Lecture 1

Friday, August 20, 2021 3:38 PM

Andrew Barnhart

Acoustics - Study of generation, transmission, and reception of energy in the form of vibrational waves in matter

Sound - Propagating fluctuations in an elastic medium (20 - 20k Hz)

for sound to propagate, medium must have mass & stiffness

Sources of sound
- vibration of solids
- interaction between flow and solids
- flow turbulence
- thermal generation (localized heat sources)

General Approach

i) derive / identify governing equations
ii) combine to form wave equation
iii) identify possible solutions
iv) Apply boundary conditions to select appropriate solution from all possible

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Types of Waves

transverse wave – displacement is perpendicular to motion of wave

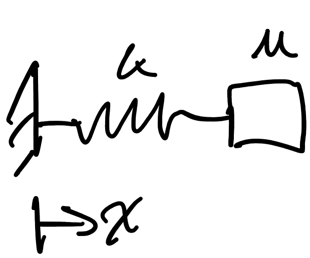
longitudinal / compressional – particles displace with the waves (parallel)

Compare & Contrast
- Wave Propagation
- Modal Approaches

Fundamentals of Vibration

SDOF – Single degree of freedom

1.1 Simple undamped Oscillator



Generally Neglect gravity

Lecture 2

Wednesday, August 25, 2021 1:28 PM

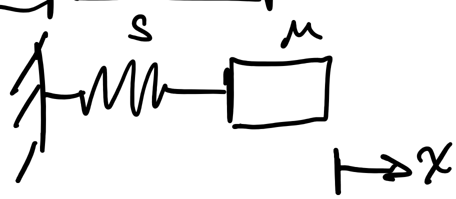
Sound - propagating fluctuations within an elastic medium

must have mass & spring

General Approach

- Derive / Identify Governing Equations
- continue → wave equation
- identify possible solutions
- apply boundary conditions

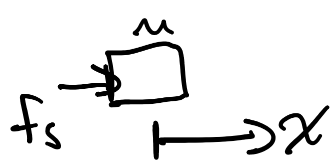
Simple Undamped Oscillator



- free vibration

- to follow Procedure -

FBD:



(i) Equation of motion:

$$F = m \cdot a = m \frac{\partial^2 x}{\partial t^2} \quad (1)$$

(ii) Restoring Force

$$F = -sx \quad (2)$$

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$$F = -sx \quad (2)$$

Sub 2->1

$$-sx = M\frac{\partial^2 x}{\partial t^2} \Rightarrow M\frac{\partial^2 x}{\partial t^2} + sx = 0$$

Normalize to mass

$$\frac{\partial^2 x}{\partial t^2} + \frac{s}{M}x = 0 \quad (3)$$

where $\frac{s}{m} = \omega_n^2$

2nd order ODE
solution has 2 constants

$$x = A_1\cos(\gamma t)$$

Sub into 3

$$\frac{\partial^2 x}{\partial t^2} = -\gamma^2 A_1\cos(\gamma t)$$

$$-\gamma^2 \cancel{A_1\cos(\gamma t)} + \omega_0^2 \cancel{A_1\cos(\gamma t)} = 0$$

$\therefore$ Satisfied if $\gamma^2 = \omega_0^2$

Assume solution is acceptable if

Assume solution is acceptable if
$\gamma^2 = \omega_0^2$ where $\omega_0 = \sqrt{s/\mu}$

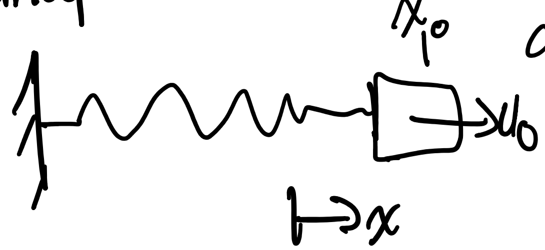
First acceptable solution
$$x = A_1 \cos(\omega_0 t)$$

also works:
$$x = A_1 \sin \gamma t \quad \text{if } \gamma^2 = \omega_0^2$$

complete solution:
$$x = A_1 \cos \omega_0 t + A_2 \sin \omega_0 t$$

Initial Conditions: boundaries in time (as opposed to space)

2 constants: A_1 & A_2 so we need 2 independent conditions



at $t=0$, $x = x_0$, $u = u_0$

e.g. at $t=0$, $x = x_0$ (initial displacement)

$\frac{\partial x}{\partial t} = \dot{x} = u_0$ velocity since $x = A_1 \cos \omega_0 t + A_2 \sin \omega_0 t$;

at $t=0$ $\quad x = A_1 \quad \therefore \quad A_1 = x_0$

Velocity:

$$u = \dot{x} = \frac{dx}{dt} = -\omega_0 A_1 \sin \omega_0 t + \omega_0 A_2 \cos \omega_0 t$$

at $t=0$

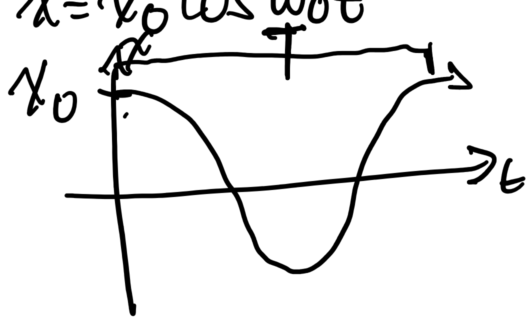
$$u = \omega_0 A_2 = u_0 \quad \text{so} \quad A_2 = \frac{u_0}{\omega_0}$$

$$x = x_0 \cos \omega_0 t + \frac{u_0}{\omega_0} \sin \omega_0 t \quad (4)$$

$$\omega_0 = \sqrt{s/m}$$

if the initial velocity $= 0 = u_0$ $\quad x_0 \neq 0$

$$x = x_0 \cos \omega_0 t$$



T = period [s]
$= 1/f_0$ [Hz]
$= \frac{2\pi}{\omega_0}$ [rad/s]

If initial displacement $= 0 = x_0$
but $u_0 \neq 0$

$$x = \frac{u_0}{\omega_0} \sin \omega_0 t$$

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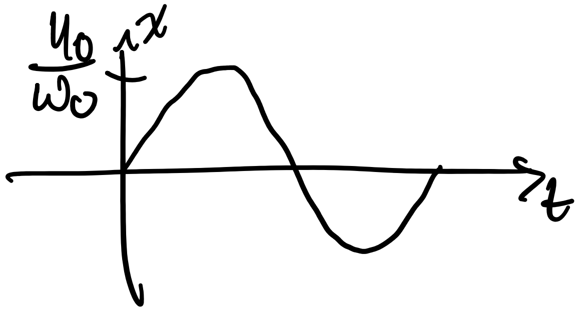


Complete Response when $x_0 \neq 0$ and $u_0 \neq 0$ is the sum of the two solutions

Complex Response

Assumed Solution: $\tilde{x} = \tilde{A} e^{\gamma t}$

$\tilde{x} = x_R + j x_i$

$\tilde{A} = a + jb$

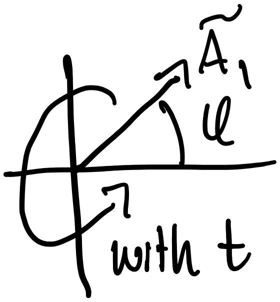
$\mathrm{Re}\{\tilde{x}\} =$ physical soln.

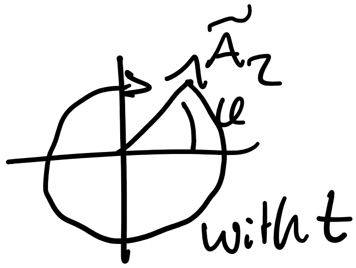
$$\frac{\partial^2 \tilde{x}}{\partial t^2} + \omega_0^2 \tilde{x} = 0 \qquad \tilde{x} = \tilde{A} e^{\gamma t}$$

$\gamma^2 = -\omega_0^2$ for acceptable solution

$$\gamma = \sqrt{-\omega_0^2} = \sqrt{-1} \cdot \omega_0 = \pm j\omega_0 = \gamma$$

$\tilde{A}e^{j\omega_0 t}$ $\tilde{A}e^{-j\omega_0 t}$

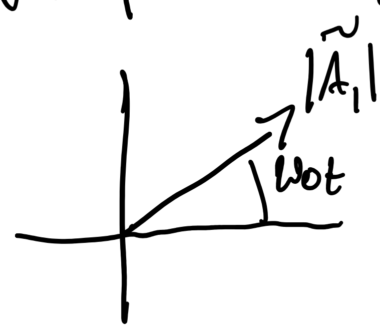


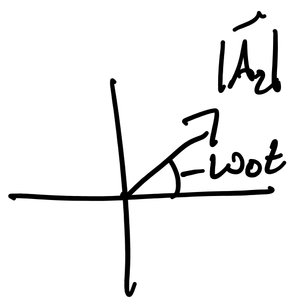


where $\varphi = \omega_0 t$ $(\pm)$

Complete Solution

$\tilde{x} = \tilde{A}_1 e^{j\omega_0 t} + \tilde{A}_2 e^{-j\omega_0 t}$





Physical soln...

projection of rotation onto real axis

$Re\{x\}$

Determine $\tilde{A}_1$ and $\tilde{A}_2$ by applying initial conditions

at $t=0$ $\tilde{x} = x_0 = \boxed{\tilde{A}_1 + \tilde{A}_2}$

at $t=0$ $\tilde{x} = x_0 = \boxed{A_1 + A_2}$

$\dot{\tilde{x}} = u_0 = \boxed{j\omega_0 \tilde{A}_1 - j\omega_0 \tilde{A}_2}$

2 equations and 2 unknowns

$\tilde{A}_1 = \frac{1}{2}\left(x_0 - j\frac{u_0}{\omega_0}\right)$

$\tilde{A}_2 \ \frac{1}{2}\left(x_0 + j\frac{u_0}{\omega_0}\right)$

$A_1 = A_2^*$

$\tilde{A}_1 = a + jb$ } only 2 real constants

$\tilde{A}_2 = a - jb$

Complete Solution

$$\tilde{x} = \frac{1}{2}\left(x_0 - j\frac{u_0}{\omega_0}\right)\underbrace{e^{j\omega_0 t}}_{\cos\omega_0 t + j\sin\omega_0 t} + \frac{1}{2}\left(x_0 + j\frac{u_0}{\omega_0}\right)e^{-j\omega_0 t}$$

$$\tilde{x} = x_0\cos\omega_0 t + \frac{u_0}{\omega_0}\sin\omega_0 t$$

Lecture 3

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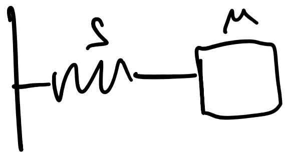


$F=Ma$

$f=sx$

$$\frac{\partial^2 x}{\partial t^2}+\omega_0^2 x=0$$

$$x=A_1\cos\omega_0 t + A_2\sin\omega_0 t$$

Determine constants by applying initial (time) boundary conditions

$\tilde{x}=\tilde{A}e^{\gamma t}$ can be expressed in this form if $\gamma=\pm j\omega_0$

$$\tilde{x}=A_1 e^{j\omega_0 t}+A_2 e^{-j\omega_0 t}$$

$$A_2=A_1^*$$

Can express $e^{j\omega_0 t}$ as

$$\cos\omega_0 t + j\sin\omega_0 t$$

$$\tilde{x}=x_0\cos\omega_0 t+\frac{x_0}{\omega_0}\sin\omega_0 t$$

To compare physical quantity

$$x=\mathrm{Re}[\tilde{x}]$$

$x^2=Ae^{j\omega_0 t}$ displacement

$\frac{\partial\tilde{x}}{\partial t}=\dot{\tilde{x}}=\tilde{u}=j\omega_0 A e^{j\omega_0 t}=j\omega_0\tilde{x}$ velocity

$\frac{\partial^2\tilde{x}}{\partial t^2}=\ddot{\tilde{x}}=\tilde{a}=-\omega_0^2 A e^{j\omega_0 t}=-\omega_0^2\tilde{x}=j\omega_0\tilde{u}$ acceleration

$$\frac{\partial^2 \tilde{x}}{\partial t^2} = \ddot{\tilde{x}} = \tilde{a} = -\omega_0^2 A e \quad = -\omega_0 \quad$$

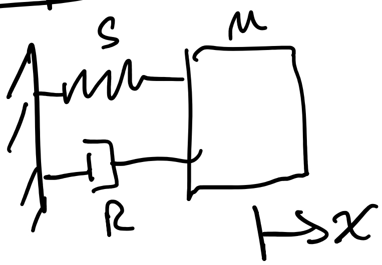
$e^{\boxed{j\omega_0 t}} \to$ phase

can be written as $e^{j\varphi}$

$$\frac{\partial(\varphi)}{\partial t} = \omega_0 = \text{time rate of change of phase}$$

1.2

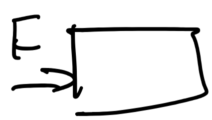
Damped Oscillations



Most realistic systems dissipate energy

1.2.1 Governing Equations

Restoring force



$$F = -sx - R\,\partial x/\partial t$$

EOM: $$M\frac{\partial^2 x}{\partial t^2} = -sx - R\frac{\partial x}{\partial t}$$

$$M\frac{\partial^2 x}{\partial t^2} + R_M\frac{\partial x}{\partial t} + sx = 0$$

$$\frac{\partial^2 x}{\partial t^2} + \frac{R_M}{M}\frac{\partial x}{\partial t} + \frac{s}{M}x = 0$$

$$\frac{\partial^2 x}{\partial t^2} + \frac{R_m}{m}\frac{\partial x}{\partial t} \ldots m$$

$$\frac{\partial^2 x}{\partial t^2} + \frac{R_m}{m}\frac{\partial x}{\partial t} + \omega_0^2 x = 0$$

Solutions

$$\tilde{x} = Ae^{\gamma t}$$

Result:

$$\gamma^2 \cancel{Ae^{\gamma t}} + \gamma(R_m/m)\cancel{Ae^{\gamma t}} + \omega_0^2 \cancel{Ae^{\gamma t}} = 0$$

$$\gamma^2 + \gamma(R_m/m) + \omega_0^2 = 0$$

$$\gamma = \frac{-\frac{R_m}{m} \pm \sqrt{(R_m/m)^2 - 4\omega_0^2}}{2}$$

let $\beta = \frac{R_m}{2m}$

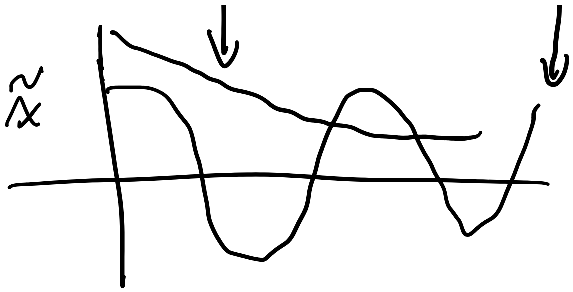
$$\gamma = -\beta \pm j\underbrace{\sqrt{\omega_0^2 - \beta^2}}_{\omega_d^2}$$

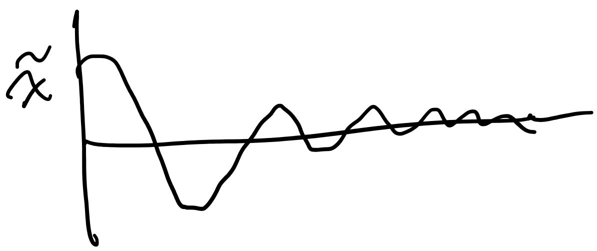
$e^{\gamma t}$ Damped Natural Fq

considering underdamped case
where $\beta^2 < \omega_0^2$

$$\gamma = -\beta \pm j\omega_d$$

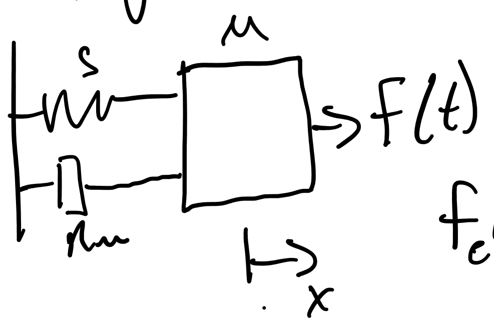
$$\tilde{x} = \tilde{A}e^{-\beta t}e^{\pm j\omega_d t}$$





oscillation at $\omega_d = \sqrt{\omega_0^2 - \beta^2}$

Now apply external Force



$f_e(t) = F\cos\omega t \quad t \geq 0$

$$m\frac{\partial^2 x}{\partial t^2} + \frac{R}{m}\frac{dx}{\partial t} + sx = f(t) \quad (3)$$

inhomogeneous ODE

LaPlace Transform to Solve

- algebraic solution

$x = x_{transient} + x_{steady\text{-}state}$
(homogeneous) (particular)

For real systems

since $R_m > 0$ always

transient solution is negligible as t increases

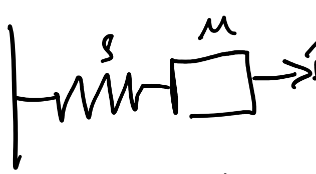
therefore; concentrate on steady state solution

Steady State Solution

Assume complex form for driving force

$\tilde{f}_e = F e^{j\omega t}$ $\quad \omega =$ driving Fq

$Re[\tilde{f}_e(t)] = F\cos\omega t$



$\tilde{f}_e(t) = F e^{j\omega t}$

Linear Systems
- at steady state the system responds at the driving frequency

ω - forcing Fq (you control)

ω_0 - natural Fq (property of system)

$\tilde{f}_e = F e^{j\omega t}$ $\quad \tilde{x} = \tilde{A} e^{j\omega t}$

substitute into (3)

ic

substitute into (3)

$$-\omega^2 m\tilde{A}\cancel{e^{j\omega t}} + j\omega R_m \tilde{A}\cancel{e^{j\omega t}} + s\tilde{A}\cancel{e^{j\omega t}} = F\cancel{e^{j\omega t}}$$

$$\tilde{A} = \frac{F}{-\omega^2 m + j\omega R_m + s}$$

Amplitude of Response is directly proportional to forcing amplitude

$$\tilde{A} = \frac{F}{j\omega(R_m + j(\omega m - \frac{s}{\omega})}$$

Steady State Response of Damped SDOF

$$x = \tilde{A}e^{j\omega t}$$

$$= \frac{Fe^{j\omega t}}{j\omega(R_m + j(\omega m - \frac{s}{\omega}))}$$

physical displacement $= Re[\tilde{x}]$

why pull $j\omega$ out?

$$\tilde{u} = j\omega\tilde{x}$$

$$\tilde{u} = \frac{Fe^{j\omega t}}{R_m + j(\omega m - s/\omega)}$$

Acceleration

$$\tilde{a} = \frac{\partial^2 \tilde{x}}{\partial t^2} = \frac{\partial \tilde{u}}{\partial t} = -\omega^2 \tilde{x} = j\omega \tilde{u}$$

Acoustic Displacements and Velocities

progressive plane waves

$$\left|\frac{p}{u}\right| = \rho_0 c \leftarrow \text{impedence of air}$$

$\rho_0 = 1.2$ kg/m³

$c = 340$ m/s

$p = 1$ Pa $\Rightarrow 94$ dB

$$|u| = \frac{|p|}{\rho_0 c} = \frac{1}{400} = 2.5 \text{ mm/s}$$

$$\omega |x| = |u|$$

$$|x| = \frac{|u|}{\omega} = \frac{2.5 \times 10^{-3}}{2\pi \cdot 1 \times 10^{3}} \approx 0.5 \times 10^{-6} \text{ m} \approx |x|$$

Lecture 4

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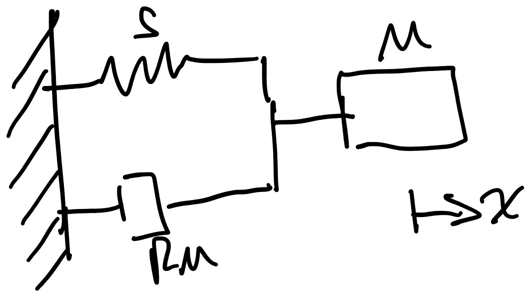
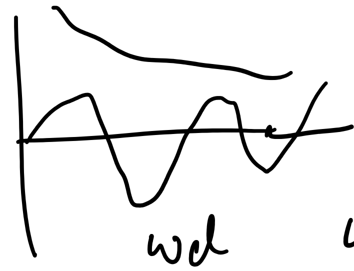




$\omega_n = \sqrt{s/m}$

$f_e = F e^{j\omega t}$ steady-state response
- system respones at driving Fq

$$\tilde{x} = \frac{F e^{j\omega t}}{j\omega\left(R_m + j\left(\omega m - \frac{s}{\omega}\right)\right)}$$

$$\dot{\tilde{x}} = \frac{F e^{j\omega t}}{R_m + j\underbrace{\left(\omega m - \frac{s}{\omega}\right)}_{=0 \text{ at } \omega=\omega_n}} = \tilde{u}$$

- large response

$$\omega m = \frac{s}{\omega}$$

$$\omega^2 = \frac{s}{m} = \omega_n^2$$

Mechanical Impedance

Define $\tilde{Z}_m = \dfrac{\text{complex driving force}}{\text{complex steady-state velocity at driving point}}$

$$\tilde{Z}_m = \frac{\tilde{f}_e}{\tilde{u}}$$

$\tilde{u}$

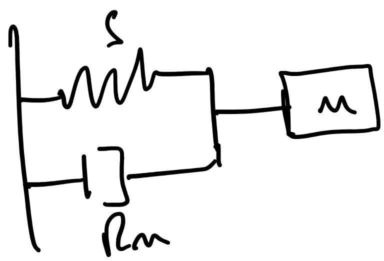
For Damped SDOF

$$\tilde{Z}_m = \frac{F\cancel{e^{j\omega t}}}{\frac{f\cancel{e^{j\omega t}}}{R_m + j(\omega M - S/\omega)}} = R_m + j(\omega M + S/\omega)$$

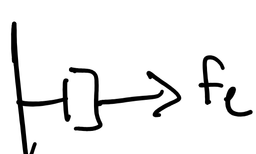
$$\underbrace{\tilde{Z}_m = \overbrace{R}^{\text{mechanical Resistance}} + j\overbrace{X}^{\text{mechanical Reactance}}}$$

$R = \mathrm{Re}\{\tilde{Z}_m\} = R_m$

$X = \mathrm{Im}\{\tilde{Z}_m\} = \omega M - S/\omega$

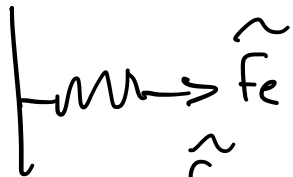


Set S and M = 0



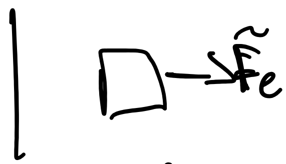
$\tilde{Z}_m = \frac{\tilde{f}_e}{\tilde{u}} = R_m$ real and pos.

Set R_m and $M = 0$



$$\tilde{Z}_m = \frac{\tilde{f}_e}{\tilde{u}_e} = -j\frac{s}{\omega} \begin{cases} \text{Imaginary} \\ \text{Negative} \\ \text{inversely } \propto \text{ to } f_e \end{cases}$$

Set R_m and $s = 0$



$$\tilde{Z}_m = j\omega m \begin{cases} \text{imaginary} \\ \text{positive} \\ \text{dir. } \propto \text{ to } f_e \end{cases}$$

Mechanical Resonance

Definition: Occurs when $\text{Im}\{\tilde{Z}_m\}$ goes to 0

$$\tilde{Z}_m = R_m + j\left(\omega m - \frac{s}{\omega}\right)$$

$$\text{Im}\{\tilde{Z}_m\} = \omega m - \frac{s}{\omega} \Rightarrow \omega^2 = \frac{s}{m} = \omega_n^2$$

Resonance occurs at $\omega = \omega_n$ (undamped natural Frequency)

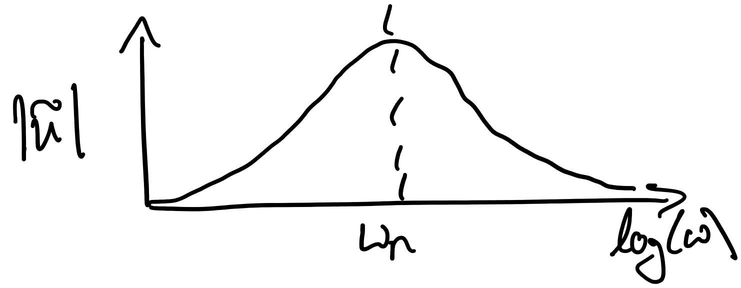
<u>drive at ω_n</u>

$$\tilde{u} = \frac{F e^{j\omega t}}{R_m} \Rightarrow \text{maximum in velocity}$$

$\tilde{Z}$ purely real

R_m

At Resonance $\tilde{Z}_m$ = purely real



3 Frequency Ranges of interest

i) $\omega < \omega_0$

ii) $\omega \simeq \omega_0$

iii) $\omega > \omega_0$

$$\tilde{Z}_m = R_m + j\left(\omega M - \frac{S}{\omega}\right)$$

i) $\omega < \omega_0$ .... stiffness control region

$$\tilde{Z}_m \simeq -j\frac{S}{\omega} \qquad \tilde{x} \simeq \frac{F}{S} e^{j\omega t}$$

$|\tilde{x}|$ is independent of frequency for a given input force

ii) near resonance

$\omega \simeq \omega_0$

$$\tilde{Z}_m = R_m \qquad \tilde{x} = \frac{F}{j\omega R_m} e^{j\omega t}$$

$|\tilde{x}|$ is independent

$\tilde{u} = j\omega\tilde{x} = \frac{F}{R_m} e^{j\omega t}$ then $|\tilde{u}|$ is independent of frequency

$\omega \simeq \omega_0$ ... Damping Controlled Region

iii) $\omega > \omega_0$ ... Mass Controlled Region

$\tilde{Z}_m \simeq j\omega m$

$\tilde{a} = j\omega\tilde{u} = \left(\frac{F}{m}\right) e^{j\omega t}$

acceleration is independent of frequency

Transducers are designed to operate in one of these regions

$\tilde{Z}_m \simeq j\omega m$

- imag
- pos
- dir. $\propto$ to Fq

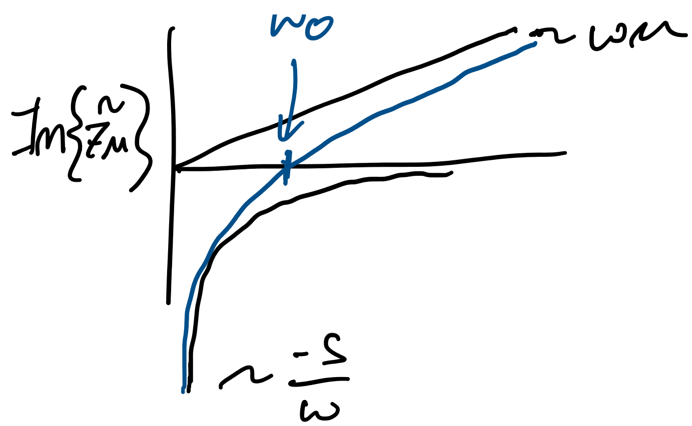
Mass Like

contrast to

$\tilde{Z}_m \simeq \frac{-js}{\omega}$

- imag
- neg
- inverse $\propto$ to Fq

Stiffness Like



$\tilde{Z}_m = \frac{\hat{f}_c}{\tilde{u}}$ acknowledge of $\tilde{Z}_m$ allows prediction of

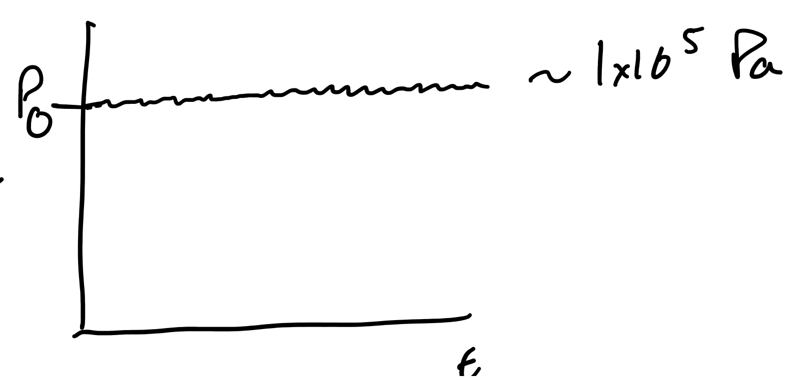
$\tilde{u} = \frac{\hat{f}_c}{\tilde{Z}_m}$ Response of a system

- might know $\tilde{Z}_m$ from measurement or from theoretical or numerical prediction

[if the system is <u>linear</u>]

Superposition -

- linear acoustics
- linear vibration
  - small amplitude fluctuations in sound pressure
  - velocity



Loud sound corresponds to 1 Pa fluctuation
(94 dB)

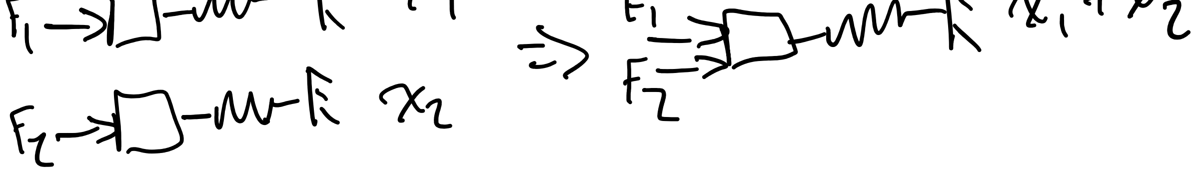
* linear systems
  - small amplitude motions

- output of a linear system is linearly proportional to the input

- double force = double response
- linear system responds at (and only at) the driving frequency

$$Fe^{j\omega t} \to \tilde{A}e^{j\omega t} \approx \tilde{x}$$

- responses to two or more inputs is the linear sum of individual responses



Lecture 5

Wednesday, September 1, 2021 1:27 PM

Steady State -

$$F_e e^{j\omega t} = \tilde{x} = \tilde{A} e^{j\omega t}$$

Mechanical Impedance

$$\frac{\tilde{F}_e}{\tilde{u}} = \tilde{Z}_m = R_m + j\left(\omega m - \frac{s}{\omega}\right)$$

$j\omega m$ -> mass like impedance

$-j\frac{s}{\omega}$ -> stiffness like impedance

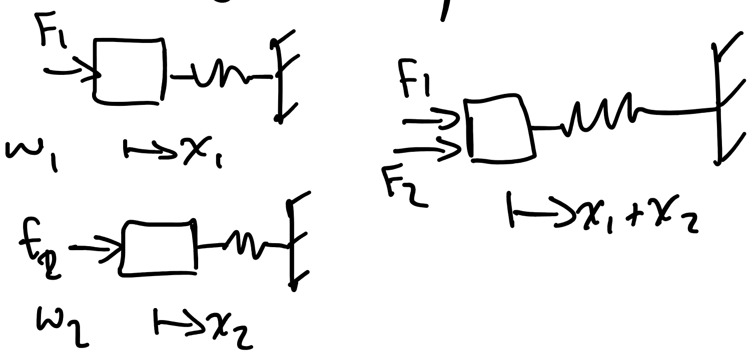
Resonance

as $\omega \to \omega_0$: Resonance

$\omega < \omega_0$: stiffness controlled

$\omega > \omega_0$: mass controlled

Superposition - output of a linear system is linearly proportional to the input



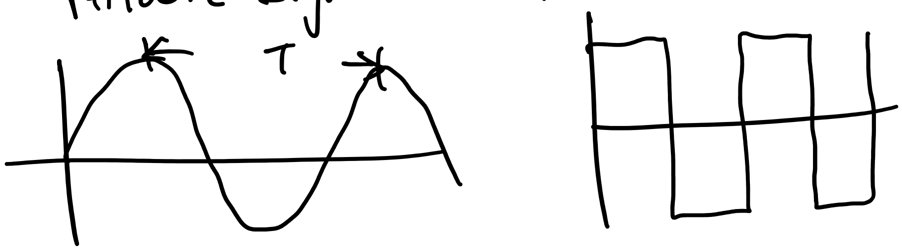
- convenient to break up input forces into individual F_e components

- Convenient to ...
forces into individual Fz components
[Frequency Analysis]

can then find response to each
component and sum responses

Fourier Analysis

Periodic Signals - repeat in T



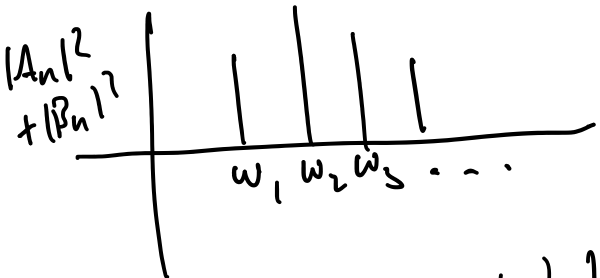
Fourier Theorem -

Any single-valued periodic function
with period T can be represented
exactly as a sum of sinusoids
periodic in T

f(t) is periodic and single-valued

$$f(t) = \frac{1}{2}A_0 + A_1\cos\omega_1 t + A_2\cos\omega_2 t + \dots$$
$$+ B_1\sin\omega_1 t + B_2\sin\omega_2 t \dots$$

$\omega_1 = \frac{2\pi}{T}$ $\quad \omega_2 = 2\omega_1$ $\quad \omega_3 = 3\omega_1$

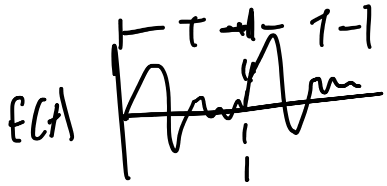


Signal is represented by contributions at discrete frequencies

$\omega_1 = \frac{2\pi}{T}$ = fundamental or first harmonic

$\omega_2 = 2\omega_1$ 2nd Harmonic

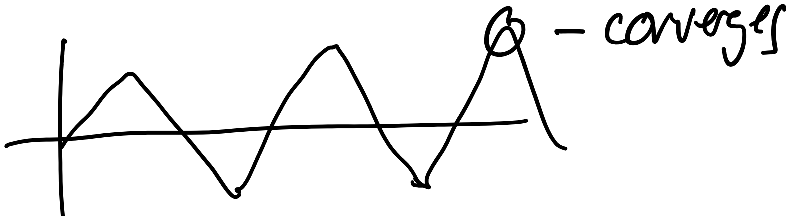
$\vdots$ $\vdots$

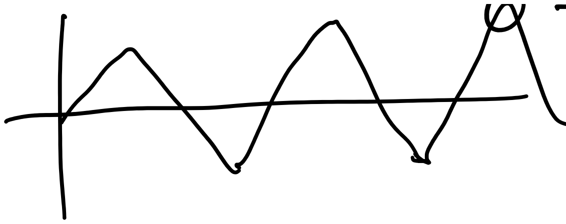


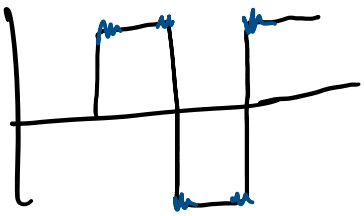
$$A_n = \frac{2}{T}\int_0^T f(t)\cos(\omega_n t)\,dt$$
$$B_n = \frac{2}{T}\int_0^T f(t)\sin(\omega_n t)\,dt$$

Fourier Series

Square Wave is not Single Valued due to undefined Region



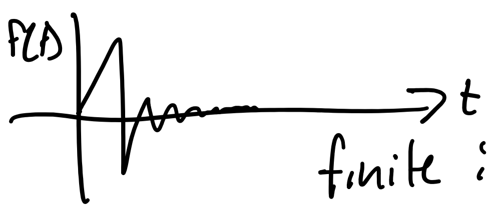




does not converge

Gibbs Phenomenon

Transient Signals

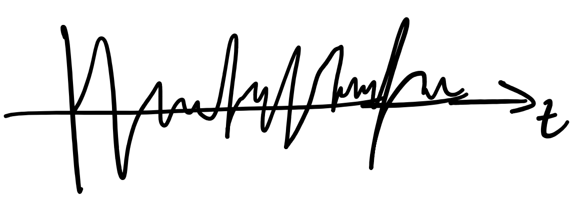


finite in time

Can apply Fourier Transform

composed of infinite number of frequencies

Continuous Non-Periodic



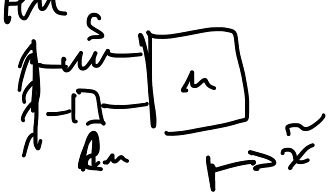
- power spectrum
- power spectral density

-power spectral density

Summary

Physical System   mathematical modelling

SDOF



* To oscillate needs Mass and stiffness

Approach to problem solving

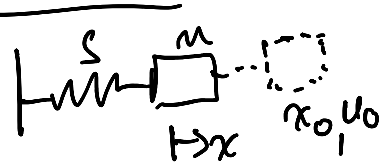
(1) Identification of governing Eqn's
  - restoring force
  - EOM (F=ma)

(2) Combine -> 2nd order ODE

(3) Identify Possible Solutions
  - express in a convenient form

(4) Find Solns that satisfy B.C.

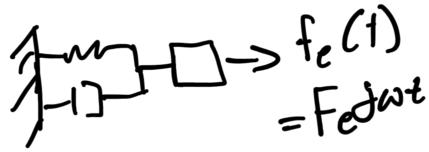
Free Response (Driven by Initial Conditions)



w_0 - system responds at Natural Frequency

Forced Response

Forced Response



- linear systems in steady-state response
- Responds at driving frequency

Resonance

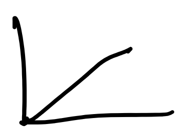
- system driven at the natural frequency

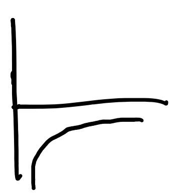
when $Im\{\tilde{Z}_m\} \to 0$ response is a maximum (velocity)

mechanical Impedance

$$\tilde{Z}_m = R_m + j\left(\omega m - \frac{s}{\omega}\right) = \frac{\tilde{f}_e}{\tilde{u}}$$

$$\tilde{u} = \frac{\tilde{f}_e}{\tilde{Z}_m}$$ if $\tilde{Z}_m$ is known, can predict response at any driving force

if $\tilde{Z}_m$  - mass ctrld

if $\tilde{Z}_m$  - stiffness ctrld

Linear Superposition -> Fourier Methods

Next: Vibrating String

- Vibration of extended Systems

Wave Propagation

Lecture 6

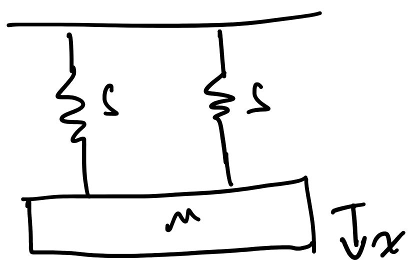
Friday, September 3, 2021 1:27 PM

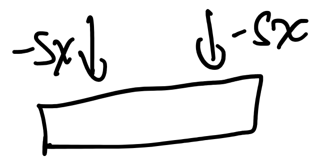
Homework Hints:

Come up with EOM and restoring force

$$\frac{\partial^2 x}{\partial t^2} + \omega_0^2 x = 0$$

Find ω_0

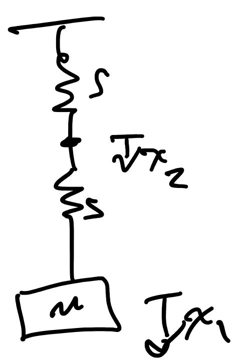


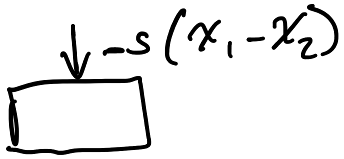


$$f = ma = m\frac{\partial^2 x}{\partial t^2} \longrightarrow f = -2sx$$

$$m\frac{\partial^2 x}{\partial t^2} + 2sx = 0$$

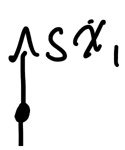
Springs in series:





$$f = m\frac{\partial^2 x_1}{\partial t^2}$$

Need to isolate x_1



$\Sigma F = 0$

1.3.C

EOM $x = x_0 \cos\omega_0 t + \frac{u_0}{\omega_0} \sin\omega_0 t$

plot $\left(\frac{x}{A}\right)$ let $A = 1$ vs. $\left(\frac{t}{T}\right)$

$\omega_0 = \frac{2\pi}{T}$

1.5.4 $A = x + jy$

$B = x + jY$

find $|A|$ . . .

