Course Content Overview

01/10/22

1. Measurement & Application of Acoustic Intensity Method
2. Acoustic Filters (Transfer Matrix Method)
3. Sound Transmission Through Barrier Systems
4. Duct Acoustics - higher order Modes
5. Room Acoustics - porous Matls.

1.0 Acoustic Intensity

Intensity Methods are used to:

- Measure sound power of a source
- Measure component sound powers
  - rank the noise sources
  - rank & identify paths
- Measure transmission loss of barrier systems
- Visualize radiation fields
  - impress boss

"Modern" Implementation - signal processing
- fft

Two-Microphone cross spectral procedure

1.1 Instantaneous Intensity

Intensity = Sound Power / Unit Area (flux)

$$= \left(\frac{\text{Force}}{\text{Area}}\right) \cdot \text{Velocity} = \text{Pressure} \cdot \text{Velocity}$$

$$\bar{I}_{inst} = p(t)\,\bar{u}(t)$$

$p(t)$: real sound pressure

$\bar{u}(t)$: real vector particle velocity

94 dB → $P_{rms} = 1\,Pa$ ... $\frac{p}{u} = \rho_0 c \Rightarrow u = \frac{p}{\rho_0 c} = 2.4\,\frac{mm}{s}$

1.2 Time-Averaged Acoustic Intensity

$$\bar{I} = \frac{1}{T}\int_0^T I_{inst}\, dt \qquad T = \text{signal period}$$

$$= \frac{1}{T}\int_0^T p(t)\,\bar{u}(t)\, dt$$

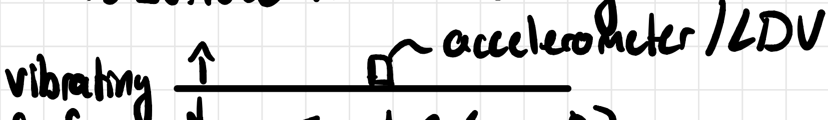
Complex harmonic Signals

$$P(\bar{r},t) = P(r)e^{j\omega t} \qquad \bar{u}(\bar{r},t) = \bar{u}(r)e^{j\omega t}$$

$$I = Re\{p \cdot u^*\} \cdot \frac{1}{2}$$

1.3 Measurement of Particle Velocity

(i) surface normal velocity



$$I = \frac{1}{2} Re\{p\, u_n^*\}$$

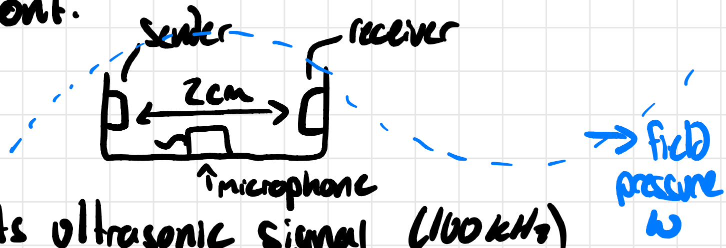
Cons (Accel)
- Temperature
- mass loading
- very local - time consuming
- cross calibration is difficult
- know intensity only at the surface

Reminder - Intensity is Time-Averaged 01/12/22

$$\bar{I} = \frac{1}{2}\mathrm{Re}\{p\,\bar{u}^*\}$$

measurement methods cont.

ii) ultrasonic probe
(Trondheim)



Sending Transducer emits ultrasonic signal (100kHz) (ω_p)

$\omega \ll \omega_p$ so $\lambda \gg \lambda_p$

- instantaneous sound speed in the probe region fluctuates as the external sound field moves air back and forth
  - causes the phase of the probe to fluctuate

$$c_x(t) = c_0 + u_x(t)$$

c_0 ~ ambient sound speed; $u_x(t)$ ~ particle velocity associated with the external sound field

$P_{receiver} \propto e^{-jk_p d}$ ~ phase ($k_p d$)

k_p ~ probe wave number ... $k_p = \frac{\omega_p}{c_x(t)}$ ~ ω_p probe freq, $c_x(t)$ ~ instantaneous sound speed

$$\varphi(t) = -k_p d = \frac{-\omega_p d}{c_0 + u_x(t)} = \frac{-\omega_p d}{c_0\left[1 + \frac{u_x(t)}{c_0}\right]}$$

$\frac{u_x(t)}{c_0}$ ~ acoustic Mach number $\ll 1$

$$\varphi(t) \approx \frac{-\omega_p}{c_0} d \left(1 - \frac{u_x(t)}{c_0}\right)$$

$$\underbrace{\phi(t) = \phi_0}_{\text{const.}} + \phi'(t) \qquad \phi'(t) = \frac{k_0 \cdot d}{c_0} u_x(t) \quad \ldots \quad k_0 = \frac{\omega_p}{c_0}$$

$$\boxed{u_x(t) = \frac{c_0}{k_0 d} \phi'(t)}$$

if phase can be continuously measured particle velocity can be found

to measure intensity, combine with microphone

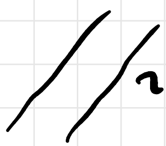
$$I_x = \frac{1}{2} Re\{p \cdot u^*\}$$

Problems: • probe tends to affect the sound field
• sensitive to ambient fluid motion
• calibration is tricky

(iii) microflown TU Wente

mems-based...

hot-wires



2 thin wires closely spaced

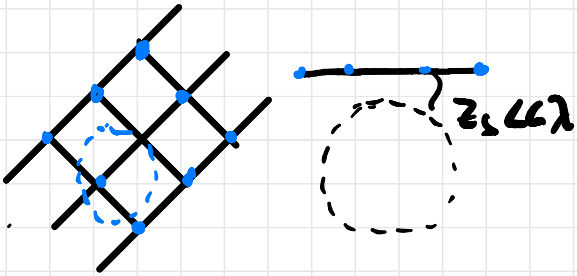
- measuring temperature fluctuations
  - infer particle velocity

