the RMSE of *model* 1 in all cases. Table V also reports the ratio of RMSE of these two models. All ratios are smaller than one, which indicates that *model* 2 creates smaller estimates errors and predicts more accurately compared to *model* 1. Data from the Federal Reserve provides even smaller RMSE ratios than data from the Fama-Bliss and even stronger evidence that the $S-shape$ factor is a useful predictor.

INSERT TABLE VI NEAR HERE

Table VI presents the out-of-sample R^2 of *model* 1 and *model* 2. The out-of-sample R^2 for *model* 1 are around 20% using the Fama-Bliss data and are around 16% using the Federal Reserve data; for *model* 2, the out-of-sample R^2 are around 25% using the Fama-Bliss data and are around 30% using the Federal Reserve data. According to the Fama-Bliss data, there are around 5% increases in the out-of-sample R^2 comparing *model* 2 with *model* 1 , and according to the Federal Reserve data, the increases are around 14%. The Federal Reserve data provides stronger evidence that the $S-shape$ factor is a useful predictor compared to the Fama-Bliss data. For example, for bonds with maturities $n=3$, according to the Fama-Bliss data, the out-of-sample R^2 increases from 18% to 24% because of the $S-shape$ factor, which is a 33% increase; according to the Federal Reserve data, the out-of-sample R^2 increases from 13% to 26%, which is a 100% increase.

V. Utility Analysis

This section applies the expected return analysis in Campbell and Thompson (2008). They develope the calculation because the out-of-sample R^2 is very small in their paper: lower than 1%. Because of this, they use a utility function to check whether the prediction is economically meaningful. It turns out that a statistically insignificant number can still be economically significant. Predictors in their paper proved to be useful economically. Similarly, we employ the expected return calculation in this section to compare the predictive ability of *model* 2 and *model* 1 and to see how much the additional $S-shape$ factor increases a typical investor's expected return.

To do this, this section considers an investor with a single-period investment horizon and mean-variance preference. The investor's objective function is:

$$E_t(r_{t+1}^W) - \frac{\gamma}{2} var_t(r_{t+1}^W),$$

where r_{t+1}^W is the return on wealth or the expected portfolio return and γ is the coefficient of relative risk aversion. The return on wealth follows

$$r_{t+1}^W = r_t^f + \alpha_t' r_{t+1}^e,$$

where r_t^f is the return on risk-free asset, α_t' is the weight invested on risky asset, and r_{t+1}^e is the excess return on long bonds with mean $E_t(r_{t+1}^e)$ and variance Σ_t. Suppose:

$$r_{t+1}^e = \mu + x_t + \varepsilon_{t+1},$$

where μ is the unconditional average excess return, x_t is a predictor variable with mean zero, and constant variance σ_x^2, and ε_{t+1} is a random shock with mean zero and constant variance

σ_ε^2. Then the investor should set the weight on risky asset in the optimal portfolio without constraints as:

$$\alpha_t = \frac{1}{\gamma}\Sigma_t^{-1}E_t(r_{t+1}^e),$$

which equals to $(\frac{1}{\gamma})(\frac{\mu}{\sigma_x^2+\sigma_\varepsilon^2})$ if the investor does not observe x_t, and equals to $(\frac{1}{\gamma})(\frac{\mu+x_t}{\sigma_\varepsilon^2})$ if the investor does observe x_t. The Euler equation implies excess return on long bonds is

$$E_t(r_{t+1}^e) = \gamma\Sigma_t\alpha_t = \gamma cov(r_{t+1}^e, r_{t+1}^W),$$

which equals to $(\frac{1}{\gamma})(\frac{\mu^2}{\sigma_x^2+\sigma_\varepsilon^2}) = \frac{S^2}{\gamma}$ if the investor does not observe x_t, and equals to $(\frac{1}{\gamma})(\frac{\mu^2+\sigma_x^2}{\sigma_\varepsilon^2}) = (\frac{1}{\gamma})(\frac{S^2+R^2}{1-R^2})$ if the investor does observe x_t. Note here the S represents the unconditional Sharpe Ratio of the risky asset and the R^2 represents the R^2 statistics for regressions of excess return on the predictor variable x_t.

The difference between two expected returns represents how much better expected return gets because of observing x_t: portfolio with larger portion of risky assets has higher expected excess return. The absolute difference between two expected returns is:

$$(\frac{1}{\gamma})(\frac{R^2}{1-R^2})(1+S^2).$$

The proportional increase of expected return because of observing x_t is:

$$(\frac{R^2}{1-R^2})(\frac{1+S^2}{S^2}).$$

Comparing *model* 1 with *model* 2, the absolute increases on expected return because of the $S - shape$ factor is:

$$(\frac{1}{\gamma})(\frac{R_2^2}{1-R_2^2} - \frac{R_1^2}{1-R_1^2})(1+S^2),$$

in which R_2^2 represents the R^2 from $model\,2$ and R_1^2 represents the R^2 from $model\,1$. The proportional increase of expected return because of the $S - shape$ factor is:

$$(\frac{R_2^2}{1-R_2^2})/(\frac{R_1^2}{1-R_1^2}) - 1.$$

INSERT TABLE VII NEAR HERE

Table VII shows what the absolute and proportional increases on expected return are for a typical investor if she uses $model\,2$ instead of $model\,1$. The calculation makes use of the out-of-sample R^2 in table VI and set the risk adverse coefficient γ to 1.