Additional Problems

#1

1.6.12 $\tilde{x} = e^{-\beta t}\left(A_1 e^{j\omega_0 t} + A_2 e^{-j\omega_0 t}\right)$

plot Real part of x $Re\{x\}$

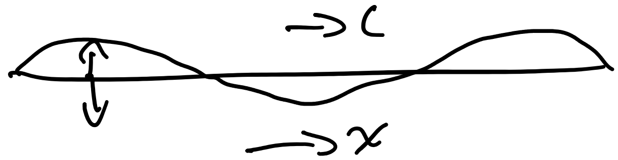
#2

$\tilde{Z}_m = R_m + j\left(\omega m - \frac{s}{\omega}\right)$

2. Vibrating String

- extended systems
- derive a wave equation

Transverse oscillation of a String

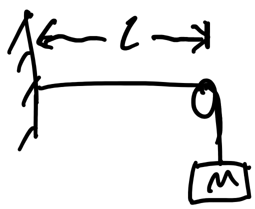


- transverse wave motion
  - motion of particles is perpendicular to direction of propogation
  - phasespeed (wave speed)
  - particle velocity
  - wave number, k, spatial frequency
  - application of B.C.'s
  - characteristic impedance
  - standing waves vs. propogating waves

2.1 Derivation of a Wave Equation

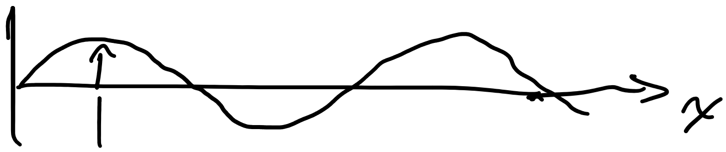
- governs transverse vibration of a tensioned string

- governs transverse vibration of a tensioned string



ρ_L = mass/length - linear density

T = tension force



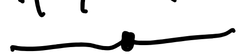
Assumptions

1. uniform static tension
   - independent of transverse displacement

2. Small Amplitude Responses
   - linear Assumptions

3. No losses

4. In each segment of the string $\rho_0 L$ is uniform ρ_{L_1} ρ_{L_2}

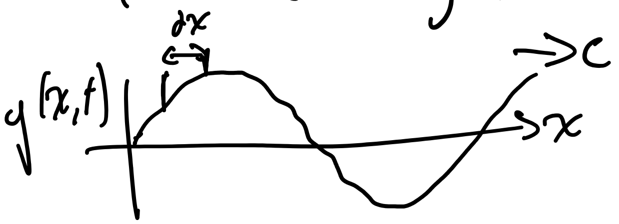


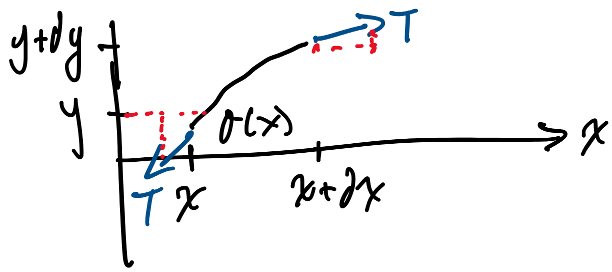
5. Gravity Forces are negligible

5. Gravity Forces are negligible
- orientation is not significant

2.1.1 Restoring Force

$t = t_1$





vertical component at x is
$$-T\sin\theta(x)$$

vertical component at $x+\partial x$
$$T\sin\theta(x+\partial x)$$

Net vertical force acting in y-direction
$$\partial f_y = T\sin\theta(x+\partial x) - T\sin\theta(x)$$

Taylor Series Expansion

Express $f(x+\partial x)$ as $f(x) + \frac{\partial f}{\partial x}\partial x + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}(\partial x^2) + \dots$

Using small amplitude assumption
can truncate series

using small amplitude assumption

- can truncate series

$$\sin\theta(x+\partial x) \simeq \sin\theta(x) + \frac{\partial \sin\theta(x)}{\partial x}\partial x + \cancel{\dots}$$

$$\partial f_y = T\left(\sin\theta x + \frac{\partial \sin\theta(x)\partial x}{\partial x}\right) - T\sin\theta(x)$$

$$\partial f_y = T\frac{\partial \sin\theta(x)}{\partial x}\partial x$$

$$\partial f_y = T\frac{\partial[\sin\theta(x)]}{\partial x}\partial x$$

use small displacements:

can argue angle will always be small

θ is always small

when θ is small

$\sin\theta(x) \approx \tan\theta$ $\left[\tan = \frac{\sin}{\cos}\right]$

$= \frac{\partial y}{\partial x}$ (slope of string)

Nathan Secador

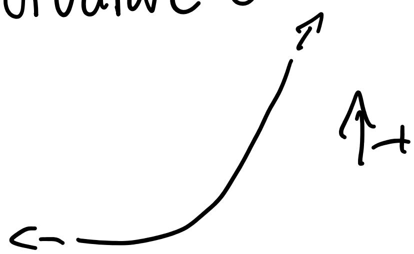
$$\partial f_y \simeq \frac{\partial(\partial y/\partial x)}{\partial x}\partial x \cdot T$$

$$\boxed{\partial f_y = T\left(\partial^2 y/\partial x^2\right)\partial x} \quad (1)$$

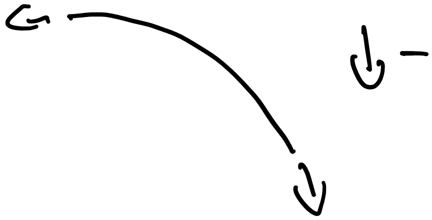
force is directly proportional

force is directly proportional to the curvature of the string

$$\frac{\partial^2 y}{\partial x^2} > 0$$



$$\frac{\partial^2 y}{\partial x^2} < 0$$



no curvature
→ no net force on string segment

has to be curvature to generate restoring force

2.1.2 EOM

$f = ma$ ρ_L = mass/unit l

$m = \rho_L \cdot dx$ $a = \frac{\partial^2 y}{\partial t^2}$

$f = dF_y =$

$$F = \partial F_y =$$

$$\boxed{\partial F_y = \rho_L \partial x \frac{\partial^2 y}{\partial t^2}} \quad (2)$$

Combine 1 & 2

$$T \frac{\partial^2 y}{\partial x^2} \cancel{\partial x} = \rho_L \cancel{\partial x} \frac{\partial^2 y}{\partial t^2}$$

$$\frac{\partial^2 y}{\partial x^2} - \frac{\rho_L}{T} \frac{\partial^2 y}{\partial t^2} = 0$$

2nd Order PDE

- function of space and time
  Independent Variables

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 y}{\partial t^2} = 0$$

$C = \sqrt{T/\rho_L}$ - Speed of wave propogation

Lecture 7

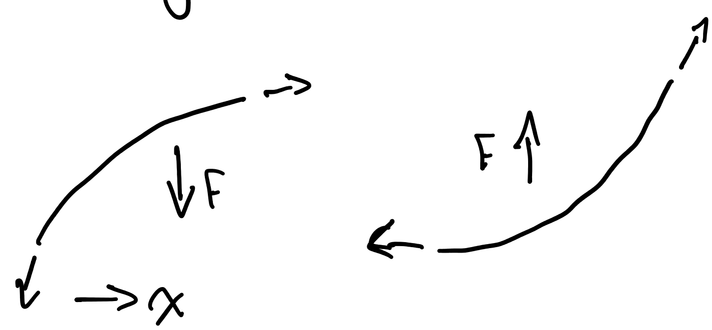
Wednesday, September 8, 2021 1:22 PM

Vibration of Strings

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0$$

Restoring Force

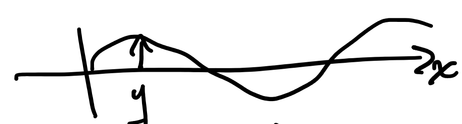
* Wave equation governing small amplitude transverse displacement of a tensioned string



Solutions to the Wave Equation

Two Independent Variables
space & time

General Solution



$$y(x,t) = y_1(\underbrace{ct-x}_{w_1}) + y_2(\underbrace{ct+x}_{w_2})$$

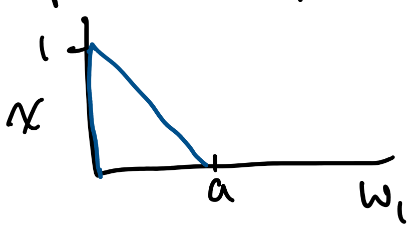
$$y(x,t) = y_1(w_1) + y_2(w_2) \qquad w_1 = ct - x$$
$$w_2 = ct + x$$

y_1 and y_2 are any functions of a single variable

y_1 and y_2 are any functions of a single variable

Prove by direct substitution that the general solution works

Simple example:



$y_1 = 1$ at $w_1 = 0$

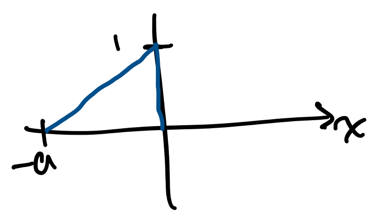
$y_1 = 0$ at $w_1 = a$

plot as a function of x at $t=0$

$w_1 = ct - x$ at $t=0$, $w_1 = -x$, $x = -w_1$

$w_1 = 0$ $x = 0$

$w_1 = a$ $x = -a$



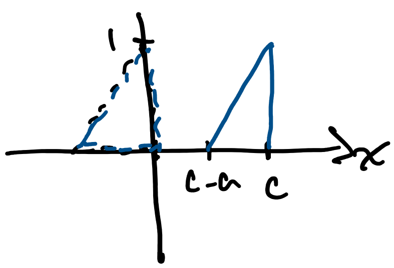
Plot as a function of x at $t = 1s$

$w_1 = c - x$ at $t = 1s$

$x = c - w_1$

$w_1 = 0 \Rightarrow x = c$

$w_1 = a \Rightarrow x = c - a$



Speed of propagation is $\frac{\text{distance}}{\text{time}} = \frac{c}{1s}$

Speed of propogation is time is

waves speed $= c = \sqrt{T/\rho}$

Shape remains the same
- linear wave propogation
- non-dispersive wave propogation

$y_1(ct-x)$ disturbance traveling in $+x$ direction without changing shape

$y_2(\underbrace{ct+x}_{w_2})$ disturbance traveling in $-x$ direction

By holding w_1 constant
- follow a particular point on the wave as time advances

$y(x,t) = y_1 + y_2$ ... General Solution

Represents Superposition of waves traveling to the right and left $(+x, -x)$

$ct - x = \text{const.} \Rightarrow x = ct - \text{const.}$

$\frac{dx}{dt} = c$

What is the transverse Velocity of the String??

$$V_t = \frac{dy_1}{dt} = \frac{dy_1(ct-x)}{d(ct-x)} \cdot \left[\frac{d(ct-x)}{\partial t}\right] \Rightarrow c$$

$$= c \cdot \frac{dy_1(ct-x)}{\partial(ct-x)} \neq c$$

V_t depends on y_1

Speed of Wave Propagation is not equal to transverse velocity of the string

Summary: $y(x,t) = y_1(ct-x) + y_2(ct+x)$
positive-going negative-going

* function propagates without changing shape

* non-dispersive solution
- all frequency components travel at the same phase speed
- preserves shape

Harmonic Single Frequency Solutions

Assume a separable form of solution

$$y(x,t) = Y(x)e^{j\omega t}$$

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0$$

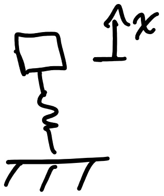
$$\frac{\partial^2 Y}{\partial x^2}\cancel{e^{j\omega t}} + \frac{\omega^2}{c^2}Y\cancel{e^{j\omega t}} = 0$$

$$\underbrace{\frac{\partial^2 Y}{\partial x^2} + \left(\frac{\omega}{c}\right)^2 Y = 0}_{\text{Scalar Helmholtz Eqn}}$$

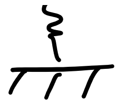
define $k^2 = \frac{\omega^2}{c^2}$ $\quad k = \frac{\omega}{c}$ ... wavenumber

$$\frac{\partial^2 Y}{\partial x^2} + k^2 Y = 0 \quad (4)$$

SHE ... governs spatial dependence of the solution

SDOF  $\frac{\partial^2 x}{\partial t^2} + \omega^2 x = 0$

$\pm j\omega_0 t$

SDOF 

$$x = A e^{\pm j \omega_0 t}$$

$$Y(x) = A e^{\pm jkx}$$ two solutions

Solution for spatial dependence of a single frequency transverse vibration of a tensioned string

$$y(x,t) = A_1 e^{jkx} e^{j\omega t} + A_2 e^{-jkx} e^{j\omega t}$$

$$= A_1 e^{j(kx + \omega t)} + A_2 e^{j(-kx + \omega t)}$$

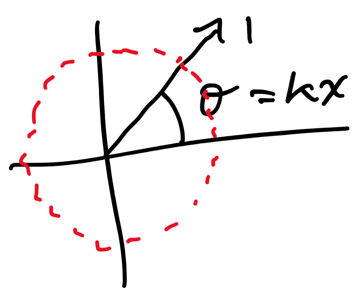
$$= A_1 e^{jk(x + ct)} + A_2 e^{jk(ct - x)}$$

$$\downarrow \qquad\qquad \downarrow$$

$y_1 = ct + x$ negative-going

$y_2 = ct - x$ positive-going

What is k?

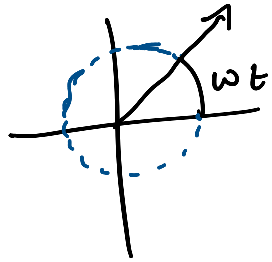


$$e^{+jkx}$$

$$e^{j\theta} \quad \text{so } \theta = kx$$

$e^{j\theta}$ so $\theta = kx$

$e^{j\omega t}$

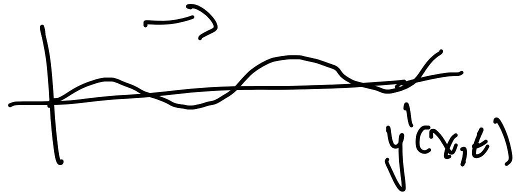


k is the spatial frequency (wavenumber)

ω is the temporal frequency

Lecture 8

Friday, September 10, 2021 1:24 PM



$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0$$

$$y(x,t) = \underbrace{y_1(ct-x)}_{\text{positive dir.}} + \underbrace{y_2(ct+x)}_{\text{negative dir}}$$

Single Freq Harmonic Solutions

$$y(x,t) = Y(x)e^{j\omega t}$$

Scaler Helmholtz Equation

$$\frac{\partial^2 Y}{\partial x^2} + k^2 y = 0 \qquad k = \frac{\omega}{c}$$

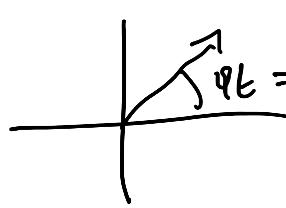
$$Y(x) = Ae^{\pm jkx}$$

$$y(x,t) = Ae^{\pm jkx}e^{j\omega t}$$

ω ... rate of change of phase with time

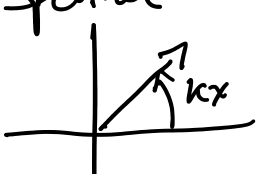
T ... period

f ... frequency $= \frac{1}{T}$

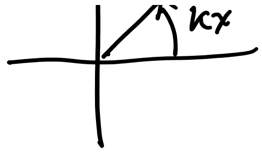


$\Rightarrow \omega = \frac{\partial \varphi}{\partial t}$

Spatial

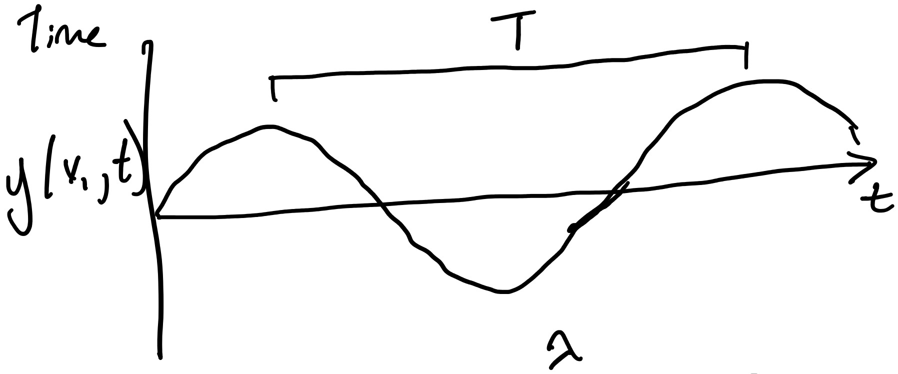


$\frac{\partial \varphi}{\partial x}$

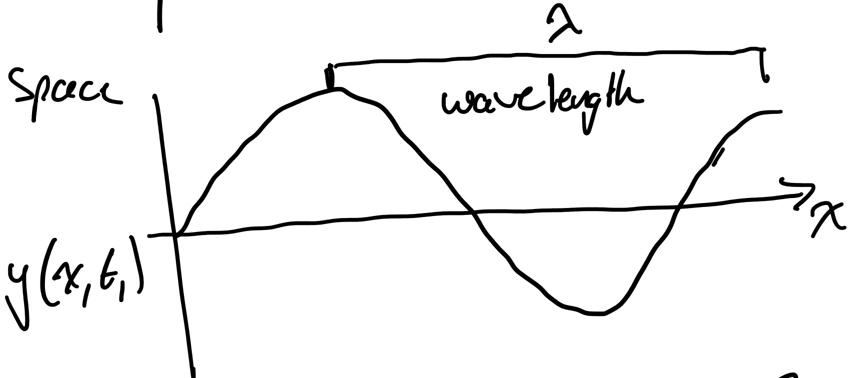


$\frac{\partial \psi}{\partial x}$

k ... rate of change of phase with position
... spatial Frequency



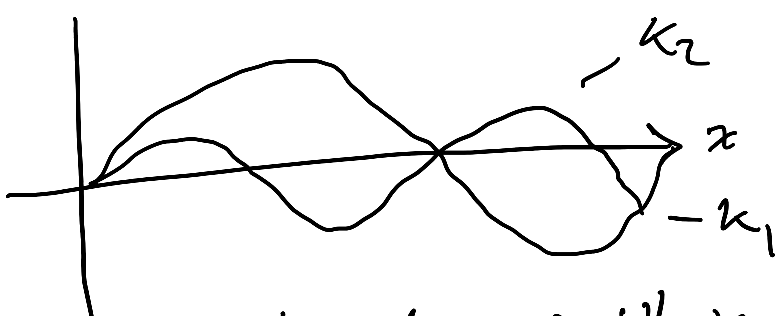
(fixed Space)



(fixed time)

$k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$

$\lambda f = c \quad \lambda = \frac{c}{f} \quad k = \frac{2\pi}{c/f} = \frac{\omega}{c}$



$k_2 > k_1$ (more rapidly in space)

Larger Wave Number ; Shorter wavelength

$y = A\cos(\omega t - kx)$

$y = Ae^{j(\omega t - kx)}$

$= Ae^{j\omega t} e^{-jkx}$

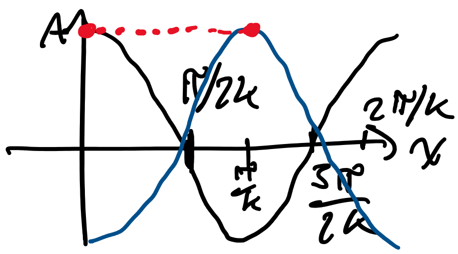
$e^{-jkx} = \cos(-kx) - j\sin(-kx)$

$\cos(-kx) = \cos(kx)$

$Re\{y\}$ at $t=0$

when A = Real Number

$Re\{y\} = A\cos(kx)$



when $kx = \frac{\pi}{2}$, $\cos = 0$

$x = \frac{\pi}{2k}$

at $t = \frac{T}{2}$

• same point on the wave

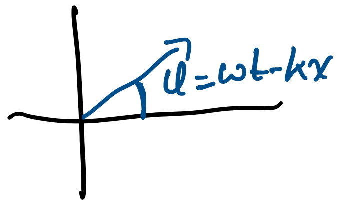
when $t = T$

wave pattern advances by one wavelength

	Time	Space
frequency	ω	k
period	T	λ

General Harmonic Solution

$y(x,t) = A_1 e^{j(\omega t - kx)} + A_2 e^{j(\omega t + kx)}$



hold u constant to follow a point on the wave

as t increases, x also increases

Transverse velocity

$$\frac{dy}{dt}(x,t) = u(x,t) = j\omega y(x,t)$$

Transverse Acceleration

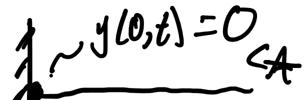
$$\frac{\partial^2 y}{\partial t^2}(x,t) = \frac{du}{dt} = a(x,t) = -\omega^2 y(x,t)$$

Boundary Conditions

General Solution: $y(x,t) = y_1(w_1) + y_2(w_2)$

$$w_1 = ct - x$$
$$w_2 = ct + x$$

Fixed Condition:



$$y(0,t) = y_1(w_1|_{x=0}) + y_2(w_2|_{x=0})$$

$$\qquad ct - x \qquad\qquad ct + x$$

$$y(0,t) = y_1(ct) + y_2(ct) = 0$$

$$y_2(ct) = -y_1(ct)$$

Generally True that $y_2(w_2) = -y_1(w_1)$ when $w_1 = w_2$

Generally true that ...

at $x=0$, y_1 and y_2 must be equal and opposite - cancelling with eachother to create 0 displacement

So at a particular time, $t=t_1$

$w_1 = c\cancel{t_1} - x_1 \quad w_2 = \cancel{c t_1} + x_2$

if $w_1 = w_2$ then $x_1 = -x_2$

image approach

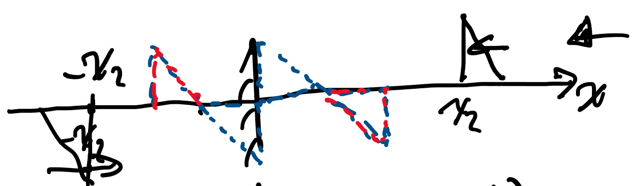


image space | real world

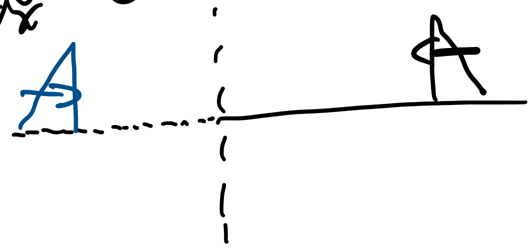
Hard Boundary Causes a Reflection that is upside down and backwords

Free end string

Zero transverse force Boundary

$\frac{dy}{dx} = 0$

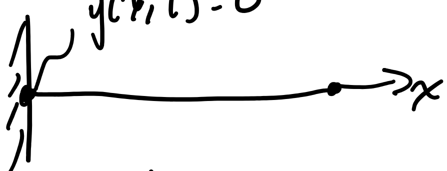
$\frac{\partial y}{\partial x} = 0$



frequency Domain

$$y(x,t) = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)}$$

Two boundary conditions needed



$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$y(0,t) = Ae^{j\omega t} + Be^{j\omega t} = 0 \Rightarrow B = -A$$

$$y(x,t) = Ae^{j\omega t}\left(e^{-jkx} - e^{jkx}\right)$$

$$= -2jA\, e^{j\omega t}(\sin kx)$$

Lecture 9

Monday, September 13, 2021 1:24 PM

SAE N&V

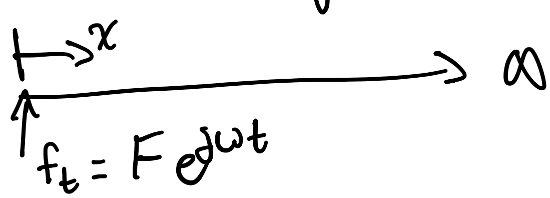
Roger Shively - Sound Design in automobiles

Perry Gu - Geely

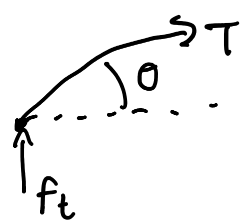
$$\frac{d^2y}{dx^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0$$

$$y(x,t) = y_1(ct - x) + y_2(ct + x)$$

2.3.2. Force Boundary Condition



FBD



$\Sigma f_y = 0$

$$f_t + T\sin\theta\Big|_{x=0} = 0$$

small displacement => small theta
(linear small amplitude vibration)

$$\sin\theta \simeq \tan\theta \simeq \frac{dy}{dx}$$

$$\sin\theta \simeq \tan\theta \simeq \frac{\partial y}{\partial x}$$

$$f_t + T\frac{\partial y}{\partial x}\bigg|_{x=0} = 0 \qquad \frac{\partial y}{\partial x}\bigg|_{x=0} = -\frac{f_t}{T}$$

In special case when $f_t = 0$

$$\frac{\partial y}{\partial x} = 0 \quad \text{at a free end}$$

no lateral constraint

$$y(x,t) = \underbrace{Ae^{j(\omega t - kx)}}_{\text{positive}} + \underbrace{Be^{j(\omega t + kx)}}_{\text{negative}}$$

$$\frac{dy}{dx} = -jkAe^{j(\omega t - kx)} + jkBe^{j(\omega t + kx)}$$

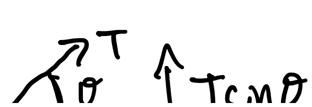
$$\frac{dy}{dx}\bigg|_{x=0} = -jkA\cancel{e^{j\omega t}} + jkB\cancel{e^{j\omega t}} = \frac{-f_t}{T} = \frac{-F\cancel{e^{j\omega t}}}{T}$$

$$\Rightarrow \boxed{-jkA + jkB = \frac{-F}{T}}$$

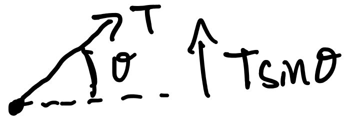
Force BC at $x=0$

2.3.3 Mass at $x=0$

FBD:



FBD:



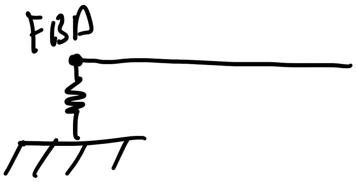
$$\sum F_y = Ma$$

$$T\sin\theta\Big|_{x=0} = M\frac{\partial^2 y}{\partial t^2}\Big|_{x=0}$$

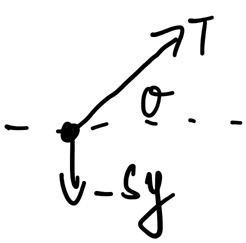
$$\frac{\partial^2 y}{\partial t^2}\Big|_{x=0} = \frac{T}{M}\frac{\partial y}{\partial x}\Big|_{x=0}$$

2.5.4 stiffness BC

FBD



FBD



$$\sum F_y = 0$$

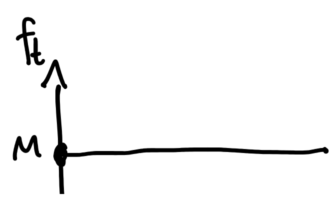
$$T\sin\theta\Big|_{x=0} - sy\Big|_{x=0} = 0$$

$$T\frac{dy}{dx}\Big|_{x=0} - sy\Big|_{x=0}$$

$$\left[\frac{dy}{dx} = \frac{s}{T}y\right]\Big|_{x=0}$$

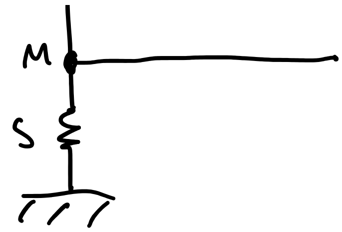
- fixed condition
- Mass condition

BC's

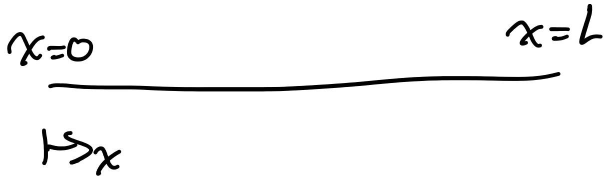


- Mass condition
- Stiffness condition
- transverse force

BC's

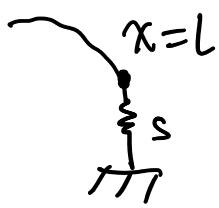


can combine above

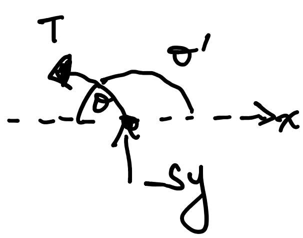


2.3.5 Boundary Condition applied at the positive-x end of a string

e.g. stiffness at the end



$$\Sigma F_y = 0$$



$$T\sin(\theta')\Big|_{x=L} - sy\Big|_{x=L} = 0$$

$$\sin\theta' = -\frac{dy}{dx}$$

$$-T\frac{dy}{dx}\Big|_{x=L} = sy\Big|_{x=L}$$

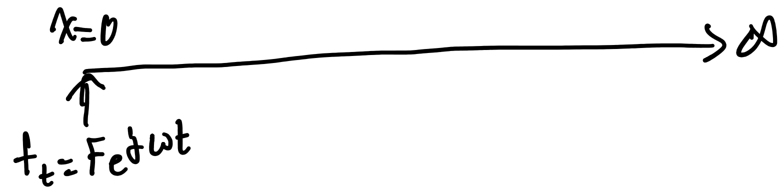
$$\frac{dy}{dx}\Big|_{x=L} = -\frac{s}{T}\, y\Big|_{x=L}$$

sign of Boundary condition depends on the position

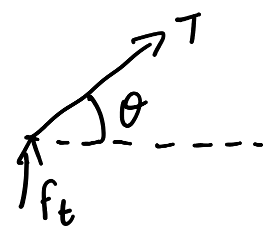
Important for Exam

2.4 Forced Vibration

2.4.1 semi infinite string



FBD:



$\Sigma f_y = 0$

$$\frac{dy}{dx}\Big|_{x=0} = \frac{-f_t}{T}$$

$$y(x,t) = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)}$$

nothing returns from infinity...
therefore negative going term cancels

$$y(x,t) = Ae^{j(\omega t - kx)}$$

$$y(x,t) = \underline{\underline{A}}\, e^{j(\omega t - kx)}$$

$$\left[\frac{dy}{dx} = -jkA e^{j(\omega t - kx)}\right]$$

$$\left.\frac{dy}{dy}\right|_{x=0} = -jkA e^{j\omega t}$$

$$-jkA \cancel{e^{j\omega t}} = -\frac{F \cancel{e^{j\omega t}}}{T}$$

$$-jkA = \frac{-F}{T} \Rightarrow A = \frac{F}{jkT}$$

$$y(x,t) = \frac{F}{jkT} e^{j(\omega t - kx)}$$

$Re\{y(x,t)\}$ = Physical Solution

Transverse Velocity

$$u(x,t) = \frac{dy}{dt} = \frac{\cancel{j}\omega F}{\cancel{j}kT} e^{j(\omega t - kx)}$$

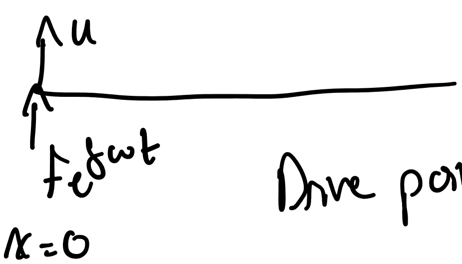
$$\frac{\omega F}{kT} e^{j(\omega t - kx)} \quad \text{and} \quad k = \frac{\omega}{c} \Rightarrow c = \sqrt{T/\rho_L}$$

$$\frac{\cancel{\omega} F}{\frac{\cancel{\omega}}{\cancel{c}} \rho_L c^2} \Rightarrow \frac{F}{\rho_L c} e^{j(\omega t - kx)}$$

characteristic impedance

Characteristic of Both Medium and of the wave type - how 'easy' it is for the medium to carry energy away from the drive point

Input Mechanical Impedance



Drive point Impedance

$$Z_{M0} = \frac{\text{Complex Driving force at } x=0}{\text{Velocity at drive point}}$$

$$= \frac{Fe^{j\omega t}}{u} = \frac{Fe^{j\omega t}}{Fe^{j\omega t}} \cdot \rho_L c$$

$$\boxed{Z_{M0} = \rho_L c}$$

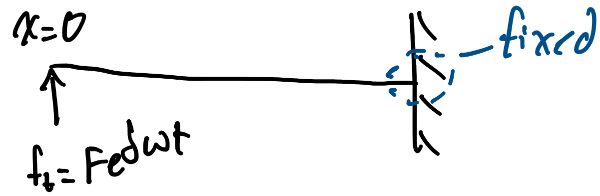
input mechanical impedance for a semi-infinite string is equal to characteristic impedance in this simple case

Purely Real

Real Input Impedance

Real Input impedance
→ implies energy carried
away to infinity

2.4.2. Finite Length String



Lecture 10

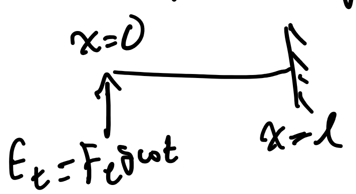
Wednesday, September 15, 2021 1:28 PM

$$y(x,t) = A e^{j(\omega t - kx)} + \cancel{B e^{j(\omega t + kx)}}$$

B.C. - Force
- Mass
- stiffness
- fixed

$$Z_{M0} = \frac{F e^{j\omega t}}{u} = \frac{F e^{j\omega t}}{F e^{j\omega t}} \cdot \rho_L c \rightarrow \text{characteristic impedance}$$

2.4.2 Finite length String



$$y(x,t) = A e^{j(\omega t - kx)} + B e^{j(\omega t + kx)}$$

(i) $y(l,t) = 0 = A e^{-jkl} + B e^{jkl} = 0$ (1)

(ii) force B.C. at $x=0$

$$f_t = F e^{j\omega t} = -T \frac{dy}{dx}\Big|_{x=0}$$

$$F \cancel{e^{j\omega t}} = -T\left(-jk A \cancel{e^{-jkx}} + jk B \cancel{e^{jkx}}\right) e^{j\omega t}$$

$$= jkT(A-B) \cancel{e^{j\omega t}}$$

$$= jkT(A-B)\cancel{e^{j\omega t}}$$

$$jkTA - jkTB = F \quad (2)$$

$$Ae^{-jkl} + Be^{jkl} = 0$$

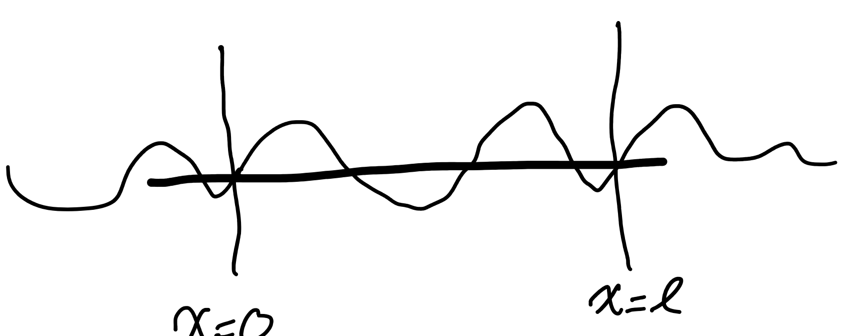
$$jkTA - jkTB = F$$

$$\begin{bmatrix} A \\ B \end{bmatrix} \begin{bmatrix} e^{-jkl} & e^{jkl} \\ jkT & -jkT \end{bmatrix} = \begin{bmatrix} 0 \\ F \end{bmatrix}$$

$$y(x,t) = \underbrace{\boxed{\frac{Fe^{jkl}}{2jkT\cos kl}}}_{A} e^{j(\omega t - kx)} \underbrace{\boxed{- \frac{Fe^{-jkl}}{2jkT\cos kl}}}_{B} e^{j(\omega t + kx)}$$

$$y(x,t) = \frac{F}{2jkT\cos kl}\left[e^{j(\omega t + k(l-x))} - e^{j(\omega t - k(l-x))}\right]$$

Expressed as a superposition of two propogating waves



Only has real meaning between $0 \le x \le l$

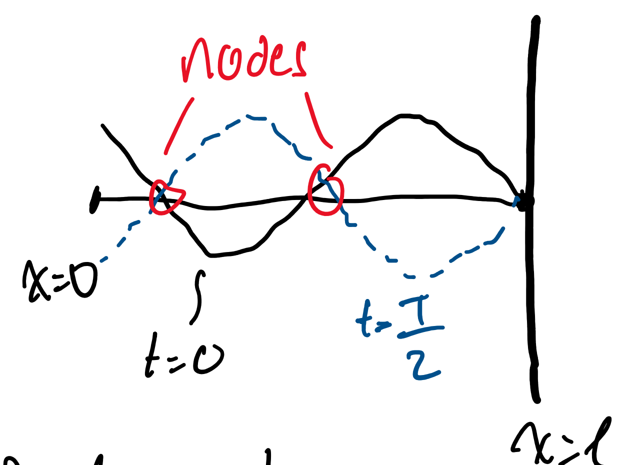
Only has real meaning between $0 \le x \le l$

Transfer to trig form:

$$y(x,t) = \frac{F e^{j\omega t} \cdot 2j \sin k(l-x)}{2jkT \cos kl} = \underbrace{\left[\frac{F}{kT}\right]\left(\frac{\sin k(l-x)}{\cos kl}\right)}_{\text{Space}} \underbrace{e^{j\omega t}}_{\text{Time}}$$

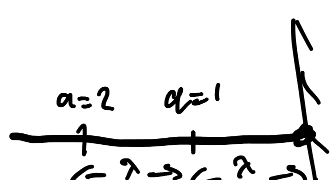
Standing Wave Representation
(separation of time and space)

- Standing Waves are created by the superposition of propagating waves that interfere with eachother

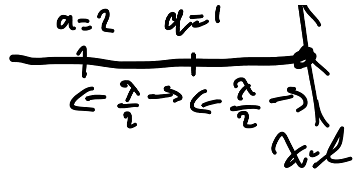


nodes - displacement is always equal to zero

(iv) Location of nodes



(IV) Location of nodes



$$\sin k(l - x_q) = 0$$

$$k(l - x_q) = q\pi \qquad q = 0, 1, 2, 3 \ldots$$

$$x_q = l - q\frac{\pi}{k} \qquad k = \frac{2\pi}{\lambda}$$

$$x_q = l - q\frac{\pi}{2\pi}\cdot\lambda = l - q\frac{\lambda}{2}$$

nodal points are at l minus an integer multiple of $\frac{\lambda}{2}$'s away from the termination

as the frequency increases nodes move to the right and get closer together

(v) Antinodes - max displacement, half between nodes

(vi) $y(x,t) = \dfrac{F}{kT\cos kl}\sin k(l-x)\, e^{j\omega t}$

Response of system is largest at the frequency at which $\cos kl \to 0$

- condition of Resonance

$$\cos kl = 0 \qquad \frac{2\pi}{\lambda} l = \frac{\pi}{2}$$

$$k_n l = \left(\frac{2n-1}{2}\right)\pi$$

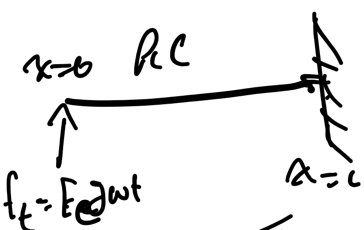
$$k_n l = \left(\frac{2n-1}{2}\right)\pi$$

$$k_n = \frac{\omega_n}{c} = \frac{2\pi f_n}{c}$$

$$2\pi f_n l = \left(\frac{2n-1}{2}\right)\pi$$

$$f_n = \left(\frac{2n-1}{4}\right)\cdot\frac{c}{l} \quad n = 1, 2, 3 \ldots$$

(vii) input mechanical impedance



$$Z_{M0} = \frac{\text{input force}}{\text{velocity}} = \frac{\cancel{F e^{j\omega t}}}{j\omega\left(\frac{\cancel{F e^{j\omega t}}}{kT}\right)\frac{\sin k(L-x)}{\cos kl}} \rightarrow x=0$$

$$\frac{\omega}{c} \qquad T = \rho_L c^2$$

$$Z_{M0} = \frac{\cos kl}{\sin kl}\cdot\frac{\omega\rho_L c}{j\omega} = \boxed{\cot(kl)\cdot -j\rho_L c = Z_{M0}}$$

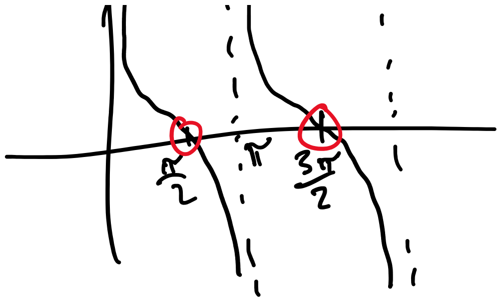
purly imagnery

geometry

Medium

$$Z_{M0} = -j\rho_L c \cot kl$$

natural frequency... when $Im\{Z_m\} = 0$



$$kl = \frac{(2n-1)}{2}\pi$$

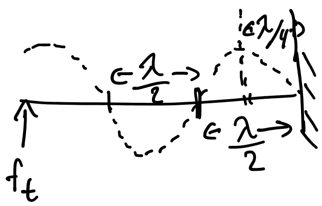
$$k_1 = \frac{\omega_1}{c} = \frac{2\pi f_1}{c}$$ can solve f_1

Lecture 11

Friday, September 17, 2021 1:24 PM

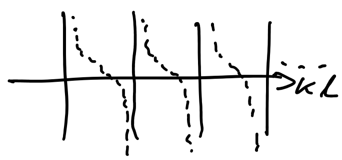
$$c = \sqrt{T/\rho_L}$$

$$Z_{M0} = \rho_L c$$



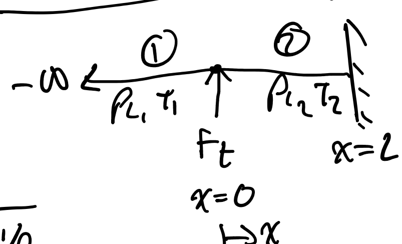
$$y(x,t) = \frac{F}{kT\cos kl}\sin k(l-x)\, e^{j\omega t}$$

$$Z_{M0} = -j\rho_L c \cot kl$$



$$f_1 = \frac{c}{4L}$$

Strings with multiple segments



$$c_1 = \sqrt{T_1/\rho_{L_1}}$$

$$c_2 = \sqrt{T_2/\rho_{L_2}}$$

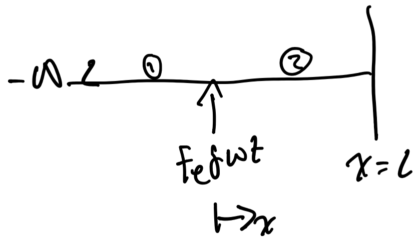
- wave eqn applies to homogeneous segments
- must treat 2 strings as distinct
    - coupled through boundary conditions

- even if physically we have one string, must treat as 2 different strings

1. $$\frac{\partial^2 y_1}{\partial x^2} - \frac{1}{c_1^2}\frac{\partial^2 y_1}{\partial t^2} = 0$$

2. $$\frac{\partial^2 y_2}{\partial x^2} - \frac{1}{c_2^2}\frac{\partial^2 y_2}{\partial t^2} = 0$$

② $\dfrac{\partial^2 y_2}{\partial x^2} - \dfrac{1}{c_2^2}\dfrac{\partial^2 y_2}{\partial t^2} = 0$



In Region 1, $x \le 0$

$$y_1(x,t) = A\cancel{e^{j(\omega t - k_1 x)}} + Be^{j(\omega t + k_1 x)}$$

$$k_1 = \frac{\omega}{c_1}$$

In Region 2, $x \ge 0$

$$y_2(x,t) = Ce^{j(\omega t - k_2 x)} + De^{j(\omega t + k_2 x)}$$

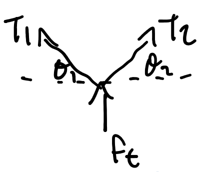
$$k_2 = \frac{\omega}{c_2}$$

3 unknowns, need 3 Boundary Conditions
- 1 Force, 2 displacement

(i) $y_2(\ell, t) = 0$
(ii) $y_1(0,t) = y_2(0,t)$] Displacement is continuous
Slope can be discontinuous due to no flexural stiffness

Force at $x=0$

(iii)

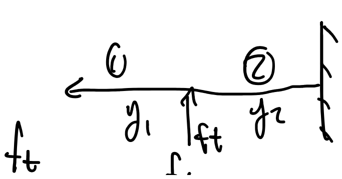


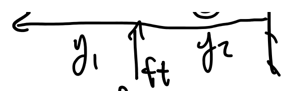
$\Sigma F_y = 0$

$$f_t + T_2 \sin\theta_2\Big|_{x=0} + T_1 \sin\theta_1\Big|_{x=0} = 0$$

$$0 = f_t + T_2 \frac{\partial y_2}{\partial x}\Big|_{x=0} - T_1 \frac{\partial y_1}{\partial x}\Big|_{x=0}$$

Sub assumed solutions into i, ii, iii
solve for B, C, + D



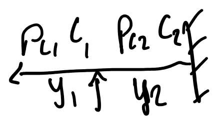


$$z_{MO} = \frac{f_t}{u_1|_{x=0}} \text{ or } \frac{f_t}{u_2|_{x=0}}$$

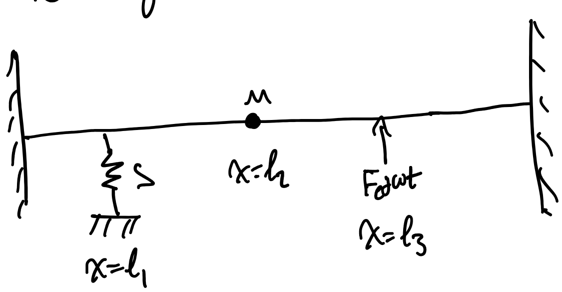
$$z_{MO} = \rho_{L_1} c_1 - j\,\rho_{L_2} c_2 \cot k_2 L \qquad k_2 = \frac{\omega}{c_2}$$

- impedances of string segments add in series
  - velocity is shared between two segments at the drivepoint.