- incorperate a microphone

Problems: • cost
• calibration

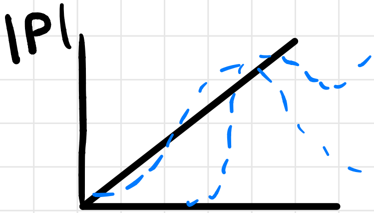
(iv) Holographic Procedure

NAH - Nearfield Acoustical Holography

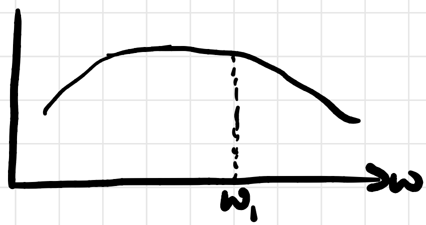


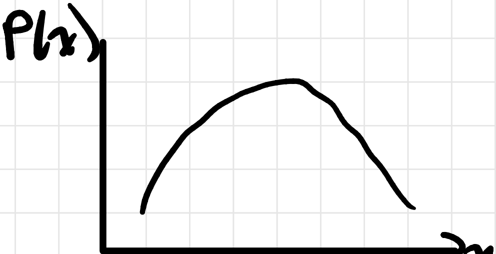
Measurements close to source

- measure spatial distribution of sound pressure close to the source

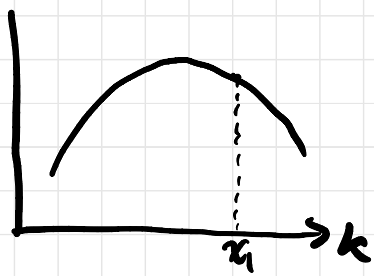


Spatial transient

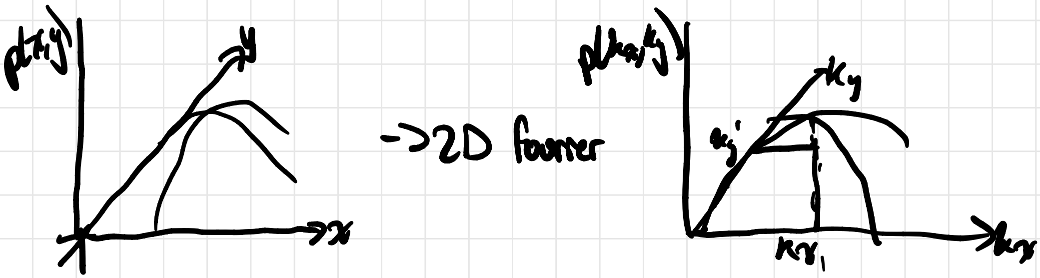




Fourier →



Create wavenumber spectrum



we are measuring sound, which satisfies the wave equation

$$k^2 = k_x^2 + k_y^2 + k_z^2 \qquad k = \frac{\omega}{c}$$

knowing k_{x_1}, k_{y_1}

$$\rightarrow k_{z_1} = \sqrt{k^2 - k_{x_1}^2 - k_{y_1}^2}$$

$$e^{-j(k_{x_1} x + k_{y_1} y + k_{z_1} z)} \sim \text{plane wave}$$

Plane wave decomposition

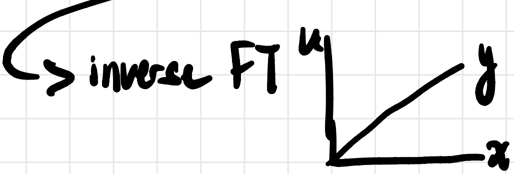
sound field is expressed as the sum of an infinite number of plane wave components traveling in all directions

for a freely prop. plane wave $\frac{p}{u} = \rho_0 c$

can find u by this relation...

$$\tilde{u}_p(k_x, k_y, k_z) = P(k_x, k_y, k_z)/\rho_0 c$$

3D particle velocity field can be found in this way



at every point can calculate intensity

Acoustic Intensity 01/14/22

$I = \frac{1}{2} Re\{p u^*\}$

Nearfield Acoustic Holography cont.

Pros - fast

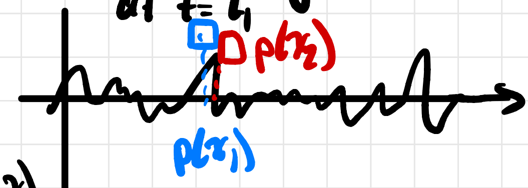
- can project the sound field towards or away from the source

Cons - expensive
- complicated

v) Two-Microphone procedure

use linearized momentum equation to relate particle velocity to the pressure gradient

$$\nabla p = -\rho_0 \frac{d\bar{v}}{dt}$$



$$\frac{dp}{dx} = \lim_{\Delta x \to 0} \frac{p(x_1 + \Delta x) - p(x_1)}{\Delta x}$$

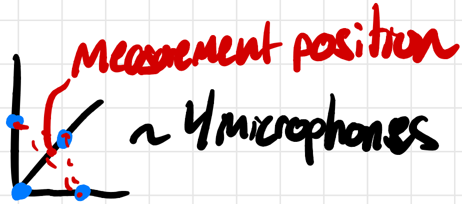
good estimate when Δx is small compared to λ

... if $\frac{\Delta x}{\lambda} < \frac{1}{10}$ less then a 5% error in the gradient

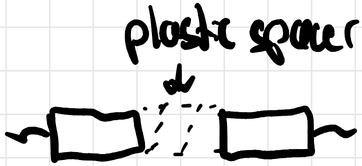
$$\frac{dp}{dx} \simeq \frac{P(x_1 + \Delta x) - P(x_1)}{\Delta x}$$

pressure gradient is estimated at the midpoint between microphones.

for 3D gradient



can use 6 mics to measure at "origin"



