ME413 HW 02

Benjamin Masters

TOTAL POINTS

95 / 100

QUESTION 1

1 Q1 **15** / **20**

- **- 0 pts** Correct
- **- 5 Point adjustment**

QUESTION 2

2 Q2 **20** / **20**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

QUESTION 3

3 Q3 **20** / **20**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

QUESTION 4

4 Q4 **20** / **20**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

QUESTION 5

5 Q5 **20** / **20**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

1 Q1 **15 / 20**

- **- 0 pts** Correct

- 5 Point adjustment

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2 Q2 **20 / 20**

- **- 0 pts** Correct

+ 1 Point adjustment

gradescope

3 Q3 **20 / 20**

- **- 0 pts** Correct

+ 1 Point adjustment

gradescope

4 Q4 **20 / 20**

- **- 0 pts** Correct

+ 1 Point adjustment

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5 Q5 **20 / 20**

- **- 0 pts** Correct

+ 1 Point adjustment

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Question 1 (20 points)
Express the following complex numbers in the complex exponential form $A\,e^{i\theta}$

(i) $\left(2-\sqrt{5}i\right)(3+2i)$

(ii) $\dfrac{1}{\sqrt{a+ib}}$

(iii) $\left(a-ib\right)^n$

(iv) $(3i)^2+i+6$

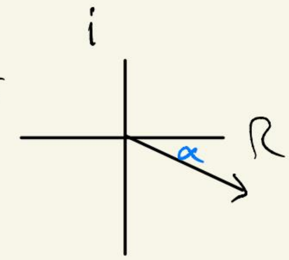
i) $(2-\sqrt{5}\,i)(3+2i) = 6+4i-3\sqrt{5}\,i+2\sqrt{5}$

$6+2\sqrt{5}+i(4-3\sqrt{5}) \approx 10.5-2.7i \approx$

$\alpha = \tan^{-1}(-2.7/10.5) = 0.252\text{ rad}$

$\theta = 2\pi - \alpha = 6.03\text{ rad}$

$r = \sqrt{10.5^2+2.7^2} = 10.84$



$\boxed{10.84\,e^{6.03i}}$

ii) $\dfrac{1}{\sqrt{a+ib}}$ $\quad r = \dfrac{1}{|(a+bi)^{.5}|}$ $\quad \theta = \tan^{-1}(b/a)\cdot -.5$

$\boxed{\dfrac{1}{|(a+bi)^{.5}|}\cdot e^{-i\tan^{-1}(b/a)/2}}$

iii) $(a-ib)^n \Rightarrow r = |(a-ib)^n|$ $\quad \theta = \tan^{-1}(b/a)n$

$\boxed{|(a-ib)^n|\,e^{i\tan^{-1}(b/a)\cdot n}}$

iv) $-9+i+6 = i-3$ $\quad \alpha = \tan^{-1}(1/-3) = -.322$

$r = \sqrt{-3^2+1^2} = \sqrt{10}$

$\theta = \pi + \alpha = 2.82$

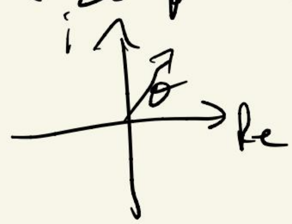
$\boxed{\sqrt{10}\,e^{2.82i}}$

Question 2 (20 points)
Describe, with the aid of a sketch when necessary, each of the following
(i) Spring force, damping force, inertia force, excitation
(ii) Initial conditions, static equilibrium position
(iii) Periodic motion, frequency, period, natural frequency, resonance
(iv) Underdamped system, critically damped system and overdamped system
(v) Amplitude, phasor, phase angle

i) Spring force: The force exerted on an object by a spring attached to it, can be either in compression or tension.
Damping force: A force exerted on an object in an attempt to restrict the motion of the object.
Excitation: A force exerted on an object that causes a response in motion.
ii) Initial conditions: The known starting quantities of a system in motion that can be used to solve for constants in the equation of motion.
Static Equilibrium: The case when the forces on an object act to cancel eachother out and the object is stationary.
iii) Periodic motion: Motion that repeats over a set amount of time.
Frequency: the amount of times a periodic motion repeats over a unit time.
Period: The amount of time it takes for a periodic motion to go through one cycle.
natural frequency: frequency at which a system oscillates with no external force applied
resonance: when a force at a frequency close to the natural frequency is applied, an increase in amplitude is observed
iv) Underdamped system: a damped oscillating system that will continue to oscillate about the equilibrium before reaching it.
Critically damped: a system that reaches equilibrium without oscillating or over shooting.
Overdamped: a system that reaches equilibrium after overshooting the equilibrium position.

v) Amplitude: the maximum displacement a system travels from equilibrium when oscillating.