Notes:



(i) Soln for y_1 only applies in region $x \le 0$
(ii) Soln for y_2 only applies in region $0 \le x \le L$
(iii) use same approach even if the two strings have the same properties
(iv) the same approach can be extended to any number of segments



$$y_1(x,t) = A e^{j(\omega t - k_1 x)} + B e^{j(\omega t - k_1 x)} \qquad k_1 = \frac{\omega}{c_1}$$
$$0 \le x \le l_1 \qquad c_1 = \sqrt{T_1/\rho_{L_1}}$$

$y_2(x,t) = C \sim + D \sim \quad k_2$

$y_3(x,t) = E \sim + F \sim \quad k_3$

$y_4(x,t) = G \sim + H \sim \quad k_4$

8 unknowns - 5 displacement
3 force [spring
mass

v

8 unknowns - 5 displacement
3 force $\begin{cases} \text{Spring} \\ \text{mass} \\ \text{Drivepoint} \end{cases}$

$$\begin{bmatrix} & & \\ & 8\times 8 & \\ & & \end{bmatrix} \begin{bmatrix} A \\ \vdots \\ 4 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

Forcing Vector

General Approch to multisegment cases

- write general solution for each segment
- write boundary conditions
- sub solutions into B.C.'s
- solve

Normal Modes of Strings

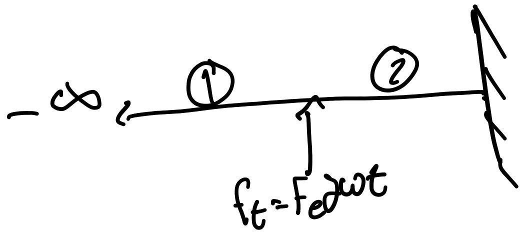
forced → $Fe^{j\omega t}$ we know the response is proportional

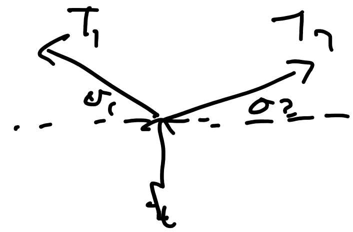
Free Vibration →

- String is forced into motion by initial conditions
- natural frequencies + Mode shapes

lecture 12

Monday, September 20, 2021 1:28 PM





$$\underbrace{\text{wave equ. 1} \quad \text{wave equ. 2}}$$

- coupled at point of applied Force - so $\sum f_y = 0$

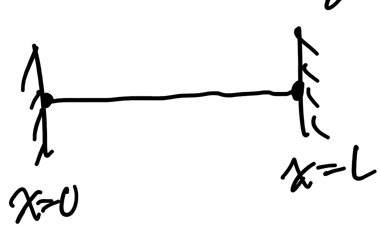
$$T_1 \underbrace{\sin\theta_1}_{-\frac{dy_1}{dx}} + T_2 \underbrace{\sin\theta_2}_{\frac{dy_2}{dx}} + f_t = 0$$

$$Z_{m0} = \rho_1 c - j \rho_1 c \cot kL$$

2.5 Normal Modes of Strings

- string forced into motion by initial conditions

2.5.1 Characteristic Equation



Zero Displ. at each end

one wave equ.

$$y(x,t) = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)}$$

$$y(x,t) = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)}$$

BC at $x=0$ $\quad y(0,t)=0$

$$A+B=0 \Rightarrow B=-A$$

BC at $x=L$ $\quad y(L,t)=0$

$$\cancel{e^{j\omega t}}\left[Ae^{-jkL} + Be^{jkL}\right] = 0$$

$$A\underbrace{\left[e^{-jkL} - e^{jkL}\right]}_{-2j\sin kL} = 0$$

$$-2jA\sin kL = 0 \quad ②$$

$\sin k_nL = 0$ – Characteristic Equation

$k_nL = n\pi \quad n=1,2,3\ldots$

$$k = \frac{\omega}{c} = \frac{2\pi f}{c}$$

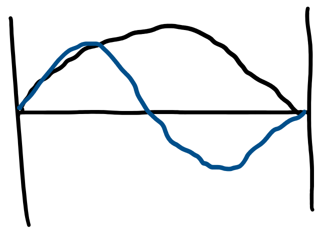
$$f_n = \frac{n}{2}\frac{c}{L} \quad n=1,2,3\ldots$$

Allowed Natural Frequencies.

$$L = \frac{n}{2}\frac{c}{f_n} \quad \ldots \quad c=f\lambda \Rightarrow L = \frac{n}{2}\lambda_n$$

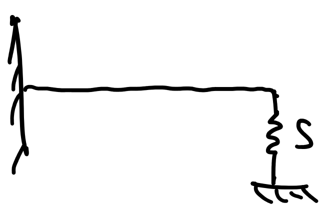
$L = \frac{n}{2}\frac{c}{f_n}$ .... $c = \dots$ $\Rightarrow$ $L = \frac{n}{2}\lambda_n$

first Mode
Second mode



At natural frequencies the string is an integer multiple of half wavelength

- fixed-fixed Case



2.5.2. Mode Shapes.

$$y(x,t) = \left(Ae^{-jkx} + Be^{jkx}\right)e^{j\omega t}$$

first B.C. $\quad y(0,t) = 0$

$$B = -A$$

$$y_n(x) = -2jA_n \sin k_n x$$ – spatial variation of vibration

$$y_n(x,t) = a_n \underbrace{\sin k_n x}_{\text{mode shape of the } n^{th} \text{ mode}}\, e^{j\omega_n t}$$

n^{th} normal mode

... individual solutions of the wave equation

Modes - individual solutions of the wave equation and satisfy boundary conditions...

k_n's - allowed wave numbers

$\omega_n = 2\pi f_n$ - natural freq

a_n = modal amplitude

Infinite number of possible solutions

Complete Solution - weighted sum of possible solutions

2.5.3. Complete Soln.

superposition of possible solutions

$$y(x,t) = \sum_{n=1}^{\infty} \tilde{a}_n \sin k_n x \, e^{j\omega_n t}$$ - weighted sum of modes

Real Displacement of the n^{th} mode

$$Re\{y_n\} = (a_n \cos\omega_n t - b_n \sin\omega_n t)\sin k_n x$$

find a_n by knowing position at $t=0$

Say that $Re\{y(x,0)\}$ = known

∞

Say that Re{y(x, 0)} ...

$$\mathrm{Re}\{y(x,0)\} = \sum_{n=1}^{\infty} a_n \sin k_n x$$

$$a_n = \frac{2}{L}\int_0^L \mathrm{Re}\{y(x,0)\} \sin k_n x \, dx$$

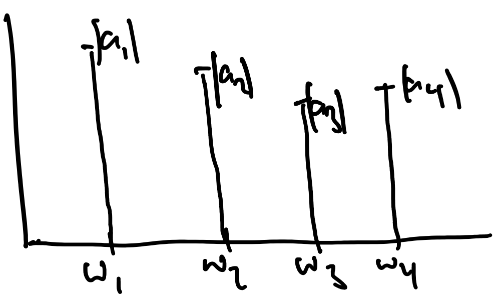
Use initial velocity of string to solve for b_n's

$$u(x,t) = \frac{dy}{dt} = \frac{d}{dt}\sum_{n=1}^{\infty} \tilde{a}_n \sin(k_n x)\, e^{j\omega_n t}$$

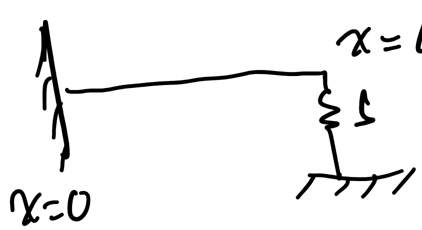
$\mathrm{Re}\{u(x,0)\} \Rightarrow$ find b_n by specifying $u(x,0)$

$\tilde{a}_n = a_n + j a_n$ now known

$|\tilde{a}_n|$



2.5.4 other BC's



$$k = \frac{\omega}{c}$$

$$c = \sqrt{T/\rho_L}$$

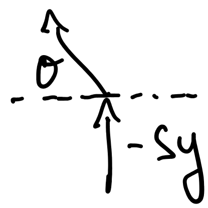
$$y(x,t) = A e^{j(\omega t - kx)} + B e^{j(\omega t + kx)}$$

$y(x,t) = Ae^{\cdots} \quad + Be^{\cdots}$

at $x=0$ $\quad y(0,t)=0 \qquad B=-A$

$$y(x,t) = -2jA\sin(kx)\cdot e^{j\omega t}$$

at $x=L$



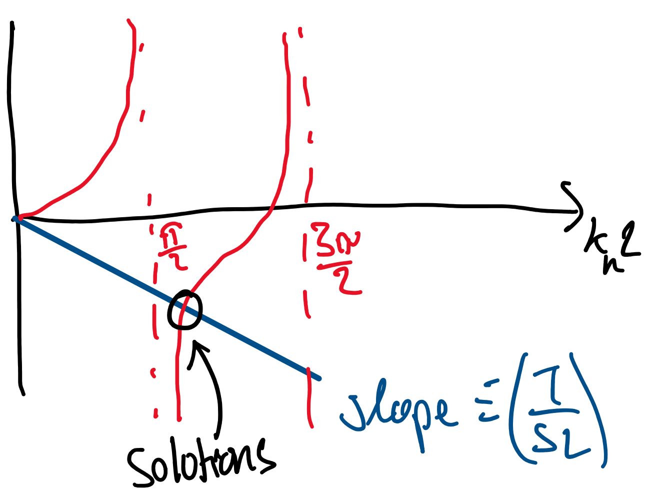
$\Sigma F_y = 0$

$$T\sin\theta\,\big|_{x=L} - Sy\,\big|_{x=L} = 0$$

$$T\left(-\frac{dy}{dx}\right)\Big|_{x=L} - Sy\,\big|_{x=L} = 0$$

$$\frac{dy}{dx} = -2jkA\cos kx\, e^{j\omega t}$$

$$\underbrace{-(k_nL)\frac{T}{SL} = \tan k_nL}_{\text{characteristic equation}}$$



Lecture 13

Wednesday, September 22, 2021 1:27 PM

Homework Hints

1) $\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0$

2... $y = a(ct-x)^2$ should cancel.

2.8.1 $y = 4\cos(3t-2x)$

$y = 4\cos(\omega t - kx)$ $\omega = 3$, $k = 2$

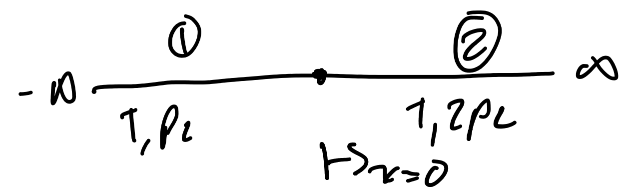
$k = \frac{\omega}{c}$

$\frac{\frac{rad}{s}}{\frac{m}{s}}$

"particle speed" - particle velocity

$u = \frac{dy}{dt}$

2.8.2 -



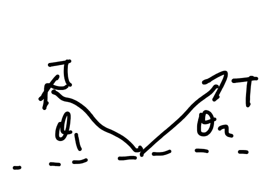
$y_1 = Ae^{j(\omega t - k_1 x)} + Be^{j(\omega t + k_1 x)}$

$y_2 = Ce^{j(\omega t - k_2 x)}$

relation between k_1 and k_2

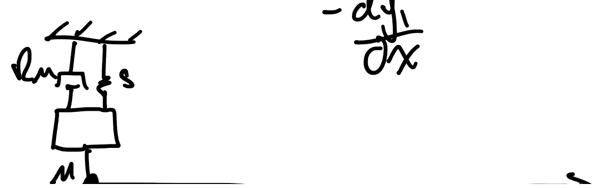
$c = \sqrt{T/\rho_L}$

$\frac{C}{A}$

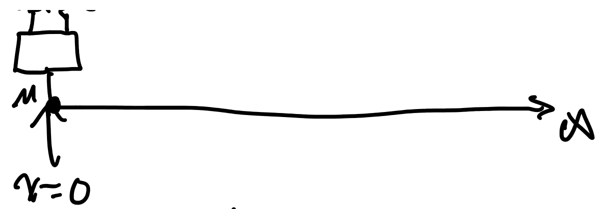


$\Sigma F_y = 0$ $\underbrace{T\sin\theta_1}_{-\frac{dy_1}{dx}} + T\sin\theta_2 = 0$

2.9.2



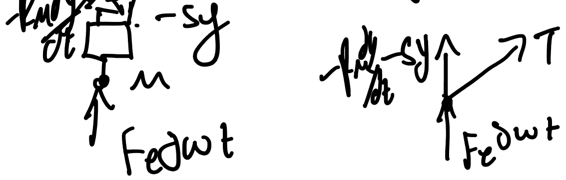
2.9.2



$$Z_{m0} = \frac{F e^{j\omega t}}{dy/dt|_{x=0}}$$

$$y = A e^{j(\omega t - kx)}$$

Force boundary

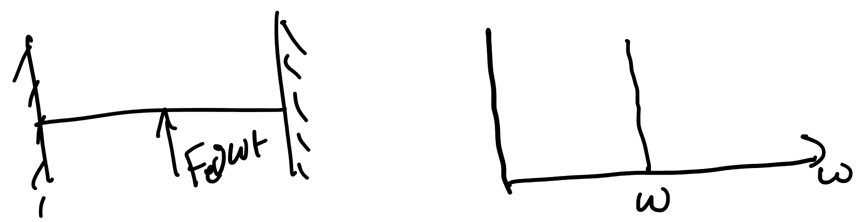


$$\Sigma F_y = m \frac{\partial^2 y}{\partial t^2}\Big|_{x=0}$$

$\rho_L c$ + SDOF {Z}

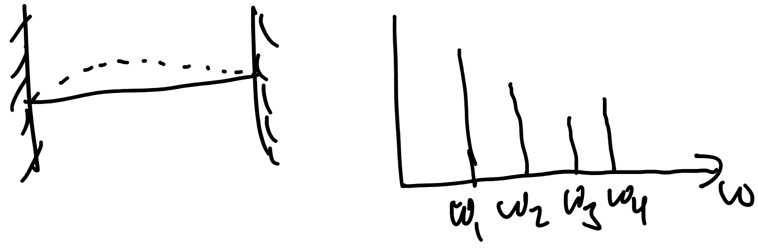
Forced Motion

- responds only at the forcing frequency



Free Motion

- responds at the natural frequencies



$\sin kL = 0$ characteristic equation for fixed-fixed string.

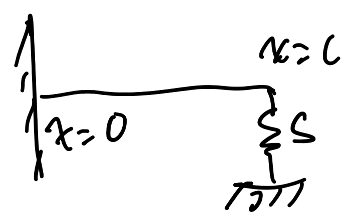
for fixed-fixed string.

infinit no. of solns.

$$k_n L = n\pi$$

$$y(x,t) = \sum_{n=1}^{\infty} a_n \sin k_n x \, e^{j\omega_n t}$$

2.5.4 other B.C.'s



$$y = -2j A \sin kx \, e^{j\omega t}$$

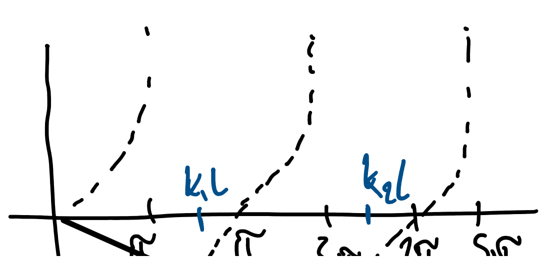
$$\left[-T\frac{dy}{dx} - sy\right]\Big|_{x=L} = 0$$

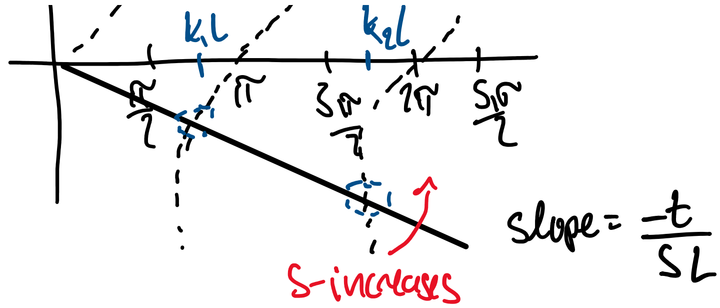
$$\cancel{2}jkT\cancel{A}\cancel{e^{j\omega t}}\cos kL + \cancel{2}js\cancel{A}\cancel{e^{j\omega t}}\sin kL = 0$$

$$kT\cos kL = s\sin kL$$

$$-\frac{k_n T}{s} = \tan kL$$

$$-k_n L\left(\frac{T}{sL}\right) = \tan kL \quad \ldots$$ characteristic equation for finite stiffness



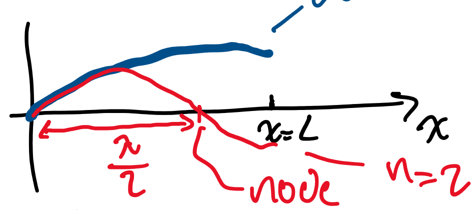


- as $s \to \infty$, behaves like fixed boundary
  - Natural frequencies decrease as s decreases
- as $s \to 0$ behaves like free end.

Mode Shape

$y_n(x,t) = \tilde{a}_n \sin k_n x \, e^{j\omega t}$ n^{th} mode

• k_1 is smaller than fixed-fixed



$$y(x,t) = \sum_{n=1}^{\infty} \tilde{a}_n \sin k_n x \, e^{j\omega t}$$

Section 2 Summary

- Derivation of wave equation (modeling)
  - restoring force
  - EOM

  ] single eqn. for y, wave eqn
- must be inertia and stiffness

Wave propagation $f(ct-x)$, $f(ct+x)$

$c = \sqrt{T/\rho_L}$ ....

$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$

Boundary Conditions:

- fixed
- stiffness
- mass
- force
- damper

} can be combined

$F = T\sin\theta \Rightarrow \sin\theta = -\frac{dy}{dx}$

Forced Response: $F e^{j\omega t}$

$y(x,t) = A e^{j(\omega t - kx)} + B e^{j(\omega t + kx)}$

Impedance: $\frac{\text{Force}}{\text{velocity}}$ ... Z_{mo}

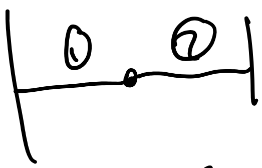
Characteristic Impedance:

$\rho_L c$

- Standing Wave Representation
- Propagating Wave

Free Response

- natural frequencies, mode shapes
  - => characteristic equation

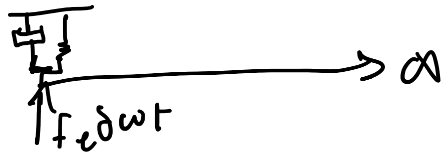


Treat as 2 strings with Diff. properties ---
connected at point of discontinuity

Lecture 14

Friday, September 24, 2021 1:29 PM

2.9.2.



Acoustic Wave Equation and Simple Sol'n's

- Sound - Propagating small amplitude fluctuations in pressure in an elastic medium
- "ideal" acoustics
  - assume inviscid, lossless, adiabatic
- small amplitude → "linear" acoustics

must have

inertia - Air: 1.2 kg/m3

stiffness - Air: 1.4×10^5 Pa

Derive a wave equation:

pressure & particle velocity

- equation of state] restoring force
- continuity equation]
- momentum] EOM

result - single eqn in pressure

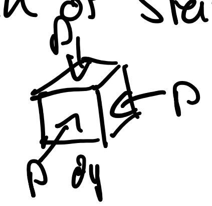
- 1-D solutions
  - plane wave
  - cylindrical and spherical waves] uniform, except r

- Acoustic Intensity
- specific acoustic impedance
- Decibels

3.2 Derivation of the Wave Equation

3.2.1 Pressure - Velocity Relation (I)

① Eqn of State



Ambient Pressure

$P_0 = 1 \times 10^5$ Pa

Ambient Air Density

$\rho_0 = 1.2$ kg/m³

for an ideal gas
- pressure is a function of density
  (density is a function of velocity)

Pressure changes from P_0 to P
as density changes $\rho_0 \to \rho$

as density changes $\rho_0 \to \rho$

$$P = P_0 + \left(\frac{\partial P}{\partial \rho}\right)_{\rho_0}(\rho - \rho_0) + \cancel{\frac{1}{2}\left(\frac{\partial^2 P}{\partial \rho^2}\right)_{\rho_0}(\rho - \rho_0)^2} \cdots$$

linear acoustics:

- ignore 2nd order terms
  - fluctuations small compared to ambient

$$\underbrace{P - P_0}_{\text{sound pressure}} = \underbrace{\rho_0\left(\frac{dP}{d\rho}\right)_{\rho_0}}_{\substack{\text{Bulk Modulus} \\ \text{(stiffness)}}} \underbrace{\frac{(\rho - \rho_0)}{\rho_0}}_{\substack{\text{condensation} \\ \text{non-dimensional}}}$$

if $\rho > \rho_0$, $s > 0$ ... compression
if $\rho < \rho_0$, $s < 0$ ... expansion

Bulk Modulus -

$$p = \rho_0 \frac{dP}{d\rho}\Big|_{\rho_0} \cdot s$$

$$p = \beta \cdot s \quad (1)$$

β = adiabatic

What is β for adiabatic compression

two states P, ρ_0 $\quad P, \rho_0$

From Thermo

$$\left(\frac{P}{P_0}\right) = \left(\frac{\rho}{\rho_0}\right)^\gamma$$

γ - ratio of specific heats

$$\left(\frac{P}{P_0}\right) = \left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

γ - ratio of specific heats

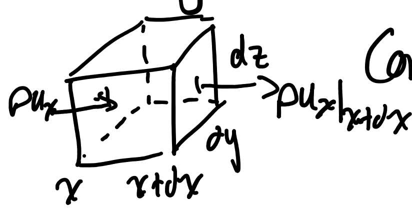
$\gamma = 1.4$ for air & adiabatic compression

$$\beta = \rho_0 \left(\frac{\partial P}{\partial \rho}\right)_{\rho_0} = \underbrace{\gamma P_0 = 1.4 \times 10^5\,\text{Pa}}_{\text{Bulk Modulus of air}}$$

$P = \beta s$ - relate to u

Continuity equation (conservation of mass)

Control Volume in Space



start with 1-D in x direction

Lecture 15

Monday, September 27, 2021 1:25 PM

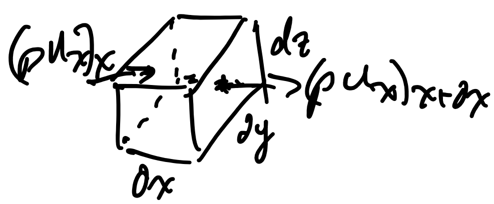
Wave Equation

3.2.1 Pressure-Velocity relation (I)

a) Equation of State

$p = \beta s \qquad s = \dfrac{\rho - \rho_0}{\rho_0}$

Continuity Equation (conservation of mass)



Rate at which mass flows in:

is $(\rho u_x)_x \, dy \, dz$ kg/s

Rate at which mass flows out:

is $(\rho u_x)_{x+dx} \, dy \, dz$ kg/s

$$= \left[(\rho u_x)_x + \frac{d(\rho u_x)}{dx}\bigg|_x dx + \dots \right] dy \, dz$$

Net rate of mass inflow

$$(a) - (b) = -\frac{\partial(\rho u_x)}{\partial x}\bigg|_x dx \, dy \, dz$$

Rate of Change of mass within controlled Volume

Rate of change of mass within controlled volume

$$\frac{\partial \rho}{\partial t}\,dx\,dy\,dz \qquad kg/s$$

$$\frac{\partial \rho}{\partial t}\,\cancel{dx\,dy\,dz} = -\frac{\partial(\rho u_x)}{\partial x}\Big|_x \,\cancel{dx\,dy\,dz}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_x)}{\partial x} = 0 \quad \ldots \text{ conservation of mass}$$

both ρ & u_x are functions of x

$$\frac{\partial \rho}{\partial t} + u_x\frac{\partial \rho}{\partial x} + \rho\frac{\partial u_x}{\partial x} = 0$$

$$S = \frac{\rho - \rho_0}{\rho_0} \Rightarrow \rho = S\rho_0 + \rho_0$$

$$\rho_0\frac{\partial (s\cancel{+1})}{\partial t} + \rho_0 u_x \frac{\partial(s\cancel{+1})}{\partial x} + \rho_0(s+1)\frac{\partial u_x}{\partial x} = 0$$

$$\rho_0\frac{\partial s}{\partial t} + \cancel{\rho_0 u_x \frac{\partial s}{\partial x}} + \rho_0(\cancel{s}+1)\frac{\partial u_x}{\partial x} = 0$$

product of two small quantities is negligible (linear acoustics)

$$\cancel{\rho_0}\frac{\partial s}{\partial t} + \cancel{\rho_0}\frac{\partial u_x}{\partial x} = 0$$

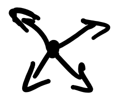
1-D: $$\frac{\partial s}{\partial t} + \frac{\partial u_x}{\partial x} = 0$$

(1)

$$\frac{\partial s}{\partial t} + \frac{\partial u}{\partial x} = 0 \quad (1)$$

3-D: $\frac{\partial s}{\partial t} + \nabla \cdot \bar{u} = 0$

$\nabla \cdot u$: divergence of particle velocity

if $\nabla \cdot u$ is positive:  density is decreasing

if $\nabla \cdot u$ is negative:  density is increasing

$$\bar{u} = u_x \hat{i} + u_y \hat{j} + u_z \hat{k}$$

$$\nabla \cdot \bar{u} = \frac{d u_x}{\partial x} + \frac{d u_y}{\partial y} + \frac{d u_z}{\partial z}$$

Recall $p = \beta s$ (2) $\Rightarrow s = \frac{p}{\beta}$

combine 1, 2

$$\frac{1}{\beta}\frac{\partial p}{\partial t} + \nabla \cdot u = 0$$] linearized continuity equation

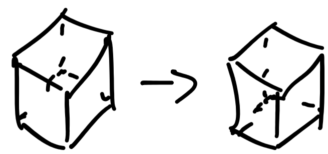
relates pressure and particle velocity

3.2.2. Pressure-Velocity (II)

EOM

EOM

apply $F = ma$ to a fixed mass moving with the fluid



1-D: $P(x) \rightarrow \square \leftarrow P(x+\partial x) = P(x) + \left.\frac{\partial P}{\partial x}\right|_x \partial x$

$x \quad x+\partial x$

net force in x-direction:

$$\partial f_x = P(x)\,\partial y\,\partial z - P(x+\partial x)\,\partial y\,\partial z$$

$$= -\left.\frac{\partial P}{\partial x}\right|_x \partial x\,\partial y\,\partial z$$

$$= -\frac{\partial P}{\partial x}\,\partial V$$ force/unit volume

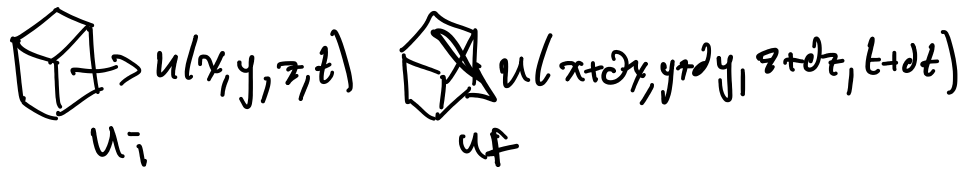
3-D: $\partial \vec{f} = -\nabla P\,\partial V$ (3)

∇P = gradient of pressure

$$\frac{\partial P}{\partial x}\hat{\imath} + \frac{\partial P}{\partial y}\hat{\jmath} + \frac{\partial P}{\partial z}\hat{k} = \nabla P$$

indicates the direction of the fastest change in pressure with position find $\underline{a}$

acceleration of moving fluid



$$a = \frac{\bar{u}_f - \bar{u}_i}{\partial t}$$

$\partial x = u_x \partial t \qquad \partial y = u_y \partial t \qquad \partial z = u_z \partial t$

$a = \frac{d\bar{u}}{dt}$ we assume convective acceleration is negligible

$$\bar{a} = \underbrace{(\bar{u} \cdot \nabla)\bar{u}}_{\text{convective accel (non-linear)}} + \frac{\partial u}{\partial t}$$

$\partial f = -\nabla P \partial V \qquad \bar{a} = \frac{d\bar{u}}{\partial t}$ Mass of the element $m = \rho \partial V$

$\partial \bar{f} = m\bar{a}$

$-\nabla P \partial V = \rho \partial V \cdot \frac{d\bar{u}}{\partial t}$

$P = P_0 + p \qquad \nabla P = \nabla p \leftarrow$ sound pressure

$\rho = \rho_0 (s+1) \qquad s = \frac{\rho - \rho_0}{\rho_0} << 1$

ρ_0

(4) $\boxed{-\nabla p = \rho_0 \frac{d\bar{u}}{\partial t}}$ Euler Equation

linearized momentum equation

2nd eqn. relating p and u

3.2.3 Linear Wave Eqn

(3) $\frac{1}{\beta}\frac{\partial p}{\partial t} + \nabla\cdot\bar{u} = 0$ $\quad \frac{\partial}{\partial t}$(3)

(4) $\nabla p + \rho_0 \frac{d\bar{u}}{\partial t} = 0$ $\quad \frac{1}{\rho_0}\nabla\cdot$(4)

Subtract (4) - (3)

$$\nabla^2 p - \frac{\rho_0}{\beta}\frac{\partial^2 p}{\partial t^2} = 0 \qquad c = \sqrt{\beta/\rho_0} = \sqrt{\gamma P_0/\rho_0}$$

$$\boxed{\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0}$$ linear wave equation

for stationary media + ideal fluids

3.2.4 Speed of Sound

c = speed of wave propagation

$= \sqrt{\gamma P_0/\rho_0} = 331.6$ m/s at 0°C

≈ 340 m/s at 20°C

C is directly proportional to absolute temp

$$C = C_0 \sqrt{T_k/273} \qquad C_0 = 331.6 \qquad T_k\ [°k]$$

$$= C_0 \sqrt{1 + T_c/273} \qquad T_c\ [°C]$$

Is C dependent on height in the atmosphere?

Lecture 16

Wednesday, September 29, 2021 1:25 PM

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

continuity: $\frac{1}{\beta}\frac{\partial p}{\partial t} + \nabla \bar{u} = 0$

linearized momentum: $\nabla p - \rho_0 \frac{\partial \bar{u}}{\partial t} = 0$

c = speed of sound = $\sqrt{\gamma P_0/\rho_0}$

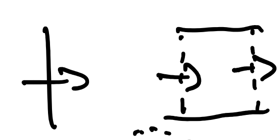
in an isothermal atmosphere:

$\frac{P_0}{\rho_0}$ is constant

$$c = c_0\sqrt{T_K/273}$$

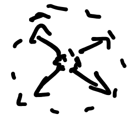
3.3 one-dimensional solutions

- plane wave



- cylindrical wave

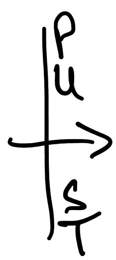
- spherical wave



radially symmetric

Plane Waves

- properties are instantaneously uniform over an infinite plane perpendicular to the direction of wave propagation



longitudinal waves

- particle motion is back and forth in direction of propagation

propagation in the x-direction:

no variation of pressure in $y + z$ directions

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \Rightarrow \frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

$$p(x,t) = p_1(ct-x) + p_2(ct+x)$$

positive $\rightarrow$ negative $\leftarrow$

$$p(x,t) = \tilde{p}(x)\, e^{j\omega t} \quad \ldots \text{ separable form}$$

sub into wave equation

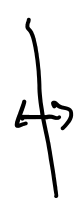
$$\frac{\partial^2 \tilde{p}}{\partial x^2} + k^2 \tilde{p} = 0 \quad \ldots \text{ Scalar-Helmholtz}$$

$$k = \frac{\omega}{c}$$

$$\tilde{p}(x) = A_1 e^{-jkx} + B_1 e^{jkx}$$

$$p(x,t) = \tilde{p}(x)\, e^{j\omega t}$$

particle velocity



can use linearized momentum

$$-\nabla p = \rho_0 \frac{du}{dt} \qquad \bar{u} = u_x\hat{i} + u_y\hat{j} + u_z\hat{k}$$

in 1-D:

$$-\frac{\partial p}{\partial x}\hat{i} = \rho_0 \frac{du_x}{dt}\hat{i}$$

$$p(x,t) = \tilde{p}(x)\, e^{j\omega t}$$

$$u_x(x,t) = \tilde{u}_x(x)\, e^{j\omega t}$$

$$-\frac{\partial p}{\partial x}\cancel{e^{j\omega t}} = \rho_0 j\omega\, \tilde{u}_x(x)\cancel{e^{j\omega t}}$$

$$-\frac{\partial p}{\partial x} \cancel{e^{j\omega t}} = \rho_0 j\omega\, u_x(x) \cancel{e}$$

$$\tilde{u}_x = -\frac{1}{j\omega\rho_0}\frac{\partial p}{\partial x}$$

Given a Pressure distribution

$$\tilde{u}_x = \frac{-1}{j\omega\rho_0}\frac{\partial p}{\partial x}$$

$$\tilde{u}_y = \frac{-1}{j\omega\rho_0}\frac{\partial p}{\partial y}$$

$$\tilde{u}_z = \frac{-1}{j\omega\rho_0}\frac{\partial p}{\partial z}$$

if no pressure gradient in a direction, there is no particle motion in that direction...

Given Pressure

$$\tilde{p}_+ = A_1 e^{-jkx}$$

$$\tilde{u}_{x_+} = -\frac{1}{j\omega\rho_0}\frac{\partial \tilde{p}_+}{\partial x} \Rightarrow \frac{\partial \tilde{p}_+}{\partial x} = -jk A_1 e^{-jkx}$$

$$= -\frac{1}{}\left(\cancel{j}k A_1 e^{-jkx}\right) = \frac{k}{\omega\rho_0} A_1 e^{-jkx} \qquad k = \frac{\omega}{c}$$

$$= \frac{-1}{-\cancel{j}\omega\rho_0}\left(\cancel{j}k A_1 e^{-jkx}\right) = \frac{k}{\omega\rho_0} A_1 e^{\cdots} \qquad k = \frac{\omega}{c}$$

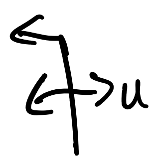
$$\tilde{u}_{x+} = \frac{1}{\rho_0 c} A_1 e^{-jkx} \qquad \rho_0 c \text{ – characteristic impedance}$$

$$\tilde{u}_{x+} = \frac{p_+}{\rho_0 c} \qquad \rho_0 c \approx 415 \text{ [Rayls]}$$

Specific Acoustic Impedance:

$$\frac{p_+}{u_{x+}} = \rho_0 c = \text{characteristic Impedance (in this simple case)}$$

$$\tilde{p}_- = A_2 e^{jkx}$$



$$\tilde{u}_x = -\frac{1}{j\omega\rho_0}\frac{\partial \tilde{p}_-}{\partial x} = \frac{-1}{j\omega\rho_0}\left(jkA_2 e^{jkx}\right) = \frac{-1}{\rho_0 c} A_2 e^{jkx}$$

$$\tilde{u}_{x-} = \frac{-A_2}{\rho_0 c} e^{jkx}$$

$$\tilde{u}_{x-} = \frac{-p_-}{\rho_0 c} \qquad \frac{\tilde{p}_-}{\tilde{u}_{x-}} = -\rho_0 c$$

Sign of impedance depends on

Sign of impedance depends on direction of wave propagation

Use these results to calculate particle velocity for any given sound field

for the harmonic case: $\boxed{\tilde{u} = \frac{-1}{j\omega\rho_0} \nabla \tilde{p}}$

Standing Wave Tube

$$\tilde{p}(x) = A_1 e^{-jkx} + A_2 e^{jkx}$$

$$\tilde{u}(x) = \frac{\tilde{p}_+}{\rho_0 c} - \frac{\tilde{p}_-}{\rho_0 c}$$

$$\tilde{u}(x) = \frac{A_1}{\rho_0 c} e^{-jkx} - \frac{A_2}{\rho_0 c} e^{jkx}$$

<u>Typical Numbers</u>

- plane wave prop in the x-direction

$$|p_+| = 1\,\text{Pa} \iff 94\,\text{dB SPL}$$

$$|u_{x_+}| = \frac{|p_+|}{\rho_0 c} = \frac{1\,\text{Pa}}{415} \approx 2.4\,\text{mm/s}$$

$c = 340 \text{ m/s}$

$|\tilde{u}_{x+}| << c$

Displacement: ξ

for harmonic fields

$j\omega \tilde{\xi}_{x+} = \tilde{u}_{x+}$

$|\tilde{\xi}_{x+}| = \dfrac{|\tilde{u}_x|}{\omega}$ ω at 1kHz ≈ 6000

ξ particle displacement

in 94 dB case:

$|\tilde{\xi}_{x+}| = \dfrac{2.4 \times 10^{-5}}{6 \times 10^{3}} = .25 \times 10^{-6} \text{ m}$

$= .25$ microns

74 dB:

$|\tilde{\xi}_{x+}| \quad O(10^{-7})$

54 dB:

$|\tilde{\xi}_{x+}| \quad O(10^{-8})$

34 dB

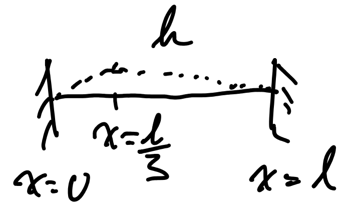
$|\tilde{\xi}_{x+}| \quad O(10^{-9}) = \text{nm}$

Lecture 17

Friday, October 1, 2021 1:27 PM

Homework Hints

2.10.1 Free Vibration



initial velocity = 0

$$y(x,t) = \sum_{n=1}^{\infty} [A_n\cos(\omega_n t) + \cancel{B_n\sin(\omega_n t)}]\sin(k_n x)$$

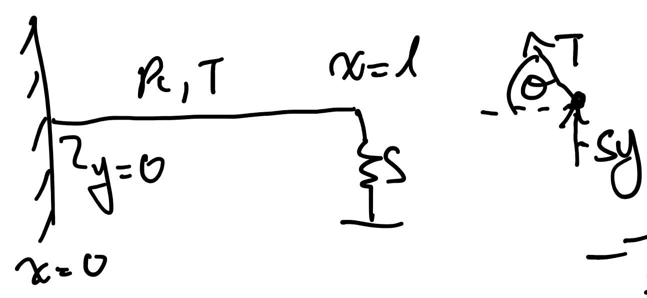
due to $y'(x,0)$

$$A_n = \frac{2}{L}\int_0^L y(x,0)\sin k_n x \, dx$$

develop equations for $y(x,0)$

$$k_n = \frac{n\pi}{L} = \frac{\omega_n}{c}$$

2.11.1



$$T\sin\theta - Sy = 0$$

$$-T\frac{dy}{dx}\bigg|_{x=l} - Sy\big|_{x=l} = 0$$

$$y = (Ae^{-jkx} + Be^{jkx})e^{j\omega t}$$

$$T = SL$$

$$y = -2jA\sin kx \, e^{j\omega t}$$

$$\cancel{S}L\frac{dy}{dx}\bigg|_{x=l} - \cancel{S}y\big|_{x=l} = 0$$

$$y = -2jA \sin kx \, e^{\cdots}$$

$$S L \left.\frac{dy}{dx}\right|_{x=L} - S y|_{x=L} = \cdots$$

$$L \frac{dy}{dx} = y$$

Plane Waves Cont.

harmonic

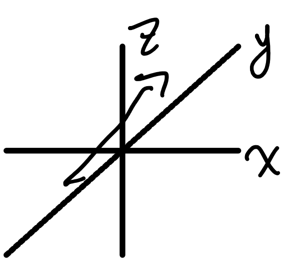
$$\nabla^2 \tilde{p} + k^2 p^2 = 0$$

$$\tilde{p}(x) = A e^{-jkx} + B e^{jkx}$$

$$\tilde{u}(x) = \frac{-1}{j\omega\rho_0} \frac{d\tilde{p}}{dx}$$

$$\frac{\tilde{p}(x)}{\tilde{u}(x)} = \pm \rho_0 c \qquad \tilde{u}_x = \frac{A}{\rho_0 c} e^{-jkx} - \frac{B}{\rho_0 c} e^{jkx}$$

3.3.1.2 Arbitrary Direction of Propagation



$$\nabla^2 p + k^2 \tilde{p} = 0 \qquad k = \frac{\omega}{c}$$

$$\tilde{p}(x,y,z) = A e^{j(\omega t - k_x x - k_y y - k_z z)}$$

Component wave numbers

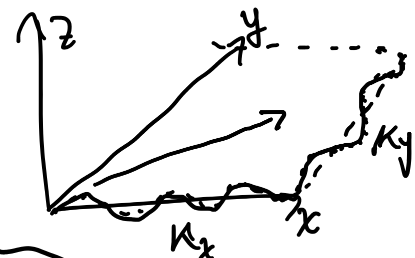
k_x] rate of change of phase

$\left.\begin{matrix} k_x \\ k_y \\ k_z \end{matrix}\right]$ rate of change of phase with position in the coord directions

$$k_x^2 + k_y^2 + k_z^2 = k^2$$

This must be true for a propagating pressure disturbance to be sound.