Minimizes directional affects due to scattering from mics + preamps

plastic spacer size

$\Delta = 12.5$ mm (high frequency, $f < 5$kHz)

$\Delta = 50$ mm (low frequency, $f < 1250$ Hz)

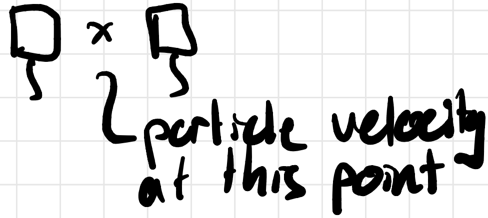
Consider the complex harmonic representation

$$p(r,t) = P(\bar{r},\omega)e^{j\omega t}$$

$$\bar{u}(r,t) = \bar{U}(\bar{r},\omega)e^{j\omega t}$$

$$-\nabla p = \rho_0 \frac{d\bar{u}}{dt} \Rightarrow -\nabla P = j\omega\rho_0 \bar{u} \qquad \bar{u} = \frac{-\nabla P}{j\omega\rho_0}$$

$$u_x \approx = \frac{P(x+\Delta x) - P(x)}{j\omega\rho_0(\Delta x)}$$

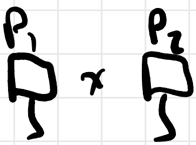


1.4 Calculation of Acoustic Intensity

1.4.1 Two-microphone cross-spectral method.

$$\bar{I}(\bar{r},\omega) = \frac{1}{2} Re\left(P(\bar{r},\omega)\, \bar{u}^*(\bar{r},\omega)\right)$$

i) estimate of pressure spectrum



$$P_x = \frac{P_1 + P_2}{2}$$

ii) estimated particle velocity spectrum

$$u_x = \frac{-(P_2 - P_1)}{j\omega\rho_0\Delta} = \frac{j}{\omega\rho_0\Delta}(P_2 - P_1)$$

iii) x-component of intensity

$$I_x = \frac{1}{2} Re\{pu^*\} = \frac{1}{2} Re\left\{\frac{P_1+P_2}{2}\left[\frac{j}{\omega\rho_0\Delta}(P_2-P_1)\right]^*\right\}$$

$$I_x = \frac{1}{2} Re\left\{-\frac{j}{2\omega\rho_0\Delta}\left(\cancel{P_2P_2^*} + P_1P_2^* - P_2P_1^* - \cancel{P_1P_1^*}\right)\right\}$$

$(P_1^* P_2) = (P_1 P_2^*)^* \Rightarrow a + jb$ and $a - jb$

$a + jb - (a - jb) = j2b$

$$\frac{1}{2}\mathrm{Re}\left\{\frac{\cancel{-j}}{\cancel{2}\omega\rho_0\Delta}\, 2\cancel{j}\,\mathrm{Im}\{P_1P_2^*\}\right\} = I_x$$

$$I_x = \frac{1}{2}\,\mathrm{Im}\{P_1P_2^*\}\cdot\frac{1}{\omega\rho_0\Delta}$$

$$= \frac{-\mathrm{Im}\{P_1^*P_2\}}{2\omega\rho_0\Delta}$$

Cross-spectrum of the two pressures

use this formula to compute Intensity in the direction joining the two microphones

Acoustic Intensity

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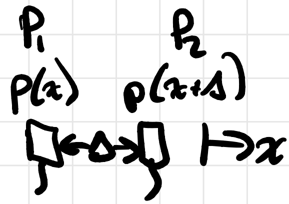
for complex harmonic signals, $e^{j\omega t}$

$$I_x = \frac{1}{2}\mathrm{Re}\{p u_x^*\}$$



particle velocity estimate

$$u_x = \frac{p(x+\Delta) - p(x)}{j\omega\rho_0\Delta}$$

pressure spectrum estimate

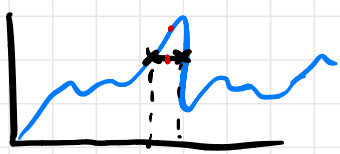
$$P(\omega) = \frac{p(x) + p(x+\Delta)}{2}$$

Acoustic Intensity Estimate

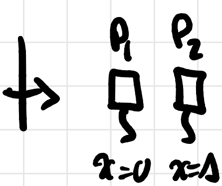
$$I_x = \frac{-\mathrm{Im}\{P_1^* P_2\}}{2\omega\rho_0\Delta}$$

1.5 Sources of Error

1.5.1 Finite Difference Error



$$\frac{dp}{dx} = \frac{P(x+\Delta) - P(x)}{\Delta}$$



$$p(x,t) = A e^{-jkx} e^{j\omega t}$$

$$u_x(x,t) = \frac{A}{\rho_0 c} e^{-jkx} e^{j\omega t}$$

Exact Intensity: $\frac{1}{2}\mathrm{Re}\{p u_x^*\} = \frac{1}{2}\mathrm{Re}\left\{\frac{AA^*}{\rho_0 c}\right\} = \frac{|A|^2}{2\rho_0 c}$, ~msp

Measured intensity: $P = Ae^{-jkx}e^{j\omega t}$

$$P_M = \frac{P_1 + P_2}{2} = \frac{A + Ae^{-jk\Delta}}{2}$$

$$u_M = \frac{j}{\omega \rho_0 \Delta}(P_2 - P_1) = \frac{j}{\omega \rho_0 \Delta}\left[Ae^{-jk\Delta} - A\right]$$

$$P_M u_M^* = \frac{-j|A|^2}{2\omega\rho_0\Delta}\underbrace{\left[e^{jk\Delta} - e^{-jk\Delta}\right]}_{2j\sin k\Delta}$$

$$= \frac{|A|^2}{\omega\rho_0\Delta}\sin k\Delta \qquad k = \frac{\omega}{c} \ \ldots \ \omega = kc$$

$$= \underbrace{\frac{|A|^2}{\rho_0 c}}_{\text{exact intensity}} \cdot \frac{\sin k\Delta}{k\Delta}$$