Phasor: a way to represent oscillations on a 2d plane consisting of a real and imaginary axis.



Phase angle: the angular displacement from equilibrium

Question 3 (20 points)

The sound field at a point has two components:

$$p = 4\cos(\omega t - \pi/3) + 3\sin(\omega t + \pi/4)$$

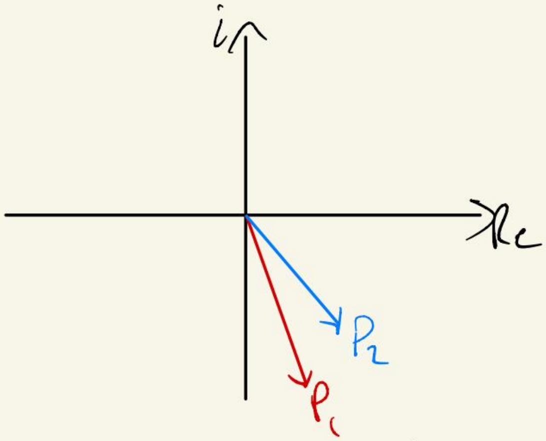
(i) Sketch a phasor diagram to represent these two components.

(ii) Determine the total sound pressure (real and imaginary parts) at the point.

(iii) What are the peak pressure and the phase of the sound field?

$$p = 4\cos(\omega t - \pi/3) + 3\sin(\omega t + \pi/4)$$

i)



$P_1 = 4\cos(\omega t - \pi/3)$

$P_2 = 3\sin(\omega t + \pi/4)$

$= 3\cos(\omega t - \pi/4)$

ii) $Re[P_1] = 4\cos(\pi/3) = 2$ $\quad Re[P_2] = 3\cos(\pi/4) = 2.12$

$P_{1i} = 4\sin(\pi/3) = -3.464$ $\quad P_{2i} = 3\sin(\pi/4) = -2.12$

$$\boxed{P_1 + P_2 = 4.12 - 5.59i}$$

iii) peak pressure $= A = \sqrt{4.12^2 + 5.59^2} = \boxed{6.94 = A}$

phase: $\tan^{-1}(5.59/4.12) = \boxed{\varphi = -0.936 \text{ rad}}$

Question 4 (20 points)

In the spring-mass-dashpot system discussed in the class, we have weight W = 50 N, spring constant k = 0.30 N/cm, and damping ratio ξ = 0.35. Find

(i) the natural frequency,

(ii) the coefficient of viscous damping C of the system.

(iii) What is the additional viscous damping needed to yield the critically damped system?

i) $M = W/g = 50N / 9.81 m/s^2 = 5.10 kg$

$\omega_n = \sqrt{k/m} = \sqrt{\frac{30 N/m}{5.10 kg}} = \boxed{2.42 rad = \omega_n}$

ii) $C = 2\xi\omega_n M = 2(.35)(2.42 rad)(5.10 kg) \boxed{= 8.64 N\text{-}s/m}$

iii) C_c when $\xi = 1$ $\quad C_c = 2\sqrt{KM} = 2\sqrt{30 N/m \cdot 5.10 kg} = 24.74$

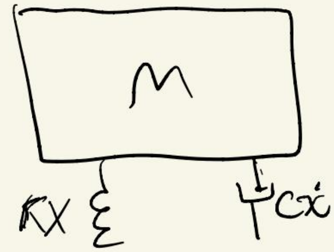
$\Delta C = C_c - C = 24.74 - 8.64 = \boxed{16.10 Ns/m = \Delta c}$

Question 5 (20 points)

In the system of spring-mass-damper, the mass m of 4 kg, the stiffness of the spring k is 50 N/cm and the coefficient of damping C is to be determined. Set up the equation for the system response, which describes the position of the mass as a function of time. If the mass is initially displayed with zero initial velocity. It is observed that the amplitude of the oscillation decreases to 0.25 of the initial value after 5 consecutive **amplitudes**. Determine

(i) the damping ratio of the damper ξ, and
(ii) the coefficient of damping C of the damper.