According to the Fama-Bliss data, the additional $S - shape$ factor can increase the absolute expected returns around 5%–15% more, which are around 13%–48% proportionally. According to the Federal Reserve data, the additional $S - shape$ factor can increase the absolute expected returns around 9%–34% more, which are around 28%–131% proportionally. Also, the statistics of 1985–2012 indicate that the $S - shape$ factor becomes less helpful during periods later than 2008, probably due to the impact of the financial crisis period starting 2008.

VI. Conclusion

In this paper, we propose a new factor to predict excess bond returns: the $S - shape$ factor. It is the fourth principal component of the yields matrix. Similar to the *level*, *slope* and *curvature* factors whose names deliver the intuition of their implications, we name the fourth factor $S - shape$ according to its shape of loadings on yields and the $S - shape$ factor represents how much S-shape like the yield curve is.

To test the predictive power of the $S - shape$ factor for excess bond returns, we fit two nested models to two datasets, the Fama-Bliss data and the Federal Reserve data. The benchmark model includes the first four yield factors (*level*, *slope*, *curvature*, and $S - shape$) while the baseline model only includes the first three factors. The in-sample and out-of-sample statistics present consistent results that the $S - shape$ factor is a significant predictor for excess bond returns.

The S-shape character is an important feature of the yield curve. Historically, market yields on U.S. Treasury securities have displayed the S-shape multiple times. The $S - shape$ factor captured the S-shape of the yield curve. It represents yields change rate under median maturities are different from yields change rates under short and long maturities.

In this paper we also demonstrate that the return-forecasting factor in Cochrane and Piazzesi (2005) is a linear combination of all yield factors in the data. The return-forecasting factor has high correlations with the second and fourth yield factors, the *slope* and $S - shape$ factors. The advantages of using yield factors to predict economic variables instead of using the return-forecasting factor cover three aspects. First, the estimation of loadings of yield factors is much less sensitive compared to the estimation of coefficients of the return-forecasting factor. Second, the estimation of yield factors faces less econometric issues (such as collinearity) than what the return-forecasting factor would face. Third, it is straightforward to check which yield factor captures the most predictive information for excess bond returns, namely the second and the fourth one.

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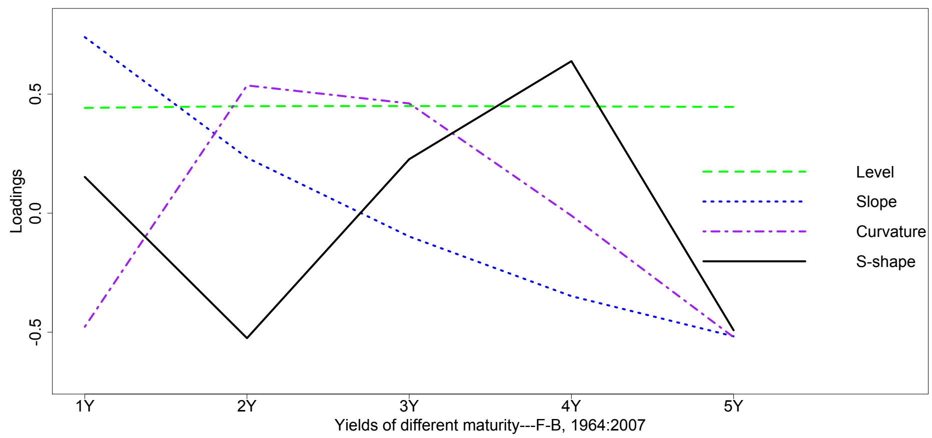
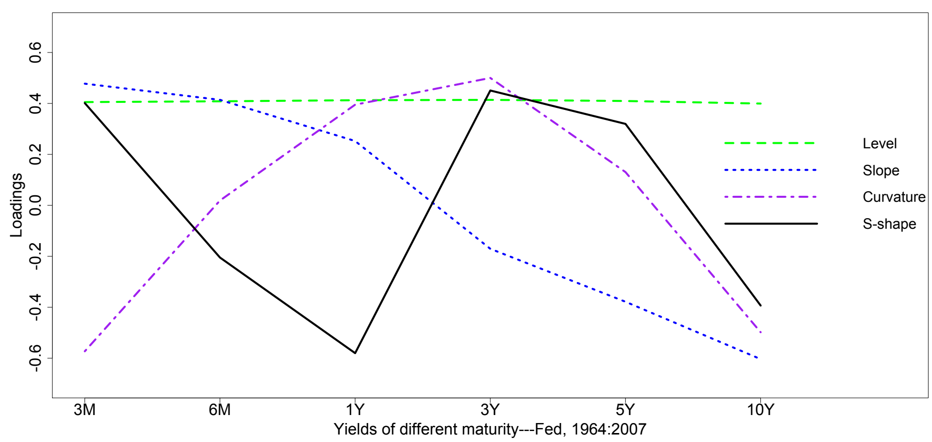
Fig. 1. The first four yield factors' loadings.
This graph uses the Fama-Bliss monthly data consisting of 1 through 5 year zero coupon bond prices and the Federal Reserve data containing market yields on U.S. treasury securies at 3-month, 6-month, 1-year, 3-year, 5-year and 10-year. Since yield factors can be denoted as linear conbinations of yields, for each factor, we connect yield factors' loadings on yields to draw a line. The first three factors are well-known as "*level*", "*slope*" and "*curvature*", and we name the fourth factor "$S - shape$" according to its shape of loadings. Data range is 1964–2007.