Only 2 of the k_x, k_y, k_z can be specified independently if the solution is to be sound



$$k_z = \sqrt{k - k_x^2 - k_y^2}$$

is sound radiated away? check

$$p(x,y,z,t) = Ae^{j(\omega t - k_x x - k_y y - k_z z)}$$

8 possible combinations

$$k_x^2 + k_y^2 + k_z^2 = k^2$$

Wave Vector:

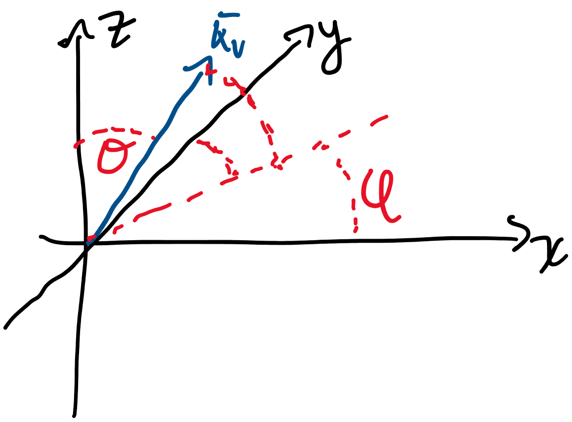
$$\bar{k}_v = k_x \hat{\imath} + k_y \hat{\jmath} + k_z \hat{k}$$

$k_v = k_x i + k_y j + k_z k$

direction of wave propagation
+ magnitude of wave number
(rate change phase in dir.
of propagation.)

$$|\bar{k}_v| = k = \frac{\omega}{c}$$

$$\tilde{p}(x,y,z,t) = A e^{j(\omega t - \bar{k}_v \cdot \bar{x})}$$



θ = polar angle

φ = azimuth angle

$$k_z = k\cos\theta$$
$$k_y = k\sin\theta \cdot \sin\varphi$$
$$k_x = k\sin\theta \cdot \cos\varphi$$

$$k_z^2 = \underset{\substack{\uparrow \\ \text{fixed by freq.}}}{k^2} \underbrace{- k_x^2 - k_y^2}_{\substack{\text{may be fixed} \\ \text{by B.C.'s} \\ \text{(vibrating plate)}}}$$

(vibrating plate)

$$k_z = \sqrt{k^2 - k_x^2 - k_y^2}$$

① $k_x^2 + k_y^2 > k^2 \Rightarrow \sqrt{-n}$

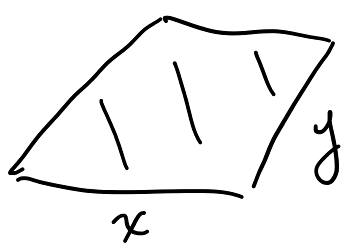
$$k_z = \pm j\sqrt{k_x^2 + k_y^2 - k^2}$$

$$k_z = \pm j\alpha$$

$$k_z = \pm j\alpha \Rightarrow e^{\pm j k_z z} \Rightarrow \underbrace{e^{\pm \alpha z}}_{\text{pure exponential}}$$

$e^{\alpha z}$ growth, $e^{-\alpha z}$ decay

$e^{\alpha z} \rightarrow$ discarded if infinite air above plate



if $k_x^2 + k_y^2 > k^2$

$$\underbrace{\tilde{p} = A e^{-jk_x x} e^{-jk_y y} e^{-\alpha z}}_{\text{pressure field above the plate}}$$

evanescent or non-propagating wave

Lecture 18

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Simple Soln's

One dimensional

$$p = e^{j(\omega t - k_x x - k_y y - k_z z)}$$

$$k^2 = k_x^2 + k_y^2 + k_z^2$$

$$k_z = \sqrt{k^2 - k_x^2 - k_y^2}$$

(i) $k_x^2 + k_y^2 > k^2$

$k_z = \pm j\alpha \quad \alpha = \sqrt{k_x^2 + k_y^2 - k^2}$

$e^{\pm j k_z z} \Rightarrow e^{\pm \alpha z}$ pure growth or decay

evanescant near-field

$$p = A e^{-jk_x x} e^{-jk_y y} e^{-\alpha z}$$

(ii) $k_x^2 + k_z^2 < k^2$ k is real

$$\tilde{p} = A e^{j(\omega t - k_x x - k_y y - k_z z)}$$

$e^{\pm j k_z z}$ oscillatory behavior in the z-direction

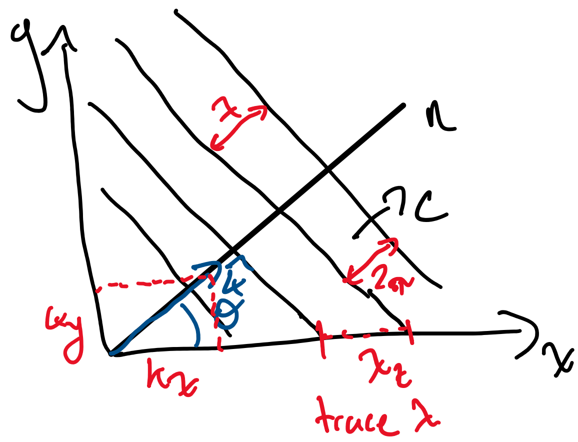
-no attenuation or decay

- sound propagating

Consider 2d case:

Consider 2d case:

- no variation in the z-direction



$$\lambda_t \cos\theta = \lambda \qquad \lambda_t = \frac{\lambda}{\cos\theta}$$

$$k_x = k\cos\theta$$
$$k_y = k\sin\theta$$

$$k_x^2 + k_y^2 + \cancel{k_z^2}^{\,0} = k^2 \qquad k = \sqrt{k_x^2 + k_y^2}$$

$$\lambda_t = \frac{\lambda}{\cos\theta} \qquad \lambda_t \to \lambda, \quad \theta \to 0$$

grazing incidence

$$\lambda_t \to \infty$$
$$\theta \to \pi/2$$

normal incidence

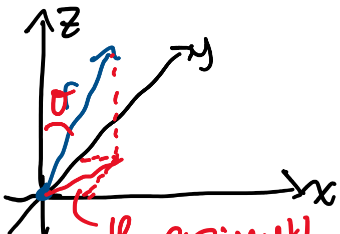
trace wave speed:

$$c_t = \frac{\lambda_t}{T} \geq c$$

$\theta = 0,\ c_t = c$ grazing

$\theta = \frac{\pi}{2},\ c_t \to \infty$ normal

Spherical Waves



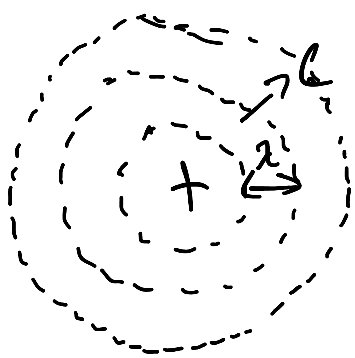
φ-azimuth

θ- polar

(r, θ, φ)

Spherically symmetric waves

-no variation in θ or φ



field created by omni-directional source

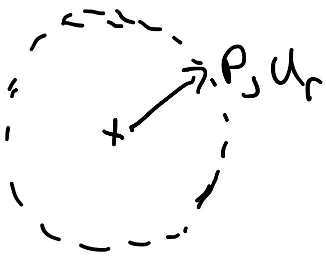
$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

In Spherical coordinates:

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \cancel{\frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial}{\partial \varphi}\right)}$$

no variation in θ or φ

$\tilde{p}$ is a function only of r



all quantities are instantaneously constant

purely radial

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} = \frac{1}{r}\frac{\partial^2 (rp)}{\partial r^2}$$

$$\frac{\partial^2 (rp)}{\partial r^2} + \frac{1}{c^2}\frac{\partial^2 (rp)}{\partial t^2} = 0$$

harmonic form:

$$\frac{\partial^2 (r\tilde{p})}{\partial r^2} + k^2 (r\tilde{p}) = 0 \quad \text{SHE}$$

harmonic soln's:

$$r\tilde{p} = Ae^{-jkr} + Be^{jkr}$$

$$\tilde{p} = \frac{A}{r}e^{-jkr} + \frac{B}{r}e^{jkr}$$

outward going   inward coming

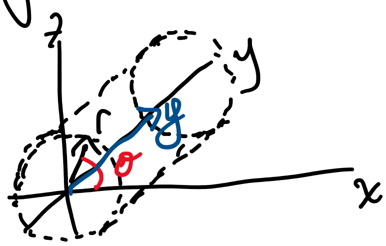
particle velocity

$$\tilde{u}_r = -\frac{1}{j\omega\rho_0}\frac{d\tilde{p}}{dr}$$ linearized momentum

$$\underbrace{u_{r+}}_{\text{expanding wave}} = \frac{1}{\rho_0 c}\Big(1 + \underbrace{1/jkr}_{\text{near-field term}}\Big)\underbrace{\tilde{p}_+}_{\text{outward going wave}}$$

plane wave: $u_+ = \frac{p_+}{\rho_0 c}$

Cylindrical Waves:



$p(r, \theta, y)$

$x = r\cos\theta$
$z = r\sin\theta$
$y = y$

Cylindrically Symmetric

$$\nabla^2 p = \frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r}$$

harmonic case: $e^{j\omega t}$

$$\frac{\partial^2 \tilde{p}}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\tilde{p} + k^2 p = 0$$

Bessel Equation of 0^{th} order

Bessel Equation of 0^{th} order

Soln's in terms of $J_0(kr) + Y_0(kr)$

$$\tilde{p}(r) = \underbrace{A H_0^{(2)}(kr)}_{\text{outward going}} + \underbrace{B H_0^{(1)}(kr)}_{\text{inward coming}}$$

$$\left.\begin{aligned} H_0^{(1)} &= J_0(kr) + j Y_0(kr) \\ H_0^{(2)} &= J_0(kr) - j Y_0(kr) \end{aligned}\right] \text{Hankel Functions}$$

$$|\tilde{p}| \simeq \frac{1}{r^{1/2}} \text{ when } kr \gg 1$$

Lecture 19

Wednesday, October 6, 2021 1:24 PM

fixed-fixed ; $\tan kL = 0$

One-Dimension:

$$\tilde{p}(x,y,z,t) = Ae^{j(\omega t - k_x x - k_y y - k_z z)}$$

$$K^2 = \sqrt{k_x^2 + k_y^2 + k_z^2} = \left(\frac{\omega}{c}\right)^2$$

$k_x^2 + k_y^2 > k^2$ $k_z \Rightarrow$ imaginery

evanescant near-field

Free Field

Plane wave: $|\tilde{p}| \propto |e^{-jkx}|$ constant for all x

Spherical wave: $|\tilde{p}| \propto \left|\frac{1}{r} e^{-jkr}\right| \Rightarrow \frac{1}{r}$

Cylindrical wave: $|\hat{p}| \propto \left|H_0^{(2)}(kr)\right| \rightarrow \frac{1}{r^{1/2}}$ $kr \gg 1$

(Far Field)

3.4. Specific Acoustic Impedance

$$\tilde{z} = \frac{\text{Acoustic Precsure}}{\text{Acoustic Particle Vel.}} = \frac{\tilde{p}}{\tilde{u}}$$

harmonic

mechanical impedance:

Mechanical impedance:

$$= \frac{\text{Force}}{\text{Velocity}}$$

plane wave in pos. direction

$$p_+(x) = Ae^{-jkx}$$

$$\tilde{u}_+(x) = -\frac{1}{j\omega\rho_0}\frac{d\tilde{p}_+}{dx} = \frac{1}{\rho_0 c}\tilde{p}_+$$

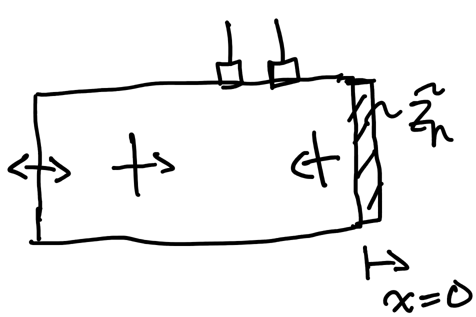
$$\tilde{z} = \frac{\tilde{p}_+}{\tilde{u}_+} = \frac{\tilde{p}_+}{\frac{1}{\rho_0 c}\tilde{p}_+} = \rho_0 c$$ ← characteristic impedance

plane wave in negative direction:

$$\tilde{z} = \frac{\tilde{p}_-}{\tilde{u}_-} = -\rho_0 c$$

$\tilde{z}$ is a function of direction

In general $\tilde{z}$ is a function of position



$$\tilde{p}(x) = Ae^{-jkx} + Be^{jkx}$$

$$\tilde{p}(x) = Ae^{-jkx} + Be^{jkx}$$

$$\tilde{u}(x) = \frac{A}{\rho_0 c} e^{-jkx} - \frac{B}{\rho_0 c} e^{jkx}$$

$$\tilde{z}(x) = \frac{\tilde{p}(x)}{\tilde{u}(x)} = \rho_0 c \frac{Ae^{-jkx} + Be^{jkx}}{Ae^{-jkx} - Be^{jkx}}$$

Reflection Coefficient: $\frac{B}{A} = R$

$$= \rho_0 c \frac{e^{-jkx} + Re^{jkx}}{e^{-jkx} - Re^{jkx}}$$

if evaluated at $x = 0$

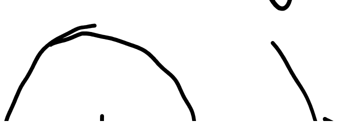
$$\tilde{z}(0) = \rho_0 c \frac{1+R}{1-R}$$

$$0 \leq |R| \leq 1$$

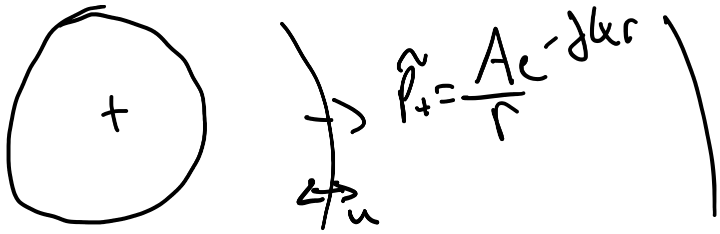
for $R = 0$ $\quad \tilde{z} \to \rho_0 c$

$R = 1$ $\quad \tilde{z} \to -j\rho_0 c \cot(kx)$

Spherically Symmetric case



$$\tilde{p} = \frac{Ae^{-jkr}}{\ }$$



$$\tilde{p}_+ = \frac{A e^{-jkr}}{r}$$

$$\tilde{u}_r = \frac{-1}{j\omega\rho_0} \cdot \frac{\partial \tilde{p}_+}{\partial r} = \frac{1}{\rho_0 c} \cdot \left(1 + \frac{1}{jkr}\right) \tilde{p}_+$$

$$\tilde{Z}_+ = \frac{\tilde{p}_+}{\tilde{u}_+} = \frac{\rho_0 c}{1 + 1/jkr} \qquad kr = \frac{2\pi}{\lambda} r = 2\pi\left(\frac{r}{\lambda}\right)$$

as $kr \to \infty$ $\quad \tilde{Z}_+ \to \rho_0 c$

at the far field, the wave front is locally plane

near fied: $\dfrac{\rho_0 c}{1 + 1/kr} \Rightarrow jkr(\rho_0 c) \Rightarrow j\omega\rho_0 r$

linear, positive, proportional to frequency, imaginary

mass-like impedance

General Spherical Case:

$$\tilde{p}(r) = \frac{A}{r} e^{-jkr} + \frac{B}{r} e^{jkr}$$

$$\tilde{u}(r) = -\frac{1}{\ } \frac{d\tilde{p}}{\ } \to 4 \text{ terms}$$

$$\tilde{u}(r) = -\frac{1}{j\omega\rho_0}\frac{d\tilde{p}}{dr} \rightarrow 4 \text{ terms}$$

$$\tilde{z}(r) = \frac{\tilde{p}(r)}{\tilde{u}(r)} \rightarrow \text{complicated}$$

Notes

(i) Specific Acoustic Impedance is usually expressed as

$$\tilde{z} = \underbrace{r}_{\substack{\text{specific}\\\text{Acoustic}\\\text{resistance}}} + j\underbrace{x}_{\substack{\text{specific Acoustic}\\\text{reactance}}}$$

(ii) $\rho_0 c$: Characteristic impedance $\neq$ specific Acoustic impedance

(iii) for outward going cylindrical & spherical waves are plane like at great distance from the source

$kr \gg 1 \quad z \rightarrow \rho_0 c$

3.5 Acoustic Intensity

in mechanics: Force · distance = work (joules)

Force · velocity = power (watts)

In acoustics:

$$\text{pressure} \cdot \text{velocity} = \text{intensity } (W/m^2) = \frac{\text{power}}{\text{unit area}}$$

Lecture 20

Friday, October 8, 2021 1:29 PM

Spherical, cylindrical, plane waves

$\frac{1}{r}$ $\frac{1}{r^{1/2}}$ ~

Specific acoustic impedance

$\frac{\tilde{P}}{\tilde{u}}$ - usually a function of position

spherical / Cylindrical: near-field terms

3.5 Acoustic Intensity

$\underbrace{\text{pressure} \times \text{velocity}}_{\text{intensity}}$ = power/unit area [W/m²]

Instantaneous Intensity:

$p(t)\bar{u}(t) = \bar{I}_t(t)$ [vector quantity]

Intensity is a vector

Time-Averaged Acoustic Intensity:

- time averaged rate of energy flow

Time Average Acoustic Intensity:

- time averaged rate of energy flow through a unit area

For a periodic Signal

$$\bar{I} = \frac{1}{T}\int_0^T \underbrace{p(t)\cdot\bar{u}(t)}_{\text{real parts}}\,\partial t \quad = \text{real quantity}$$

T = period

+ve going plane wave $\quad \rightarrow x$

$$\frac{p_+}{u_+} = \rho_0 c \qquad u_+ = \frac{p_+}{\rho_0 c}$$

$$\bar{I}_x = \frac{1}{T}\int_0^T p_+^2/\rho_0 c \,\partial t$$

$$\bar{I}_x = \frac{1}{\rho_0 c}\underbrace{\left[\frac{1}{T}\int_0^T p_+^2 \,\partial t\right]}_{\text{mean-squared pressure}}$$

$$\bar{I}_x = \frac{p_{rms}^2}{\rho_0 c}$$

Consider harmonic Cases:

$$\tilde{p}_+ = A e^{j(\omega t - kx)}$$
$$\tilde{u}_+ = \frac{A}{\rho_0 c} e^{j(\omega t + kx)}$$

$$\tilde{u}_+ = \frac{A}{\rho_0 c} e^{j(\omega t - kx)}$$

at $x=0$:

$$\tilde{p}_+ = (A_r + jA_i)(\cos\omega t + j\sin\omega t)$$

$$\tilde{u}_+ = \frac{(A_r + jA_i)}{\rho_0 c}(\cos\omega t + j\sin\omega t)$$

$$Re\{\tilde{p}_+\} = A_r\cos\omega t - A_i\sin\omega t$$

$$Re\{\tilde{u}_+\} = \frac{A_r\cos\omega t - A_i\sin\omega t}{\rho_0 c}$$

$$I_x = \frac{1}{T}\int_0^T Re\{\tilde{p}_+\}\cdot Re\{\tilde{u}_+\}\,\partial t$$

evaluates to:

$$I_x = \frac{1}{2\rho_0 c}\left[A_r^2 + A_i^2\right] = \frac{|A|^2}{2\rho_0 c} = \frac{|\tilde{p}_+|^2}{2\rho_0 c}$$

also $\dfrac{|\tilde{p}_+|^2}{2}$ = mean squared pressure = p_{rms}^2

$$I_x = \frac{p^2_{rms}}{\rho_0 c} \qquad p^2_{rms} = \frac{\tilde{p}\cdot\tilde{p}^*}{2}$$ ~ conjugate

for a freely propagating plane wave:

$+x$ $\quad I_x = p^2_{rms}$

$\overset{+}{\rightarrow} x \ldots \; I_x = \dfrac{p^2_{rms}}{\rho_0 c}$

$\overset{+}{\leftarrow} x \ldots \; I_x = -\dfrac{p^2_{rms}}{\rho_0 c}$

$$\frac{\tilde{p}_-}{\tilde{u}_-} = -\rho_0 c$$

Complex Harmonic Signals: $e^{j\omega t}$

$$\boxed{\bar{I} = \frac{1}{2} \mathrm{Re}\{\tilde{p}_+ \tilde{u}_+^*\}}$$ - use this to calculate intensity

for a freely propagating plane wave:

$$\bar{I} = \frac{1}{2} \mathrm{Re}\left\{ A \cancel{e^{-jkx}} \cancel{e^{j\omega t}} \cdot \frac{A^* \cancel{e^{jkx}} \cancel{e^{-j\omega t}}}{\rho_0 c} \right\}$$

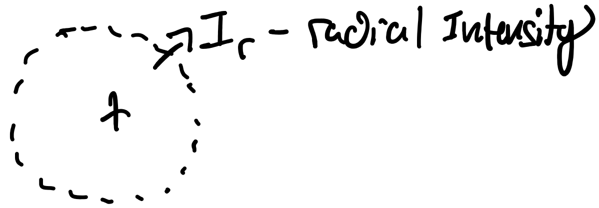
$$= \frac{1}{2} \mathrm{Re}\{ A A^* / \rho_0 c \} = \frac{1}{2} \cdot \frac{p^2_{rms}}{\rho_0 c}$$

General Expression for time-averaged acoustic intensity is

$$\bar{I} = \frac{1}{2} \mathrm{Re}\{ \tilde{p} \tilde{u}^* \}$$

when using complex harmonic representations...

Spherical Case:



Spherically symmetric case...
free space (only outward waves)

$$I_r = \frac{1}{2}\,\mathrm{Re}\{\tilde{p}_+ \tilde{u}_{r+}^*\} \qquad u_{r+} = \frac{1}{\rho_0 c}\cdot\left(1+\frac{1}{jkr}\right)\hat{p}_+$$

$$u_{r+}^* = \frac{1}{\rho_0 c}\left(1-\frac{1}{jkr}\right)\tilde{p}_+^*$$

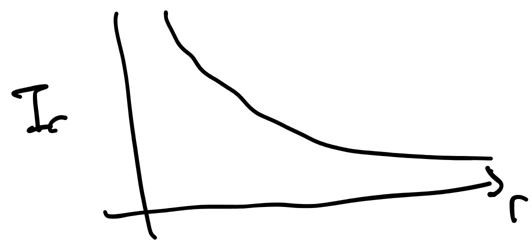
$$I_r = \frac{1}{2}\,\mathrm{Re}\left\{p_+ p_+^* \frac{1}{\rho_0 c}\left(1-\underbrace{\frac{1}{jkr}}_{\text{near-field}}\right)\right\}$$

$$I_r \Rightarrow \frac{|\tilde{p}_+|^2}{2\rho_0 c}$$

near-field term does not contribute to the acoustic intensity...

$$\tilde{p}_+ = \frac{A}{r}e^{-jkr} \qquad |\tilde{p}_+| \propto \frac{1}{r}$$

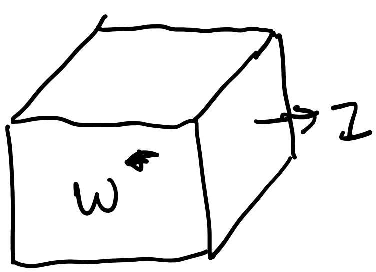
$$I_r = \frac{|\tilde{p}_+|}{2\rho_0 c} \qquad I_r \propto \frac{1}{r^2}$$



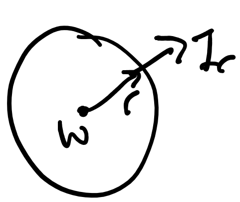
In contrast with particle velocity case
There is no intensity near-field

Intensity = Sound power per unit area

Sound Power = Intensity * Area
... obtained by integrating the normal intensity over a surface enclosing a source



Spherically symmetric .... freely propagating (outward)



$$W = \int_S I_r \, ds$$

$$W = \int_S I_r \, ds \qquad I_r \rightarrow \text{only a function of } r$$

$I_r \int ds$

$\bar{S}$

$$= I_r \underbrace{\int_S dS}_{\text{area of the sphere}}$$

$$W = I_r 4\pi r^2$$

$$\left. I_r = \frac{W}{4\pi r^2} \right\} \text{ inverse square law}$$

in a free field: $I_r = \dfrac{p^2_{rms}}{\rho_0 c}$

Lecture 21

Wednesday, October 13, 2021 1:25 PM

Acoustic Intensity -

pressure x particle velocity $\frac{\text{watts}}{m^2}$

Time Averaged

harmonic: $e^{j\omega t}$ $= \frac{1}{2} Re\{\tilde{p}\,\tilde{u}^*\} = I$ ~ vector quantity

plane wave $I_x = \frac{\tilde{p}\tilde{p}^*}{2\rho_0 c}$ ~ p^2_{rms}

$\rightarrow x$

$$I_x = \frac{p^2_{rms}}{\rho_0 c}$$

Spherical:

$|\tilde{p}_r| \propto \frac{1}{r}$ ... $I_r \propto \frac{1}{r^2}$

$$W = \int_S I \qquad I_r = \frac{W}{4\pi r^2}$$

Decibels

- sound pressure covers an enormous range

Very loud: 120 dB → 20 Pa

30 dB → 6×10^{-4} Pa

Humans respond to sound in a logarithmic way

Humans respond to sound in a logarithmic way

Levels: always referring to a decibel quantity

Sound Pressure Level

$$L_p = 10\log_{10}(p_{rms}^2/p_{ref}^2) = 20\log_{10}(p_{rms}/p_{ref})$$

$p_{ref} = 2\times10^{-5}$ Pa, $20\,\mu$Pa

minimum audible sound pressure at 1kHz
for nominal hearing.

e.g. 87dB re $20\,\mu$Pa

$10\log_{10}$(ratio of power related quantities)

$10\log_{10}(p^2_{rms}/p^2_{ref})$

$20\log_{10}(p_{rms}/p_{ref})$

Sound Intensity Level

$$L_I = 10\log_{10}(I/I_{ref}) \text{ dB re } I_{ref} = 1\times10^{-12}\ W/m^2$$

plane wave:

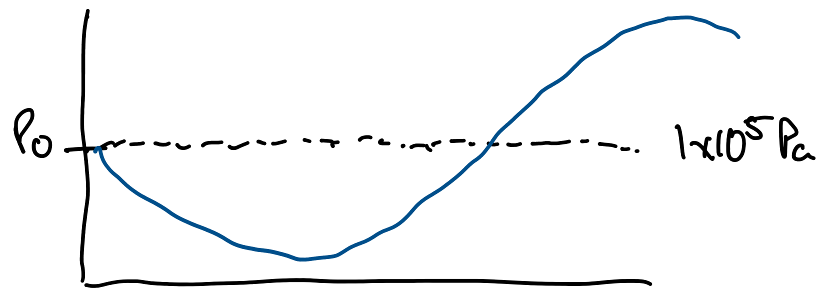
$$I = \frac{p^2_{rms}}{\rho_0 c} \Rightarrow \frac{p^2_{ref}}{\rho_0 c} = I_{ref} = \frac{(2\times10^{-5})^2}{415} \simeq 1\times10^{-12}$$

for a freely propagating wave:
choice of references makes $L_p = L_I$

Sound Power Level

$$L_W = 10\log_{10}(W/W_{ref}) \quad \text{dB re } W_{ref}$$

$$W_{ref} = 1\times10^{-12} \text{ watts}$$



$$L_p \simeq 10\log_{10}(1\times10^{-10}/4\times10^{-10}) \simeq 194 \text{ dB re } 20\,\mu\text{Pa}$$

$0 \le L_p \le 194$ range of possible SPL's

above $\simeq$ 120 dB... non-linear effects introduced

Adding dB's:

$$70\text{ dB} + 70\text{ dB} = 73\text{ dB}$$

assume uncorrelated...

1. ~~~~~~~~ $L_{p_1} \rightarrow p^2_{rms_1}$

2. ~~~~~~~~ $L_{p_2} \rightarrow p^2_{rms_2}$

$$L_p = 10\log_{10}(p^2_{rms}/p^2_{ref})$$

$$p^2_{rms} = 10^{L_p/10} \cdot p^2_{ref}$$

$$p^2_{rms} = 10^{L_p/10} \cdot p^2_{ref}$$

sum $p^2_{rms_t} = (p^2_{rms})_1 + (p^2_{rms})_2 + \ldots$

$$L_{p,total} = 10\log_{10}\left(p^2_{rms,total} / p^2_{ref}\right)$$

Section 3

Development of the Wave Equation

- assumptions
  - linear (small fluctuations)
  - ideal (no heat xfer, viscosity)

- equation of state
- equation of continuity
- equation of motion

$\Big]$ 2nd Order PDE

- wave like solution

  $x-ct$, $x+ct$

- linearized momentum equation

if pressure field is known, can find particle velocity

e^{-jkx} $\qquad$ $e^{-j(k_x x + k_y y + k_z z)}$

to be sound:

$$k^2 = k_x^2 + k_y^2 + k_z^2 \ldots \quad k = \frac{\omega}{c}$$

One-dimensional Solutions:

plane wave $\quad |p| \neq f(x)$

One-dimensional Solutions:

plane-wave $|p| \neq f(x)$

cylindrical source $|p| \propto \frac{1}{\sqrt{r}}$

spherical source $|p| \propto \frac{1}{r}$

Specific Acoustic Impedance: z

$\tilde{z} = \frac{\tilde{p}}{\tilde{u}}$ $\qquad$ $\tilde{u} = \frac{\tilde{p}}{\tilde{z}}$

Lecture 22

Friday, October 15, 2021 1:27 PM

Intensity - direction and magnitude

- time averaged product of $\tilde{p}$ & $\tilde{u}$

$$\bar{I} = \frac{1}{2} Re\{\tilde{p}\,\tilde{u}^*\} \quad W/m^2$$

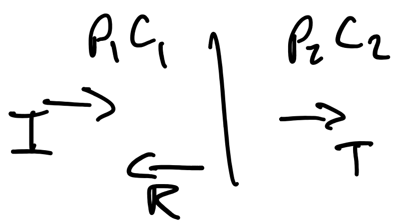
integrate over an area surrounding a source

4.0 Fundamentals of Reflection and Transmission

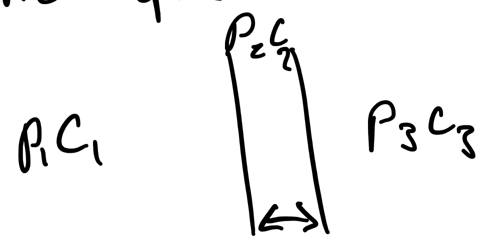
Chapter 6.

4.1 Introduction

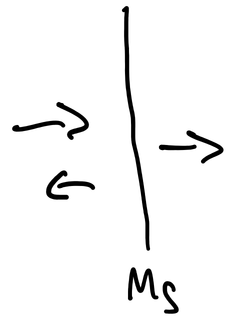
① Sound in a semi-infinite medium hitting a second (different) semi-infinite medium



② Finite-Depth intermediate layer

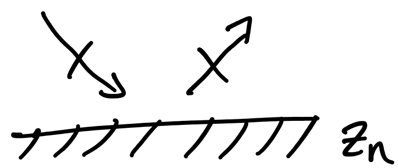


③ Sound transmission through a limp barrier



Mass law

(4) Sound reflection from an impedance surface

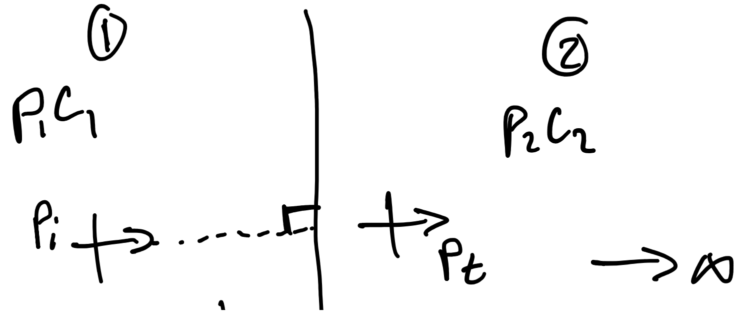


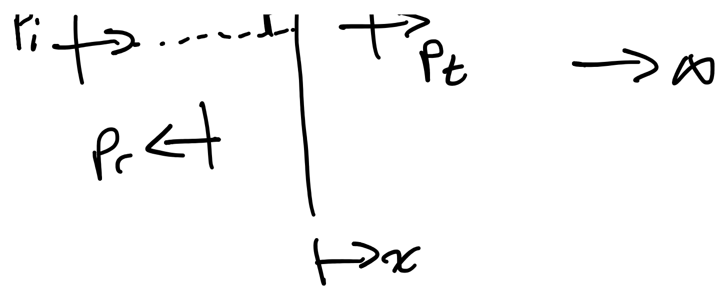
Applications:

Transmission

- aircraft fuselage
- ships
- walls
- control rooms
- dash panel

4.2 Normal Incidence reflection and transmission (two fluids)





$$\nabla^2 \hat{\tilde{p}}_1 + k_1^2 \hat{\tilde{p}}_1 = 0 \qquad \nabla^2 \hat{\tilde{p}}_2 + k_2^2 \hat{\tilde{p}}_2 = 0$$

$$k_1 = \frac{\omega}{c_1} \qquad k_2 = \frac{\omega}{c_2}$$

$$\hat{\tilde{p}}_1 = P_i e^{-jk_1 x} + P_r e^{jk_1 x} \qquad \hat{\tilde{p}}_2 = P_t e^{-jk_2 x}$$

$$\hat{\tilde{u}}_1 = \frac{-1}{j\omega\rho_0} \frac{\partial \hat{\tilde{p}}_1}{\partial x} \qquad \hat{\tilde{u}}_2 = \frac{-1}{j\omega\rho_0} \hat{\tilde{p}}_2$$

$$\hat{\tilde{u}}_1 = \frac{P_i}{\rho_1 c_1} e^{-jk_1 x} - \frac{P_r}{\rho_1 c_1} e^{jk_1 x} \qquad \hat{\tilde{u}}_2 = \frac{P_t}{\rho_2 c_2} e^{-jk_2 x}$$

3 constants ... P_i, P_r, P_t

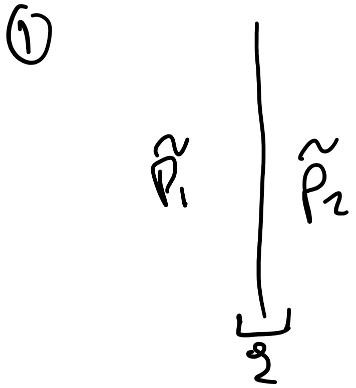
$$\frac{P_r}{P_i} = R \qquad \frac{P_t}{P_i} = T$$

R... plane wave pressure reflection coefficient.

T... plane wave pressure transmission coefficient.

2 boundary conditions at the interface...

(i) Pressure Balance/Continuity (Analogy to Force)



$$\tilde{p}_1 - \tilde{p}_2 = M_s \tilde{a}$$

$$M_s = \frac{\rho_1 \varepsilon}{2} + \frac{\rho_2 \varepsilon}{2}$$

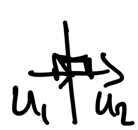
$$\tilde{a} = \frac{\tilde{p}_1 - \tilde{p}_2}{M_s} \quad \text{as } \varepsilon \to 0,\ M_s \to 0,\ \tilde{a} \to \infty$$

$\tilde{p}_1$ must equal $\tilde{p}_2$ to avoid infinite accelerations at the interface...

$$\tilde{p}_1\big|_{x=0} = \tilde{p}_2\big|_{x=0}$$

pressure continuity boundary condition

(ii) Velocity Continuity Equation (Boundary Condition)



if $u_1 \neq u_2$, fluids seperate (cavitation)

$u_{1n}(0) = u_{2n}(0)$ for fluids to remain in contact

slip is allowed in the tangential direction.
since fluid is assumed inviscid

in harmonic case $j\omega\xi_{1n} = j\omega\xi_{2n}$

so displacements are also continuous at $x=0$

Notes:

Since $p_1(0) = p_2(0)$

and $u_{1n}(0) = u_{2n}(0)$

then $\frac{p_1}{u_1} = \frac{p_2}{u_2}$ so... normal specific Acoustic Impedance is continuous at the interface

$$\left.\frac{\tilde{p}_1}{\tilde{u}_{1n}}\right|_{x=0} = \left.\frac{\tilde{p}_2}{\tilde{u}_{2n}}\right|_{x=0} = \tilde{z}_1\big|_{x=0} = \tilde{z}_2\big|_{x=0}$$

Lecture 23

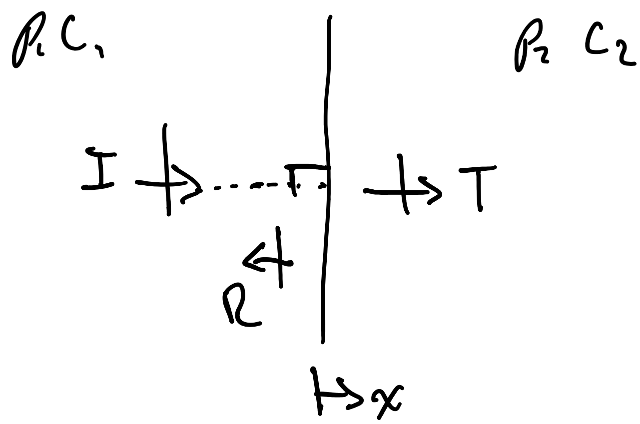
Monday, October 18, 2021 1:30 PM

Acoustic vs. Acoustical

Acoustic - contains units
(Acoustic Pressure, Impedance)

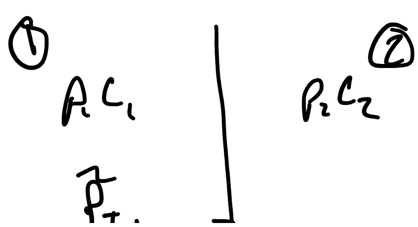
Acoustical - does not contain units
(Acoustical Society, Engineering etc.)

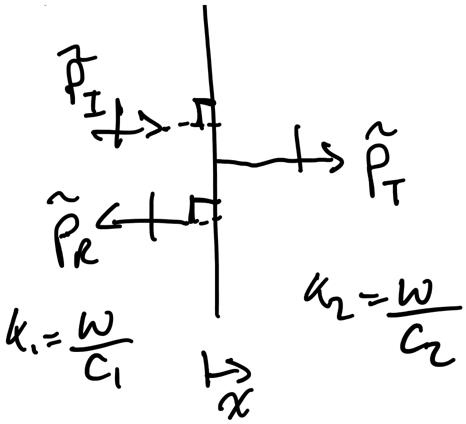
Fundamentals of Reflection and transmission



$$\tilde{p}_1(0) = \tilde{p}_2(0)$$
$$\tilde{u}_{1_n}(0) = \tilde{u}_{2_n}(0)$$

Application of Boundary Conditions





$$\tilde{p}_1 = P_I e^{-jk_1 x} + P_R e^{jk_1 x}$$
$$\tilde{p}_2 = P_T e^{-jk_2 x}$$

Pressure Boundary Condition

$$\tilde{p}_1(0) = \tilde{p}_2(0)$$

$$P_I + P_R = P_T \quad (1)$$

Velocity Boundary Condition

$$\tilde{u}_1 = \frac{P_I}{\rho_1 c_1} e^{-jk_1 x} - \frac{P_R}{\rho_1 c_1} e^{jk_1 x}$$

$$\tilde{u}_2 = \frac{P_T}{\rho_2 c_2} e^{-jk_2 x}$$

$$\tilde{u}_1(0) = \tilde{u}_2(0)$$

$$\frac{P_I}{\rho_1 c_1} - \frac{P_R}{\rho_1 c_1} = \frac{P_T}{\rho_2 c_2} \quad (2)$$

Remember that P_I is known ✱

Remember that P_I is known

(1) → (2)

$$R = \frac{P_R}{P_I} \quad \text{and} \quad T = \frac{P_T}{P_I}$$

(1) $1 + R = T$

(2) $1 - R = \frac{\rho_1 c_1}{\rho_2 c_2} \cdot T$ $\qquad \xi_{21} = \frac{\rho_2 c_2}{\rho_1 c_1}$

$$1 - R = \frac{T}{\xi_{21}}$$

$$T = \frac{2\xi_{21}}{\xi_{21}+1} \qquad R = \frac{\xi_{21}-1}{\xi_{21}+1}$$

Notes:

1) if no dissipation in regions 1 and 2

$\frac{\rho_2 c_2}{\rho_1 c_1} = \xi_{21}$ is real $\qquad T = \frac{2\xi_{21}}{\xi_{21}+1}$

then both R and T are real $\qquad R = \frac{\xi_{21}-1}{\xi_{21}+1}$

R can be positive or negative

if $\xi_{21} > 1$, R is positive ... in phase reflection ... "hard" surface

if $\zeta_{21} > 1$, R is positive ... in phase reflection ... "hard" surface
if $\zeta_{21} < 1$, R is negative ... out of phase reflection ... "soft" surface

(ii)

Air ①

$(\rho_1 c_1)_{air} = 415$ Rayls

Water ②

$(\rho_2 c_2)_{water} = 1000 \cdot 1500 \approx 1.5 \times 10^6$

$\zeta_{21} >> 1$

$$R = \frac{\zeta_{21} - 1}{\zeta_{21} + 1} \rightarrow R = 1 \text{ when } \zeta_{21} >> 1$$

$$\tilde{p}_1(0) = \tilde{p}_2(0) = P_I + P_R = P_I(1 + R) \Rightarrow P_T = 2P_I$$

when $\zeta_{21} >>$, pressure is doubled

$R = 1 \quad T = 2 \qquad R + T \neq 1$

Intensity: $I_2 = \dfrac{p^2_{rms\,2}}{\rho_2 c_2} \simeq 0$

Essentially no energy transmission at a hard surface...
- almost all energy is reflected.

$$I_{1_I} = \frac{(p^2_{rms})_{1I}}{\rho_1 c_1}$$

iii) $$R = \frac{\zeta_{21} - 1}{\zeta_{21} + 1}$$

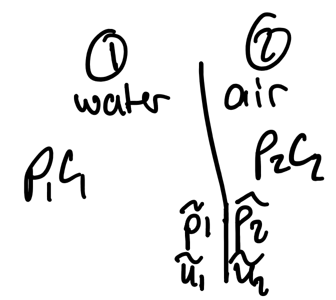
no sound reflected ... $R \to 0$ ... $\zeta_{21} \to 1$ $\quad T \to 1$

$$\rho_1 c_1 = \rho_2 c_2 \qquad \left(\frac{\rho_1}{\rho_2}\right) = \left(\frac{c_2}{c_1}\right)$$

if this condition is satisfied...

zero reflection, perfect transmission

iv) $\zeta_{21} << 1$



$$\zeta_{21} = \frac{\rho_2 c_2}{\rho_1 c_1} << 1$$

$$R = \frac{\zeta_{21} - 1}{\zeta_{21} + 1}$$ $R \to -1$ ... perfect reflection ... phase inversion

$$\tilde{p}_1(0) = P_I + P_R \Rightarrow P_I(1 + R) \simeq 0$$

$$\tilde{p}_2(0) \simeq 0$$

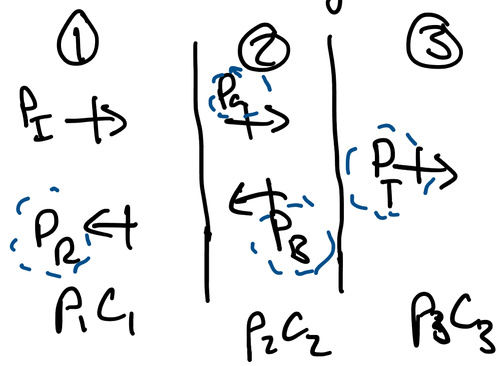
$T \to 0$

~~#~~ pressure release surface

$\tilde{p} = 0$ ... pressure release b.c.