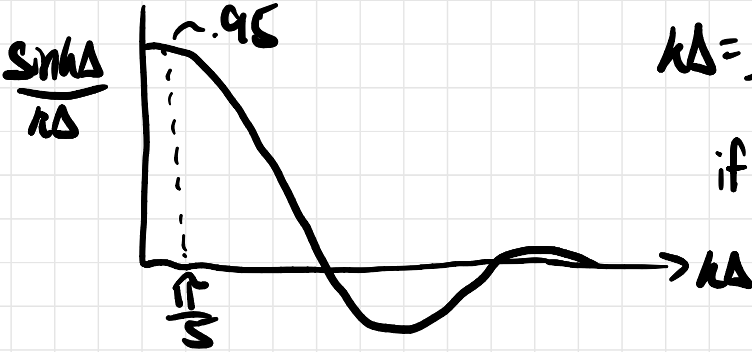
finite difference error

$$k\Delta = \frac{2\pi}{\lambda}\Delta = 2\pi\left(\frac{\Delta}{\lambda}\right)$$

non-dimensional mic separation

$$\frac{I_{mx}}{I_{bx}} = \frac{\sin k\Delta}{k\Delta}$$

- finite difference for a plane wave
- independent of position
- frequency dependent
- always less than 1
- bias error (always one direction)

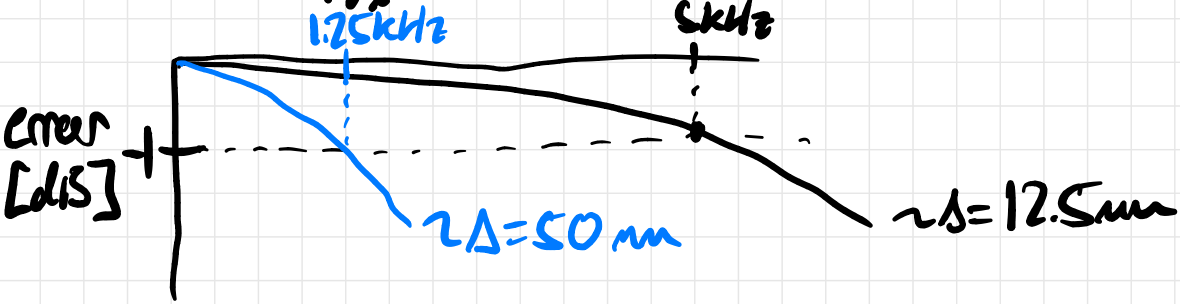


$$k\Delta = \frac{\pi}{5} = \frac{2\pi}{\lambda}\Delta \Rightarrow \frac{\Delta}{\lambda} = \frac{1}{10}$$

if mic spacing is less than $\frac{\lambda}{10}$, error is 5% or less

- magnitude is always less than the true intensity
- error increases with frequency for a fixed Δ
- establishes a high frequency limit on an accurate measurement.

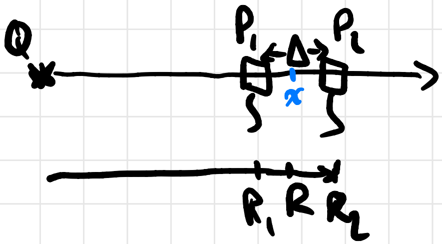
$$\text{Error} = \frac{I_{tx} - I_{mx}}{I_{tx}} = 1 - \frac{\sin k\Delta}{k\Delta}$$



1.5.2 Near field Error

- proportional to distance from the source

Monopole in free space



$$R = R_1 + \frac{\Delta}{2} = R_2 - \frac{\Delta}{2}$$

i) exact intensity: $p(R,t) = jk\rho_0 c\, Q \frac{e^{-jkR}}{4\pi R} e^{j\omega t}$

$$u_r(R,t) = \frac{j}{\omega\rho_0}\frac{\partial p}{\partial r} = \frac{Q}{4\pi r}\left[\frac{1}{R} + jk\right] e^{-jkR} e^{j\omega t}$$

near field term

$$I_{tr} = \frac{1}{2}\mathrm{Re}\{p u_r^*\} = \frac{1}{2}\frac{k^2\rho_0 c|Q|^2}{(4\pi R)^2} \propto \left(\frac{1}{R}\right)^2$$

ii) measured intensity

$$p_m = \frac{p_1 + p_2}{2} \qquad u_{mr} = \frac{j}{\omega\rho_0\Delta}(p_2 - p_1)$$

$$I_{mr} = \frac{1}{2}\mathrm{Re}\{p_m u_{mr}^*\} = \frac{1}{2}\frac{k^2\rho_0 c|Q|^2}{(4\pi)^2 R_1 R_2}\left(\frac{\sin k\Delta}{k\Delta}\right)$$

2 finite difference error

$$\frac{I_{mr}}{I_{tr}} = \frac{R^2}{(R_1 R_2)} \frac{\sin k\Delta}{k\Delta} \qquad R_1 = R - \frac{\Delta}{2} \ldots R_2 = R + \frac{\Delta}{2}$$

$$= \underbrace{\frac{1}{1 - \left(\frac{\Delta^2}{4R^2}\right)}}_{\text{near field error (always > 1)}} \frac{\sin k\Delta}{k\Delta}$$

in a good measurement

$\frac{\Delta^2}{4R^2} \ll 1$ ... can approximate as

$$\frac{I_{mr}}{I_{tr}} = \left(1 + \frac{\Delta^2}{4R^2}\right) \frac{\sin k\Delta}{k\Delta}$$

- depends on distance from the source
  - large errors close to the source
  - not frequency dependent
  - depends on wave front curvature
- depends on source type
- increases with source complexity