(a) Loadings using the Fama-Bliss data.

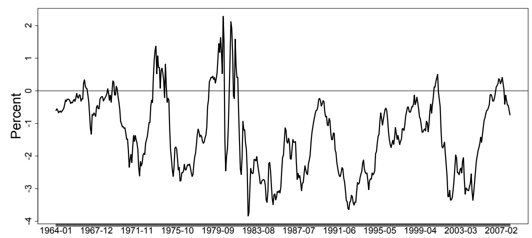


(b) Loadings using the Federal Reserve data.

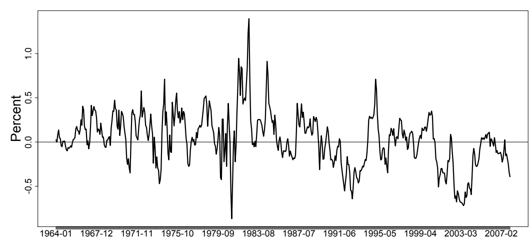
Figure 1: Loadings of yield factors.

Fig. 2. Time series plots of yield factors.
This figure plots the time series of the first four yield factors using monthly U.S. Treasury data from the Federal Reserve. Data range is 1964–2007.

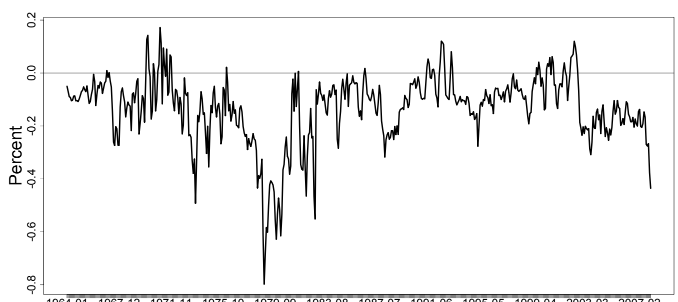
(a) Level.



(b) Slope.



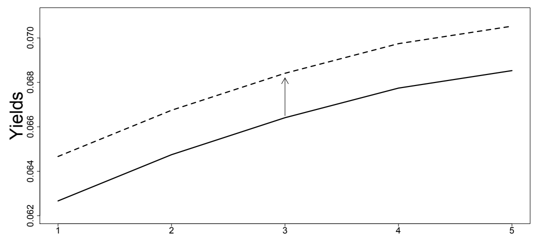
(c) Curvature.



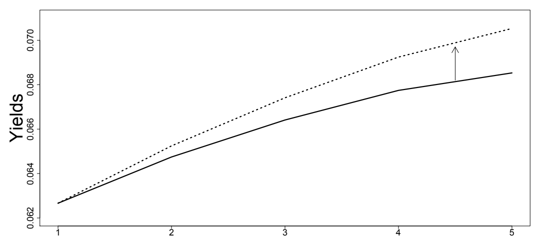
(d) S-shape.

Figure 2: Time series plots of yield factors.

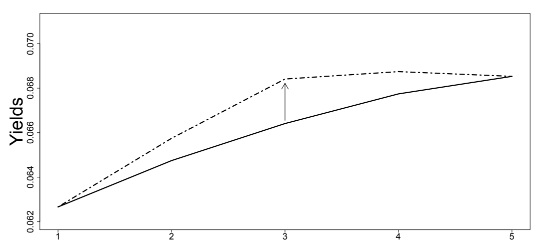
Fig. 3. The first four yield factors' effect.
$PC1$ represents the first yield factor, $PC2$ represents the second yield factor, etc. The solid lines represent yield curves at a certain time. The dashed lines represent yield factors' effects on the shape of the yield curve.



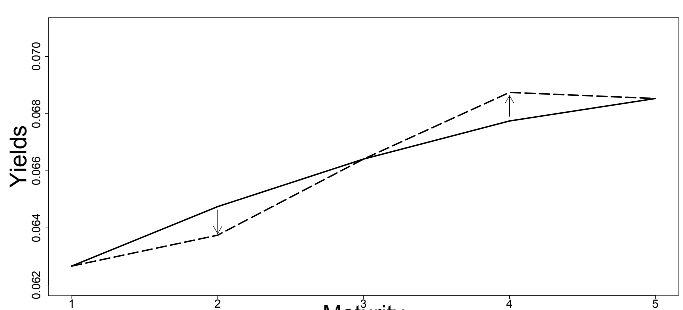
(a) PC1: "Level" effect.



(b) PC2: "Slope" effect.



