ME413 HW 07

Benjamin Masters

TOTAL POINTS

147 / 150

QUESTION 1

1 Q1 **25 / 25**

- **- 0 pts** Correct
- **+ 1** Point adjustment

QUESTION 2

2 Q2 **22 / 25**

- **- 0 pts** Correct
- **- 3** Point adjustment

QUESTION 3

3 Q3 **25 / 25**

- **- 0 pts** Correct
- **+ 1** Point adjustment

QUESTION 4

4 Q4 **25 / 25**

- **- 0 pts** Correct
- **+ 1** Point adjustment

QUESTION 5

5 Q5 **50 / 50**

- **- 0 pts** Correct
- **+ 1** Point adjustment

1 Q1 **25 / 25**

- **- 0 pts** Correct

+ 1 Point adjustment

2 Q2 **22 / 25**

- **- 0 pts** Correct

- 3 Point adjustment

3 Q3 **25 / 25**

- **- 0 pts** Correct

+ 1 Point adjustment

4 Q4 **25 / 25**

- **- 0 pts** Correct

+ 1 Point adjustment

5 Q5 **50 / 50**

- **- 0 pts** Correct

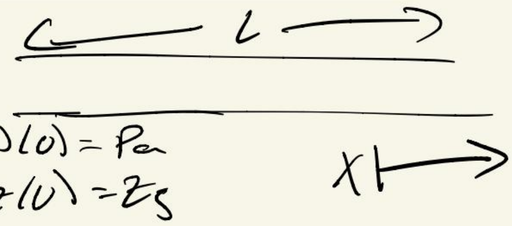
+ 1 Point adjustment

Question 1 (25 points)
A pipe of length L has a source located at $x = 0$. It is known that the source has a characteristic of Z_s (i.e. $Z(0) = Z_s$). It is also known by measurement that the pressure is $p(0) = p_a$. Find the pressure and particle velocity at $x = L$. Express your answer in terms of Z_s and p_a.

Benjamin Mastos

boundary conditions:

$$p(0) = p_a, \quad z(0) = Z_s$$



$$p(x) = Ae^{-ikx} + Be^{ikx}$$

$$u(x) = \frac{A}{\rho_0 c_0} e^{-ikx} - \frac{B}{\rho_0 c_0} e^{ikx}$$

$$uZ = p \qquad u = p/Z$$

(1) $p(0) = p_a = Ae^{-ik(0)} + Be^{ik(0)} \Rightarrow p_a = A + B$

(2) $u(0) = \frac{p_a}{Z_s} = \frac{A}{\rho_0 c_0} e^{-ik(0)} - \frac{B}{\rho_0 c_0} e^{ik(0)} = \frac{A}{\rho_0 c_0} - \frac{B}{\rho_0 c_0} = p_a/Z_s$

(1)->(2) $A = p_a - B \Rightarrow \frac{p_a}{Z_s} = \frac{(p_a - B)}{\rho_0 c_0} - \frac{B}{\rho_0 c_0}$

$$\frac{p_a}{Z_s}\rho_0 c_0 = p_a - 2B$$

$$B = \frac{p_a}{2}\left(1 - \frac{\rho_0 c_0}{Z_s}\right) \Rightarrow A = p_a - \frac{p_a}{2}\left(1 - \frac{\rho_0 c_0}{Z_s}\right)$$

$$A = p_a\left(1 - \frac{1}{2} + \frac{\rho_0 c_0}{2Z_s}\right) = \frac{p_a}{2}\left(1 + \frac{\rho_0 c_0}{Z_s}\right) = A$$

$$p(x) = \frac{p_a}{2}\left(1 + \frac{\rho_0 c_0}{Z_s}\right)e^{-ikx} + \frac{p_a}{2}\left(1 - \frac{\rho_0 c_0}{Z_s}\right)e^{ikx}$$

$$u(x) = \frac{p_a}{2\rho_0 c_0}\left(1 + \frac{\rho_0 c_0}{Z_s}\right)e^{-ikx} - \frac{p_a}{\rho_0 c_0}\left(1 - \frac{\rho_0 c_0}{Z_s}\right)e^{ikx}$$

$$p(L) = \frac{P_a}{2}\left(1 + \frac{\rho_0 c_0}{Z_s}\right)e^{-ikL} + \frac{P_a}{2}\left(1 - \frac{\rho_0 c_0}{Z_s}\right)e^{ikL}$$

$$u(L) = \frac{P_a}{2}\left(\frac{1}{Z_s} + \frac{1}{\rho_0 c_0}\right)e^{-ikL} - \frac{P_a}{2}\left(\frac{1}{\rho_0 c_0} - \frac{1}{Z_s}\right)e^{ikL}$$

Question 2 (25 points)
An acoustic signal consisting of 400 Hz plane wave is normally incident to an acoustical tile surface having a complex impedance of 1500 – i 3000 MKS Rayls.
(a) Find the standing wave ratio in the resulting pattern of standing wave.
(b) Determine the location of first four nodes.