$R > 1.1\Delta$ for nfe < 1 dB

for dipole... $R > 1.6\Delta$ for nfe < 1 dB

for lat. quadrupole ... $R > 2.3\Delta$ for nfe < 1 dB

for $\Delta = 50$ mm $\quad R \to 115$ mm

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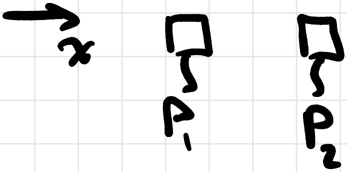


$$I_x = \frac{\mathrm{Im}\{P_1 P_2^*\}}{2\omega\rho_0\Delta}$$

$$\frac{I_{m_x}}{I_{0x}} = \underbrace{\left(1+\frac{\Delta^2}{4R^2}\right)}_{\text{purely geometric}}\frac{\sin k\Delta}{k\Delta}$$

nearfield error

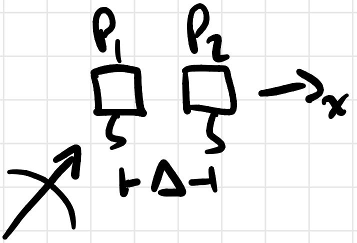
More complex sources

+	-	+	-
-	+	-	+
+	-	+	-
-	+	-	+

vibrating panel

- (much more complicated than quadrupole)
- exercise caution when measuring close to real sources

1.5.3 Phase MisMatch Errors



i) Estimate Intensity in the absence of Phase & Amplitude errors

$$P_1 = |P_1| e^{j\phi_1} \quad P_2 = |P_2| e^{j\phi_2}$$

$$Im_{tx} = \frac{Im\{P_1 P_2^*\}}{2\omega\rho_0\Delta} = \frac{|P_1||P_2|}{2\omega\rho_0\Delta} Im\{e^{j(\phi_1-\phi_2)}\}$$

$$= \frac{|P_1||P_2|}{2\omega\rho_0\Delta} \sin\delta\phi \sim \phi_1 - \phi_2$$

Intensity is very sensitive to the phase of the sound field.

ii) Estimate intensity in the presence of phase & amplitude errors

$$P_{1e} = a_1 |P_1| e^{j(\phi_1 + \phi_{1e})}$$
$$P_{2e} = a_2 |P_2| e^{j(\phi_2 + \phi_{2e})}$$

a_1, a_2 ... amplitude error
ϕ_1, ϕ_2 ... phase errors

- mic characteristics
- Measurement electronics

use these expressions to calculate intensity

$$Im_{px} = \frac{Im\{P_1 P_2^*\}}{2\omega\rho_0\Delta} = a_1 a_2 |P_1||P_2| \sin(\delta\phi + \delta\alpha)$$

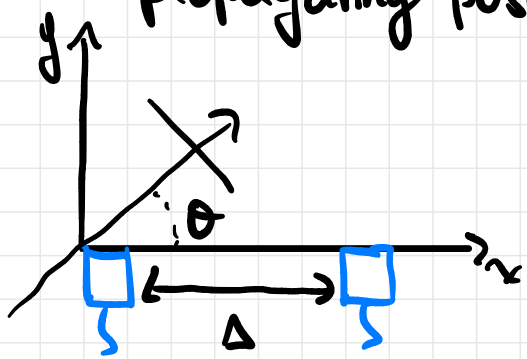
$\delta\phi = \phi_1 - \phi_2 \qquad \delta\alpha = \phi_{1e} - \phi_{2e}$ } phase mismatch

Error $\dfrac{Im_{ex}}{Im_{tx}} = a_1 a_2 \dfrac{\sin(\delta\phi + \delta\alpha)}{\sin(\delta\phi)}$

$a_1, a_2 \rightarrow$ amplitude calibration is easy...

a_1 & $a_2 \simeq 1$

iii) consider $\delta\phi$ - plane wave in free space propagating past the mics



$$p \propto e^{-jk_x x} e^{-jk_y y}$$

$$k_x = k\cos\theta$$
$$k_y = k\sin\theta$$

$\phi_1 = 0$ since mic is at the origin

$$\phi_2 = -k_x\Delta = -k\Delta\cos\theta = -\frac{\omega}{c}\Delta\cos\theta$$

e.g. 100 Hz, $\Delta = 50$ mm, $c = 340$ m/s,

$\phi_2 = 0.1\cos\theta = 6°\cos\theta$

$\theta = 0 \quad \Phi_2 = \frac{\omega}{c}$ ... largest possible phase diff.

$\theta = \pi/2 \quad \Phi_2 = 0$ ... minimum phase difference

Actual phase difference depends on the nature of the sound field.

iv) Error

if $|\delta\phi + \delta\alpha| \ll 1$ (true in a good measurement)

$$\frac{I_{m_{ex}}}{I_{m_{tx}}} = \frac{\delta\phi + \delta\alpha}{\delta\phi} = 1 + \frac{\delta\alpha}{\delta\phi}$$

$\delta\alpha$ ~ assume approx constant

$\delta\phi$ ~ continuously decreases as frequency decreases + depends on the sound field

low frequency region where $\delta\phi$ is small result is dominated by $\delta\alpha$...