Normal Incidence Sound

Transmission through a fluid layer

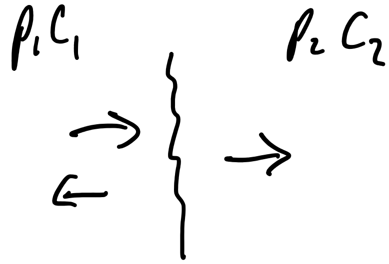


4 unknowns

3 wave Equations

Lecture 24

Wednesday, October 20, 2021 1:28 PM



$$R = \frac{\zeta_{21} - 1}{\zeta_{21} + 1}$$

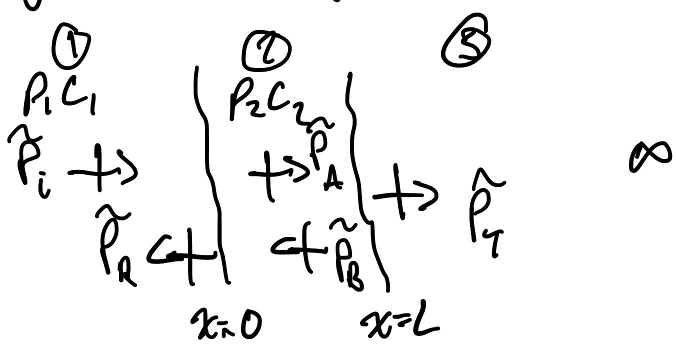
$$T = \frac{2\zeta_{21}}{\zeta_{21} + 1}$$

$$\zeta_{21} = \frac{\rho_2 c_2}{\rho_1 c_1}$$

when $\rho_2 c_2 > \rho_1 c_1$ R is positive
when $\rho_2 c_2 < \rho_1 c_1$ R is negative

$\rho_2 c_2 >> \rho_1 c_1$ $R \to 1 \ldots T \to 2$
$\rho_2 c_2 << \rho_1 c_1$ $R \to -1 \ldots T \to 0$

4.2.2. Normal incidence sound transmission through a fluid layer



$$\frac{\partial^2 p_1}{\partial x^2} + k_1^2 \tilde{p}_1 = 0 \qquad \frac{\partial^2 p_2}{\partial x^2} + k_2^2 \tilde{p}_2 = 0 \qquad \frac{\partial^2 p_3}{\partial x^2} + k_3^2 \tilde{p}_3 = 0$$

$$\tilde{p}_1 = P_i e^{-jk_1 x} + P_R e^{jk_1 x}$$
$$\tilde{p}_2 = P_A e^{-jk_2 x} + P_B e^{jk_2 x}$$
$$\tilde{p}_3 = P_T e^{-jk_3 x}$$

B.C.'s at $x=0$ $\quad [\tilde{p}_1 = \tilde{p}_2]_{x=0}$ (1)

$[\tilde{u}_{1n} = \tilde{u}_{2n}]_{x=0}$ (2)

at $x = L$ $\quad [\tilde{p}_2 = \tilde{p}_3]_{x=L}$ (3)

$[\tilde{u}_{2n} = \tilde{u}_{3n}]_{x=L}$ (4)

$P_i + P_R = P_A + P_B$ (1) $\Rightarrow \boxed{1 + R = A + B}$ $\quad A = \frac{P_A}{P_i}$, $B = \frac{P_B}{P_i}$

ii) $1 - R = \frac{1}{z_{21}}(A - B)$ (2) $\quad z_{21} = \frac{\rho_2 c_2}{\rho_1 c_1}$

iii) $A e^{-jk_2 L} + B e^{jk_2 L} = T e^{-jk_3 L}$ (3) $\quad T = P_T / P_i$

iv) $A e^{-jk_2 L} - B e^{jk_2 L} = \frac{1}{z_{32}} T e^{-jk_3 L}$ (4) $\quad z_{32} = \frac{\rho_3 c_3}{\rho_2 c_2}$

Solves to:

$$R = \frac{\left(1 - \frac{\rho_1 c_1}{\rho_3 c_3}\right)\cos k_2 L + j\left(\frac{\rho_2 c_2}{\rho_3 c_3} - \frac{\rho_1 c_1}{\rho_2 c_2}\right)\sin k_2 L}{\left(1 + \frac{\rho_1 c_1}{\rho_3 c_3}\right)\cos(k_2 L) + j\left(\frac{\rho_2 c_2}{\ldots} + \frac{\rho_1 c_1}{\ldots}\right)\sin k_2 L}$$

$$\overline{\left(1+\frac{\rho_1c_1}{\rho_3c_3}\right)\cos(k_2L) + j\left(\frac{\rho_2c_2}{\rho_3c_3}+\frac{\rho_1c_1}{\rho_2c_2}\right)\sin k_2L}$$

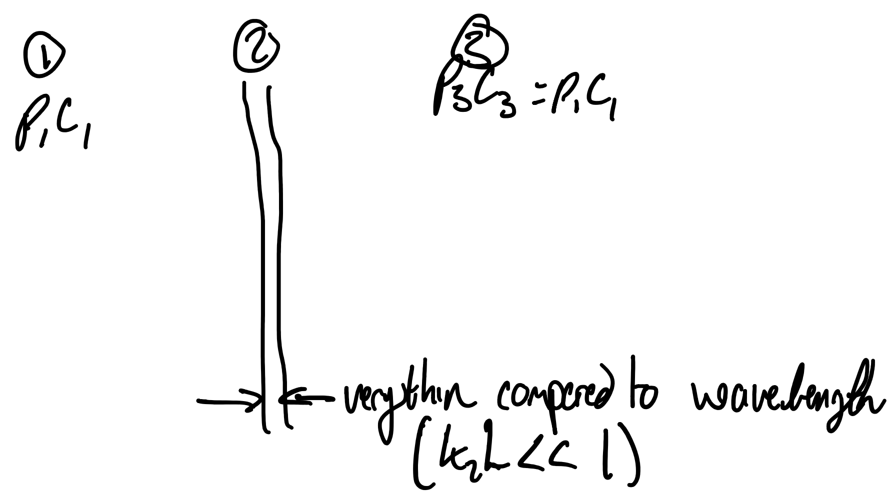
$k_2L = \frac{2\pi L}{\lambda_2} = 2\pi\left(L/\lambda_2\right)$ ... how deep the layer is with respect to wavelength

if $R \to 0$ $\rho_1c_1=\rho_3c_3$
and $\sin k_2L = 0$

if $R \to 0$, perfect transmission
$\rho_1c_1=\rho_3c_3$
$\sin k_2L = 0$
$k_2L = n\cancel{\pi} = 2\cancel{\pi}\left(L/\lambda_2\right)$

$\boxed{L = \frac{n\lambda_2}{2}}$ perfect transmission

Thin, heavy barrier case...



[illegible]
$(k_2 L << 1)$

Say $\sin k_2 L \approx k_2 L$
$\cos k_2 L \approx 1$

- limp barrier (no flexural stiffness)
- heavy) $\rho_2 c_2 >> \rho_1 c_1$

$$R = \frac{\cancel{\left(1 - \frac{\rho_1 c_1}{\rho_3 c_3}\right)} \cdot 1 + j\left(\frac{\rho_2 c_2}{\rho_3 c_3} - \cancel{\frac{\rho_1 c_1}{\rho_2 c_2}}\right) k_2 L}{\left(1 + \cancel{\frac{\rho_1 c_1}{\rho_3 c_3}}^{\,1}\right) \cdot 1 + j\left(\frac{\rho_2 c_2}{\rho_3 c_3} + \cancel{\frac{\rho_1 c_1}{\rho_2 c_2}}\right) k_2 L}$$

$$R = \frac{j \frac{\rho_2 \cancel{c_2}}{\rho_3 c_3} \frac{\omega}{\cancel{c_2}} \cdot L}{2 + j \frac{\rho_2 \cancel{c_2}}{\rho_3 c_3} \frac{\omega}{\cancel{c_2}} \cdot L}$$

$\rho_2 L = m_s$ (mass per unit area)

$\rho_3 c_3 = \rho_1 c_1 = \rho_0 c$

$$R = \frac{j\omega m_s}{2\rho_0 c + j\omega m_s}$$

if ωm_s is small, pure transmission
if ωm_s is large, reflection

physical parameter of importance is mass/unit area (m_s)

$\omega \to \infty$, $R \to 1$

$\omega \to 0$, $R \to 0$

$m_s \to \infty$, $R \to 1$, $\tau \to 0$
$m_s \to 0$, $R \to 0$

$M_s \to \infty,\ R \to 1,\ T \to 0$

$M_s \to 0,\ R \to 0$

$$T = \frac{2e^{jk_3L}}{\left(1+\frac{\rho_1 c_1}{\rho_3 c_3}\right)\cos k_2L + j\left(\frac{\rho_2 c_2}{\rho_3 c_3} + \frac{\rho_1 c_1}{\rho_2 c_2}\right)\sin k_2L}$$

Sonar Case $\quad \sin k_2L \to 0 \quad \cos k_2L \to 1$

$\rho_1 c_1 = \rho_3 c_3$

$$T = \frac{2e^{jk_3L}}{\pm 2} \qquad T = \pm e^{jk_3L}$$

perfect transmission
except for phase shift

thin heavy) limp barrier

$\rho_1 c_1 = \rho_3 c_3 \qquad \frac{\rho_2 c_2}{\rho_1 c_1} \gg 1 \qquad k_2L \ll 1$

$$T \to \frac{2e^{jk_3L}}{2 + j\left(\frac{\rho_2 c_2}{\rho_3 c_3}\right)k_2L} = \boxed{\frac{2\rho_0 c\, e^{jkL}}{2\rho_0 c + j\omega M_s} = T}$$

(below critical frequency)

$\omega \to 0 \quad T \to 1$

$\omega \to \infty \quad \omega M_s \gg 2\rho_0 c \quad \therefore$ high frequency region

$\boxed{|T| = \frac{2\rho_0 c}{\dots}}$ mass law

$$\boxed{|T| = \frac{2\rho_0 c}{\omega M_s}}$$ Mass law

$$|T| \propto \frac{1}{M_s}$$

$$|T| \propto \frac{1}{\omega}$$

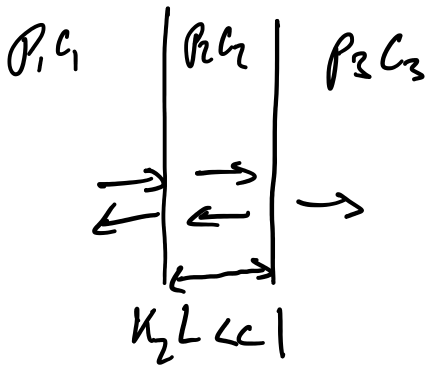
"limp" for sheet of aluminum

thickness of aluminum .05"

limp below 10 kHz

Lecture 25

Friday, October 22, 2021 1:27 PM



$\rho_2 c_2 \gg \rho_1 c_1 = \rho_0 c = \rho_3 c_3$

$$T = \frac{2\rho_0 c\, e^{jkL}}{2\rho_0 c + j\omega M_s}$$

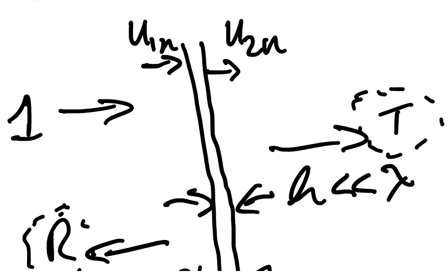
$$R = \frac{j\omega M_s}{2\rho_0 c + j\omega M_s}$$

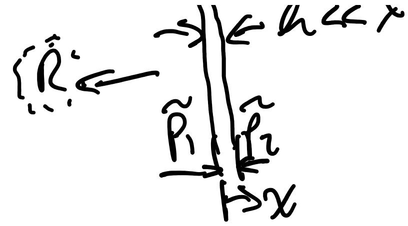
$\omega M_s \gg 2\rho_0 c \quad |T| \to \dfrac{2\rho_0 c}{\omega M_s}$ ... Mass law

$T \alpha\, \omega\ ,\ T \propto M_s$

Transmission Loss: $T_L = 10\log_{10}\left(\frac{1}{|T|^2}\right)$

Alternative approach



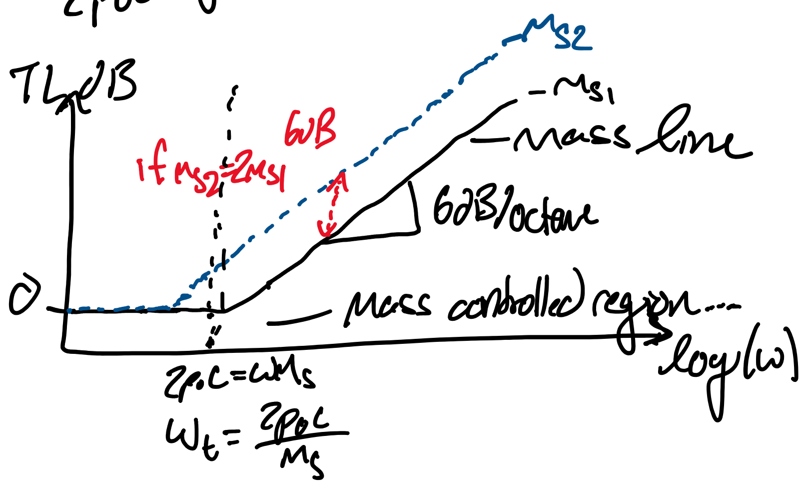


① $u_{1n} = u_{2n}$ at $x=0$ assume incompressible

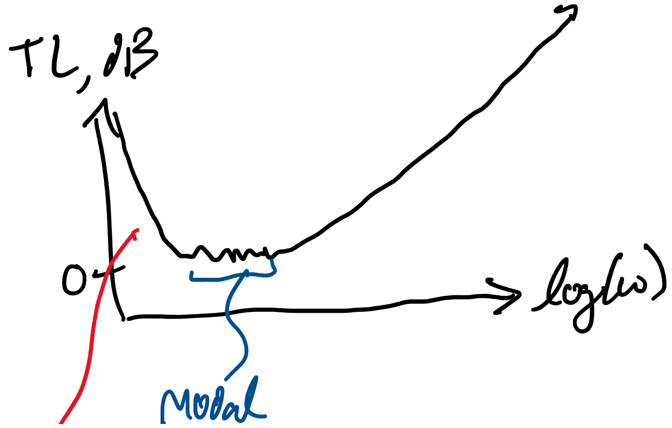
② $\tilde{p}_1 - \tilde{p}_2 = M_s \dfrac{d\tilde{u}_n}{dt}$

$$= j\omega M_s \tilde{u}_{2n} = j\omega M_s \tilde{u}_{1n}$$

$$T = \frac{2\rho_0 c\, e^{jkL}}{2\rho_0 c - j\omega M_s} \qquad TL = 10\log_{10}\left(\frac{1}{|T|^2}\right)$$



for a finite panel



stiffness line
- flexural + mount stiffnesses

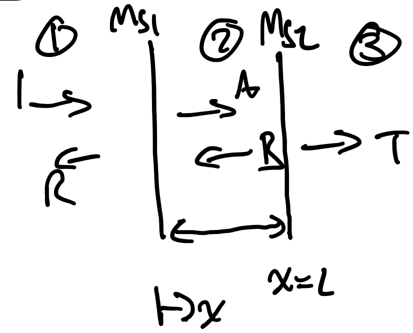
Modal response

happens when

$\boxed{D}$ $D \ll 1$

if panels are large compared to λ, can treat them as infinite

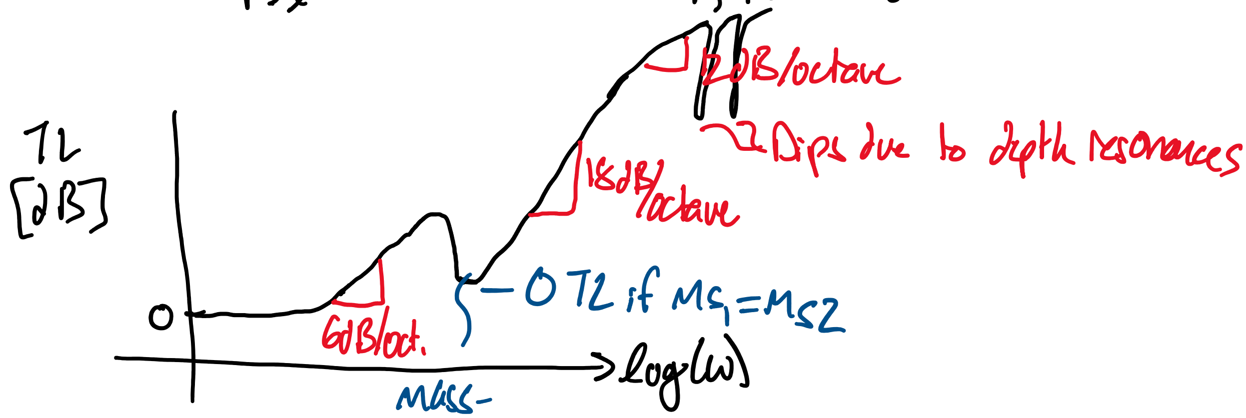
Double Panel

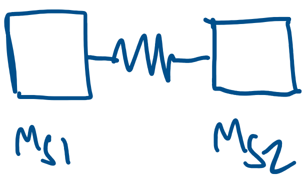


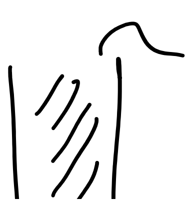
B.C's

1) $u_{1n} = u_{2n}$
2) $\tilde{p}_1 - \tilde{p}_2 = j\omega M_{s1} u_{1n}$] at $x=0$

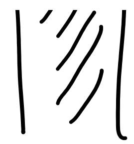
3) $u_{2n} = u_{3n}$
4) $\tilde{p}_2 - \tilde{p}_3 = j\omega M_{s2} u_{2n}$] at $x=L$







decouples the two panels



~ absorbing material
- glass fiber
to defeat standing wave
depth resonances.

Decoupled double panel

$$TL \simeq TL_1 + TL_2 + 6\,dB$$

Single panel... double mass $30\,dB \rightarrow 36\,dB$

but using second panel...

$$30 \Rightarrow 60\,dB + 6\,dB$$

Relation to Acoustic Intensity...

<u>Pressure</u> $R = \dfrac{P_r}{P_i}$ $T = \dfrac{P_T}{P_i}$

<u>in 2-fluid...</u>

$$R = \frac{\xi_{21} - 1}{\xi_{21} + 1} \quad \text{where } \xi_{21} = \frac{\rho_2 c_2}{\rho_1 c_1}$$

$$T = \frac{2\xi_{21}}{\xi_{21} + 1}$$

Intensity coefficients:

$$R = \frac{I_r}{I_i} \quad T = \frac{I_T}{I_i}$$

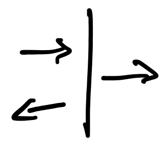
for freely propagating plane waves...

for freely propagating plane waves...

$$I = \frac{p^2_{rms}}{\rho_0 c}$$

harmonic $\quad p^2_{rms} = \frac{\tilde{p}\tilde{p}^*}{2} = \frac{|\tilde{p}|^2}{2}$

Two fluid case...

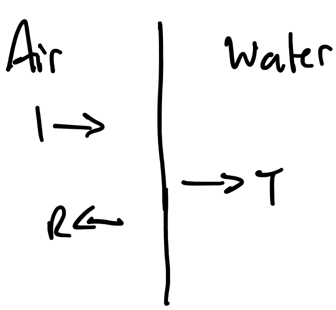


$$R_I = \frac{I_R}{I_i} = \frac{\dfrac{p^2_{rms,r}}{\rho_1 c_1}}{\dfrac{p^2_{rms,i}}{\rho_1 c_1}}$$

$$R_I = \frac{p^2_{rms,r}}{p^2_{rms,i}} = |R^2| = R_I$$

$$T_I = \frac{I_T}{I_i} = \frac{\dfrac{p^2_{rms,T}}{\rho_2 c_2}}{\dfrac{p^2_{rms,i}}{\rho_1 c_1}} = \frac{1}{\varsigma_{21}} \frac{p^2_{rms,T}}{p^2_{rms,i}}$$

$$T_I = \frac{1}{\varsigma_{21}} |T^2|$$



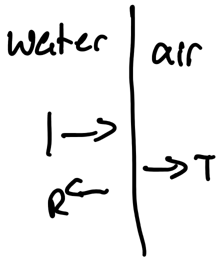
$\rho_2 c_2 \gg \rho_1 c_1 \qquad \varsigma_{21} \to \infty$

$|T| = 2 \qquad |T_I| \to \frac{4}{\infty} = 0$

$|R| = 1 \qquad |R_I| = 1$

$|T| = 2$ $\qquad$ $|T_I| \to \frac{1}{\infty} = 0$

$|R| = 1$ $\qquad$ $|T_R| = 1$



$\rho_1 c_1 \gg \rho_2 c_2 \qquad \xi_{21} \to 0$

$|T| = 0$

$|R| = 1$ (out of phase)

$|T_I| = 0$

$|R_I| = 1$

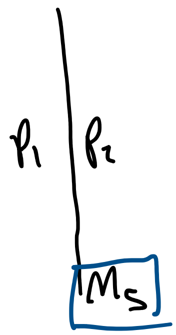
$R + T \neq 1$

$$\underline{R_I + T_I = 1}$$

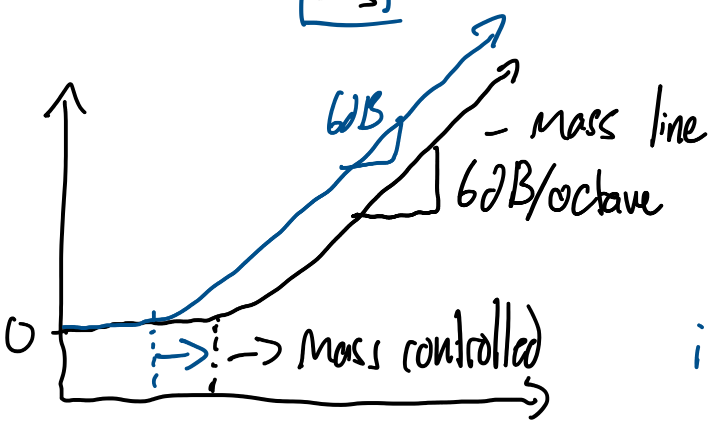
Lecture 26

Monday, October 25, 2021 8:32 AM

Thin heavy limp panel



$$T = \frac{2\rho_0 c\, e^{jkL}}{2\rho_0 c + j\omega M_s}$$



ωM_s ...

increasing ωM_s ....

Double Panel system:

$$TL_{total} = TL_1 + TL_2 + 6dB$$

Double Panel gives much higher performance...

$$T + R \neq 1$$

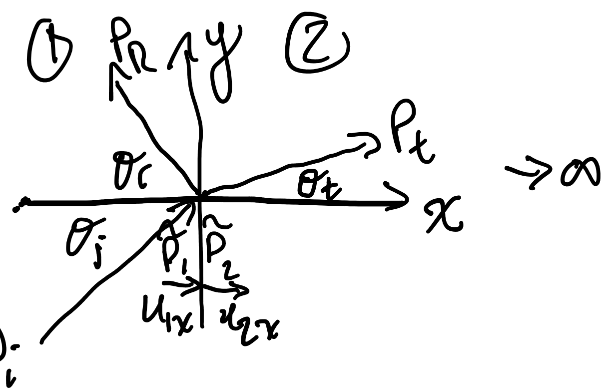
$T + R \neq 1$

but it is always true that

$$\boxed{T_i + R_i = 1}$$

hard to transmit acoustic energy across a large impedance difference...

4.3 Oblique incidence reflection and transmission in two fluid case



$\tilde{u}_{1y} \neq \tilde{u}_{2y}$ Slip is allowed because no viscosity...

2-D - no propogation in the z-direction...

2-1) - no propogation in the z-direction...

$$\nabla^2 \tilde{p}_1 + k_1^2 \tilde{p}_1 = 0 \qquad k_1 = \frac{\omega}{c_1}$$

$$\nabla^2 \tilde{p}_2 + k_2^2 \tilde{p}_2 = 0 \qquad k_2 = \omega / c_2$$

① $$\tilde{p}_1 = P_i e^{-j(k_{ix} \cdot x + k_{iy} \cdot y)} + P_r e^{j(k_{rx} x - k_{ry} y)}$$

$$k_{ix}^2 + k_{iy}^2 = k^2 \qquad k_{ix} = k_1 \cos\theta_i$$

$$k_{iy} = k_1 \sin\theta_i$$

$$k_{rx} = k_1 \cos\theta_r$$

$$k_{ry} = k_1 \sin\theta_r$$

$$\tilde{u}_{1x} = \frac{-1}{j\omega\rho_1} \frac{d\hat{p}_1}{dx}$$

$$= \frac{P_i}{\rho_1 c_1} \cos\theta_i \, e^{-j(k_{ix} x + k_{iy} \cdot y)}$$

$$- \frac{P_r}{\rho_1 c_1} \cos\theta_r \, e^{j(k_{rx} x - k_{ry} y)}$$

② $$\tilde{P}_t = P_t e^{-j(k_{tx} x + k_{ty} y)}$$

$$k_{tx} = k_t \cos\theta_t \qquad k_{ty} = k_t \sin\theta_t$$

$$k_{tx}^2 + k_{ty}^2 = k_t^2$$

$$\hat{\tilde{u}}_{2x} = \frac{-1}{j\omega\rho_2}\frac{\partial \tilde{p}}{\partial x} = \frac{P_t}{\rho_2 c_2}\cos\theta_t e^{-j(k_{tx}x + k_{ty}y)}$$

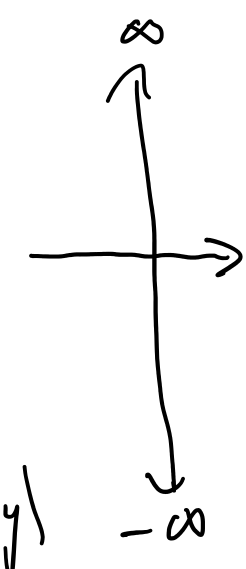
Two boundary Conditions...

i) $\hat{\tilde{p}}_1(0) = \hat{\tilde{p}}_2(0)$

ii) $\hat{\tilde{u}}_{1x}(0) = \hat{\tilde{u}}_{2x}(0)$

looking at i) Pressure Equality

$$\hat{\tilde{p}}_1(0,y) = \hat{\tilde{p}}_2(0,y)$$



$$P_i e^{-j(k_{ix}\cdot 0 + k_{iy}\cdot y)} + P_r e^{-j(k_{ix}\cdot 0 - k_{iy}\cdot y)} = P_t e^{-j(k_{tx}\cdot 0 + k_{ty}y)}$$

$$P_i e^{-jk_{iy}y} + P_r e^{-jk_{iy}y} = P_t e^{-jk_{ty}y}$$

Must be independent of position in,

Must be independent of position in the y-direction...

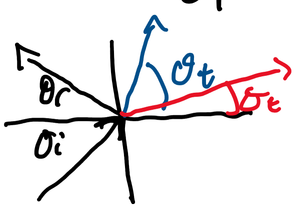
only true if $\boxed{k_{ry} = k_{iy} = k_{ty}}$

$k_1 \sin\theta_r = k_1 \sin\theta_i$ so $\sin\theta_r = \sin\theta_i$

and $\boxed{\theta_r = \theta_i}$

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{k_2}{k_1} = \frac{c_1}{c_2}$$ Snell's Law

$$\sin\theta_t = \frac{c_2}{c_1}\sin\theta_i$$



$c_2 > c_1$ sound refracts away from normal

$c_2 < c_1$ sound refracts towards the normal

Pressure B.C. $\hat{p}_1(0) = \hat{p}_2(0)$

$$\boxed{\frac{P_i + P_r}{P_i} = \frac{P_t}{P_i} \Rightarrow 1 + R = T}$$

$R = \frac{P_r}{P_i}$

$T = \frac{P_t}{P_i}$

Velocity B.C. $\tilde{u}_{1x}(0,y) = \tilde{u}_{2x}(0,y)$

$$\frac{P_i}{\rho_1 c_1}\cdot\cos\theta_i - \frac{P_r}{\rho_1 c_1}\cos\theta_i = \frac{P_t}{\rho_2 c_2}\cos\theta_t$$

$$\Rightarrow \boxed{1-R = \frac{\cos\theta_t}{\cos\theta_i}\cdot\frac{1}{\xi_{21}}\cdot T}$$

$$\xi_{21} = \frac{\rho_2 c_2}{\rho_1 c_1}$$

using 2 equations

$$R = \frac{\xi_{21} - \frac{\cos\theta_t}{\cos\theta_t}}{\xi_{21} + \frac{\cos\theta_t}{\cos\theta_i}} \qquad T = \frac{2\xi_{21}}{\xi_{21} + \frac{\cos\theta_t}{\cos\theta_i}}$$

when $\theta_i = 0$ ... normal incidence ...
results go to previous results.

$$\sin\theta_t = \frac{c_2}{c_1}\sin\theta_i$$

write R and T in terms of θ_i only

Snells Law

$$\sin\theta_t = \frac{c_2}{c_1}\sin\theta_i$$

$\sin^2\theta + \cos^2\theta = 1$

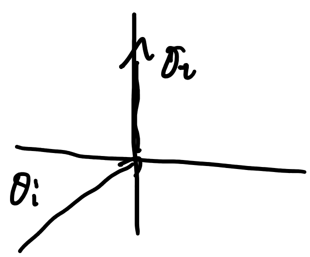
$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{c_2}{c_1}\right)^2\sin^2\theta_i} = \cos\theta_t$$

when $1-\left(\frac{c_2}{c_1}\right)^2\sin^2\theta_i = 0$

defines θ_c... critical angle
or angle of total reflection

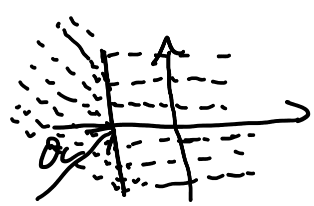
Critical incidence angle, θ_c

$\theta_i = \theta_c$ when $\theta_t = \pi/2$



$$\sin\theta_c = \frac{c_1}{c_2}\sin\theta_t \quad 1$$

$R \to 1$



in region 2, sound props parallel
to the interface

What if $\theta_i > \theta_c$

$$\sin\theta_t = \frac{c_2}{c_1}\sin\theta_i \quad > 1$$

if θ_t is complex

if θ_t is complex

$\sin\theta_t$ can be > 1

$$\sin\theta_t = \frac{e^{j\theta_t} - e^{-j\theta_t}}{2j}$$

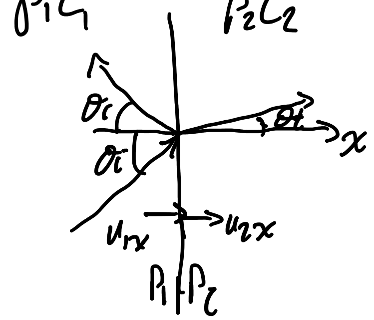
when $\theta_i > \theta_t$

k_{tx} is imaginary

- exponential decay of pressure away from the interface

Lecture 27

Wednesday, October 27, 2021 1:24 PM

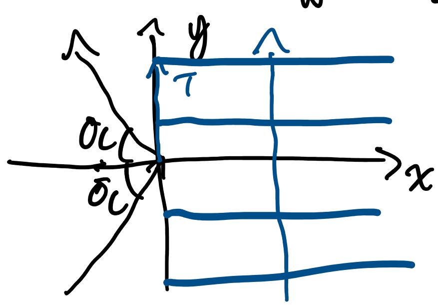


Snell's law: $\sin\theta_t = \frac{c_2}{c_1}\sin\theta_i$

$$R = \frac{\zeta_{21} - \frac{\cos\theta_t}{\cos\theta_i}}{\zeta_{21} + \frac{\cos\theta_t}{\cos\theta_i}}$$

$$\cos\theta_t = \underbrace{\sqrt{1 - \left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i}}_{=0 \text{ when } \theta_i = \theta_c}$$

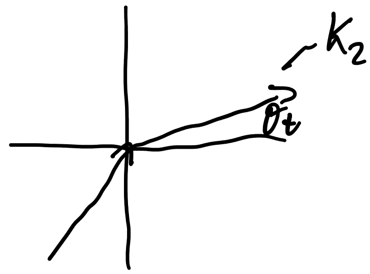
when $c_2 > c_1$



$$R = \frac{\zeta_{21}\cos\theta_i - \sqrt{1 - \cancel{\left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i}^{\to 0}}}{\zeta_{21}\cos\theta_i + \sqrt{1 - \cancel{(c_2/c_1)^2 \sin^2\theta_i}_{0}}}$$ at $\theta_i = \theta_c$

$R = 1$ Total Reflection

What happens if $\theta_i > \theta_c$
what is the propagation direction in ②



k_{tx}? k_{ty}?

$$k_{ty} = k_2 \sin\theta_t = k_1 \sin\theta_i$$

forcing spatial pattern ... real number

✗ Wave numbers parallel to interface must be equal...

true because of pressure continuity...

$$k_{tx} = k_2 \cos\theta_t \qquad \cos^2\theta_t + \sin^2\theta_t = 1$$

$$k_{tx} = \pm k_2 \sqrt{1 - \sin^2\theta_t}$$

$$k_{tx} = \pm k_2 \sqrt{\underbrace{1 - \underbrace{\left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i}_{>1 \text{ when } \theta_i > \theta_c}}_{<0,\ \text{purely imaginary}}}$$

$$= \pm\sqrt{-1}\, k_2 \sqrt{\underbrace{\left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i - 1}_{\text{positive for } \theta_c > \theta_i}}$$

$$k \sqrt{\left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i - 1}$$

$\theta_c > \theta_i$

So $k_{tx} = \pm j k_2 \sqrt{\left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i} = \pm j\gamma$ where $\gamma = k_2\sqrt{\left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i - 1}$

Sound field in region ②:

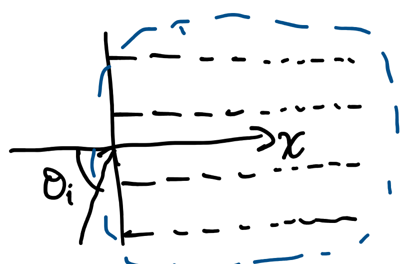
$$\tilde{P}_t = P_t \underbrace{e^{-jk_{tx}x}}_{e^{-j(\pm j\gamma)x}} e^{-jk_{ty}y}$$

k_{ty} ... real

$$\tilde{P}_t = P_t \underbrace{e^{\pm\gamma x}}_{\substack{\text{pure}\\\text{exponential}\\\text{growth or decay}}} \underbrace{e^{-jk_{ty}y}}_{\text{propagating component}}$$

if ① $\to \infty$, discard $e^{+\gamma x}$ ∴ leaves pure exponential decay

if ② has finite depth ... include both



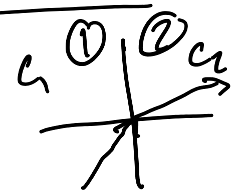
propagates in y direction

decays with x exponentially

evanescent near field

Consequences of Snell's Law

1. From snell's law



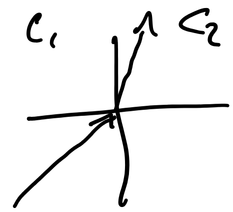
when $c_2 < c_1$

$\sin\theta_t < \sin\theta_i$

$\theta_t < \theta_i$

Sound refracts towards the normal

2. if $c_2 > c_1$



$\theta_t > \theta_i$

Sound refracts away from the normal

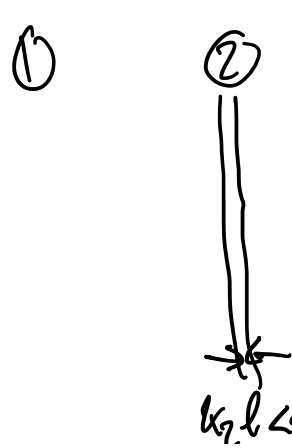
$\sin\theta_c = \frac{c_1}{c_2}$

3. if $\theta_i > \theta_c$

exponential decay into the second medium

θ_t is a complex number

Reflection and Transmission at a thin limp panel — no flexural stiffness



no free wave propagation within the panel in the y-direction

→ local reaction surface

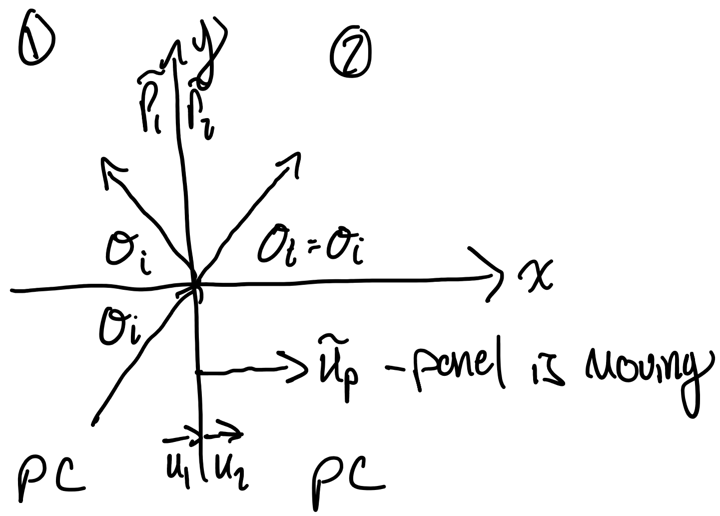
$k_2 l \ll 1$ — local reaction surface

$l \ll \lambda$

no wave propagation within panel in the x-direction due to thin-ness

-panel does not compress in x-direction

- thickness remains constant.



assume same medium in ① and ②

panel is limp - no free wave prop in y-direction

① $$\tilde{P}_1 = \tilde{P}_i e^{-j(k_x x + k_y y)} + \tilde{P}_r e^{j(k_x x - k_y y)}$$

$$\theta_i = \theta_r = \theta_t$$

$$k_x = k\cos\theta_i \qquad k_y = k\sin\theta_i \qquad k = \frac{\omega}{c}$$

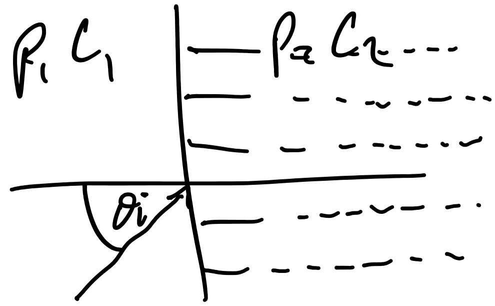
$$\tilde{u}_{1x} = \frac{\tilde{P}_i}{\rho_0 c}\cos\theta_i \, e^{-j(k_x x + k_y y)} - \frac{\tilde{P}_r}{\rho_0 c}\cos\theta_i \, e^{j(k_x x - k_y y)}$$

② $\tilde{P}_2 = \tilde{P}_t e^{-j(k_x x + k_y y)}$

$$\tilde{u}_{2x} = \frac{\tilde{P}_t}{\rho_0 c} \cos\theta_i \, e^{-j(k_x x - k_y y)}$$

Lecture 28

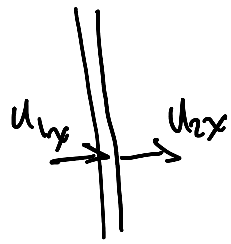
Friday, October 29, 2021 1:30 PM



$k_{tx} \rightarrow$ imaginary
nearfield

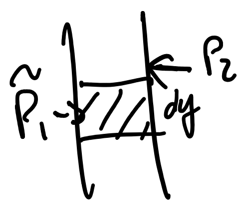
Boundary Conditions

i)



$u_{1x}(0) = u_{2x}(0)$

ii) EOM of the Panel



$F = ma$

$$\underbrace{(\tilde{p}_1 - \tilde{p}_2)\big|_{x=0} \cdot dy}_{F}$$

$$(\tilde{p}_1 - \tilde{p}_2)\cancel{dy}\big|_{x=0} = \overbrace{M_s\, d\cancel{y}}^{M}\ \frac{\partial \tilde{u}_p}{\partial t}$$

$$\tilde{p}_1 - \tilde{p}_2\big|_{x=0} = M_s \frac{\partial \hat{\tilde{u}}_{1x}}{\partial x} = M_s \frac{\partial \tilde{u}_{2x}}{\partial x}$$

$$\left[\tilde{p}_i - \tilde{p}_t = j\omega M_s u_{1x}\right]\Big|_{x=0}$$

Sub soln's into B.C.'s 1 and 2

i) $P_i - P_r = P_t$

ii) $P_i + P_r - P_t = j\frac{\omega M_s}{\rho_0 c} \cdot \cos\theta \cdot P_t$

normalize to P_i (with respect to incident wave)

$$R = \frac{P_r}{P_i} = \frac{j\omega M_s}{\frac{2\rho_0 c}{\cos\theta} + j\omega M_s}$$

$$T = \frac{P_t}{P_i} = \frac{\frac{2\rho_0 c}{\cos\theta}}{\frac{2\rho_0 c}{\cos\theta} + j\omega M_s}$$

Special Cases

i) $\theta = 0$ normal incidence

$\cos\theta = 1$ $\quad R \to \frac{j\omega M_s}{2\rho_0 c + j\omega M_s}$ $\quad T \to \frac{2\rho_0 c}{2\rho_0 c + j\omega M_s}$

Exactly the same as before (3 medium)

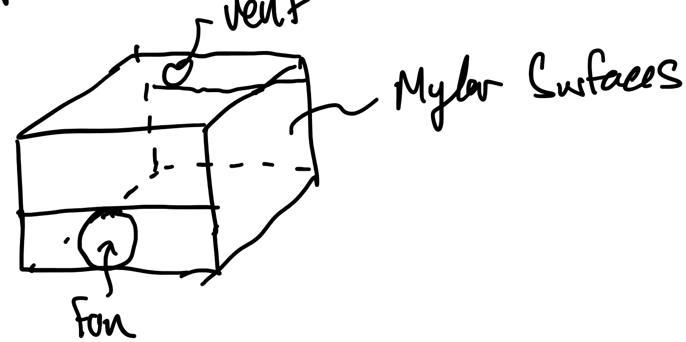
$k_2 L \ll 1 \quad \rho_2 c_2 \gg \rho_1 c_1 \gg \rho_3 c_3$

$k_2 L << 1 \quad \rho_2 c_2 >> \rho_1 c_1$
$>> \rho_3 c_3$

ii) $M_s \to \quad T \to 1 \quad R \to 0$

panel disappears

INCE Plenum

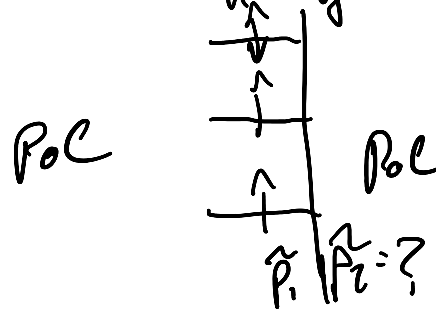


iii) $M_s \to$ large $\quad R \to 1 \quad T \to \underbrace{\frac{2\rho_0 c}{\cos\theta} \cdot \frac{1}{j\omega M_s}}$ mass law

$$T \propto \frac{1}{M_s} \propto \frac{1}{\omega}$$

"Massive"... $\omega M_s >> \frac{2\rho_0 c}{\cos\theta}$

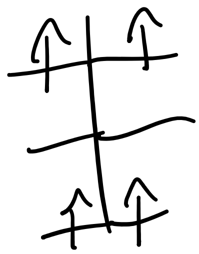
iv) Grazing Incidence



$\theta_i = \frac{\pi}{2}$

no motion of panel

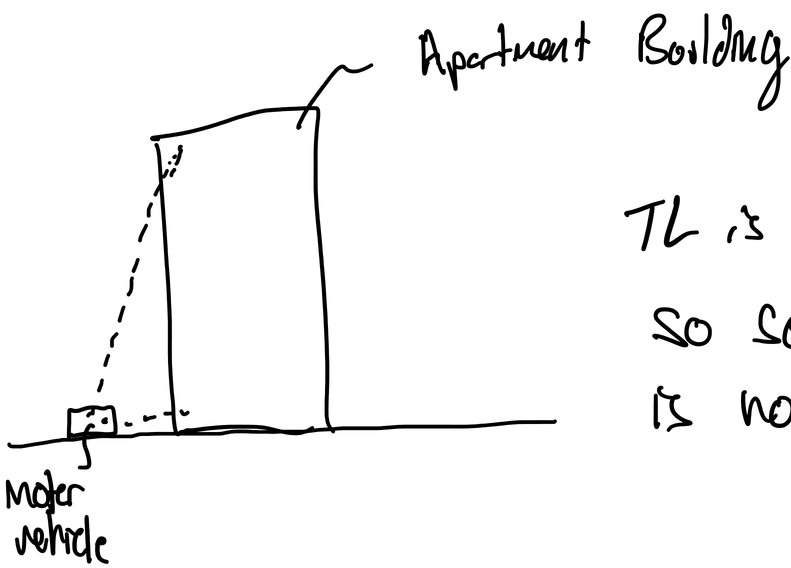
$\tilde{p}_1$ must equal $\tilde{p}_2$ to prevent motion of panel