You may take the characteristic impedance of air as 400 MKS rayl and the sound speed in air as 340 m/s.

$\rho_0 c_0 = 400$ MKS Rayls
$c_0 = 340$ m/s

$f = 400$ Hz $\quad Z_s = 1500 - i\,3000$ MKS Rayls

$$\delta_s = 2\left(\frac{2\pi f}{c_0}\right)\cdot y_{max,1}$$

$$\zeta = \frac{Z_s}{\rho_0 c_0} = \frac{1500}{400} - i\frac{3000}{400} \quad \frac{\text{MKS Rayls}}{\text{MKS Rayls}}$$

$$\zeta_s = 3.75 - i\,7.5$$

$$R_s = \frac{\zeta_s - 1}{\zeta_s + 1} = \frac{(2.75 - i\,7.5)}{4.75 - i\,7.5}\cdot\frac{(4.75 + i\,7.5)}{4.75 + i\,7.5}$$

$$R_s = \frac{13.0625 + 20.625i - 35.625i + 56.25}{22.5625 + 56.25}$$

$$R_s = \frac{69.3125 - i15}{78.8125} = 0.88 - i(.19)$$

$$|R_s| = \sqrt{.88^2 + .19^2} = .90 \qquad \delta_s = \tan^{-1}(-.19/.88) = -0.213 \text{ rad}$$

$$S^2 = \frac{(1+.9)^2}{(1-.9)^2} = 361 \qquad \boxed{S = 19}$$

$$-0.213 \text{ rad} = 4\pi\frac{(400 \text{ Hz})}{340 \text{ m/s}}\cdot y_{max,1} \Rightarrow y_{max,1} = -.0144 \text{ rad}$$

location of first four nodes:

$$\boxed{\begin{array}{ll} 1: -.0144 \text{ rad} & 2: -.0144 + \pi \text{ rad} \\ 3: -.0144 + 2\pi \text{ rad} & 4: -.0144 + 3\pi \text{ rad} \end{array}}$$

Question 3 (25 points)
The speed of sound in water is 1480 m s^{-1} and the density of water is 1000 kg m^{-3}. Consider a series of plane waves of frequency of 2960 Hz normally incident to a concrete wall. The pattern of standing waves results in a peak pressure amplitude of 30 Pa and a pressure amplitude of 10 Pa at the nearest pressure node at a distance of 50 cm from the wall.
(a) What is the ratio of the intensity of the reflected waves to that of the incident wave?
(b) Find the specific acoustic impedance of the wall.

$C_0 = 1480 m/s \quad \rho_0 = 1000 Kg/m3 \quad f = 2960 Hz \quad A_{pk} = 30 Pa = P_{max}$

$A_r = 10 Pa = P_{min}$

node 1 = 50cm = .5m = X

$Z = \rho_0 C_0 = 1480 m/s \cdot 1000 Kg/m^3 = 1480 \frac{kN \cdot s}{m^3}$

$S^2 = \frac{P_{max}^2}{P_{min}^2} = 9 \qquad S = 3 \qquad |R_s| = \frac{S-1}{S+1} = |R_s| = \frac{2}{4} = \frac{A}{B}$

$I = \frac{P^2}{\rho_0 c_0} \Rightarrow \frac{I_R}{P_R^2} = \frac{I_I}{P_I^2} \Rightarrow \frac{P_R^2}{P_I^2} = \frac{I_R}{I_I}$

using $|R_s| = \left(\frac{A}{B}\right) = \left(\frac{P_2}{P_1}\right)$: $\left(\frac{A}{B}\right)^2 = \boxed{\frac{I_R}{I_I} = \frac{1}{4}}$

b) $Z_s = \zeta_s \rho_0 c_0 \qquad \zeta_s = \frac{1+R_s}{1-R_s}$

$R_s = |R_s| e^{i\delta_s} \qquad \delta_s = 2kx \qquad k = \frac{2\pi f}{c_0} = 12.57 rad/m$

$\delta_s = 2(12.57 rad/m)(0.5m) = 12.57 rad$

$R_s = \frac{1}{2} e^{12.57i} \qquad \zeta = \frac{1 + \frac{1}{2}e^{12.57i}}{1 - \frac{1}{2}e^{12.57i}}$