Error always becomes large at low frequencies or when the actual phase diff is small

Phase mismatch provides low frequency limit to measurements

Lower Limit: Phase Diff
Upper Limit: Finite Difference Error

- Space Mics per apert
- Precision Microphones
- Phase Calibration

v) Precision Microphones

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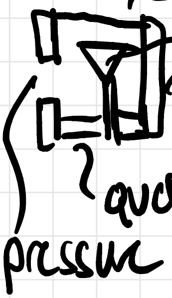
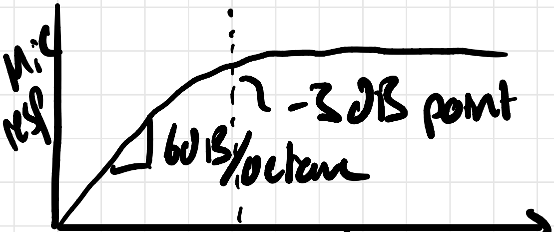
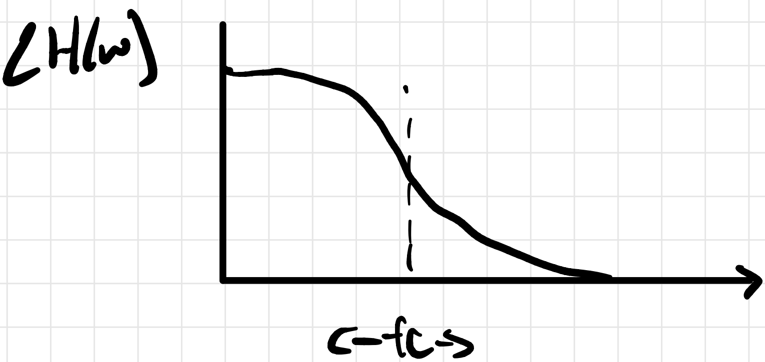




first-order high pass filter



Significant phase difference extending well beyond fc because of slow phase change

PS. 1

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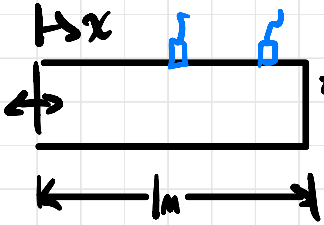


$Z_n = 1245$ Rayls $= 3\rho_0 c$

$Z_n = \infty$

$$p = Ae^{-jkx} + Be^{jkx}$$

$$u = \frac{A}{\rho_0 c} e^{-jkx} - \frac{B}{\rho_0 c} e^{jkx}$$

B.C.'s at $x = 0$ $\quad u(0) = 0.001$ m/s

at $x = 1$m $\quad u(l) = 0 \quad (\infty)$

or $\dfrac{p(l)}{u(l)} = Z_n$ (finite impedance)

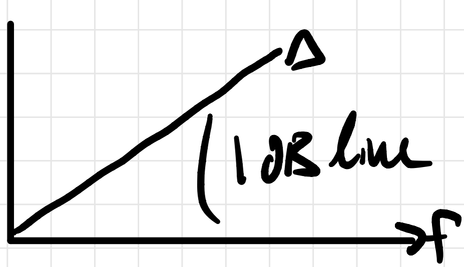
$$I = \frac{1}{2} \mathrm{Re}\{pu^*\}$$

$$\frac{\mathrm{Im}\{P_1 P_2^*\}}{2\omega\rho_0\Delta} \leftarrow \text{Two-Mic intensity...}$$

$$MSP = \frac{pp^*}{2} \qquad MSU = \frac{uu^*}{2}$$

plot $10\log\left(\frac{I_m}{I_t}\right)$

finite impedance case only...



True Radial Intensity $= \frac{1}{2} \mathrm{Re}\{p u_r^*\}$

$$u_r = -\frac{1}{j\omega\rho_0}\frac{\partial p}{\partial r}$$

- Phase mismatch error - Low freq limit to an accurate measurement.

Calibration Procedure
- can use less expensive microphones

b) Switching Procedure

Take Geometric Mean of the Cross Spectra

(Noise Source Must be Stationary)

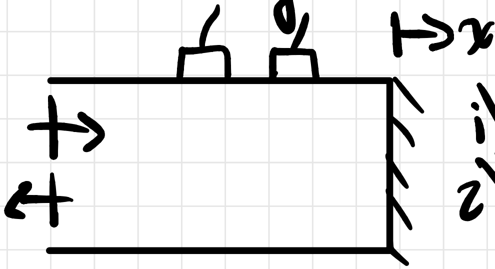
Problems
- lack of Stationarity
- Twice the number of measurements

c) Perfectly Matched Mics + Instrumentation
- easy approach
- expensive approach

} still necessary to calibrate to ensure an accurate result

FDE: High freq limit
Phase Mismatch: Low freq Limit
NFE: How Close to source

1.5.4 Reactivity Error



1) hard (rigid) termination
2) perfectly absorbing
$Z_n = \rho_0 c$

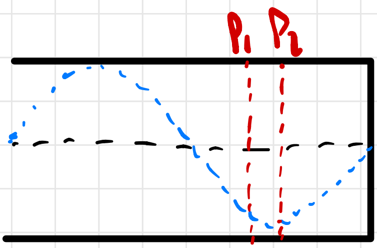
$$p(x) = Ae^{-jkx} + Be^{jkx}$$
$$u(x) = \frac{A}{\rho_0 c}e^{-jkx} - \frac{B}{\rho_0 c}e^{jkx}$$

1) hard termination

$u(x=0) = 0$ (at termination)

so $B = A$... perfect reflection

$$p(x) = A(e^{jkx} + e^{-jkx})$$
$$= 2A\cos kx$$



Pure Standing Wave

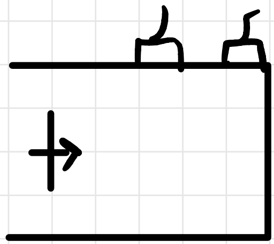
$I \propto \sin(\phi_1 - \phi_2)$
$\Rightarrow I = 0$

Phase difference in this case is either 0 or 180 because of a pure standing wave

ii) Perfectly Absorbing Case ($\rho_0 c$ termination)

no reflection...