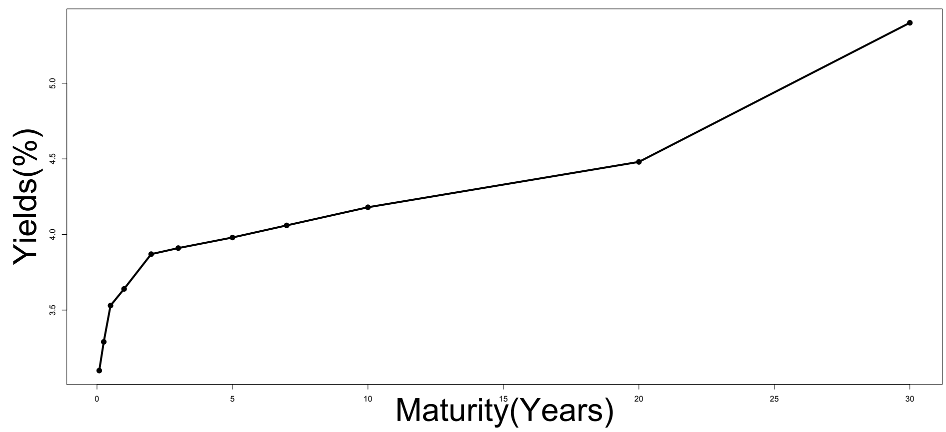
(c) PC3: "Curvature" effect.



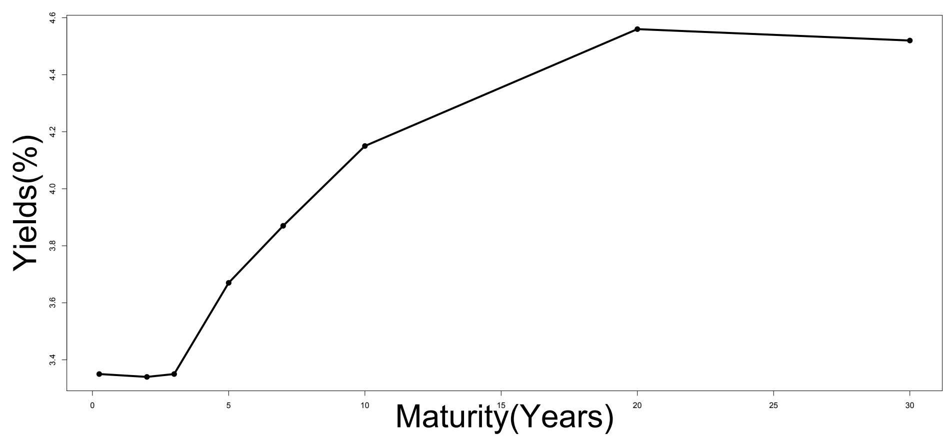
(d) PC4: "S-factor" effect.

Figure 3: Function of yield factors.

Fig. 4. Historical yield curve.
This figure plots historical yield curves at two different times, July 2005 and Nov. 2007. The purpose of this figure is to show that yield curve did appear S-shape in history. Data is from the Federal Reserve.



(a) July 2005.



(b) Nov. 2007.

Figure 4: Historical yield curves.

Fig. 5. Average excess bond returns and lagged $S-shape$ factor using the Fama-Bliss data.
This figure plots the average excess bond returns $\overline{rx}$ of four bonds with different maturities, $n = 5, 4, 3, 2$, and the lagged fourth yield factor derived from the Fama-Bliss data. The data consists of 1 through 5 year zero coupon government bond prices. The sample period is set to 1964-2007. The correlation between these two time series is 0.28.

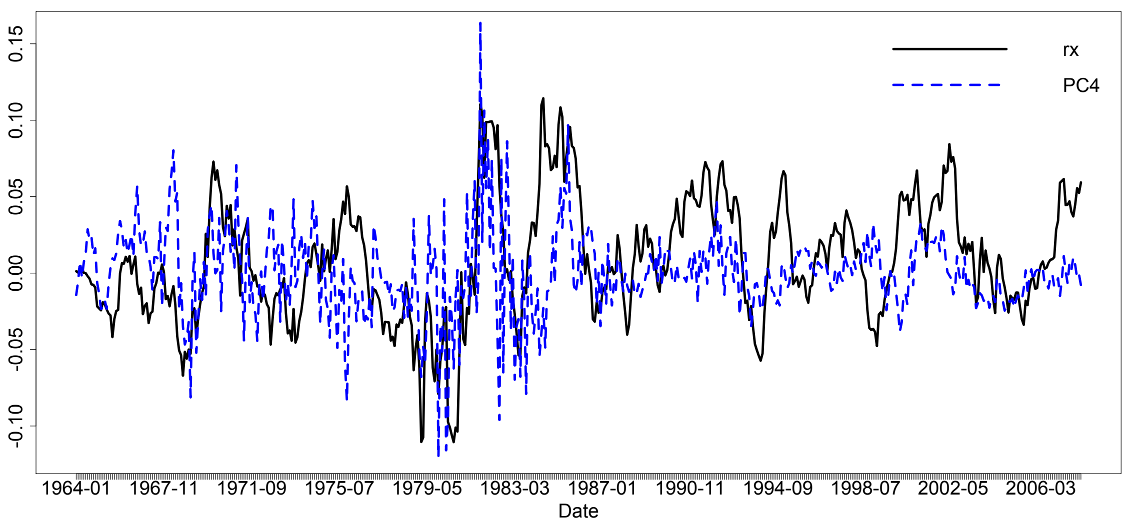


Figure 5: Excess bond returns and lagged $S - shape$ factor.