$\zeta = \frac{1 + \frac{1}{2}(\cos(12.57) + i\sin(12.57))}{1 - \frac{1}{2}(\cos(12.57) + i\sin(12.57))} = \frac{1 + .5 + .004i}{1 - .5 - .004i}$

$$\Re = \frac{1.5+.004i}{0.5-.004i}\cdot\frac{(0.5+.004i)}{(0.5+.004i)} = \frac{.75+.008i-0}{.25+0}$$

$$\frac{.75+.008i}{.25} = 3+.032i = \Re_s$$

$$Z_s = \Re_s \rho_0 c_0 = (3+.032i)(1480\,kN{\cdot}s/m^3)$$

$$\boxed{Z_s = 4.44\times10^6+(4.736\times10^4)\,i \text{ MKS Rayls}}$$

Question 4 (25 points)
The total sound field is due to two coherent sound sources (p_1 and p_2) of same frequency. Suppose

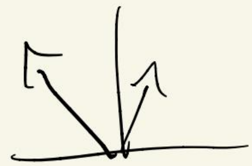
$p_1 = (2 + 3\,i) \times 10^{-4}$ Pa,

and

$p_2 = 4\,e^{i\,3\pi/4} \times 10^{-4}$ Pa.

(a) What is the total pressure? Express your answer in both rectangular and polar forms.

(b) What is the total rms pressure and its sound pressure level?



$P_1 = (2+3i)\times10^{-4}\,Pa$ $\quad P_{1,Re} = 2\times10^{-4}\,Pa$

$P_{1,I} = 3\times10^{-4}i\,Pa$

$P_2 = 4e^{i3\pi/4}$ $\quad |P_2| = 4$ $\quad \varphi_{P_2} = 3\pi/4$

$P_2 = 4(\cos(3\pi/4) + i\sin(3\pi/4)) = -2.83 + 2.83i = P_2$

$P_{2,Re} = -2.83\times10^{-4}$ $\quad P_{2,I} = 2.83\times10^{-4}$

$P_{t,Re} = 2\times10^{-4} - 2.83\times10^{-4} = -.83\times10^{-4}\,Pa$

$P_{t,I} = 3\times10^{-4}i + 2.83\times10^{-4}i = 5.83\times10^{-4}\,i$

$|P_t| = \sqrt{-.83^2 + 5.83^2} = 5.89\times10^{-4}\,Pa$

$\varphi_{pt} = \tan^{-1}(5.83/-.83) = 1.43\,rad$

$\boxed{P_t = (-.83 + 5.83i)\times10^{-4}\,Pa}$

or $\boxed{P_t = (5.89e^{1.43i})\times10^{-4}\,Pa}$

or $\boxed{P_t = 5.89\times10^{-4}(\cos(1.43) + i\sin(1.43))\,Pa}$

b) $P_{rms} = \frac{|P_t|}{\sqrt{2}} = \boxed{4.16\times10^{-4}\,Pa = P_{rms}}$ $\quad P_{re} = 20\,\mu Pa$

$L_p = 20\log(4.16\times10^{-4}\,Pa / 2\times10^{-5}\,Pa) = \boxed{26.4\,dB = L_p}$

Question 5: design of a reactive silencer (50 points)
A reactive muffler is to be designed to produce a transmission loss of 20 dB at 150 Hz. The inlet and outlet and outlet pipes are 60 mm in diameter and the average temperature of the exhaust gas is 100°C.

(a) Show that the transmission loss of the reactive muffler is given by

$$TL = 10\log\left[1+\tfrac{1}{4}\left(m-1/m\right)^2 \sin^2(kL)\right]$$

where m is the area ratio of the expansion chamber to the inlet/outlet duct.

(b) Determine the size of the expansion chamber – the cross-sectional area A_2 and the length L. You may take the speed of sound at 20°C as 343 m s^{-1}.

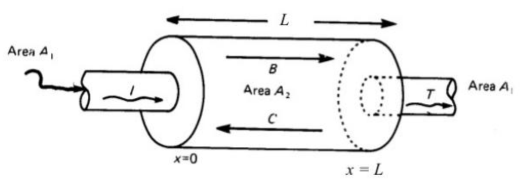


Figure: A typical reactive muffler

(c) If the average air temperature of the exhaust gases has been increased to 150°C, what is the reduction in TL at the same frequency of 150 Hz if the expansion chamber of the same size (as calculated in (b)) is used?

The following formula may be useful in your calculations in part (b):

$$\frac{R}{I}=\frac{i\left[(A_1/A_2)-(A_2/A_1)\right]\sin(kL)}{2\cos(kL)+i\left[(A_1/A_2)+(A_2/A_1)\right]\sin(kL)};\quad \frac{T}{I}=\frac{2e^{ikL}}{2\cos(kL)+i\left[(A_1/A_2)+(A_2/A_1)\right]\sin(kL)}.$$

The symbols used in the above equations have their usual meanings.