$$p(x) = Ae^{-jkx}$$



$$\delta\phi = kx_1 - kx_2 = k\Delta$$

Phase difference $\phi_1 - \phi_2 = \delta\phi$
depends on the environment

$$I_m = \frac{Im\{P_1 P_2^*\}}{2\omega\rho_0\Delta} = \frac{|P_1||P_2|}{2\omega\rho_0\Delta}\sin(\delta\phi + \underbrace{\delta\alpha}_{\text{phase mismatch}})$$

- Sound field itself affects the measurement accuracy
- difficult to make a good measurement when $\delta\phi$ is small

Reactivity Index

Reactivity is the degree to which the sound field consists of non-propagating components

Residual Intensity

$L_I = 10 \log_{10}(I/I_{ref})$ if $I = 0 \Rightarrow L_I = -\infty$

but L_p = something ← hard termination

Absorbing termination

$$L_p = L_I$$

Residual Intensity, $R_I = L_p - L_I$

for hard termination... $R_I = \infty$ bad
for absorbing termination... $R_I = 0$ good

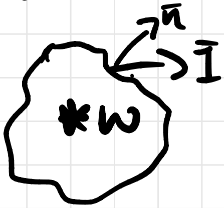
Rule of Thumb... $R_I > 15$ dB
it is difficult to make good measurement

Example of a "Quality Indicator"
in ASTM/ANSI

Conclusion – Check Quality Indicators to
ensure a valid and defensible
Measurement.

1.6 Applications of Acoustic Intensity Method

1.6.1 Sound Power Measurement



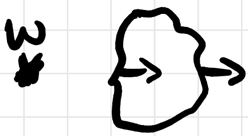
$$W = \int_S \bar{I}\cdot\bar{n}\, dS$$

$$= \int_S I_n\, dS$$

normal component of intensity

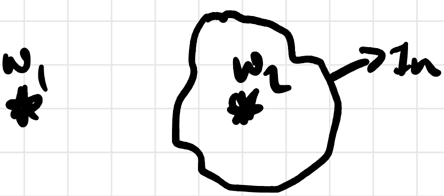
Sound Power, W
when surface completely encloses the source

if surface does not enclose

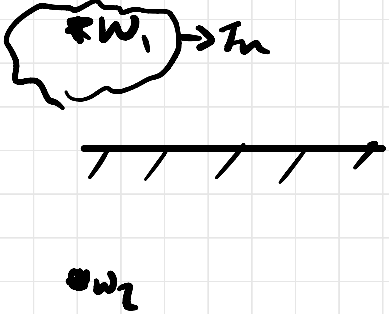


=> $W = \int_S I_n\, dS = 0$

if the source is exterior

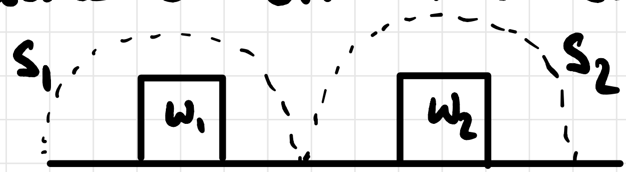


$$\int_S I_n\, dS = W_2$$



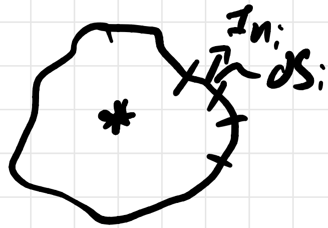
$$\int_S I_n \, dS = W_1$$

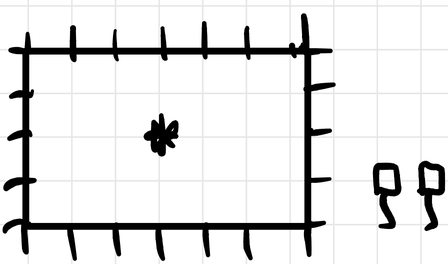
- Can measure sound power even in the presence of reflections
- Can measure sound power even in the presence of other noise sources



i) point wise measurement

$$W = \int_S I_n \, dS = \sum_{i=1}^{n} I_{n_i} \, dS_i$$

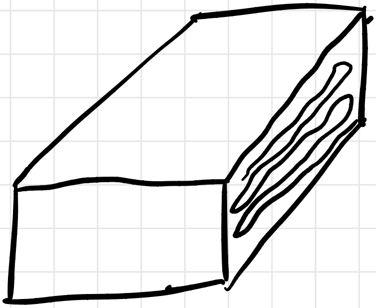




Check accuracy by progressively increasing number of measurements until convergence is achieved

Pro - accuracy
Con - time-consuming

ii) Scanning Approach
hold probe normal to surface and continually average the cross-spectra

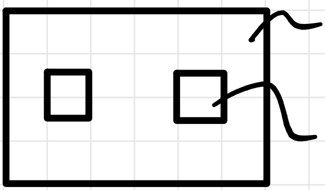


Pro - faster
Con - Difficult to quantify accuracy

1.6.2 Source Ranking + Identification

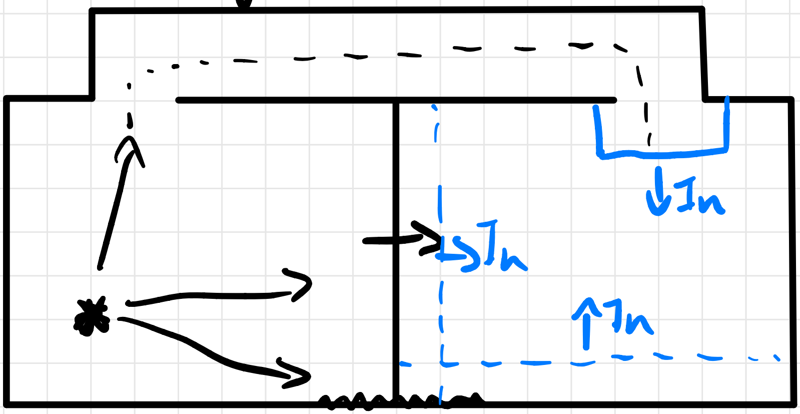
form surfaces that enclose individual sources of a composite source