Fig. 6. Average excess bond returns and lagged $S-shape$ factor using the Federal Reserve data.
This figure plots the average excess bond returns $\overline{rx}$ of bonds with four different maturities, $n = 5, 4, 3, 2$, and the lagged fourth yield factor derived from the Federal Reserve data. The data consists of monthly observations of market yields on U.S. treasury securities at 3-month, 6-month, 1-year, 3-year, 5-year and 10-year. The sample period is set to 1964-2007. The correlation between these two time series is 0.34.

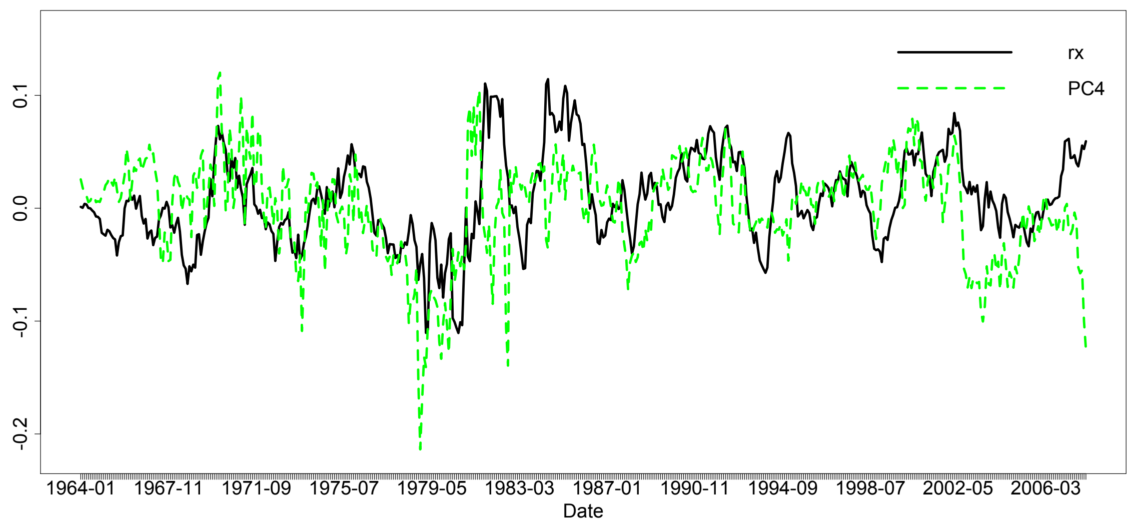


Figure 6: Excess bond returns and lagged $S-shape$ factor.

Table I
Regression results of the return-forecasting factor on yield factors derived from the Fama-Bliss data.
Yield factors are principal components derived from 1 through 5 year zero coupon bond prices from CRSP. The sample period is set to 1964-2007. This table presents statistics of univariate and multivariate regressions with the return-forecasting factor (CP) in Cochrane and Piazzesi (2005) as the dependent variable and principal components (PCs) of yields as regressors. The regression equation for the last line of this table is: $CP_t = \beta_0 + \beta_1 PC1_t + \beta_2 PC2_t + \beta_3 PC3_t + \beta_4 PC4_t + \varepsilon_t$. We use OLS to estimate the coefficients and method in Newey and West (1994) to fix the standard error estimation problem due to heteroskedasticity and autocorrelation among errors. $\overline{R}^2$ reports adjusted R^2. Standard errors are in parentheses. The significant codes are "***" for the level of 0.01, "**" for 0.05, and "*" for 0.10.

Table I: Regressions of the return-forecasting factor on principal components

$PC1$	$PC2$	$PC3$	$PC4$	$PC5$	$\overline{R}^2$
0.003**					9%
(NW)(0.00)					
	−0.06***				57%
	(0.01)				
		0.13***			6%
		(0.03)			
			0.39***		27%
			(0.07)		
				−0.04	0%
				(0.08)	
0.003***	−0.06***	0.13***			72%
(0.00)	(0.00)	(0.02)			
0.003***	−0.06***	0.13***	0.39***		100%
(0.00)	(0.00)	(0.00)	(0.00)		

Table II

Regression results of the return-forecasting factor on yield factors derived from the Federal Reserve data.

Yield factors are principal components derived from monthly data of U.S. Treasury securities at 3-month, 6-month, 1-year, 3-year, 5-year and 10-year. The sample period is set to 1964-2007. This table presents statistics of univariate and multivariate regressions with the return-forecasting factor (CP) in Cochrane and Piazzesi (2005) as the dependent variable and principal components (PCs) of yields as regressors. The regression equation for the last line of this table is: $CP_t = \beta_0+\beta_1 PC1_t+\beta_2 PC2_t+\beta_3 PC3_t+\beta_4 PC4_t+\beta_5 PC5_t+\beta_6 PC6_t+\varepsilon_t$. We use OLS to estimate the coefficients and the method in Newey and West (1994) to fix the standard error estimation problem due to heteroskedasticity and autocorrelation among errors. $\overline{R}^2$ reports adjusted R^2. Standard errors are in parentheses. The significant codes are "***" for the level of 0.01, "**" for 0.05, and "*" for 0.10.