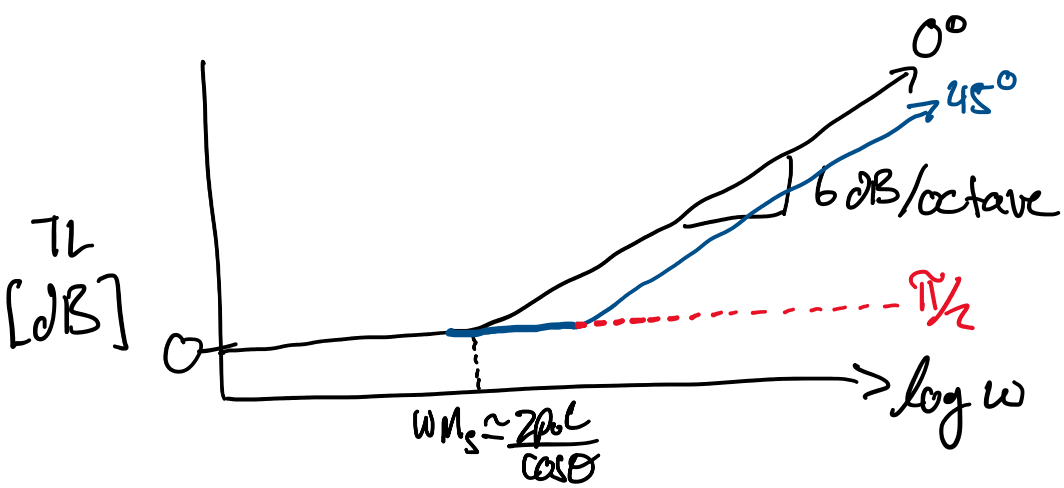
so $T=1$

$R=0$

grazing T approaches 1 as a limiting case



TL is greater at normal incidence so sound level due to traffic is not as much an issue



$TL = 10 \log_{10} \left(1/|T|^2 \right)$

transition shifts to higher frequencies as the
angle increases

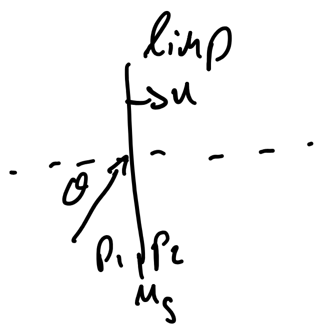
Lecture 29

Monday, November 1, 2021 1:28 PM

Fall '17 String

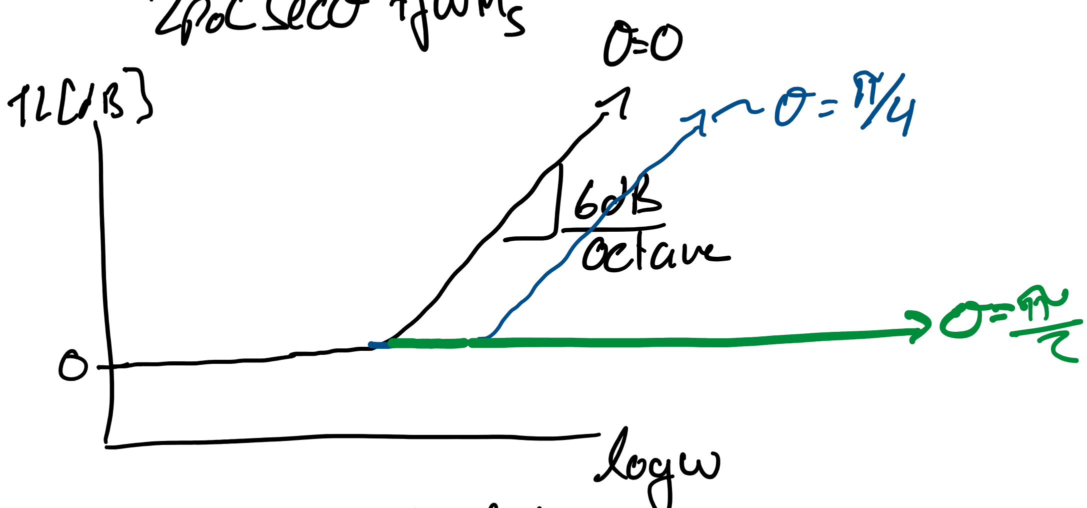
Fall '19 Acoustic

Before:



$$T = \frac{\frac{2\rho_0 c}{\cos\theta}}{\frac{2\rho_0 c}{\cos\theta} + j\omega M_s} \qquad R = \frac{j\omega M_s}{\frac{2\rho_0 c}{\cos\theta} + j\omega M_s}$$

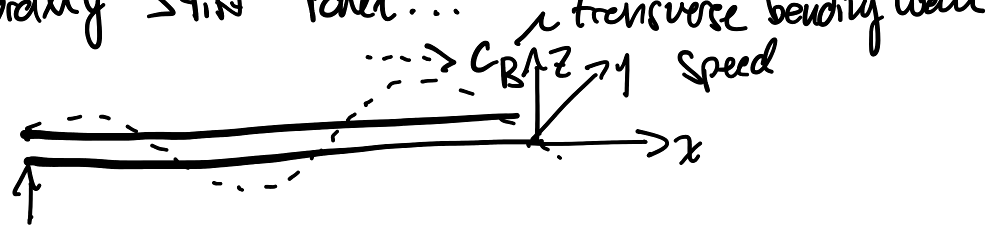
$$T = \frac{2\rho_0 c \sec\theta}{2\rho_0 c \sec\theta + j\omega M_s}$$

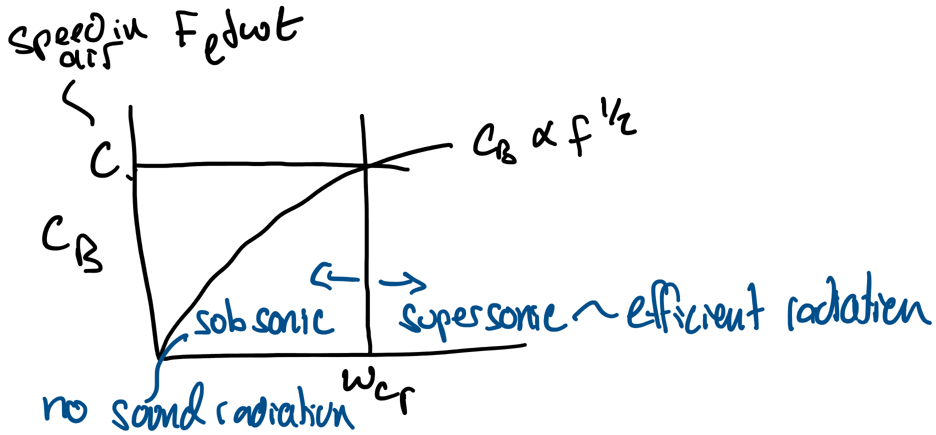


* remember that double panels are more weight efficient in a practical situation

are more weight efficient in a practical situation.

Flexurally Stiff Panel...





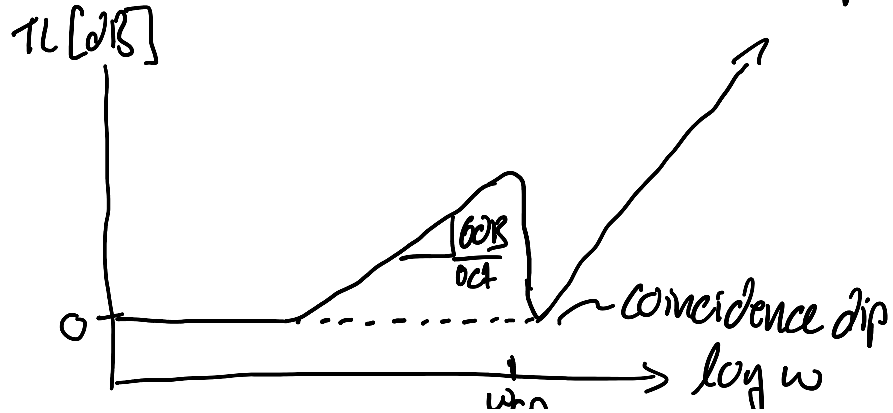
ω_{cr} - critical frequency

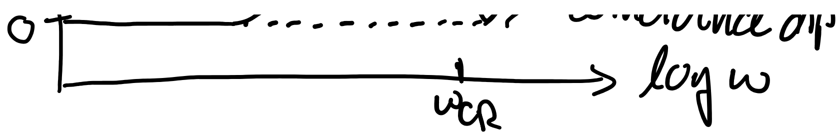
$c_B < c$ ... $k_B > k$

$k_z = \sqrt{k^2 - k_x^2}$

k_z is imaginary ... exponential decay in z-direction

Spatial resonance because of wavelength matching





as flexural stiffness increases,
critical frequency decreases

This is why we prefer low stiffness barriers.

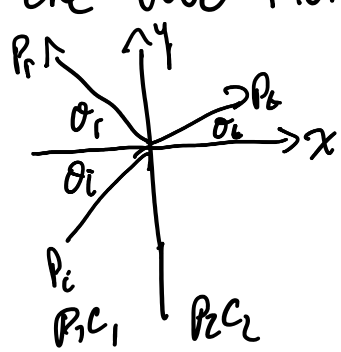
for .05" Al , $f_{cr} = 10kHz$
Acoustically limp until ~10k

for Carbon Fiber Composite ... $f_{cr} \simeq 100Hz$

4.4. Reflection Coefficient Calculations
by using surface normal impedance

- convenient when not concerned with sound transmission

Recall the two-fluid case



- assume solutions for two sound fields
- define b.c.'s
→ R, T soln's

at $x=0$ $\quad \hat{p}_1 = \hat{p}_2$

at $x=0$ $\quad \dfrac{\tilde{p}_1}{\tilde{u}_1} = \dfrac{\tilde{p}_2}{\tilde{u}_2}$

so impedance is continuous at the surface

Define $\left.\dfrac{\tilde{p}_1}{\tilde{u}_{1_x}}\right|_{x=0} = \tilde{Z}_{1n}$ ... specific surface normal impedance

$$= \tilde{Z}_{2n} = \left.\frac{\tilde{p}_2}{\tilde{u}_{2_x}}\right|_{x=0}$$

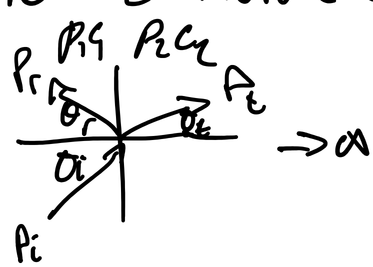
directed into the surface

Assume $\tilde{Z}_n$ is known ... $\underbrace{\dfrac{\tilde{p}}{\tilde{u}_x} = \tilde{Z}_n}_{\text{impedance b.c.}}$ ~ use to calculate reflection coefficient R

by...

- model
- measurement
- theory

Example 2 fluid case - what is $\tilde{Z}_{2n}$?

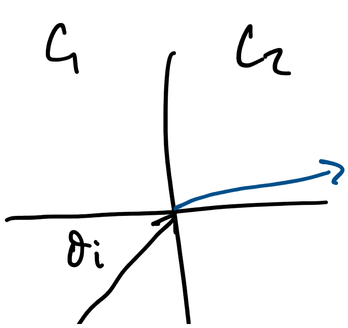


$$p_2|_{x=0} = P_t e^{-jk_{2y}y}$$

$$u_{2x}|_{x=0} = \frac{P_t}{\rho_2 c_2}\cos\theta_t\, e^{-jk_{2y}y}$$

$$\tilde{Z}_{2n}|_{x=0} = \frac{\rho_2 c_2}{\cos\theta_t} \Rightarrow \frac{\rho_2 c_2}{\sqrt{1-\left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i}}$$

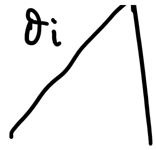
specific normal surface impedance for a semi-infinite fluid region



$\boxed{c_1 \ll c_2}$

$$\sin\theta_t = \frac{c_2}{c_1}\sin\theta_i$$

$\theta_t \to 0$



$\theta_t \to 0$

$$\tilde{Z}_{2n}\Big|_{x=0} = \frac{\rho_2 c_2}{\sqrt{1-\cancel{\left(\frac{c_2}{c_1}\right)^2} \sin^2\theta_i}} \Rightarrow \tilde{Z}_{2n}\Big|_{x=0} \simeq \rho_2 c_2$$

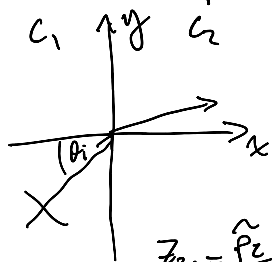
Surface of local reaction

$$Z_m = \rho c + j\left(M\omega - \frac{s}{\omega}\right)$$

Lecture 31

Friday, November 5, 2021 1:26 PM

Reflection from an impedance surface



if $c_2 << c_1 \ldots \theta_t << \theta_i$

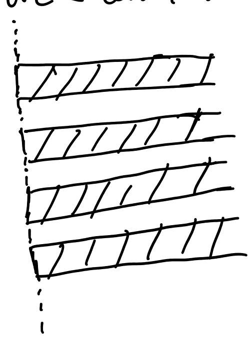
$$Z_{2n} = \frac{\tilde{p}_2}{\tilde{u}_{2n}}\Big|_{x=0}$$

$$Z_{2n} = \frac{\rho_2 c_2}{\sqrt{1-\underbrace{\left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i}_{\text{negligible at limit}}}}$$

Z_{2n} independent of incidence angle

local reaction surface

Z_{2n} is locally reacting when the second medium is slow

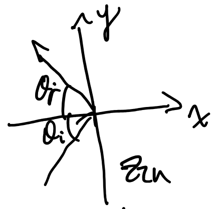


propagation prevented parallel to the surface

continuity of surface normal impedance is true regardless of the complexity of the surface (second) medium ... whether or not the surface is locally reacting.

can R be found if Z_{2n} is known?

$$\frac{\tilde{p}_1}{\tilde{u}_{1x}}\Big|_{x=0} = Z_{2n}$$



$$\tilde{p}_1\big|_{x=0} = P_i e^{-jk_y y} + P_r e^{-jk_y y}$$

$$\tilde{u}_x = \frac{P_i}{\rho_1 c_1}\cos\theta_i\, e^{-jk_y y} - \frac{P_r}{\rho_1 c_1}\cos\theta_i\, e^{-jk_y y}$$

$$Z_{1n} = \frac{\tilde{p}_1}{\tilde{u}_{1x}}\bigg|_{x=0} = \frac{P_i + P_r}{P_i\cos\theta_i - P_r\cos\theta_i}\cdot\rho_1 c_1 = \rho_1 c_1 \frac{1+R}{\cos\theta_i(1-R)} = \frac{\rho_1 c_1}{\cos\theta_i}\frac{(1+R)}{(1-R)} = Z_{1n}$$ ← Assumed known

Solve for R...

$$\boxed{R = \frac{\zeta_{2n}\cos\theta_i - 1}{\zeta_{2n}\cos\theta_i + 1}} \quad \text{where} \quad \zeta_{2n} = \frac{Z_{2n}}{\rho_1 c_1}$$

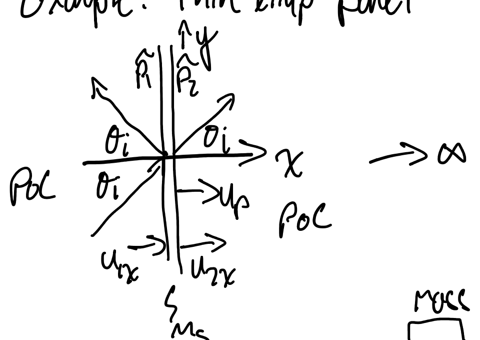
plane wave reflection coefficient of a plane surface having a known Z_{2n}

Absorption Coefficient

$$\alpha = 1 - |R|^2$$

- fraction of the incident Energy absorbed at the surface.

Example: Thin limp panel



at $x=0$ $\quad \tilde{p}_1 - \tilde{p}_2 = \overbrace{M_s}^{\text{mass}}\cdot\overbrace{j\omega\tilde{u}_p}^{\text{accel}}$

$$u_p = u_{1x}|_{x=0} = u_{2x}|_{x=0}$$

at $x=0$ ...

$$\tilde{p}_1 = j\omega M_s \tilde{u}_x + \tilde{p}_2 \qquad \div\, u_{1x}$$

$$\tilde{p}_1 \quad j\omega M_s + \tilde{p}_2$$

$$\frac{\tilde{p}_1}{\tilde{u}_{1x}} = j\omega m_s + \frac{\tilde{p}_2}{\tilde{u}_{2x}}$$

→ specific surface normal impedance of <u>backing space</u>

$$\tilde{Z}_{1n} = j\omega m_s + \tilde{Z}_b$$

$$\frac{\tilde{p}_1 - \tilde{p}_2}{\tilde{u}_p}$$ ... in vacuo specific mechanical transfer impedance of the panel

Impedances add in series because particle velocity is shared

<u>Notes</u>

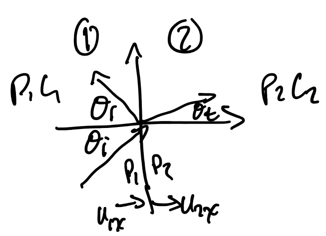
i) For a Complex System ... Z_{1n} can be found by measurement, or theory, or numerical

ii) when Z_{2n} is independent of incidence angle
→ "surface of local reaction"
- occurs when $c_2 \ll c_1$, or when $\theta_t = 0$
(forced by physical structure .... Capillary tubes)

iii) Z_{1n} is normally complex
$$\tilde{Z}_{1n} = R_n + j X_n$$

<u>Concludes Section 4</u>

Book 6.1 → 6.7



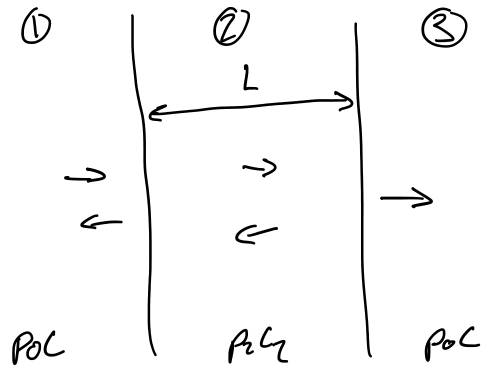
<u>Snell's Law</u>

* $\theta_i = \theta_r$

* $\frac{\sin\theta_t}{\sin\theta_i} = \frac{c_2}{c_1}$

$R + T \neq 1$

$R_I + T_I = 1$ ... Energy Conservation

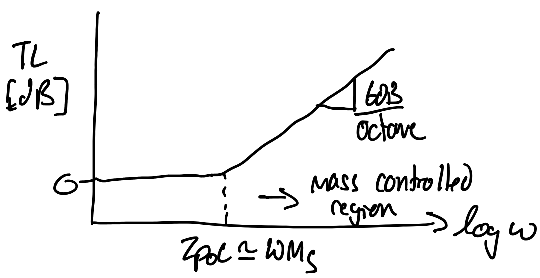


Sonar dome ... if $L = n\frac{\lambda_2}{2}$ ... perfect transmission

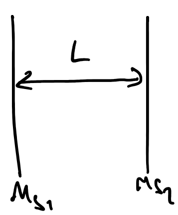
Thin limp barrier

Mass law...

$\tau \propto \frac{1}{M_s}$, $\tau \propto \frac{1}{f}$



Double Panel

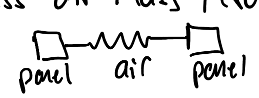


$TL = TL_1 + TL_2 + 6\ dB$

if the panels are decoupled

can decouple with absorptive material

- mass-air-mass resonance



lecture 32

Monday, November 8, 2021 1:30 PM

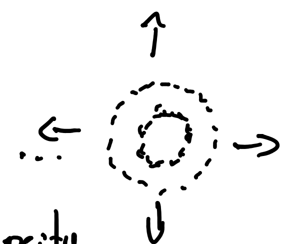
Sound Generation and Radiation

Chapter 7

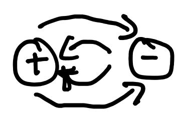
Compact Sources: small compared to a wavelength...

- loudspeakers at low frequencies
- Exhaust pipe openings

"Simple Sources"

- monopole 

  volume velocity source

- dipole  ~ no net volume change ~ point force applied to the fluid

  ⇔ unbaffled loudspeaker

  small axial fans

- Quadrupole

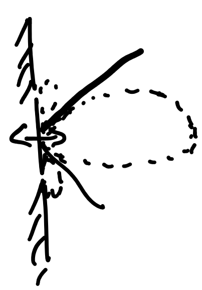
  ⊕ → ⊖
  ⊖ ← ⊕

  lateral Quadrupole
  - no volume change
  - no net force applied to the fluid.

- an oscillatory moment applied to the fluid.
- turbulence ... jet flow

⊕ ⊖ ⊕ ⊖ longitudinal quadrupole

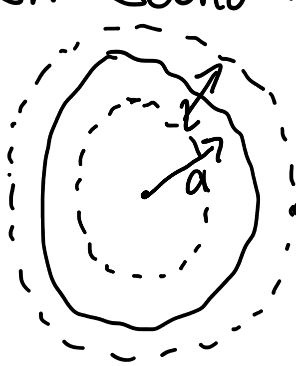
- Non-Compact Sources
  - not small compared to a wavelength
  - extended surface



beaming of Sound

piston in a baffle

5.1 Sound Radiating in a Pulsating Sphere

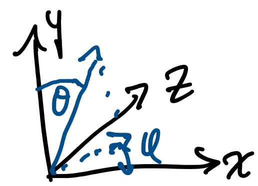


$\leftrightarrow u_0 e^{j\omega t}$

purely radial velocity.

- Wave Equ.

- Wave Equ.
- Assume Soln.
- Apply Velocity B.C. at $r=a$



Spherically Symmetric
- Volume Velocity Source

↳ no variations in θ or φ

$P(r)$ ...

Scalar Helmholtz Equation

$$\nabla^2 p + k^2 p = 0 \quad \ldots \quad k = \frac{\omega}{c}$$

$$\tilde{p}(r,t) = \tilde{p}(r) e^{j\omega t}$$

$$\nabla^2 \to \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}$$

Due to spherical symmetry $\frac{\partial}{\partial \theta}, \frac{\partial}{\partial \varphi} \to 0$

$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r} + k^2 p = 0$$

$$\frac{\partial^2 (rp)}{\partial r^2} + k^2 rp = 0$$

$$p(r,t) = \left[\frac{A}{r} e^{-jkr} + \frac{B}{r} e^{jkr}\right] e^{j\omega t}$$

$$p(r,t) = \left[\underbrace{\frac{A}{r}e^{-jkr}}_{\text{outward}} + \underbrace{\frac{B}{r}e^{jkr}}_{\text{inward}}\right]e^{j\omega t}$$

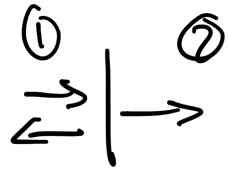
if anechoic or free space... neglect inward component

Lecture 33

Wednesday, November 10, 2021 1:30 PM

Homework Hints...

6.2.6C

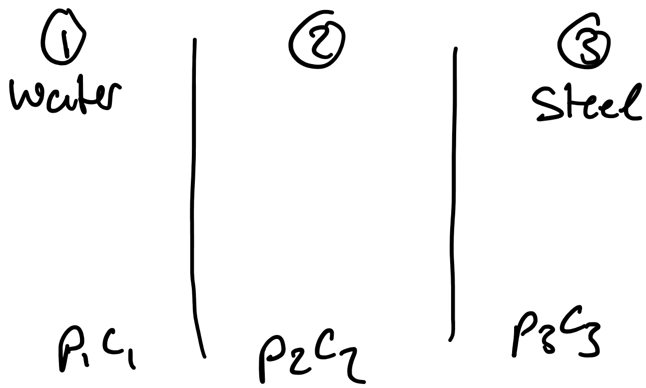


$r_1 = \rho_1 c_1$

$r_2 = \rho_2 c_2$

$$R = \frac{r_2 - r_1}{r_2 + r_1} = \frac{1 - r_1/r_2}{1 + r_1/r_2}$$

6.3.4



$$T_I = \frac{4}{2 + \left(\frac{r_3}{r_1} + \frac{r_1}{r_3}\right)\cos^2 k_2 L + \left(\frac{r_2^2}{r_1 r_3} + \frac{r_1 r_3}{r_2^2}\right)\sin^2 k_2 L}$$

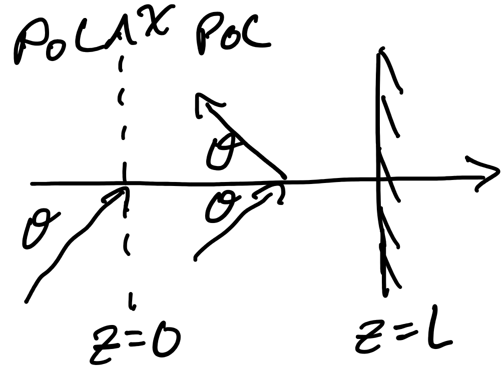
T_I must equal 1, make $\sin k_2 L = 1$ since only r_2 of interest

read comment on pg. 155

Additional Problem 1

$\rho_0 c_1 x$ $\rho_0 c$

$\tilde{p}_i = A e^{-jk_x x} e^{-jk_z z}$



$$\tilde{P}_1 = A e^{-jk_x x} e^{-jk_z z} + B e^{-jk_x x} e^{+jk_z z}$$

$$u_{1z} = \frac{-1}{j\omega\rho_0} \frac{\partial \tilde{P}_1}{\partial z}$$

$$\tilde{u}_{1z}\Big|_{z=L} = 0$$

asked for $Z_n\Big|_{z=0} = \dfrac{\tilde{P}_1\big|_{z=0}}{\tilde{u}_{1z}\big|_{z=0}}$

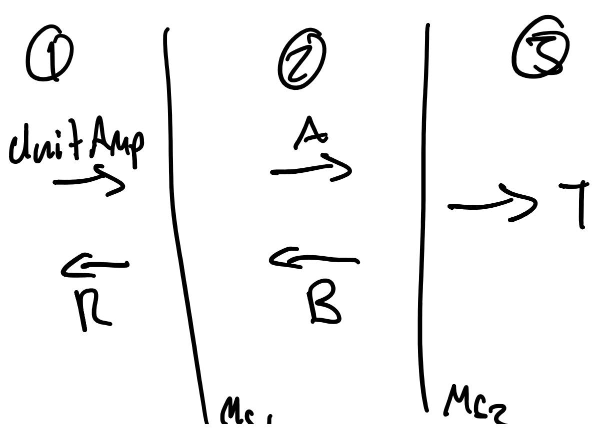
$$R = \frac{Z_n \cos\theta - \rho_0 c}{Z_n \cos\theta + \rho_0 c}$$

Should always be 1

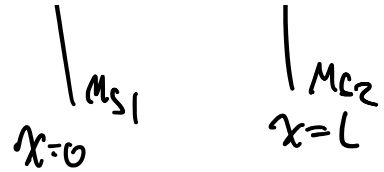
$Z_t = Z_n + R_f$ $\quad\quad \alpha = 1 - |R|^2$

as $\mathrm{Im}\{Z_t\} \to 0$ $\alpha \to$ max

Additional Problem 2



identical Panels



b.c.'s : velocity conditions (2)

EOM $p_1 - p_2 = j\omega m_s$ (2)

4 eqn's and 4 unknowns

can solve in matrix form

$$\begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} R \\ A \\ B \\ T \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

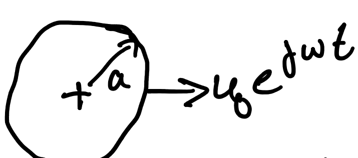
Solve for transmission loss

$$TL = 10 \log_{10} \frac{1}{|T|^2}$$

Plot from 1 Hz to 10 kHz ... log scale

Start of Lecture

sources – monopole, dipole, quadrupole.



spherically symmetric case

$\tilde{p}(r) = \frac{A}{r} e^{-jkr} + \cancel{\frac{B}{r} e^{jkr}}$ free space

$$\tilde{p}(r) = \underbrace{\frac{A}{r} e^{-jkr}}_{\text{outward}} + \underbrace{\cancel{\frac{B}{r} e^{jkr}}}_{\text{inward}} \quad \text{free space}$$

Apply B.C. at $r=a$ to solve A

... at $r=a$ $\quad \tilde{u}_r(a) = u_0$

$$\hat{u}_r(r) = \frac{-1}{j\omega\rho_0}\frac{d\tilde{p}}{dr} = \frac{A}{\rho_0 c}\frac{e^{-jkr}}{r}\left(1 - \underbrace{\frac{j}{kr}}_{\text{near field}}\right)$$

u_0 at $r=a$

$\tilde{u}_r(a) = u_0$

$$A = \rho_0 c a u_0 e^{jka} \cdot \frac{1}{1 - j/ka}$$

$$A = \rho_0 c a u_0 e^{jka} \frac{ka}{ka - j} \qquad ka - j \sim \sqrt{(ka)^2+1}\, e^{-j\theta_a}$$

$$\theta_a = \tan^{-1}(1/ka)$$

$$ka = \frac{2\pi a}{\lambda} = 2\pi\left(\frac{a}{\lambda}\right) \text{ non-dimensional size of the source radius}$$

$$A = \rho_0 c a u_0 e^{jka} \frac{ka}{\sqrt{(ka)^2+1}\, e^{-j\theta_a}} \qquad \frac{ka}{\sqrt{(ka)^2+1}} \sim \cos\theta_a$$

$$= \rho_0 c a u_0 e^{jka} \cos\theta_a\, e^{j\theta_a}$$

$$\tilde{p}(r) = \rho_0 c a U_0 e^{jka} e^{j\theta_a} \cos\theta_a \frac{e^{-jkr}}{r}$$

pulsating sphere of radius a

Impedance, Intensity, Sound Power

$$\frac{\tilde{p}(r)}{\tilde{u}_r(r)} = \tilde{Z}(r) = \rho_0 c \frac{kr}{kr-j}$$

$$\tilde{Z}(r) = \rho_0 c \cos\theta_r e^{j\theta_r} \qquad \cos\theta_r = \frac{kr}{\sqrt{(kr)^2+1}}$$

far field: $kr \gg 1$

$$\lim_{kr \to \infty} \tilde{Z}(r) = \rho_0 c$$

if $kr \ll 1$

$$\tilde{Z}(r) \simeq j\rho_0 c(kr) \qquad k = \frac{\omega}{c}$$

$$\tilde{Z}(r) \simeq j\omega\rho_0(r)$$

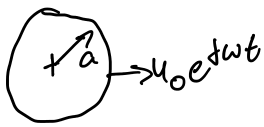
Intensity...

$$I_r = \frac{1}{2} \mathrm{Re}\{\tilde{p} \cdot \tilde{u}^*\}\Big|_r$$

$$= \frac{1}{2}\left(\frac{a}{r}\right)^2 \rho_0 c u_0^2 \cos^2 \varphi_a$$

Lecture 34

Friday, November 12, 2021 1:29 PM



$P_1(r) = \frac{A}{r} e^{-jkr}$ ← outward wave expansion.

B.C. → A

$Z_r = \frac{\tilde{p}}{\tilde{u}} = \frac{\rho_0 c\, kr}{kr - j} = \rho_0 c \frac{1}{1 - j\frac{1}{kr}}$ ... in far field → $\rho_0 c$

Intensity: $\frac{1}{2} \mathrm{Re}\left\{ \tilde{p}(r) \cdot \tilde{u}(r)^* \right\} = I_r$

$I_r = \frac{1}{2}\left(\frac{a}{r}\right)^2 \rho_0 c\, u_0^2 \cos^2 \theta_a$

purely radial

inverse square law

$I_r \propto \frac{1}{r^2}$

Sound power:

$W = \int_S I_r \, dS$

$W = 4\pi r^2 \cdot I_r(r) = \boxed{2\pi a^2 \rho_0 c\, u_0^2 \cos^2 \theta_a = W}$

large sources radiate more effectively than small sources

5.3 Simple Sources...

5.3.1 point monopole source

- represents compact source that changes volume

$\tilde{p} = \frac{A}{r} e^{-jkr}$

$A = j\rho_0 c a u_0 e^{-jka} \frac{ka}{ka - j}$

for the sphere case;

$4\pi a^2 u_0 =$ volume displaced by the

For the sphere case:

$4\pi a^2 u_0$ = Volume displaced by the sphere per unit time [m^3/s]

(volume velocity, Q) ← monopole source strength

$$A = \omega \rho_0 \frac{Q}{4\pi k a} e^{jka} \frac{ka}{ka - j}$$

$$\lim_{ka \to 0} A = j\omega \rho_0 \frac{Q}{4\pi}$$ "point source"

$$\tilde{p}(r) = j\omega \rho_0 \frac{Q}{4\pi r} e^{-jkr}$$

$$\tilde{p}(r) = j\rho_0 c \frac{kQ}{4\pi r} e^{-jkr}$$ ... point monopole

Holography: replicates sound field by representation as a collection of point monopoles.

$$\lim_{ka \to 0} I_r = \frac{\rho_0 c}{2} \frac{k^2 Q^2}{(4\pi r)^2}$$

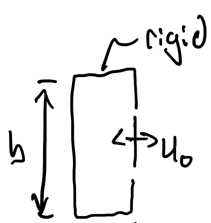
$$W = \frac{\rho_0 c k^2 Q^2}{8\pi}$$ Small sources are poor radiators.

represent a compact source that changes volume

... Q ...

if $d \ll \lambda$
can be treated as compact

$$\int_S u_n \, dS \quad [m^3/s]$$



circular diaphragm

$b \ll \lambda$

$Q = (\text{Area})(\text{Velocity})$

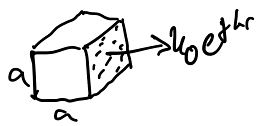
$Q = \pi a^2 \cdot u_0$

replace real source by point monopole

$$\tilde{p}(r) = j\rho_0 c \frac{kQ}{4\pi r} e^{-jkr} \quad \ldots \quad Q = \pi a^2 \cdot u_0$$

Simple Volume Source

Any source that displaces Q [m^3/s] and is compact is said to be a simple volume source where source strength $Q = \int_S A\, u_n\, ds$



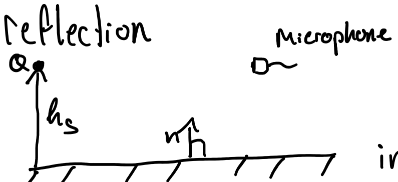
$Q = a^2 u_0$

Some important sources do not exhibit a volume change.

fan
unbaffled loudspeakes
} not volume sources

5.3.3 reflection at a hard surface

Single reflection



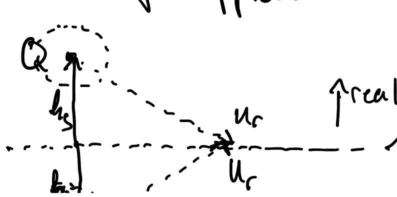
infinite plane surface

unit vector normal to the surface

$\bar{u} \bullet \bar{n} = 0$ ... zero normal particle velocity boundary condition

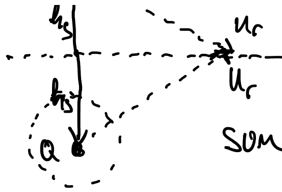
Equivalent System in free space that satisfies surface normal b.c.

- image approach



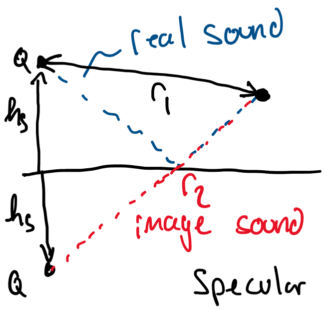
real space

nominal surface location



Sum of normal particle velocities $= 0$ everywhere on the nominal surface, $\bar{u} \cdot \bar{n} = 0$ is satisfied

Image Source Arrangement is equivalent mathematically to the real case



Solution applies in the region $z \geq 0$

Specular reflection
$\theta_i = \theta_r$

$$P_t = P_1 + P_2$$

$$P_t = j\rho_0 c \frac{kQ}{4\pi r_1} e^{-jkr_1} + j\rho_0 c \frac{kQ}{4\pi r_2} e^{-jkr_2}$$

Direct Field — Reflected Component

$$\tilde{P}_t = j\rho_0 c \frac{kQ}{4\pi r_1} e^{-jkr_1} \left[1 + \left(\frac{r_1}{r_2}\right) e^{-jk(r_2 - r_1)} \right]$$

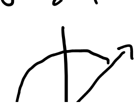
$\left(\frac{r_1}{r_2}\right)$ - relative spherical spreading attenuation

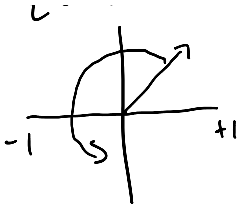
$(r_2 - r_1)$ - path length difference

$k(r_2 - r_1)$ - phase difference between direct and reflected

$0 \leq \left(\frac{r_1}{r_2}\right) \leq 1$ — reflected sound is attenuated with respect to the direct sound due to spherical spreading

$e^{-jk(r_2 - r_1)}$

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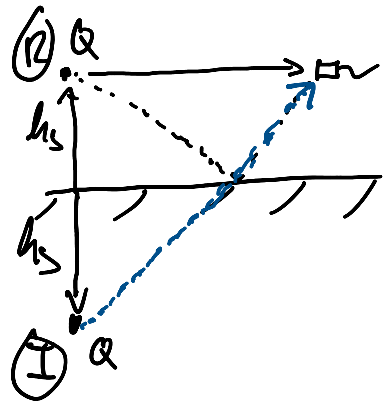
Lecture 35

Monday, November 15, 2021 1:30 PM

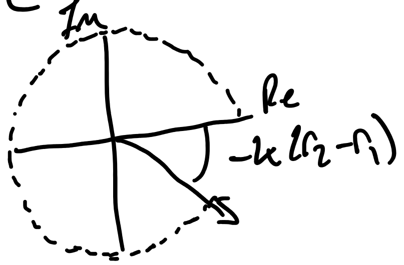
Reflection at a hard Surface



$$\tilde{p}_t = j\rho_0 c \frac{kQ}{4\pi r_1} e^{-jkr_1}\Big[1 + \underbrace{\left(\frac{r_1}{r_2}\right)}_{\text{spherical attenuation}} e^{-jk\overbrace{(r_2-r_1)}^{\text{path length difference}}}\Big]$$

pLd:

$e^{-jk(r_2-r_1)}$



$k(r_2-r_1) = 0, 2\pi, 4\pi \ldots \quad e^{-jk(r_2-r_1)} = 1$ ... reinforcement

$\frac{2\pi}{\lambda}(r_2-r_1) = 2\pi \Rightarrow r_2-r_1 = \lambda$ ...in-phase reinforcement

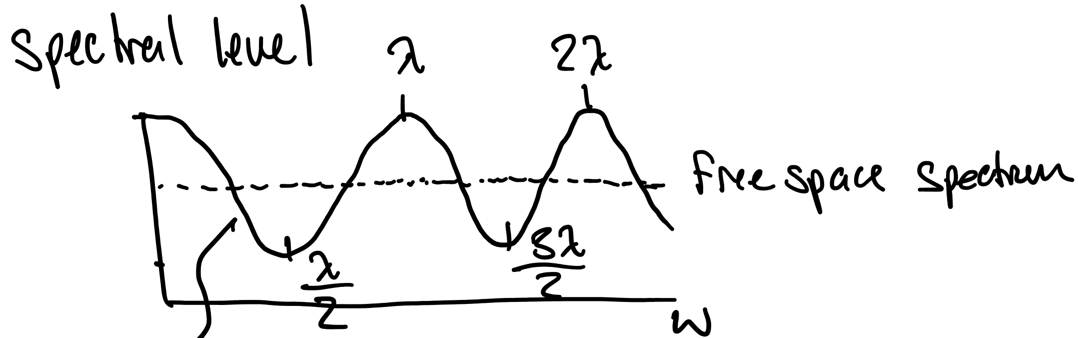
approximate maximum

if $k(r_2-r_1) = \pi, 3\pi, 5\pi \ldots \quad e^{-jk(r_2-r_1)} = -1$

$r_2 - r_1 = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$ .. cancellation

$r_2 - r_1 = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \ldots$ cancellation

Minimum out-of-phase cancellation.

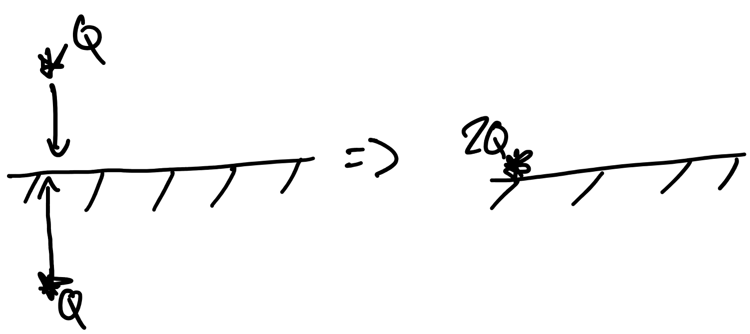


Magnitude of the ripple depends on the spherical spreading ratio.

Effect of a single coherent reflection is to add a ripple to the received spectrum.

- rate of ripple depends on $(r_2 - r_1)$

  larger path length difference -> faster rate of ripple



$r_1 = r_2 = r$ true source + image source coalesce at the surface.

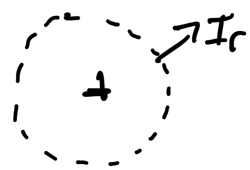
$$\tilde{p}_t = j\rho_0 c \frac{kQ}{4\pi r} e^{-jkr}\left[1 + \left(\frac{r}{r}\right) e^{-jk(r-r)}\right]$$

$$\tilde{p}_t = j\rho_0 c \frac{kQ}{4\pi r} e^{-jkr} \underbrace{\left[1 + \left(\frac{r}{r}\right) e^{-jk(r-r)}\right]}_{=2}$$

$$\tilde{p}_t = j\rho_0 c \frac{k\,2Q}{4\pi r} e^{-jkr}$$

Sound pressure doubled at the receiver location with respect to a monopole in free space.