Suppose now the mass is displaced 15 cm and then suddenly released from rest with zero initial velocity. What are the position, velocity and acceleration of the mass at time $t = 1$ s? (Hint: Find the equation for the velocity and acceleration as a function of time.)



$m = 4\,kg \quad k = 50\,N/cm = 5000\,N/m \qquad C = ? \quad \xi = ?$

i) $\Delta = \dfrac{2\pi\xi}{\sqrt{1-\xi^2}} \qquad \Delta = \ln\left(\dfrac{x_1}{x_2}\right) = \ln\left(\dfrac{1}{.25}\right) = \ln(4) = 1.386$

5 amplitudes = 2 periods?? $\Rightarrow T = 2T_d$ so $\Delta = 2\left(\dfrac{2\pi\xi}{\sqrt{1-\xi^2}}\right)$

$\left(1.386\sqrt{1-\xi^2}\right)^2 = (4\pi\xi)^2 \Rightarrow 157.91\,\xi^2 = 1.921(1-\xi^2)$

$\xi^2(157.91 + 1.921) = 1.921 \qquad \xi^2 = \dfrac{1.921}{159.8} \Rightarrow \boxed{\xi = 0.1096}$

ii) $C = 2\xi\omega_n m = 2(.1096)(\sqrt{5000/4})(4) = \boxed{31.0\ N\text{-}s/m = C}$

b) $X_0 = 15\,cm = .15\,m \qquad \dot{x}(0) = 0 \qquad$ at $t=1$: $x = ?, \dot{x} = ?, \ddot{x} = ?$

$\omega_n = \sqrt{5000/4} = 35.36\,rad/s$

$\omega_d = \omega_n\sqrt{1-\xi^2} = 35.15\,rad/s$

$x(t) = X_0 e^{-\xi\omega_n t}\cos(\omega_d t - \varphi)$

$x(0) = .15\,m = .15\, e^{-\xi\omega_n t}\cos(\omega_d t - \varphi) = 1$

$\cancel{e^{-(.1096)(35.36)(0)}} \cdot \cos(\omega_d(0) - \varphi) = 1$

$\cos(-\varphi) = 1 \Rightarrow \varphi = 0$

$x(1) = .15\,m\, e^{-(.1096)(35.36)(1)} \cdot \cos(35.15) = \boxed{x(1) = -.017\,m}$

$\dot{x}(t) = -\xi\omega_n X_0 e^{-\xi\omega_n t}\cos(\omega_d t) - \omega_d X_0 e^{-\xi\omega_n t}\sin(\omega_d t)$

$\dot{x}(1) = -(.1096)(35.36)(.15)e^{-.1096\cdot 35.36}\cos(35.15) - 35.15(.15)e^{-.1096\cdot 35.36}\sin(35.15)$

$\boxed{\dot{x}(1) = 1.48\,m/s}$

$$\ddot{x}(t) = \zeta^2 \omega_n^2 X_0 e^{-\zeta\omega_n t}\cos(\omega_d t) + \zeta\omega_n\omega_d X_0 e^{-\zeta\omega_n t}\sin(\omega_d t)$$
$$+ \zeta\omega_d\omega_n X_0 e^{-\zeta\omega_n t}\sin(\omega_d t) - \omega_d^2 X_0 e^{-\zeta\omega_n t}\cos(\omega_d t)$$

$$\ddot{x}(1) = (.1096)^2(35.36)^2(.15)e^{-.1096\cdot 35.36}\cos(35.15) + 2(.1096)(35.36)(35.15)(.15)e^{-.1096\cdot 35.36}$$
$$\cdot\sin(35.15) - (35.15)^2(.15)e^{-(.1096)(35.36)}\cos(35.15)$$
$$= -.039 - .473 + 5.19 = \boxed{\ddot{x}(1) = 2.67\,m/s^2}$$