Table II: Regressions of the return-forecasting factor on principal components

$PC1$	$PC2$	$PC3$	$PC4$	$PC5$	$PC6$	$\overline{R}^2$
0.002*						5%
(NW)(0.00)						
	−0.03***					54%
	(0.00)					
		0.01				0%
		(0.02)				
			0.13***			8%
			(0.04)			
				0.02		0%
				(0.09)		
					−0.04	0%
					(0.09)	
0.002***	−0.03***	0.01				60%
(0.00)	(0.00)	(0.01)				
0.002***	−0.03***	0.01	0.13***			68%
(0.00)	(0.00)	(0.01)	(0.01)			
0.002***	−0.03***	0.01	0.13***	0.02	−0.04	68%
(0.00)	(0.00)	(0.01)	(0.01)	(0.03)	(0.04)	

Table III
In-sample regression results of predictive models.
The regression equation of *model* 1 is: $rx_{t+1}^{(n)} = \delta_{1,0} + \delta_{1,1}PC1_t + \delta_{1,2}PC2_t + \delta_{1,3}PC3_t + \varepsilon_{t+1}^{(n)}$, in which $PC1$ stands for the first yield factor (principal component), etc. The regression equations of *model* 2 is *model* 1 plus an additional right-hand-side variable: $PC4$. *Model* 3 is a regression with a single predictor: CP—the return-forecasting factor in Cochrane and Piazzesi (2005). This table records the coefficients, standard errors and significant codes of the last right-hand-side variable in each model during different sample periods. 1964-2002 is the sample period used in Cochrane and Piazzesi (2005). The first two columns use yield factors derived from the Fama-Bliss data. The third and fourth columns use yield factors derived from the Federal Reserve data. For the first four columns, we use OLS to estimate the coefficients and the method in Newey and West (1994) to fix the standard error estimation problem due to heteroskedasticity and autocorrelation among errors. For the last column, we use two step OLS to estimate the coefficients and the method in Hansen (1982) to fix the standard error estimation problem due to generated regressors. The significant codes are "***" for the level of 0.01, "**" for 0.05, and "*" for 0.10.

Table III: Forecasts of 1-year excess returns using 3 models: coefficients

Panel A:

	PCs (F-B)		PCs (Fed)		CP (F-B)	
$n = 2$	*Model* 1	*Model* 2	*Model* 1	*Model* 2	*Model* 3	
Periods	$PC3$	$PC4$	$PC3$	$PC4$	CP	
1964-2007	0.07*	0.16***	0.01	0.15**	0.45***	
(NW)	(0.03)	(0.05)	(0.02)	(0.07)	(0.13)	(2 OLS)
t-stat	1.91	3.16	0.37	2.27	3.45	t-stat
1964-2002	0.08***	0.15***	0.005	0.19***	0.45***	
	(0.03)	(0.04)	(0.02)	(0.05)	(0.16)	
	2.81	3.44	0.27	4.90	2.83	
1964-1984	0.14***	0.15***	−0.05	0.23***	0.49	
	(0.04)	(0.05)	(0.03)	(0.05)	(0.80)	
	3.70	2.72	−1.64	4.91	0.62	
1985-2007	−0.02	0.16**	0.02	0.11	0.45***	
	(0.05)	(0.08)	(0.02)	(0.20)	(0.07)	
	−0.43	2.00	1.02	0.55	> 5.00	
1985-2012	−0.04	0.13*	0.02	0.07	0.43***	
	(0.05)	(0.08)	(0.02)	(0.05)	(0.07)	
	−0.77	1.75	0.95	1.29	> 5.00	

Panel B:

$n=3$	PCs (F-B)		PCs (Fed)		CP (F-B)	
Periods	$Model\,1$ $PC3$	$Model\,2$ $PC4$	$Model\,1$ $PC3$	$Model\,2$ $PC4$	$Model\,3$ CP	
1964-2007	0.12**	0.36***	0.03	0.28*	0.86***	
(NW)	(0.06)	(0.09)	(0.03)	(0.16)	(0.06)	(2 OLS)
t-stat	1.98	4.12	0.82	1.77	> 5.00	t-stat
1964-2002	0.14***	0.36***	0.02	0.34***	0.85***	
	(0.05)	(0.08)	(0.04)	(0.09)	(0.06)	
	2.71	4.39	0.65	3.86	> 5.00	
1964-1984	0.25***	0.37***	-0.11*	0.42***	0.89	
	(0.07)	(0.10)	(0.06)	(0.08)	(0.77)	
	3.41	3.74	-1.84	4.96	1.26	
1985-2007	-0.03	0.33**	0.03	0.20	0.84***	
	(0.10)	(0.15)	(0.04)	(0.46)	(0.04)	
	-0.29	2.27	0.69	0.43	> 5.00	
1985-2012	-0.07	0.25*	0.03	0.11	0.80***	
	(0.09)	(0.14)	(0.05)	(0.10)	(0.04)	
	-0.73	1.79	0.71	1.04	> 5.00	

Panel C:

$n=4$	PCs (F-B)		PCs (Fed)		CP (F-B)	
Periods	$Model\,1$ $PC3$	$Model\,2$ $PC4$	$Model\,1$ $PC3$	$Model\,2$ $PC4$	$Model\,3$ CP	
1964-2007	0.16*	0.52***	0.04	0.35**	1.24***	
(NW)	(0.08)	(0.12)	(0.05)	(0.18)	(0.03)	(2 OLS)
t-stat	1.91	4.46	0.85	1.99	> 5.00	t-stat
1964-2002	0.18**	0.52***	0.03	0.44***	1.24***	
	(0.07)	(0.11)	(0.05)	(0.10)	(0.04)	
	2.44	4.78	0.67	4.22	> 5.00	
1964-1984	0.32***	0.52***	-0.15**	0.53***	1.21***	
	(0.10)	(0.13)	(0.07)	(0.10)	(0.38)	
	3.17	4.06	-2.08	> 5.00	3.23	
1985-2007	-0.03	0.42**	0.03	0.25	1.26***	
	(0.16)	(0.19)	(0.06)	(0.71)	(0.03)	
	-0.21	2.22	0.56	0.35	> 5.00	
1985-2012	-0.11	0.23*	0.05	0.12	1.28***	
	(0.12)	(0.19)	(0.06)	(0.14)	(0.02)	
	-0.89	1.75	0.78	0.87	> 5.00	

Panel D:

$n=5$ Periods	PCs (F-B) $Model\,1$ $PC3$	PCs (F-B) $Model\,2$ $PC4$	PCs (Fed) $Model\,1$ $PC3$	PCs (Fed) $Model\,2$ $PC4$	CP (F-B) $Model\,3$ CP	
1964-2007	0.18*	0.54***	0.06	0.40	1.45***	
(NW)	(0.10)	(0.15)	(0.05)	(0.27)	(0.03)	(2 OLS)
t-stat	1.76	3.66	1.02	1.50	> 5.00	t-stat
1964-2002	0.21**	0.54***	0.05	0.52***	1.46***	
	(0.09)	(0.13)	(0.06)	(0.13)	(0.03)	
	2.48	4.03	0.88	4.08	> 5.00	
1964-1984	0.34***	0.55***	-0.18*	0.63***	1.40***	
	(0.12)	(0.16)	(0.10)	(0.12)	(0.09)	
	2.92	3.44	-1.93	> 5.00	> 5.00	
1985-2007	-0.03	0.39*	0.02	0.26	1.46***	
	(0.25)	(0.24)	0.07	0.62	(0.02)	
	-0.12	1.64	0.27	0.42	> 5.00	
1985-2012	0.16	0.23	0.05	0.09	1.49***	
	(0.15)	(0.24)	(0.08)	(0.17)	(0.02)	
	-1.06	0.97	0.65	0.50	> 5.00	

Table IV
In-sample adjusted R^2 of predictive models.
The regression equation of *model* 1 is: $rx_{t+1}^{(n)} = \delta_{1,0} + \delta_{1,1}PC1_t + \delta_{1,2}PC2_t + \delta_{1,3}PC3_t + \varepsilon_{t+1}^{(n)}$, in which $PC1$ stands for the first yield factor (principal component), etc. The regression equations of *model* 2 is *model* 1 plus an additional right-hand-side variable, $PC4$. *Model* 3 is a regression with a single predictor, CP: the return-forecasting factor in Cochrane and Piazzesi (2005). This table reports the in-sample adjusted R^2 of each model during different sample periods. 1964-2002 is the sample period used in Cochrane and Piazzesi (2005). The first two columns use yield factors derived from the Fama-Bliss data. The third and fourth columns use yield factors derived from the Federal Reserve data.

Table IV: Forecasts of 1-year excess returns: in-sample $\overline{R}^2$

	PCs (F-B)		PCs (Fed)		CP (F-B)
Periods/ In-sample $\overline{R}^2$	*Model* 1	*Model* 2	*Model* 1	*Model* 2	*Model* 3
$n = 2$					
1964-2007	21%	26%	17%	30%	25%
1964-2002	26%	31%	21%	39%	31%
1964-1984	32%	36%	30%	52%	36%
1985-2007	16%	21%	17%	29%	22%
1985-2012	12%	16%	12%	17%	15%
$n = 3$					
1964-2007	18%	27%	15%	28%	27%
1964-2002	24%	33%	20%	38%	34%
1964-1984	32%	36%	27%	51%	36%
1985-2007	14%	20%	13%	23%	21%
1985-2012	11%	14%	10%	13%	14%
$n = 4$					
1964-2007	20%	29%	17%	28%	30%
1964-2002	26%	37%	23%	39%	37%
1964-1984	32%	36%	28%	49%	36%
1985-2007	17%	22%	15%	23%	24%
1985-2012	13%	16%	12%	13%	18%
$n = 5$					
1964-2007	20%	27%	19%	28%	27%
1964-2002	27%	34%	25%	39%	34%
1964-1984	27%	34%	29%	49%	34%
1985-2007	17%	19%	15%	20%	21%
1985-2012	13%	14%	12%	12%	16%