$f = 150\,Hz \quad TL = 20\,dB$

$d_1 = 60\,mm \quad T = 100°C$

$A_1 = \pi\left(\frac{60mm}{2}\right)^2 = 900\pi\,mm^2$

$K = \frac{\omega}{C_0} = \frac{2\pi f}{C_0}$

$m = \frac{A_2}{A_1}$

$e^{ikL} = \cos kL + i\sin kL$

a) $$\frac{I}{T} = \frac{2e^{ikL}}{2\cos(kL) + i\left[(A_1/A_2) + (A_2/A_1)\right]\sin kL}$$

$TL = 10\log(W_{inc}/W_t)$

$TL = 10\log\left(\left|\frac{I}{T}\right|^2\right)$ (A)

$$\left|\frac{I}{T}\right|^2 = \left|\frac{\sqrt{\cos^2(kL) + \frac{\sin^2(kL)}{4}(m+1/m)^2}}{\sqrt{\cos^2 kL + \sin^2 kL}}\right|^2$$

$$\left|\frac{I}{T}\right|^2 = \frac{\cos^2 kL + \frac{\sin^2 kL}{4}(m^2+2+1/m^2)}{1}$$

$$= \boxed{\cos^2 kL + \frac{1}{2}\sin^2 kL} + \frac{\sin^2 kL}{4}(m^2 + 1/m^2) \quad (1)$$

looking at highlighted portion only for now $(kL = x)$

$$\cos^2 kL + 1/2\sin^2 kL = \frac{1}{2}\left[1+\cos(2x)\right] + \frac{1}{4}\left[1-\cos(2x)\right]$$

$$= 1/2 + 1/4 + 1/2\cos(2x) - 1/4\cos(2x) = 3/4 + 1/4\cos(2x) =$$

$$= \frac{3}{4} + \frac{1}{4}\left(1 - 2\sin^2 x\right) = 1 - \frac{1}{2}\sin^2 x = \cos^2 kL + \frac{1}{2}\sin^2 kL \quad (2)$$

(2) -> (1)

$$1 - \tfrac{1}{2}\sin^2(kL) + \frac{\sin^2 kL\, m^2}{4} + \frac{\sin^2 kL}{4m^2} = \left|\frac{I}{T}\right|^2$$

$$1 + \frac{\sin^2(kL)}{4}\left(m^2 - 2 + \tfrac{1}{m^2}\right)$$

$$\left|\frac{I}{T}\right|^2 = 1 + \frac{\sin^2(kL)}{4}\left[m - \tfrac{1}{m}\right]^2$$

plugging into (A) $\quad TL = 10\log\left(|I/T|^2\right)$

$$\boxed{TL = 10\log\left(1 + \frac{\sin^2(kL)}{4}\cdot\left[m - \tfrac{1}{m}\right]^2\right)}$$

b) $TL = 20\,dB$

using $c_0 = 343\,m/s + 0.6\,\frac{m/s}{°C}\cdot(100°C - 20°C) = 391\,m/s$

$c_0 = 391\,m/s \qquad k = 2\pi f/c_0 = 2\pi\cdot 150Hz/391\,m/s = 2.41\,rad/m$

$$20\,dB = 10\log\left[1 + \tfrac{1}{4}(m - \tfrac{1}{m})^2 \sin^2(kL)\right]$$

$$10^2 = 1 + \tfrac{1}{4}(m - \tfrac{1}{m})^2\sin^2(kL) = \sqrt{99} = \sqrt{\tfrac{1}{4}(m - \tfrac{1}{m})^2 \sin^2 kL}$$

$\sqrt{99} = \frac{1}{2}(m - \tfrac{1}{m})\sin kL \qquad$ Max TL is where $\sin kL = 1$

$2\sqrt{99} = m - \tfrac{1}{m} \qquad m^2 - 2\sqrt{99}\,m - 1 = 0$

$$\frac{2\sqrt{99} \pm \sqrt{4(99)+4}}{2} \Rightarrow m = 20$$

$$\frac{A_2}{A_1} = 20 \qquad A_2 = 20(900\pi \text{ mm}^2)$$

$$\boxed{A_2 = 18000\pi \text{ mm}^2}$$

$$\sin kL = 1 \qquad kL = \sin^{-1}(1) = 1.57 \text{ rad}$$

$$L = \frac{1.57 \text{ rad}}{2.41 \text{ rad/m}} = \boxed{0.65 \text{ m} = L}$$ for 20 db TL

c) $C_0 = 343 \text{ m/s} + .6 \frac{\text{m/s}}{C}(150C - 20C) = C_0 = 421 \text{ m/s}$

$$k = \frac{2\pi f}{C_0} = \frac{2\pi(150\text{Hz})}{421C} = 2.23 \text{ rad/m}$$

$$TL = 10\log\left[1 + \tfrac{1}{4}\left(20 - \tfrac{1}{20}\right)^2 \sin^2(2.23 \text{ rad/m} \cdot 0.65\text{m})\right)$$

$$\boxed{TL = 19.98 \text{ dB}}$$