- component sound
- guide noise control



transportation
- planes

1.6.3 Ranking of Noise Paths



- measure sound power transmitting through various paths...
  - to guide noise control application

1.6.4 Measurement of Impedance and absorption

$$Z_n = \frac{p(0)}{u(0)}$$

Assume Sound field is planer near the surface

$$Z_n(\delta) = \frac{p(\delta)}{u(\delta)}$$ δ is negative...

$$p(x) = Ae^{-jkx} + Be^{jkx}$$
$$u(x) = \frac{A}{\rho_0 c} e^{-jkx} - \frac{B}{\rho_0 c} e^{jkx}$$
$$p(\delta) = Ae^{-jk\delta} + Be^{jk\delta}$$
$u(\delta)$ ...
$$Z_n(\delta) = \frac{(Ae^{-jk\delta} + Be^{jk\delta})}{(Ae^{-jk\delta} - Be^{jk\delta})} \cdot \rho_0 c \cdot \frac{\frac{1}{A}}{\frac{1}{A}}$$
$$Z_n(\delta) = \frac{e^{-jk\delta} + Re^{jk\delta}}{e^{-jk\delta} - Re^{jk\delta}} \cdot \rho_0 c$$
$$R = \frac{Z_n(0) - \rho_0 c}{Z_n(0) + \rho_0 c}$$
Algebra ...
$$Z_n(0) = \frac{\rho_0 c Z_n(\delta) + j \rho_0 c \tan k\delta}{\rho_0 c + j Z_n(\delta) \tan k\delta}$$
Acoustic impedance transfer formula

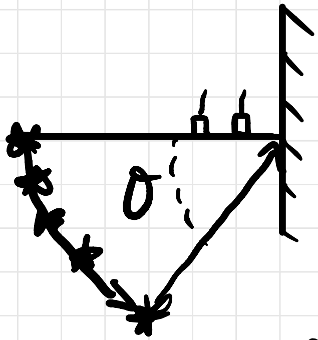
Procedure - Measure $p(\delta)$ + $u(\delta)$
using two-mic probe ... the transfer
$Z_n(\delta) \rightarrow Z_n(0)$

Then find reflection coefficient by

$$R = \frac{Z_n(0) - \rho_0 c}{Z_n(0) + \rho_0 c} \Rightarrow \alpha = 1 - |R|^2$$

... and find absorption...

Allows in situ measurement



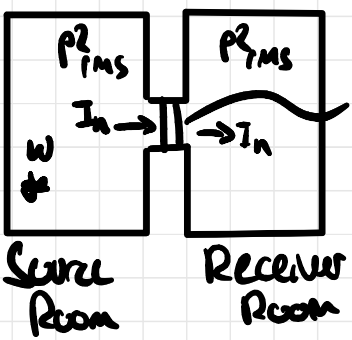
$Z_n(\theta)$

if Z_n is a function of angle of incidence

can vary angle of source from sample

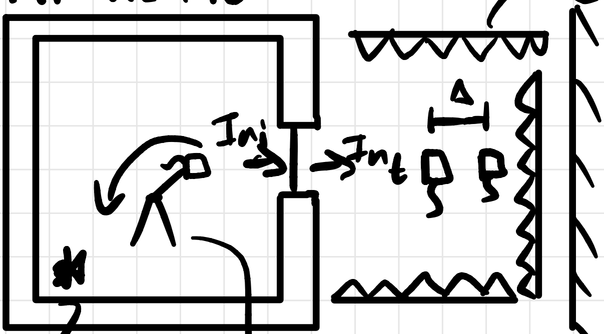
if $\theta \leq 60°$, this simple approach works

1.6.5 Measurement of transmission loss



barrier under test

At Herrich



foam wedges create an anechoic space to reduce reactivity of that field

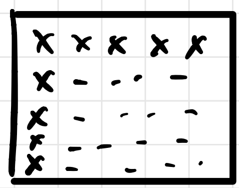
source

mic on rotating boom

Incident Sound Power

$$W_i = \frac{P_{rms}^2}{4\rho_0 c} \cdot S_p$$

panel area



5x5 array

Transmitted Sound Power

25 intensity measurements to give transmitted sound power

space averaged transmitted intensity

$$W_t = \tilde{I}_{n_t} S_p$$

Power Transmission Coefficient

$$T_\pi = \frac{\tilde{I}_{n_t} S_p}{\frac{P_{rms}^2 S_p}{4\rho_0 c}} \Rightarrow T_L = 10\log_{10}\left(\frac{1}{T_\pi}\right)$$

Quick Summary -

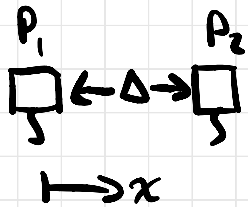
- Acoustic Intensity commonly means
  Time-Averaged Acoustic Intensity

$$I = \frac{1}{2}\mathrm{Re}\{p\bar{u}^*\} \Rightarrow I_x = \frac{1}{2}\mathrm{Re}\{p u_x^*\}$$

  vector quantity

- Particle Velocity
  - direct surface measurements
  - ultrasonic procedure
  - Holographic procedure
  - two-microphone cross-spectral approach
    - finite difference approximation to pressure gradient

$$I_x = \frac{\mathrm{Im}\{P_1 P_2^*\}}{2\omega\rho_0\Delta}$$



- Sources of Error in 2 Mic Method
  - finite difference error
    - high frequency limit given fixed Δ

$$\Delta < \frac{1}{10}\lambda$$

- nearfield error
    - limits how close you can be to the source
- Phase Mismatch
    - low frequency limit
    - can be corrected by careful calibration
- Reactivity Error
    - Residual Intensity

      $R_I = L_p - L_I$

Applications
- Sound Power