Monopole in free space:

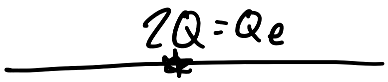


$$I_r = \frac{\rho_0 c}{2} \frac{k^2 Q^2}{(4\pi r)^2}$$

$$W = \int_S I_r \, dS$$

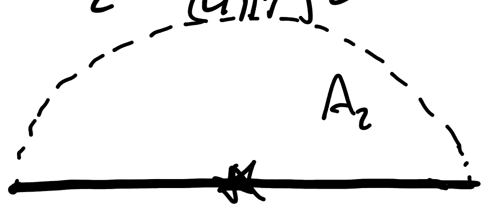
Monopole on hard surface:

$h_s = 0$



$$I_r = \frac{\rho_0 c}{2} \frac{k^2 Q_e^2}{(4\pi r)^2}$$ in the presence of a hard surface with 0 source height

$$I_r = \frac{\rho_0 c}{2} \frac{k^2 4Q^2}{(4\pi r)^2} = 2\rho_0 c \frac{k^2 Q^2}{(4\pi r)^2}$$ ... 4x free space value



$A_2 = \frac{A}{2}$ ~ free space value

Sound Power is doubled from free space

$I_{r2} = 4I_r$

$I_{r2} = 4 I_r$

$A_2 = \frac{A}{2}$

$W_2 = 2W$

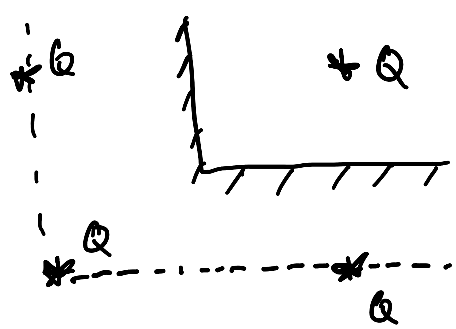
hard surface has increased sound power radiated by the source

Assuming infinite output impedance of the source

- if a source is mounted in a rigid baffle
  - then a high impedance source radiates twice the sound power

    (radiation resistance is increased)

Multiple Reflections ...



place a source near the junction of two rigid planes

3 image sources necessary to satisfy u·n boundary condition.

if source is moved into the corner then $Q^e = 4Q$

If source is moved into the corner

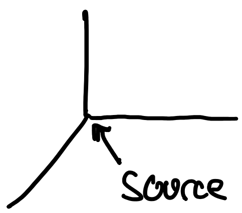
then $Q^e = 4Q$

$I_3 = 16 I_1$

Sound Power (1/4 sphere)

$W_3 = 4W_1 = 2^2 W_1$

3 walls



7 images are required

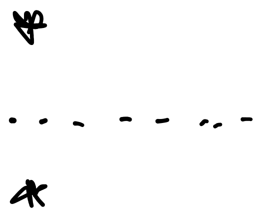
$Q^e = 8Q$

$I_4 = 64 I_1 \Rightarrow W_4 = 8W_1 = 2^3 W_1$

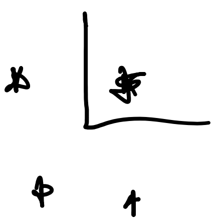
- monopoles are used to represent "small" sources aka compact sources that exhibit a volume change. $ka \ll 1$ small compared to a wavelength
- Assume Q is independent of acoustic loading.
- Sound Power of a source is affected by the environment

Lecture 36

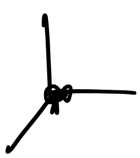
Wednesday, November 17, 2021 1:28 PM



$2Q$ $4I_c$ $2W$



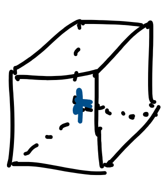
$4Q$ $16I_c$ $4W$



$8Q$ $64I_c$ $8W$

Sound power is affected by the environment in which it is placed

Closed box

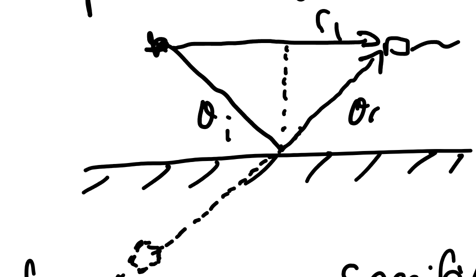


$W=0$

Partially Reflecting Surfaces

Partially Reflecting Surfaces

Impedance Surfaces



$Z_n = \frac{p}{u_n}\Big|_{surface}$

Specific surface normal impedance

Cannot Satisfy impedance boundary condition with a simple point source.

Fuzzy image works for locally reacting surfaces (surface where Z is not a function of θ_i)

Approx Solution

$$\tilde{p}(r) = A\Big[\underbrace{\frac{e^{-jkr_1}}{r_1}}_{\text{direct}} + \underbrace{\overbrace{R(\theta)}^{\text{Plane Wave Reflection Coefficient}}\frac{e^{-jkr_2}}{r_2}}_{\text{Specular Reflection}} + \underbrace{(1-R(\theta))F(w)\frac{e^{-jkr_2}}{r_2}}_{\text{ground + surface waves}}\Big]$$

$$R(\theta) = \frac{Z_n\cos\theta - \rho_0 c}{Z_n\cos\theta + \rho_0 c}$$

At grazing incidence $\theta \to \pi/2$

$R(\theta) \Rightarrow -1$

At grazing $r_1 = r_2$ ... direct and specular terms cancel

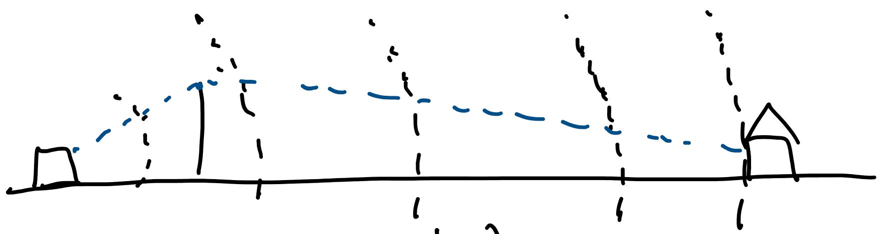
$$\tilde{p}(r) = A\Big[(1-R(\theta))F(w)\frac{e^{-jkr_2}}{\ldots}\Big]$$

$$\tilde{p}(r) = A\left[(1-R(\theta))F(w)\frac{e^{-jkr_2}}{r_2}\right]$$

Sound field consists of ground and surface waves.

Ground Wave - Propagating Wave
but $|\tilde{p}| \propto \frac{1}{r^2}$

Surface Wave - Decays exponentially w/ distance + height

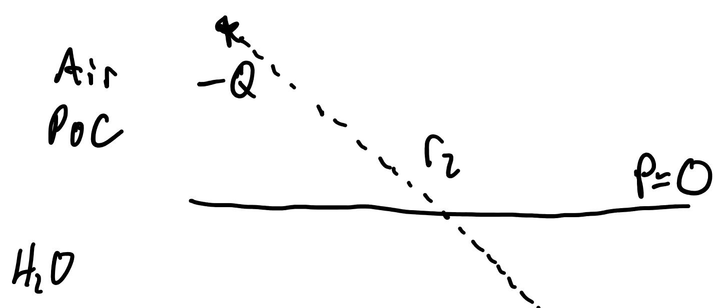


grassland
plowed fields
snow

when $\theta < 75°$

$$\tilde{p}(r) \simeq A\left[\frac{e^{-jkr_1}}{r_1} + R(\theta)\frac{e^{-jkr_2}}{r_2}\right]$$

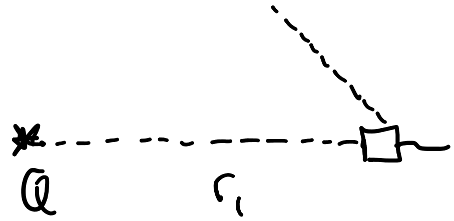
Pressure Release Surfaces



$\rho_1 c_1 \simeq 1.5 \times 10^6$
$\rho_0 c \simeq 415$

$\rho_1 c_1 \gg \rho_0 c$

H_2O
$\rho_1 c_1$

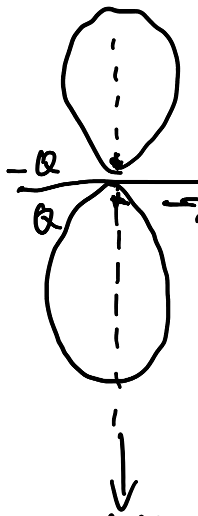


$\rho_1 c_1 >> \rho_0 c$

negative Image is required to Satisfy $p=0$

$$p(r) = A\left\{\frac{e^{-jkr_1}}{r_1} - \frac{e^{-jkr_2}}{r_2}\right\} \quad k=\frac{\omega}{c_1} \text{ (water)}$$

let source approach the surface



→ minimum radiation parallel to the surface.

figure 8 radiation

Maximum radiation perpendicular to surface

Dipole - 2 closely spaced monopoles of equal Strength running out-of-phase

alternating expansion and contraction.

Dipole axis

Monopoles are mass or volume sources

Dipole - point force
- no net change in volume

Dipole point force

- no net change in volume

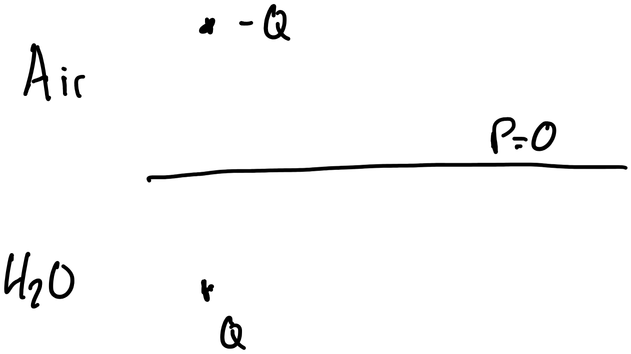
point force acts on the fluid to create oscillatory acceleration at a point

unbaffled loudspeaker

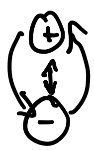
3 orthoganal dipoles can be used to represent a point force in an arbitary direction.

Lecture 37

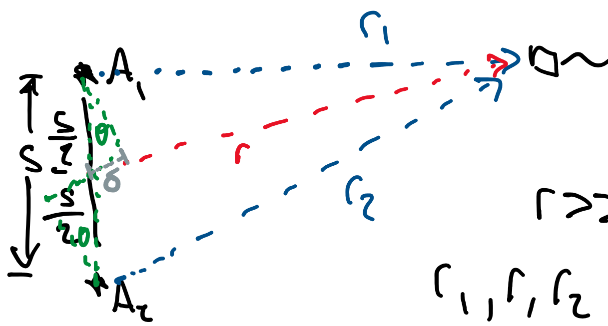
Friday, November 19, 2021 1:26 PM



Represent as Dipole



5.3.5 Dipole



$r \gg s$

r_1, r, r_2 approx. parallel

$$\delta = \frac{s}{2}\cos\theta$$

θ - angle from the dipole axis directed towards the positive monopole

$$r_1 \simeq r - \delta = r - \frac{s}{2}\cos\theta$$

$$r_2 \simeq r + \delta = r + \frac{s}{2}\cos\theta$$

$$\tilde{p}(r) = A_1 \frac{e^{-jkr_1}}{r_1} + A_2 \frac{e^{-jkr_2}}{r_2}$$

Assume:

$r \gg s$

$A_2 = -A_1 \ldots A_1 = A$

$$\tilde{p}(r) = A\left[\frac{e^{-jk(r-\delta)}}{r-\delta} - \frac{e^{-jk(r+\delta)}}{r+\delta}\right]$$

$$\tilde{p}(r) = Ae^{-jkr}\left\{\frac{r\left(e^{+jk\delta} - e^{-jk\delta}\right) + \cancel{\delta\left(e^{+jk\delta} + e^{-jk\delta}\right)}^{\to 0}}{r^2 - \cancel{\delta^2}}\right\}$$

receiver in the far field...

$r \gg s$

$\delta \ll r$

$$\tilde{p}(r) = \frac{A}{r} e^{-jkr}\left(e^{jk\delta} - e^{-jk\delta}\right)$$

$$= \frac{2jAe^{-jkr}}{r} \sin k\delta$$

$$k\delta = k\frac{s}{2}\cos\theta$$

Compact source approximations:

$ks \ll 1 \ldots \frac{2\pi}{\lambda} s \ll 1$

approximations:

$$ks \ll 1 \quad \ldots \quad \frac{2\pi}{\lambda} s \ll 1$$

$$\sin k\delta = \sin\Big(\underbrace{\frac{ks}{2}\cos\theta}_{\text{very small}}\Big) \qquad \sin\theta \to \theta \text{ (when } \theta \text{ is small)}$$

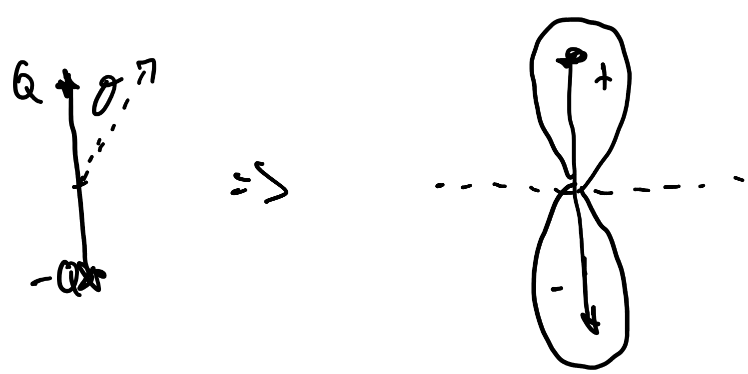
$$\sin k\delta = \frac{ks}{2}\cos\theta$$

$$\tilde{p}(r) = 2jA\,\frac{e^{-jkr}}{r}\,\frac{ks}{2}\cos\theta$$

$$\left.A = \frac{j\rho_0 c k Q}{4\pi}\right] \text{ source strength of individual monopoles}$$

Q = volume source strength

$$\tilde{p}(r) = -\rho_0 c k^2 (Qs)\,\frac{e^{-jkr}}{4\pi r}\cos\theta$$ ~ sound field radiated by a point dipole



Qs - dipole source strength.

$Qs = D$ ... dipole moment.

Assumptions -

- compact source $ks \ll 1$
- receiver in far field $s \ll r$ ... $\delta \ll r$

for a point monopole:

$$\tilde{p}(r) = j\rho_0 c \frac{kQ}{4\pi r} e^{-jkr}$$

for a point dipole:

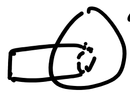
$$\tilde{p}(r) = -\rho_0 c k^2 (Qs) \frac{e^{-jkr}}{4\pi r} \cos\theta$$

$$\frac{|\tilde{p}(r)|_{dipole}}{|\tilde{p}(r)|_{monopole}} = \frac{ks}{\cos\theta}$$

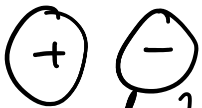
for a given monopole source strength Q,
dipole is a much weaker source

$$|\tilde{p}(r)|_{dipole} \ll |\tilde{p}(r)|_{monopole}$$

Exhaust pipe:



~ acts as Monopole



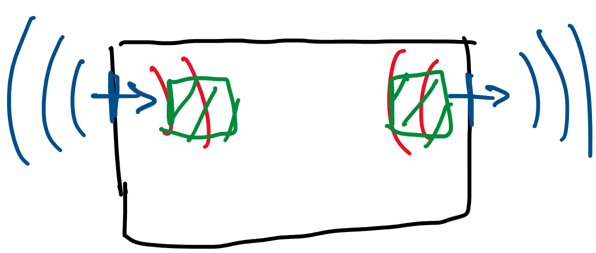
Pipe   loud Speaker

-> creates dipole

Increase Radiation Efficiency

loud speaker in baffle

dipole -> monopole



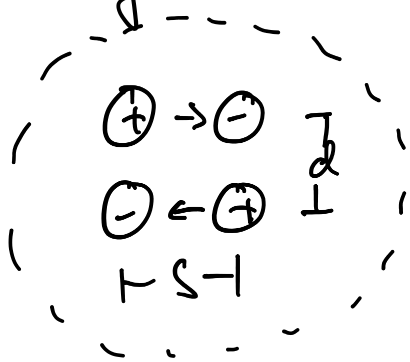
Monopole when case is closed

dipole when case is open

grate on surface

5.3.6 Quadrupole etc.

- Array of 2 dipoles



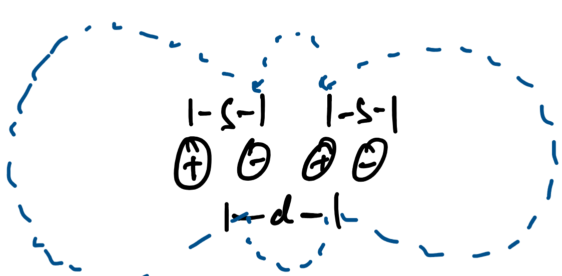
– no net force if source is compact ∴ $ks \ll 1$, $kd \ll 1$

results in a rotation of fluid.

lateral quadrupoles are used to represent compact sources that apply an oscillatory moment to the fluid

M. J. Lighthill –
represent sound generated by homogeneous turbulence as quadrupole

longitudinal quadrupole



$ks \ll 1$
$kd \ll 1$

Lecture 38

Monday, November 22, 2021 1:24 PM

Ps 5 problem 1

$$\tilde{p}(r) = j\rho_0 c \frac{kQ}{4\pi r} e^{-jkr} \quad \ldots \text{ find } Q$$

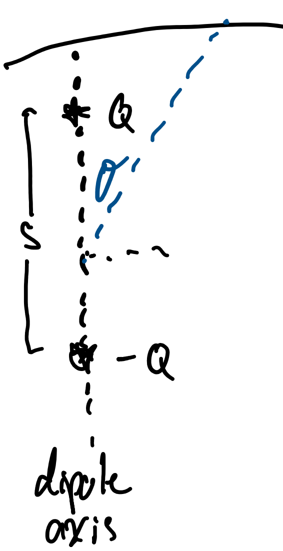
$$L_p = 10\log_{10}\left(\frac{p^2_{rms}}{p^2_{ref}}\right) = 90$$

$$10^9 = \frac{p^2_{rms}}{p^2_{ref}} \qquad p^2_{rms} = \frac{p(r)p(r)^*}{2}$$

$$p^2_{rms} = \frac{p(r)p(r)^*}{2} = 10^9 \cdot p^2_{ref}$$

$$\tilde{p}(r) = j\rho_0 c \frac{kQ}{4\pi r} e^{-jkr} \qquad \tilde{p}(r)^* = -j\rho_0 c \frac{kQ}{4\pi r} e^{jkr}$$

$$Q = 4\pi r^2 \underline{u} \qquad j\omega\tilde{\xi} = \tilde{u} \qquad \underbrace{\tilde{\xi}}_{\text{displacement}} = \frac{\tilde{u}}{j\omega}$$



$$\tilde{p}(r) = -\rho_0 c k^2 (Qs) \frac{e^{-jkr}}{4\pi r} \cos\theta$$

Quadrupole

⊕ ⊖
⊖ ⊕ lateral Quadrupole

⊖ ⊕ ⊕ ⊖ longitudinal Quadrupole

Compact Source - small compared to λ

- monopole - volume change
- dipole - force
- lateral Quad - Moment applied to fluid.

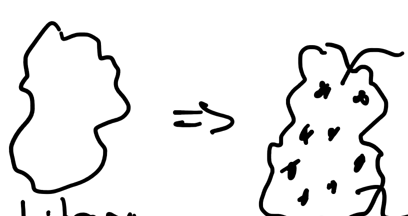
radiation efficiency drops

per unit Q (source strength)

$$|p_{monopole}| \gg |p_{dipole}| \gg |p_{quad}|$$

Multipole Decomposition -

Equivalent Source Methods -



Arbitrary Source

equivalent source array

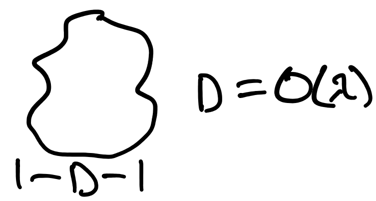
adjust strengths of simple sources to achieve same b.c. as real source

$$\tilde{p}(r) = \Sigma \text{ monopoles } + \Sigma \text{ dipoles } + \Sigma \text{ quadrupoles } + \ldots$$

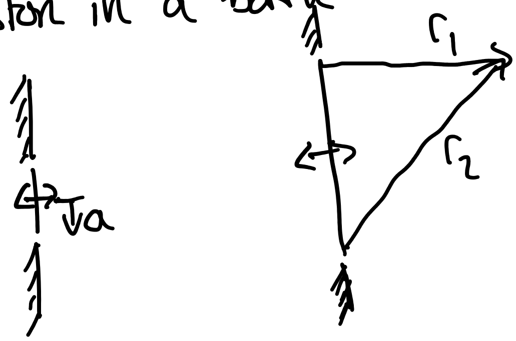
$\tilde{p}(r) = \sum \text{monopoles} + \sum \text{dipoles} + \sum \text{quadrupoles} + \ldots$

5.4 Sound radiation from extended sources

- finite size
- not small compared to λ

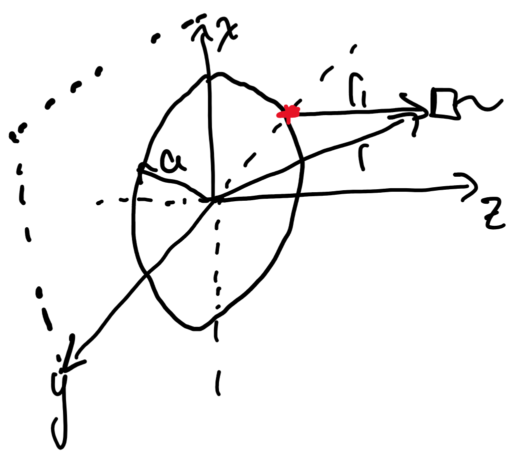


Piston in a baffle



$(r_2 - r_1) \sim O(\lambda)$

reinforcement & cancellations as a result of large path length differences.



piston flush with x-y plane, remainder of plane is baffled.

incremental source strength

$$\partial Q = u_0 \partial S \quad \text{incremental source region}$$

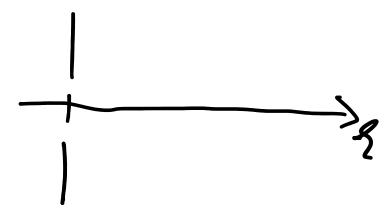
$$d\tilde{p} = j\rho_0 c k (\partial Q) \frac{e^{-jkr_1}}{2\pi r'}$$

monopole on a hard surface ...

integrate over surface of the piston...

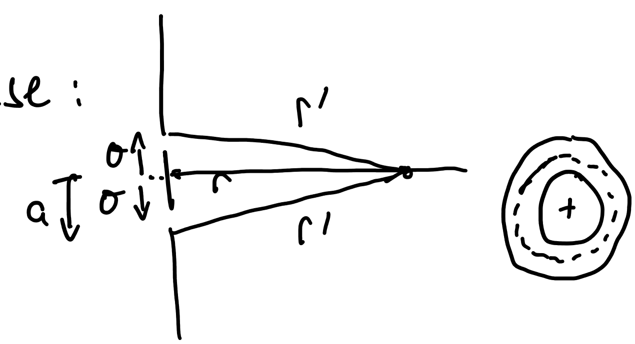
$$\tilde{p}(r,\theta) = j\rho_0 c \frac{u_0 k}{2\pi} \int_S \frac{e^{-jkr'}}{r'}\, dS$$

i) on-axis case.



ii) far-field case

on-axis case:



$$r' = \sqrt{r^2 + \sigma^2}$$

$$\tilde{p}(r,0) = j\rho_0 c \frac{k u_0}{2\pi} \cdot \int_0^a \frac{e^{-jk(r^2+\sigma^2)^{1/2}}}{(r^2+\sigma^2)^{1/2}}\, 2\pi\sigma\, d\sigma$$

$$\tilde{p}(r,0) = \underbrace{2j\rho_0 c u_0 e^{-jkr}}_{\sim \text{monopole}} \Big(\underbrace{e^{-jk\frac{r}{2}\left[\sqrt{1+a^2/r^2}\; - 1\right]}}_{\text{phase}} \Big) \underbrace{\sin\left[\frac{kr}{2}\left(\sqrt{1+\frac{a^2}{r^2}}\; - 1\right)\right]}_{\text{spatial variation}}$$

Special Cases

i) $k << 1$ compact

ii) farfield $\frac{a}{r} << 1$

$\underbrace{\hphantom{xxxxxxxxxxxxxxxx}}_{\text{monopole on a hard Surface}}$

farfield condition

$\sqrt{1 + \frac{a^2}{r^2}}$ $\quad \frac{a^2}{r^2} << 1$ $\qquad \simeq (1+x)^{1/2}$

expand...

$= 1 + \frac{x}{2} + \cancel{\frac{x^3}{3}} \ldots$

$\simeq 1 + \frac{a^2}{2r^2}$

$\sin \frac{kr}{2}\left[1 + \frac{a^2}{2r^2} - 1\right] \Rightarrow \sin\left(\frac{ka}{4} \cdot \frac{a}{r}\right)$ $\quad ka << 1$ & $\frac{a}{r} << 1$

So arg of sin is small

$\sin\left(\frac{ka}{4} \cdot \frac{a}{r}\right) \Rightarrow \frac{ka}{4} \cdot \frac{a}{r}$ $\quad \frac{1}{r}$ dependence in far field

Sub into complete solution

$|\tilde{p}(r,0)| \simeq 2\rho_0 c U_0 \frac{ka}{4}\left(\frac{a}{r}\right)$ $\quad \ldots \quad Q = U_0 \pi a^2$

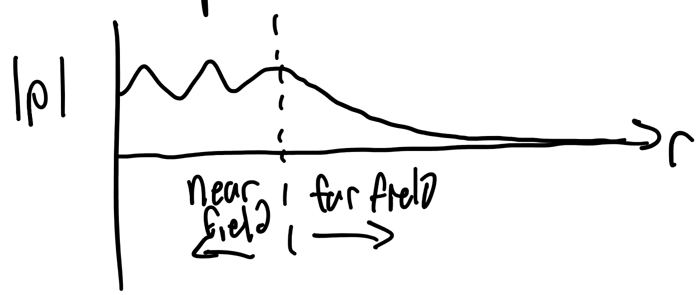
$U_0 = \frac{Q}{\pi a^2}$

$\simeq \frac{\rho_0 c\, k Q}{2\pi r}$

loud speaker flush mounted on a wall

loud speaker flush mounted on a wall
can be modeled as a monopole
on a hard surface

Complete Solution



Important when making measurements
close to a large source

Lecture 39

Monday, November 29, 2021 1:25 PM

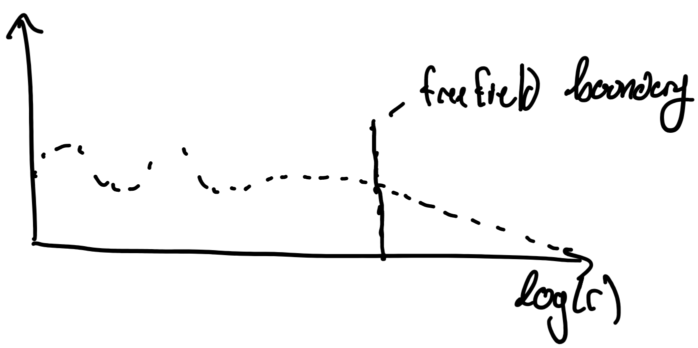
HW Hints PS5

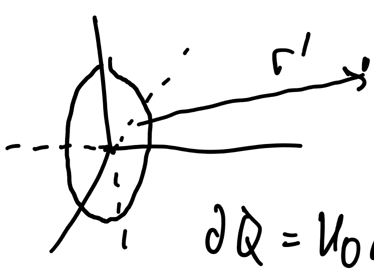
#2

$$\tilde{p}(r,0) = 2j\rho_0 c U_0 e^{-jkr} \left(e \qquad \right) \sin\left[\frac{kr}{2}\left(\sqrt{1+\frac{a^2}{r^2}} - 1\right)\right]$$

$$p^2_{rms}(r,0) = \frac{\tilde{p}(r,0) \cdot \tilde{p}(r,0)^*}{2}$$

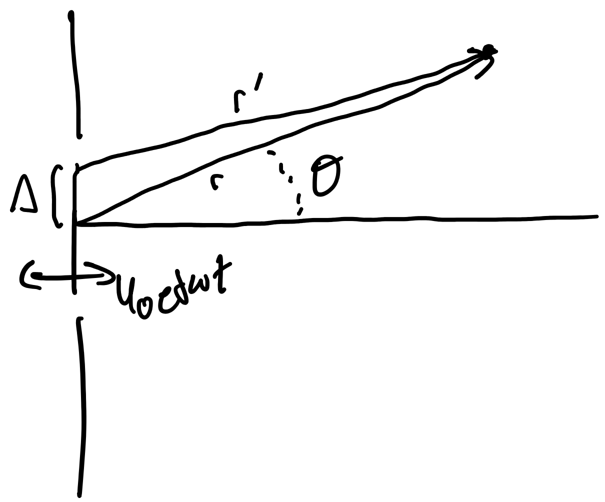
$$L_p = 10 \log_{10} \frac{p^2_{rms}(r,0)}{p^2_{ref}}$$





$$\partial Q = U_0 \, \partial s$$

$$\partial \tilde{p} = j\rho_0 c k (\partial Q) \frac{e^{-jkr'}}{2\pi r'} \Rightarrow p(r,\theta) = \frac{j\rho_0 c k U_0}{\pi} \int \frac{e^{-jkr'}}{r'} ds$$



$\frac{a}{r} \ll 1$

a = piston radius

- replace r' by r in the spherical term
- replace r' by r + Δ in phase term

$$\tilde{p}(r,\theta) = \underbrace{\frac{j\rho_0 c}{2} u_0 \left(\frac{a}{r}\right) e^{-jkr}}_{\text{monopole}} \underbrace{\left[2\,\frac{J_1(ka\sin\theta)}{ka\sin\theta} \right]}_{\text{directivity factor} \ldots O(1)}$$

J_1 ... Bessel function of the first kind.

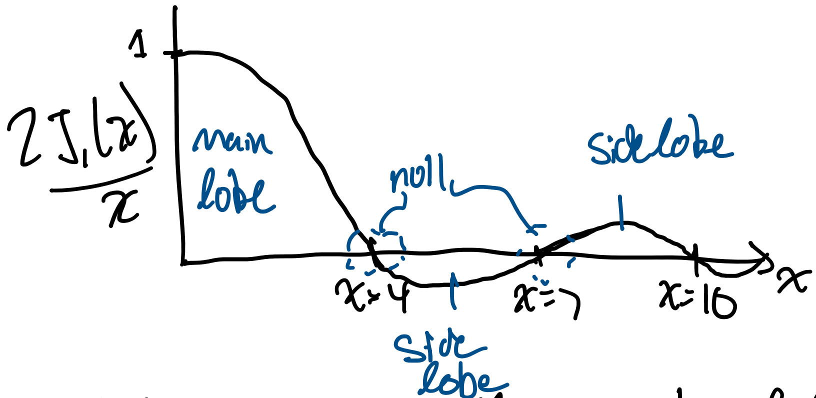
a ... piston radius

k ... ω/c

ka ... non-dimensional piston radius

θ ... polar angle

$$2\,\frac{J_1(ka\sin\theta)}{ka\sin\theta} \Rightarrow \frac{2J_1(x)}{x}$$



if ka is very small, similar behaviors to monopole

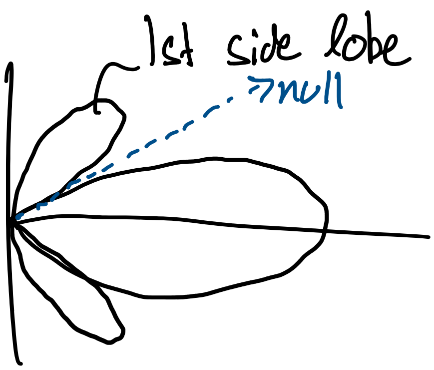
Special case $ka \ll 1$:

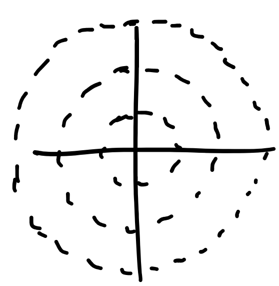
omnidirectional radiation...

directivity factor ≈ 1 for all angles

$ka \gg 1$ high freq or large piston...

- main lobe, sidelobes + nulls





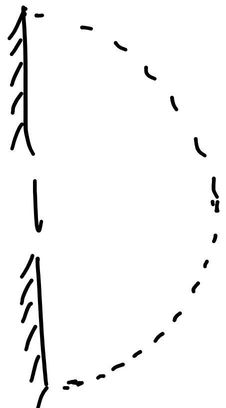
main lobe becomes narrower as ka increases

$ka \sin\theta = 4$ ... first null

solve for θ, given ka.

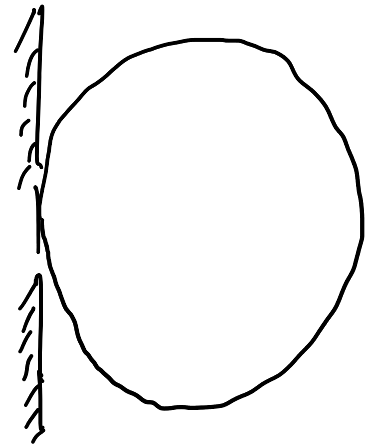
$ka \gg 1$ ... highly directional radiation

$ka \gg 1$ ... highly directional radiation



$ka \ll 1$

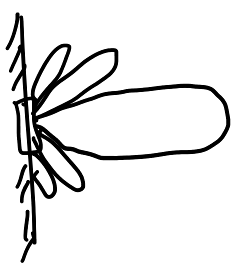
- omni
- monopole slice



$ka = 4$

- single null parallel w/ surface
-

$ka \gg 1$



First null happens at θ_1

$$ka \sin\theta_1 = j_{11}$$ .... zeroes of Bessel function.

Appendix A5

solve for θ_1 given.

for $ka < 4$, there are no nulls

$$J_1(j_{11}) = 0$$

$J_1(j_{12}) = 0 \qquad ka\sin\theta_2 = j_{12}$

At high frequencies... $ka \gg 1$

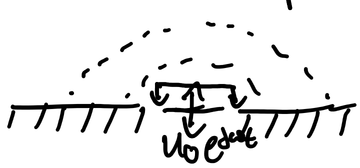
j_{11} is reached at progressively smaller angles as the freq increases...

...and beam width becomes narrower

in Public Address Systems...

- many small high frequency drivers pointing in various directions.
- relatively uniform coverage
  - omni directional

Radiation Impedance



$$Z_r = \frac{\text{force exerted on the radiator by the sound field}}{\text{source velocity}}$$

$$Z_r = \frac{\int_S P_{surface}\, dS}{u_0}$$

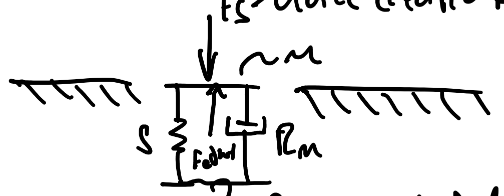
rigid piston velocity...

loudspeakers... finite internal impedances

f_r force created by sound field

loudspeakers... finite internal impedances

f_s ~ force created by sound field



simple model of a l/s diaphragm

force applied by the voice coil

EOM $\quad f - f_s = m\frac{\partial^2 \xi}{\partial t^2} - R_m\frac{\partial \xi}{\partial t} - s\xi \qquad \xi$... displacement of the diaphragm.

Assume harmonic motion.... $e^{j\omega t}$

$u = j\omega\xi$ ... velocity.

$$f - f_s = j\omega u + R_m u + \frac{s}{j\omega u}$$

$$= \underbrace{\left[R_m + j\left(\omega m + \frac{s}{\omega}\right)\right]}_{\text{in vacuo impedance of a loudspeaker}} u$$

$$f - f_s = Z_m u \Rightarrow f = Z_m u + f_s \qquad Z_r = \frac{f_s}{u}$$

$$f = Z_m u + Z_r u = u(\underbrace{Z_m}_{\text{mech. impedance}} + \underbrace{Z_r}_{\text{radiation impedance.}})$$

$$u = \frac{f}{(Z_m + Z_r)}$$ f ~ driving force.

loudspeaker response is determined by both
the mech impedance and the radiation
impedance.

Lecture 40

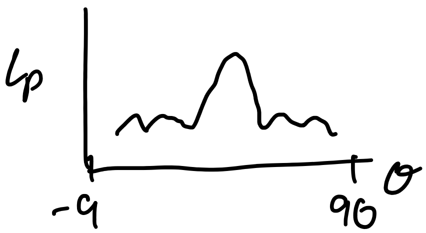
Wednesday, December 1, 2021 1:30 PM

HW hints:

#3

$$\tilde{p}(r,\theta)\, j\frac{\rho_0 c}{2}\, u_0\left(\frac{a}{r}\right) e^{-jkr}\left[2\cdot\frac{J_1(ka\sin\theta)}{ka\sin\theta}\right]$$

$$p^2_{rms} = \frac{p(r,\theta)\cdot p(r,\theta)^*}{2} \qquad L_p = 10\log_{10}(p^2_{rms}/p^2_{ref})$$



matlab(besselj) ★

$j_{11} = 3.83 = ka\sin\theta_1$

$\theta_1 = 90$, a = known, $k = \frac{\omega}{c}$

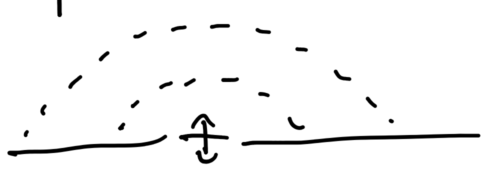
$j_{13} = 90°$ ... 2 side lobes

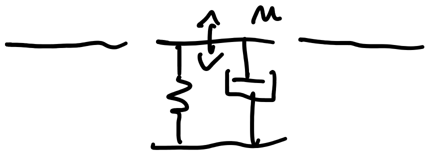
#5 $\tilde{p}(r)_{monopole} = j\rho_0 c \frac{kQ}{4\pi r} e^{-jkr}$

$$\underset{dipole}{\tilde{p}(r,\theta)} = -\rho_0 c k^2 (Qs)\frac{e^{-jkr}}{4\pi r}\cos\theta$$

$$L_{p,reduction} = 10\log_{10}\left(\frac{|p(r)_{mono}|}{|p(r)_{dipole}|}\right)^2$$

Radiation Impedance



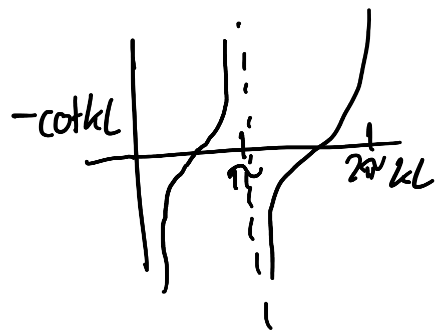


$$f = (Z_r + Z_M)\,u \quad \Rightarrow \quad u = \frac{f}{(Z_r + Z_M)}$$

movable piston

$$Z_r = -j S_p \rho_0 c \cot kL$$

$$u = \frac{f}{Z_r + Z_M}$$



$$kL = \pi$$

$$\frac{2\pi}{\lambda} L = \pi \Rightarrow \frac{L}{\lambda} = \frac{1}{2} \Rightarrow L = \frac{1}{2}\lambda$$ ... loudspeaker stops moving.

Power Radiation by the piston

$$\Pi = \frac{1}{T}\int_0^T \mathrm{Re}\{f_s\}\cdot \mathrm{Re}\{u\}\,dt$$

$$\Pi = \frac{1}{2}\mathrm{Re}\{f_s \cdot u^*\}$$

$$\Pi = \frac{1}{2}\mathrm{Re}\{Z_r \cdot u \cdot u^*\}$$

$$\Pi = \underbrace{\frac{|u|^2}{2}}_{\text{mean square velocity}} \mathrm{Re}\{Z_r\} \sim R_r$$

Circular rigid piston in a baffle $\rightarrow$ free space

$$R_r = \pi a^2 \rho_0 c \left(1 - \frac{2J_1(2ka)}{2ka}\right) \quad \text{Prob. \#4}$$

$$Z_r = R_r + jX_r$$

$$X_r = 2\pi a^2 \rho_0 c \left(\frac{H_1(2ka)}{2ka}\right)$$

Hankel function

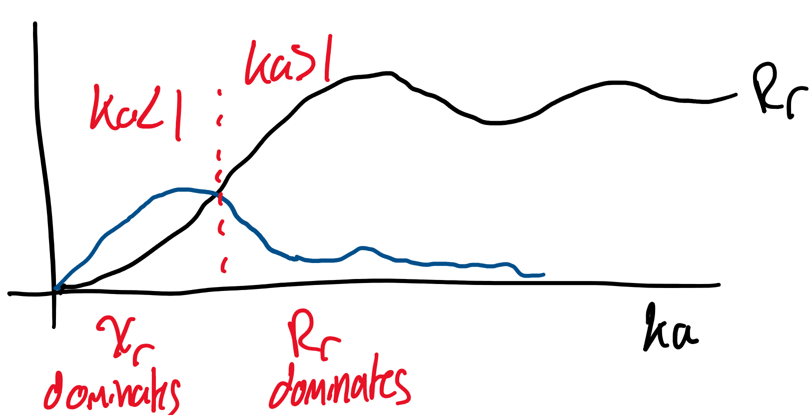
Struve function

when $ka \ll 1$

$$R_r \simeq \frac{\pi a^2}{2} \rho_0 c (ka)^2$$

$$X_r \simeq \pi a^2 \rho_0 c (ka) \frac{8}{3\pi}$$

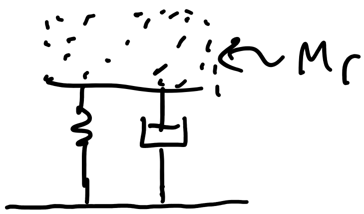
When $ka \ll 1$ X_r dominates - mass like impedance



χ_r dominates R_r dominates ka

if we assume $\chi_r = \omega M_r$ ~ added mass

$$M_r = \pi a^2 \rho_0 \left(\frac{8a}{3\pi}\right) \text{ ... effective mass}$$



$$\omega_0 = \sqrt{\frac{s}{M + M_r}}$$

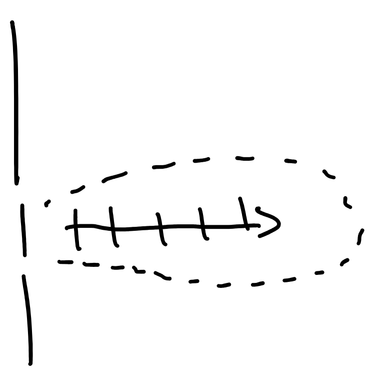
fluid loading reduces a loud speaker's natural frequency...