Table V
Out-of-sample Root Mean Squared Error (RMSE) of predictive regressions.
The calculation method is to regress the true excess bond returns on fitted excess bond returns. The fitted excess return at time t is estimated by multiplying coefficients estimated through time $t-1$ with predictors at time t. The out-of-sample Root Mean Squared Error (RMSE) is estimated by taking the square root of the mean of the squared errors. The regression equation of *model* 1 is: $rx_{t+1}^{(n)} = \delta_{1,0} + \delta_{1,1}PC1_t + \delta_{1,2}PC2_t + \delta_{1,3}PC3_t + \varepsilon_{t+1}^{(n)}$, in which $PC1$ stands for the first yield factor (principal component), etc. The regression equations of *model* 2 is *model* 1 plus an additional right-hand-side variable: $PC4$. This table reports the RMSE of each model during period 1964-2007 and 1964-2012 using recursive method starting from 20 years. The forecast samples are 1985-2007 and 1985-2012 and data is from the Fama-Bliss and the Federal Reserve.

Table V: Out-of-sample RMSE using recursive regressions

RMSE	PCs (F-B)			PCs (Fed)		
Forecast sample	$Model\,1$	$Model\,2$	$Model\,2/Model\,1$	$Model\,1$	$Model\,2$	$Model\,2/Model\,1$
n=2						
1985-2007	0.014	0.013	0.97	0.014	0.013	0.91
1985-2012	0.013	0.013	0.98	0.013	0.013	0.94
n=3						
1985-2007	0.027	0.026	0.96	0.027	0.025	0.92
1985-2012	0.025	0.024	0.97	0.026	0.024	0.95
n=4						
1985-2007	0.037	0.036	0.96	0.038	0.036	0.93
1985-2012	0.035	0.034	0.97	0.036	0.034	0.96
n=5						
1985-2007	0.045	0.044	0.97	0.046	0.044	0.95
1985-2012	0.043	0.043	0.99	0.044	0.043	0.97

Table VI
Out-of-sample R^2 of predictive regressions.
The calculation method is to regress the true excess bond returns on fitted excess bond returns and extract the R^2 of fitting. The fitted excess return at time t is estimated by multiplying coefficients estimated through time $t-1$ with predictors at time t. The regression equation of *model* 1 is: $rx_{t+1}^{(n)} = \delta_{1,0} + \delta_{1,1}PC1_t + \delta_{1,2}PC2_t + \delta_{1,3}PC3_t + \varepsilon_{t+1}^{(n)}$, in which $PC1$ stands for the first yield factor (principal component), etc. The regression equations of *model* 2 is *model* 1 plus an additional right-hand-side variable: $PC4$. This table reports the out-of-sample R^2 of each model during period 1964-2007 and 1964-2012 using recursive method starting from 20 years. The forecast samples are 1985-2007 and 1985-2012 and data is from the Fama-Bliss and the Federal Reserve.

Table VI: Out-of-sample R^2 using recursive regressions

R^2	PCs (F-B)			PCs (Fed)		
Forecast sample	*Model* 1	*Model* 2	Increase	*Model* 1	*Model* 2	Increase
n=2						
1985-2007	22%	26%	18%	17%	31%	82%
1985-2012	21%	24%	14%	16%	27%	69%
n=3						
1985-2007	18%	24%	33%	13%	26%	100%
1985-2012	17%	22%	29%	13%	22%	69%
n=4						
1985-2007	20%	26%	30%	15%	26%	73%
1985-2012	19%	24%	26%	16%	22%	38%
n=5						
1985-2007	21%	25%	19%	17%	26%	53%
1985-2012	20%	22%	10%	18%	22%	22%

Table VII
The absolute/proportional increases on expected return because of the additional $S - shape$ factor.
This table calculates the absolute/proportional increases on expected returns for an investor with a single-period horizon and mean-variance preference because of the additional $S - shape$ factor. The calculation makes use of the method in Campbell and Thompson (2008), which makes use of the out-of-sample R^2. The risk adverse coefficient γ is set to 1. The regression equation of $model\,1$ is: $rx_{t+1}^{(n)} = \delta_{1,0} + \delta_{1,1}PC1_t + \delta_{1,2}PC2_t + \delta_{1,3}PC3_t + \varepsilon_{t+1}^{(n)}$, in which $PC1$ stands for the first yield factor (principal component), etc. The regression equations of $model\,2$ is $model\,1$ plus an additional right-hand-side variable: $PC4$. Data is from the Fama-Bliss and the Federal Reserve.

Table VII: Absolute/Proportional increases on expected return: comparing $model\,2$ with $model\,1$

	Absolute increases: E(r)		Proportional increases: E(r)	
Forecast sample	PCs (F-B)	PCs (Fed)	PCs (F-B)	PCs (Fed)
n=2				
1985-2007	0.11	0.34	30%	122%
1985-2012	0.08	0.24	22%	85%
n=3				
1985-2007	0.14	0.27	48%	131%
1985-2012	0.10	0.18	33%	83%
n=4				
1985-2007	0.15	0.24	45%	102%
1985-2012	0.10	0.14	30%	54%
n=5				
1985-2007	0.09	0.19	28%	70%
1985-2012	0.05	0.09	13%	28%