$ka \gg 1$

$R_r \to \pi a^2 \rho_0 c$

$\chi_r \to 0$

$$\Pi = \frac{1}{2} \rho_0 c \pi a^2 |u_0|^2$$



plane wave like radiation at high frequencies

Book ... 7.1, 7.2, 7.4, 7.5, 7.10

① Compact Sources $kD \ll 1$

- simple sources
  - monopole – volume change
  - dipole – point force
  - quadrupole – point moment

② Extended Sources $kD \cancel{\ll} 1$

- directional characteristics
  - interference
- near field behavior

$ka \ll 1$ ... omnidirectional

$ka \gg 1$ ... highly directional

6.0 Room Acoustics

Wallace Sabine ...

$$\underbrace{T}_{\text{reverb time}} \propto \frac{\overbrace{V}^{\sim \text{space volume}}}{\underbrace{A}_{\sim \text{absorption area}}}$$

Lecture 41

Friday, December 3, 2021 1:29 PM

Room Acoustics -

Introduction... $T \propto \frac{V}{A}$

Sufficient Reflections to create a diffuse Sound field

- Sound equally likely to arrive from any direction

- instantaneously uniform energy density

- total absorption cannot be too large
- does not work well at low frequencies

Reverberation

- sensation created by the superposition of many reflections
- long rev times - direct sound is reinforced by many closely spaced reflections
    - level increase

- rev time is too long... intelligibility is

- rev time is too long... intelligibility is reduced...
  - speech masks itself

Short rev times - clarity, low sound level
long rev times - masking, high sound level

6.2 Energy Density

- sound propogates past a point in space

- fluid moves locally
- expands and contracts

Fluid element - kinetic
of fixed mass - Potential

i) kinetic energy $E_k = \frac{1}{2} \rho_0 u^2 V_0$ ~ volume
particle speed

ii) Potential energy

- associated with a change in volume

$$E_p = -\int_{V_0}^{V} p\,dV = \frac{1}{2} \frac{p^2}{\rho_0 c^2} V_0$$

iii) total energy... $E = E_k + E_p$

$$= \frac{1}{2} \rho_0 \left(u^2 + \frac{p^2}{(\rho_0 c)^2} \right) V_0$$

u = instenteneous real particle speed...
p = instenteneous real sound pressure...

iv) energ density

$$\mathcal{E}_i = \frac{E}{V_0}$$ ... instenteneous energy density

Time-averaged energy density

$$\mathcal{E} = \frac{1}{T} \int_0^T \mathcal{E}_i \, dt$$

performed over a short time..
over 1 or a few cycles of sound

$\mathcal{E}$ itself is a function of time
temporal fluctuations in $\mathcal{E}$ are much slower than $\mathcal{E}_i$

for plane harmonic waves...

$$p = \pm \rho_0 c u$$

$$\mathcal{E}_i = \rho_0 u^2 = \frac{p^2}{\ldots}$$

$$\mathcal{E}_i = \rho_0 u^2 = \frac{p^2}{\rho_0 c^2}$$

when the sound field is harmonic

$$p = \hat{p} e^{j\omega t} \qquad u = \tilde{u} e^{j\omega t}$$

Time averaged energy density

$$\mathcal{E} = \frac{1}{2} \frac{|p|^2}{\rho_0 c^2} = \frac{1}{2} \rho_0 |u|^2$$

$$\mathcal{E} = \frac{p_{rms}^2}{\rho_0 c^2}$$

- these relations are true for plane waves
- assume sound field is diffuse

plane waves arriving from all directions simultaneously

- each plane wave is randomly phased
- when randomly phased signals are added

$$p_{Tot}^2 = p_1^2 + p_2^2 + p_3^2 + \dots$$

because cross terms are zero due to a lack of correlation

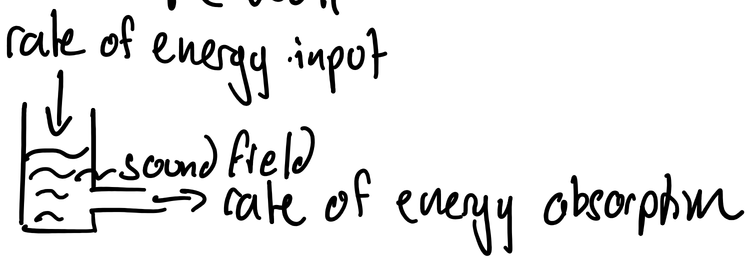
$$\mathcal{E}_t = \frac{1}{2} \frac{p_t^2}{\rho_0 c^2} = p_{rms,tot}^2 \cdot \frac{1}{\rho_0 c^2}$$

when the sound field consists of a superposition
of randomly phased plane waves...

6.3 energy model for Sound in a room

Sound Source is turned on

- due to reflections...
  energy density in the space increases
  until the rate of energy absorption
  at the walls
  equals
  the rate of energy absorption by
  the wall



Π... rate of energy delivery to the space

= Sound power of the source

Rate at which energy delivered ... Π =

Rate at which energy is stored in
the space. + Rate at which

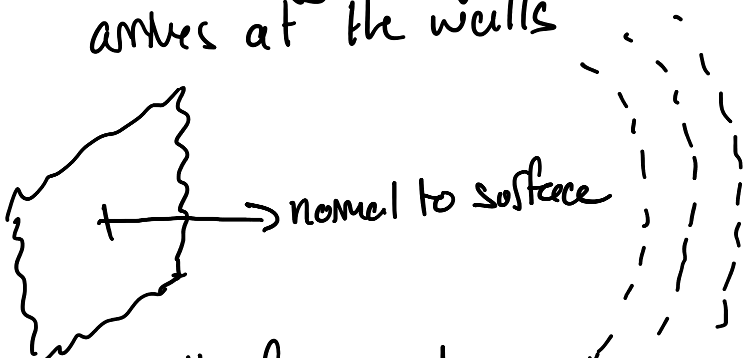
the space. + Rate at which energy is lost by absorption.

$$\Pi = \underbrace{V}_{\text{room Volume}} \frac{\partial \mathcal{E}}{\partial t} + \text{rate of energy loss}$$

$\mathcal{E}$ – space and short time averaged energy density in the room

last term = rate at which energy arrives at the surfaces * absorption at the surfaces

Relate energy density to the rate at which energy arrives at the walls



ΔS - small surface segment

$$\frac{\Delta E}{\Delta t} = \frac{\mathcal{E} c \Delta S}{4}$$ rate at which energy falls on the area ΔS

÷ΔS to get per unit area

$$\frac{\partial E}{\cdot} = \frac{\mathcal{E} c}{\cdot}$$

$$\frac{\partial E}{\partial t} = \frac{\mathcal{E} c}{4}$$

Rate at which energy is absorbed

$$\frac{\mathcal{E} c}{4} A$$

$$\Pi = \frac{V \partial \mathcal{E}}{\partial t} + \frac{A c}{4} \mathcal{E}$$

ODE governing space and short-time averaged energy density in a space

Lecture 42

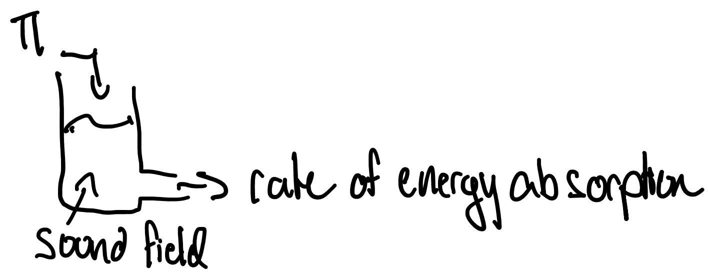
Monday, December 6, 2021 1:27 PM

Room Acoustics...

Energy Acoustics – high frequencies
reverberant = diffuse

Energy Densities - Space and Short-time averaged

$$\varepsilon_t = \frac{1}{2}\frac{|P_t|^2}{\rho_0 c^2} = P_{rms}^2 \cdot \frac{1}{\rho_0 c^2}$$



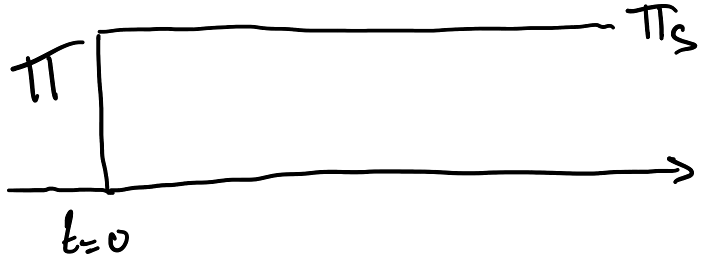
$$\Pi = V\frac{d\varepsilon}{dt} + \frac{\varepsilon c}{4}A$$

absorption area (metric sabins) [m^2]

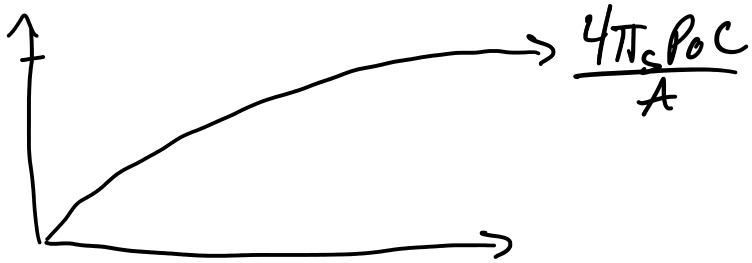
Steady-State level

when $\frac{d\varepsilon}{dt} = 0$ ... $\Pi = \frac{Ac}{4}\varepsilon$

Solve for time variation of the sound field by using LaPlace Transforms



$$p_r^2 = \frac{4\,\Pi_s \rho_0 c}{A}\left(1 - e^{-t/\tau_e}\right)$$



$\tau_e = \frac{4V}{AC}$ ~ volume of the space

< absorption area

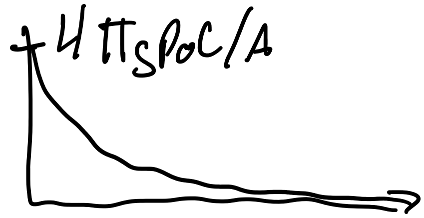
= time constant governing rate of growth or decay

Small A, large V -> large τ_e ... slow transitions
long reverberation time

Source is turned off

$$p_r^2 = \frac{4\pi_s \rho_0 c}{A}\, e^{-t/\tau_e}$$

A



Development assumes sound field is diffuse at all times

we cannot use this until a certain number of reflections has occured.

6.4 Reverberation Time

$$P_r^2(t) = P_r^2(0)\, e^{-t/\tau_e} \qquad \tau_e = \frac{4V}{Ac}$$

$\quad$ ↳ Steady State Value

$$10\log_{10}\left(\frac{P_r^2(t)}{P_{ref}^2}\right) = 10\log_{10}\left(\frac{P_r^2\, 0}{P_{ref}^2}\right) + 10\log_{10}\left(e^{-t_0/\tau_e}\right)$$

$$L_p(0) - L_p(t) = \Delta SPL = -10\log_{10}\left(e^{-t/\tau_e}\right)$$

Change in SPL from $t=0$ to t $= 4.34\left(\frac{t}{\tau_e}\right)$

$$t = \frac{\Delta SPL\, \tau_e}{4.34}$$ time taken to decay by ΔSPL

$\overline{\quad 4{,}34}$

Rev time - time taken for sound level to drop
by 60 dB after the source is turned
off

$$\overset{\text{rev time}}{T} = \frac{60}{4{,}34} \cdot \frac{4V}{AC} \Rightarrow \boxed{0.161\frac{V}{A} = T}$$

applies in proportionate spaces.

Rev time $\propto$ to Volume
$\propto$ to A

A ... absorption area

$A = \underbrace{\overline{a}}_{\text{average Sabine absorptivity (random incidence absorption coefficient)}} S \sim$ surface area of the room interior

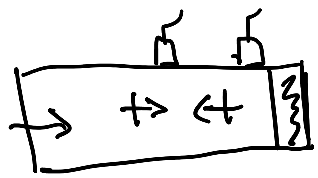
$$A = \sum_i A_i = \sum (S_i \cdot a_i)$$

$$\overline{a} = \frac{\sum S_i a_i}{S} = \text{average abs. coeff.}$$

$A = \bar{\alpha} \cdot S$

a_i's are frequency dependent

a_i's are determined using standardized tests



α_n - absorption coefficient at normal incidence

can convert to random incidence α value using the Paris formula assuming a local reaction

Test in a Reverberation Room -

i) Rev time of Empty Space
(to measure absorption of bare walls)

$$T_0 = 0.161 \frac{V}{S a_0}$$

Lecture 43

Wednesday, December 8, 2021 1:29 PM

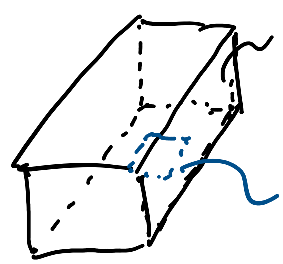
2015 format

$$\Pi = V\frac{\partial \mathcal{E}}{\partial t} + \frac{Ac}{4}\mathcal{E}$$

$$p_r^2 \propto e^{-t/\tau_e} \qquad \tau_e = \frac{4V}{Ac}$$

$$T = 0.161\,\frac{V}{A} \qquad A = \bar{\alpha} S$$

Measure absorption in a rev room



i) measure T in empty chamber

$$T_0 = 0.161 \cdot \frac{V}{S\cdot\bar{\alpha}_0}$$ ~ avg. abs. coeff of bare walls

ii) measure T with sample in place

sample area, S_e

Total absorption:

$$(S-S_e)\bar{\alpha}_0 + S_e\bar{\alpha}_e$$

$$T_e = 0.161\,\frac{V}{(S-S_e)\bar{\alpha}_0 + S_e\bar{\alpha}_e}$$

$$a_e = \frac{0.161V}{S_e T_e} - \frac{(S - S_e)\bar{\alpha}_0}{S_e}$$

$$\bar{a}_e = \bar{\alpha}_0 + 0.161\frac{V}{S_e}\left(\frac{1}{T_e} - \frac{1}{T_0}\right)$$

Sabine (random incident) absorption coefficient

$\bar{a}_e$ tabulated for many materials
normally functions of frequency

Use $\bar{a}_e$'s in combo with source sound power to estimate steady-state sound levels in a space

- developed rev-time formula
  - ignored air absorption

As a plane wave travels in air,

$$p \propto P e^{-\alpha x}$$

exponential decay with distance

$$p_r^2(t) \propto p_r^2(0)\, e^{-2\alpha c t} \qquad m = 2\alpha$$

$$p_r^2(t) \ \ p_r^2(0)\, e^{-\left[\frac{A}{4V} + m\right]ct}$$

$$P_r^2(t) = P_r^2(0)\, e^{-\left[\frac{A}{4V} + m\right]ct}$$

Rev time: $T = \dfrac{0.161V}{A + 4mV}$

$20 \le h \le 70$ h... humidity

$$m = 5.5 \times 10^{-4} \left(\frac{50}{h}\right)\left(\frac{f}{1000}\right)^{1.7}$$

molecular absorption

air absorption can be neglected at low fq's

can control rev time at 4kHz and above

Rev time formula

- low absorption, $\bar{a}$ is small, proportionate space
-

6.5 Absorbing Matrials

Acoustic Environment can be controlled by adding or removing absorbing elements

4 major types of absorbants:

1) porous materials

- acoustic tiles, glass fiber, foams, carpets
- good at high freq

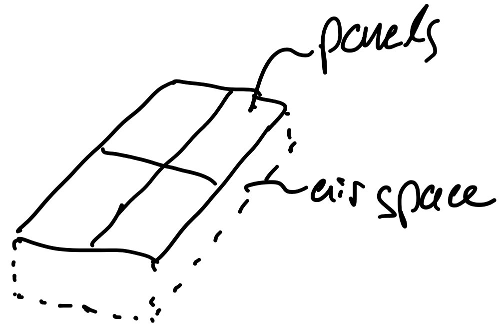
$$\text{depth} > \frac{\lambda}{10}$$

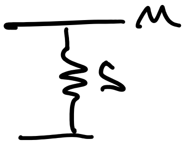
1" of good glass fiber absorbs well at 1kHz and above

Lecture 44

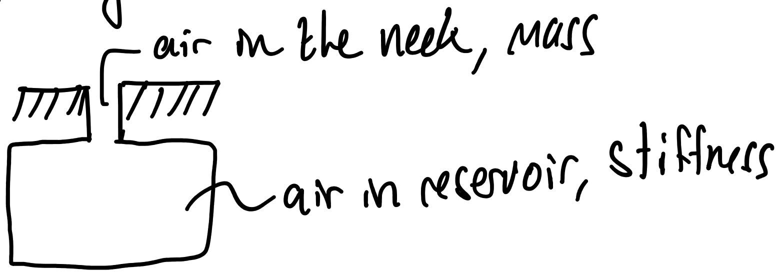
Friday, December 10, 2021 1:26 PM

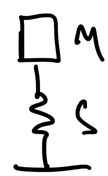
ii) Panels





iii) Cavity Absorbers (Helmholtz Resonator)





arrays of resonators of different sizes to cover a range of frequencies

iv) people and furniture

- 1 person $\simeq$ 1 m^2 of absorption at 1 kHz
- machines also absorb sound

$$A = \sum_i s_i a_i + \sum A_i$$ ~ individual elements {machine or people

Direct + Reverberant Sound fields

total = sum of direct and reverberant components

Direct MSP - assume omnidirectional monopole

$$P^2_{d_{rms}} = \rho_0 \Pi_s / 4\pi r^2$$

source power

Reverberant MSP -

$$P^2_{r_{rms}} = \rho_0 c \, \Pi_s \, 4/A$$

$$(P^2_{rms})_{total} = \rho_0 c \, \Pi_s \left(\underbrace{\frac{1}{4\pi r^2}}_{\text{direct}} + \underbrace{\frac{4}{A}}_{\text{Reverberant}} \right)$$

assuming incoherent addition

r = distance from the source to receiver

ratio of the reverberant to direct MSP

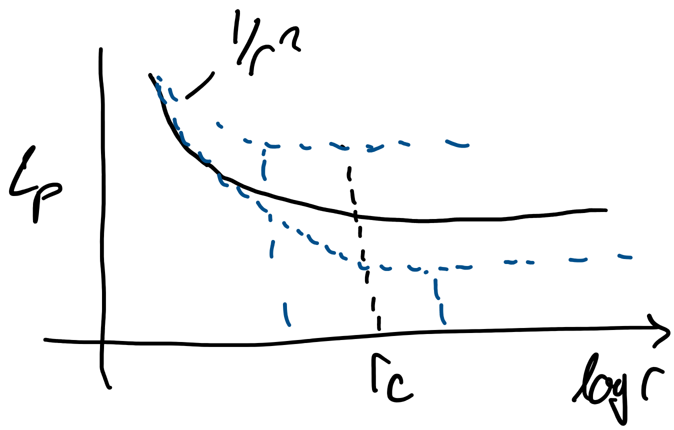
is $\dfrac{16\pi r^2}{A}$

critical radius, r_c when direct and reverberant levels are the same

$$\frac{16\pi r_c^2}{A} = 1 \Rightarrow r_c^2 = \frac{A}{16\pi}$$

$$\frac{16\pi r_c^2}{A} = 1 \Rightarrow r_c^2 = \frac{A}{16\pi}$$

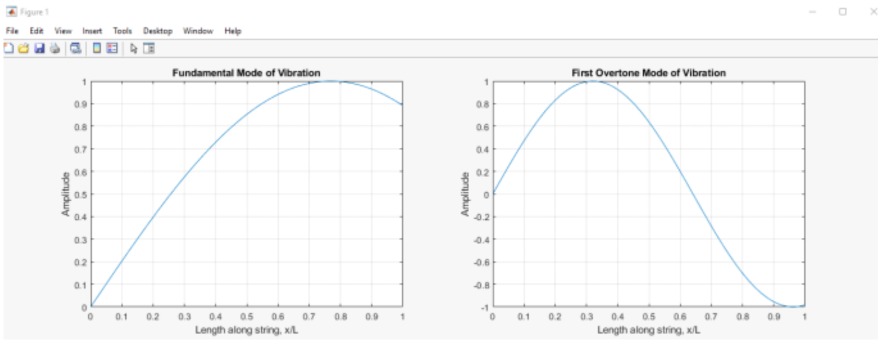
r_c - transition from direct to reverberant

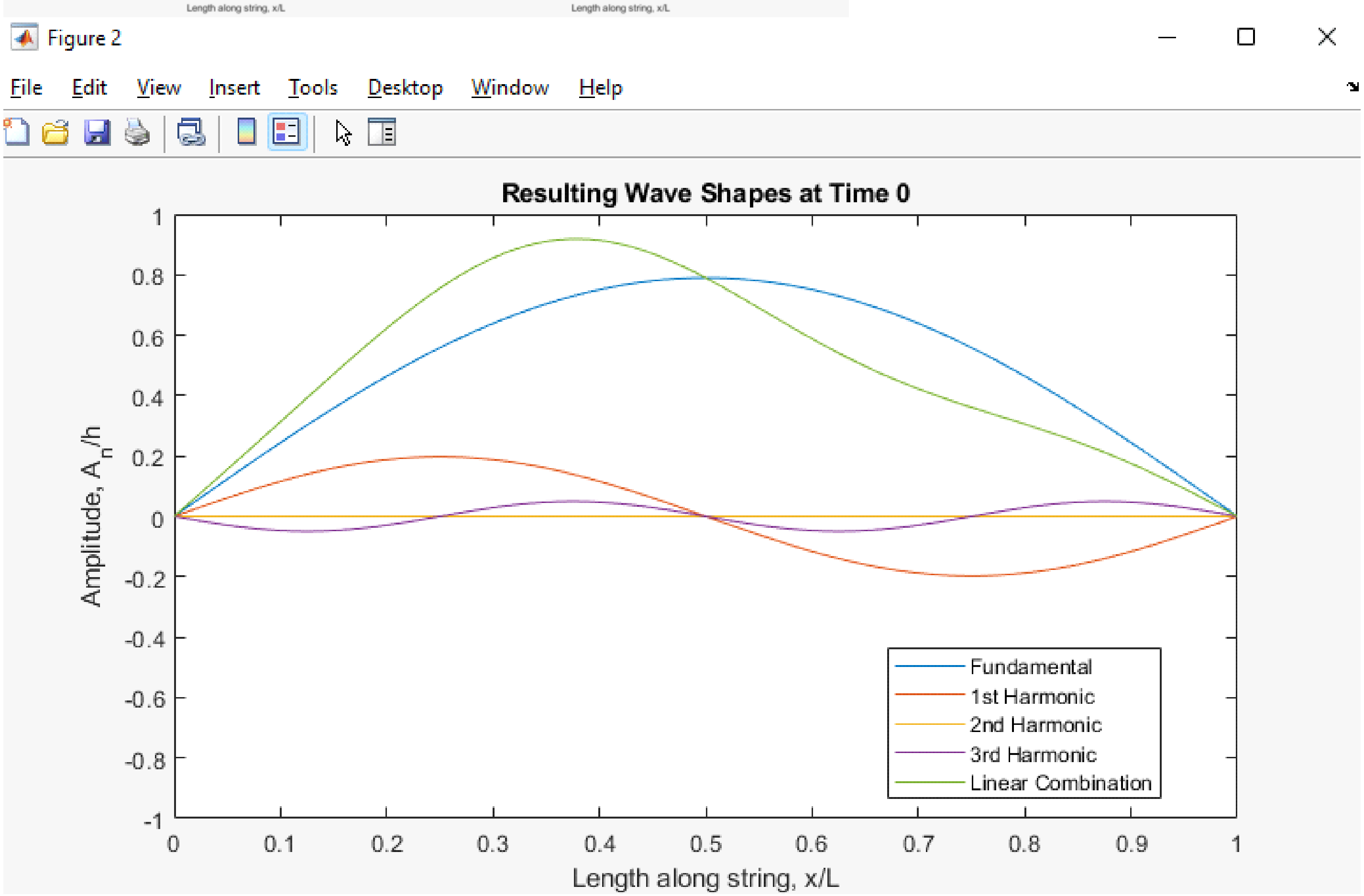


- if you're in the reverb field, adding absorption helps
- if in direct field, use barriers/enclosures

pics

Thursday, October 7, 2021 12:08 PM





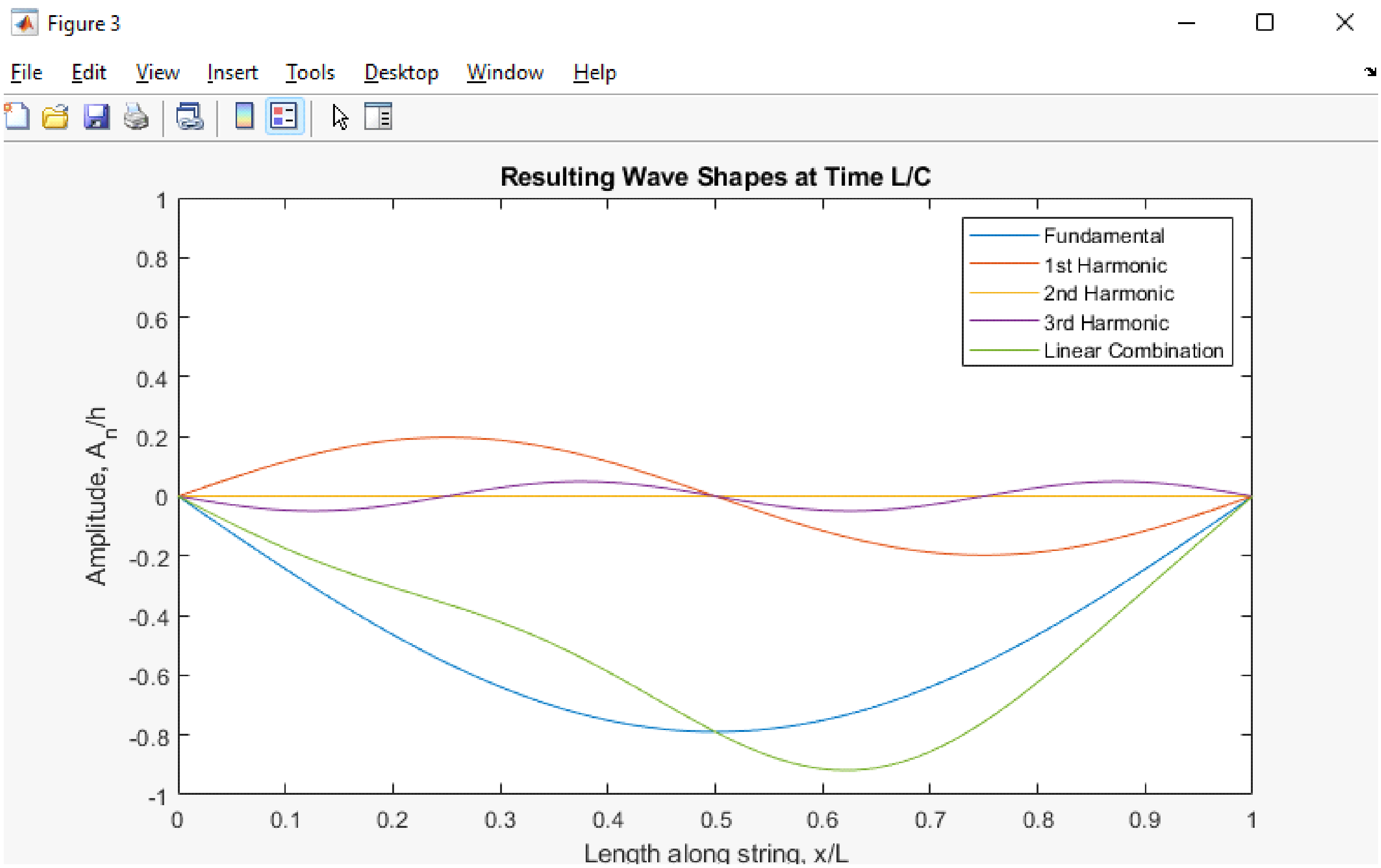
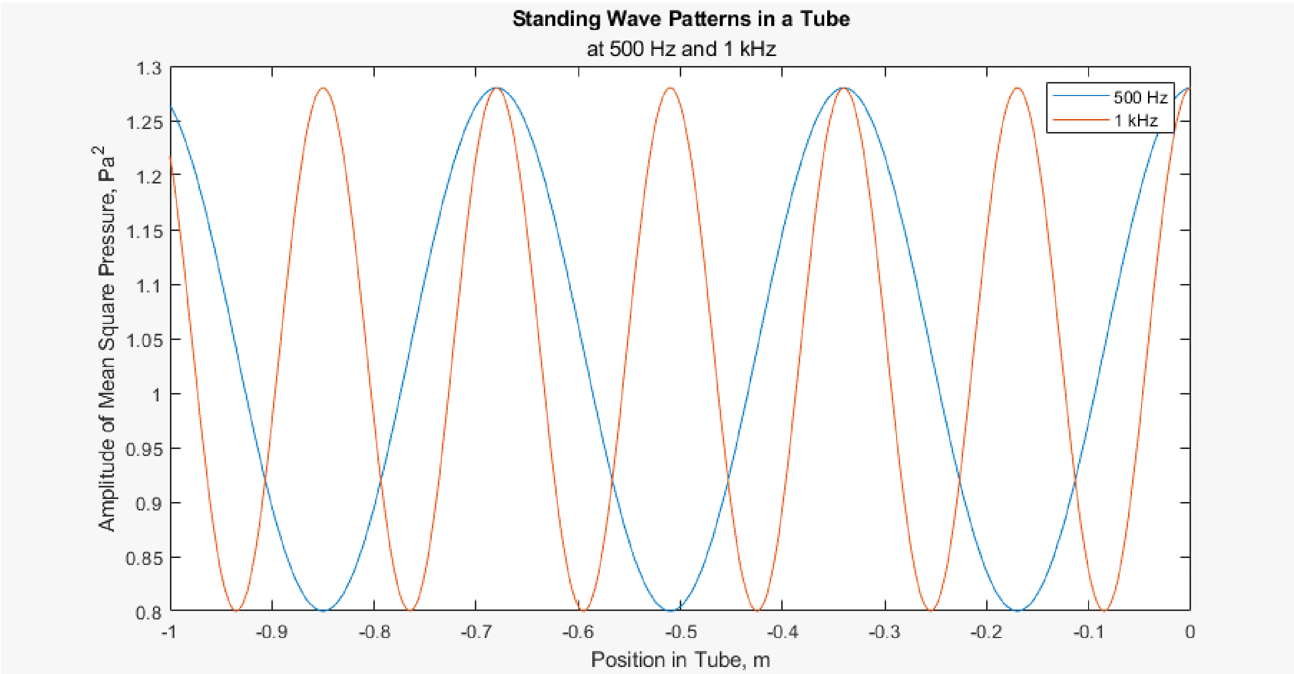
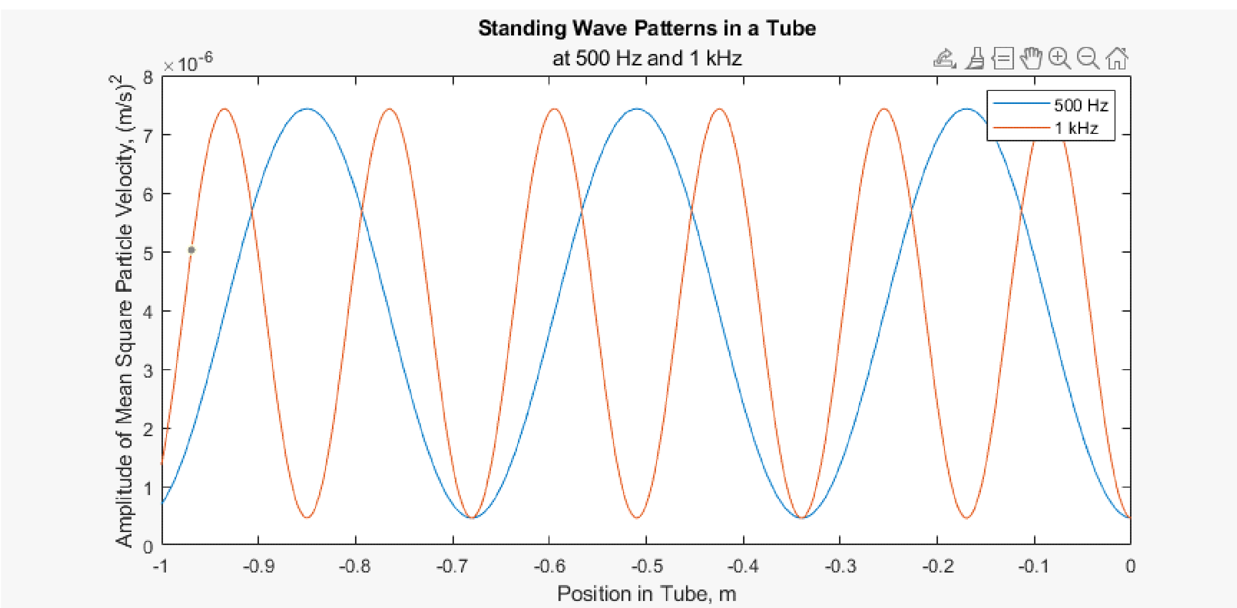


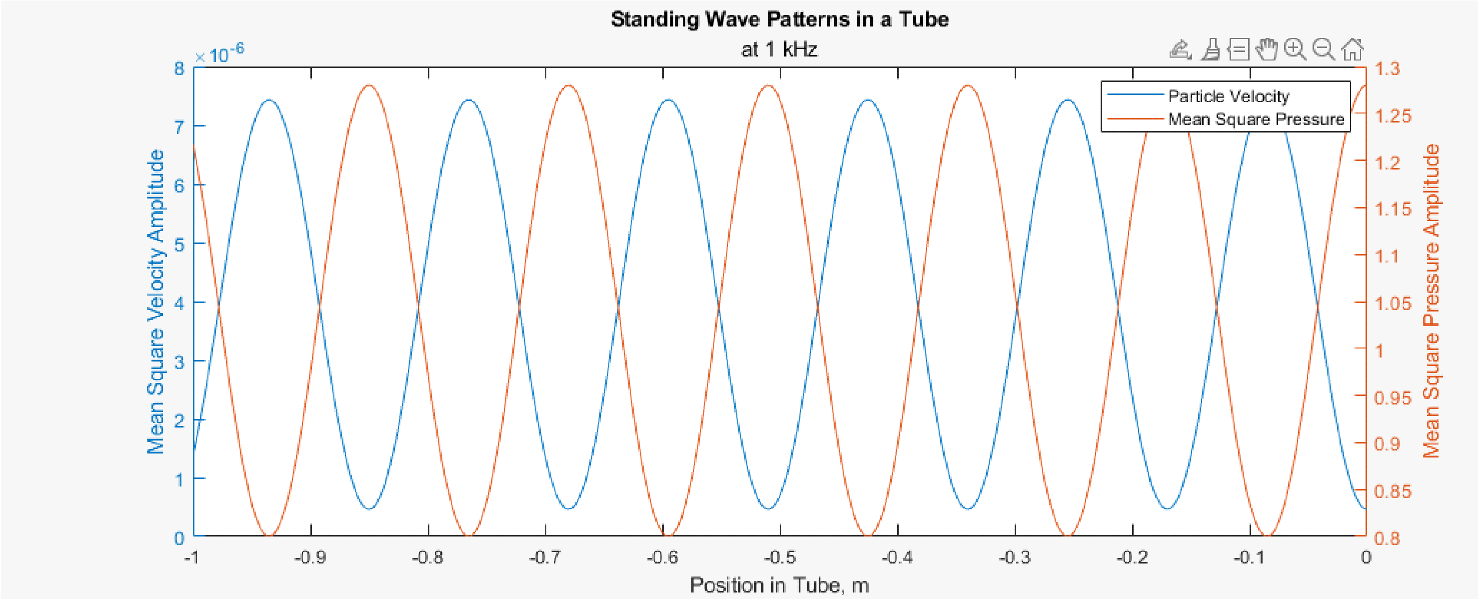
Figure 3
File Edit View Insert Tools Desktop Window Help
Resulting Wave Shapes at Time L/C
Fundamental
1st Harmonic
2nd Harmonic
3rd Harmonic
Linear Combination
Amplitude, A_n/h
Length along string, x/L
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Ps3

Monday, October 18, 2021 9:23 PM



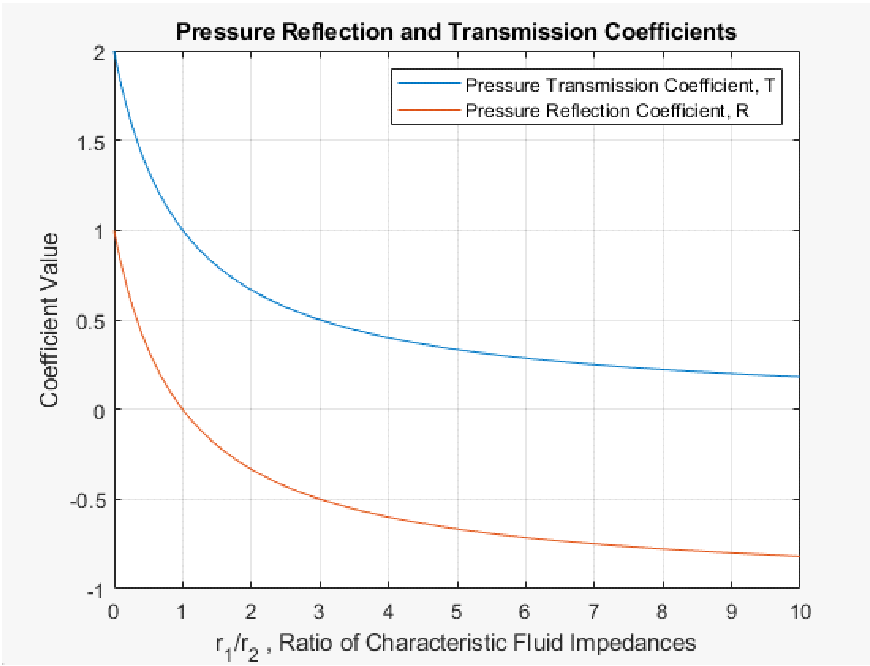


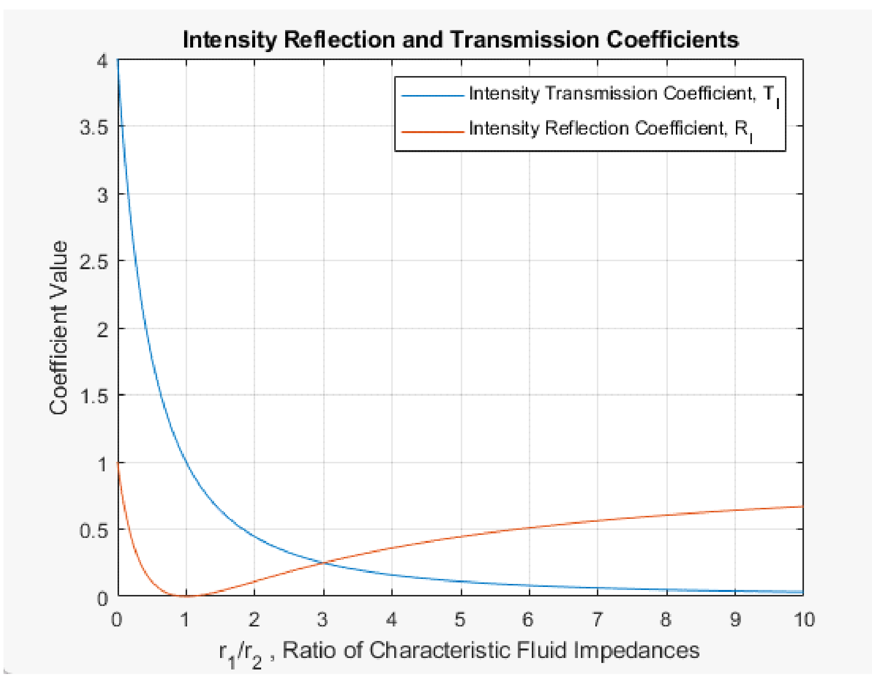


Standing Wave Patterns in a Tube
at 1 kHz
Particle Velocity
Mean Square Pressure
Mean Square Velocity Amplitude
Mean Square Pressure Amplitude
Position in Tube, m

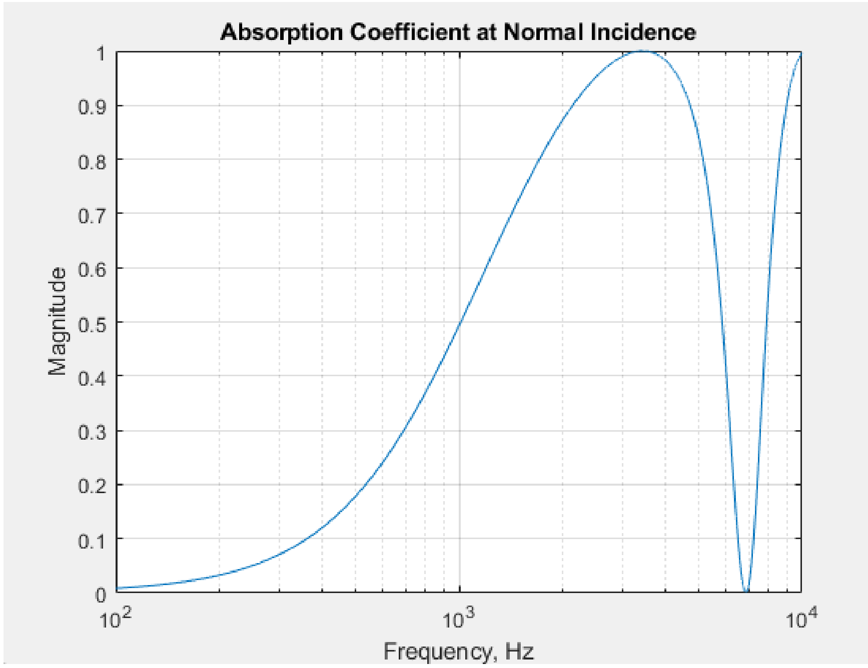
Ps4

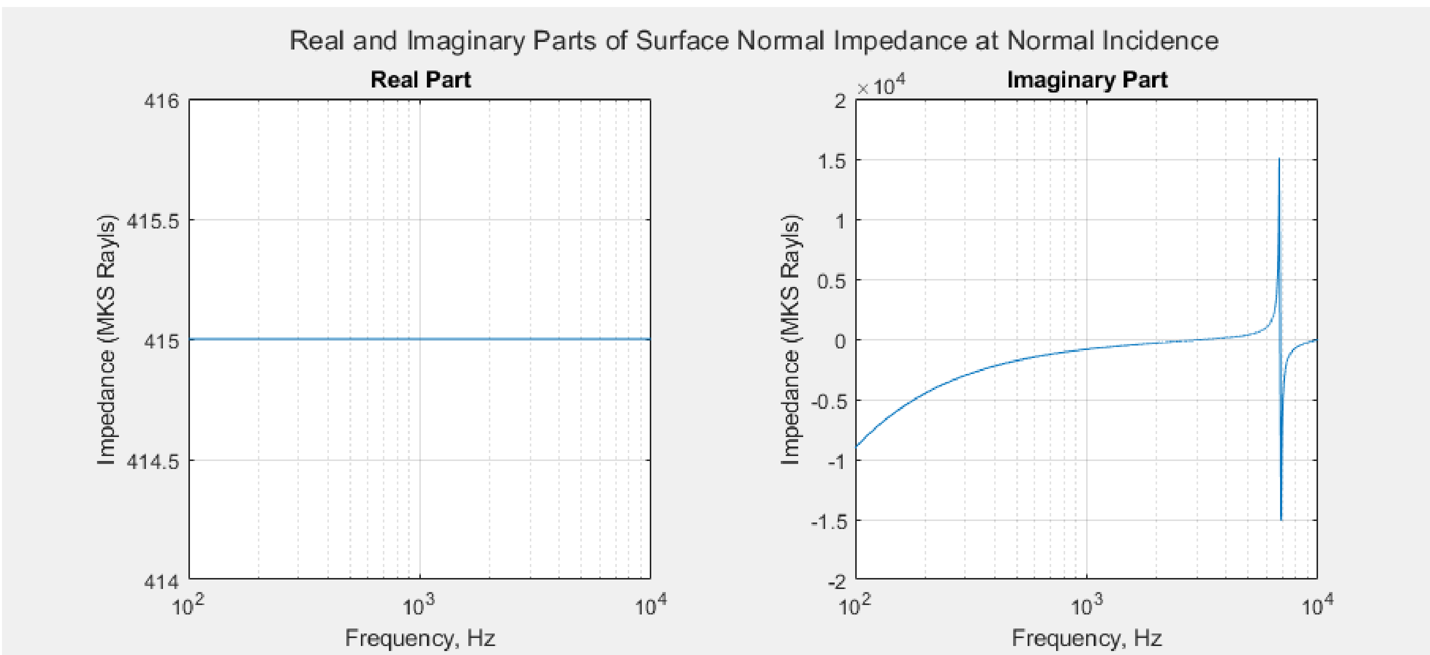
Sunday, November 14, 2021 6:27 PM

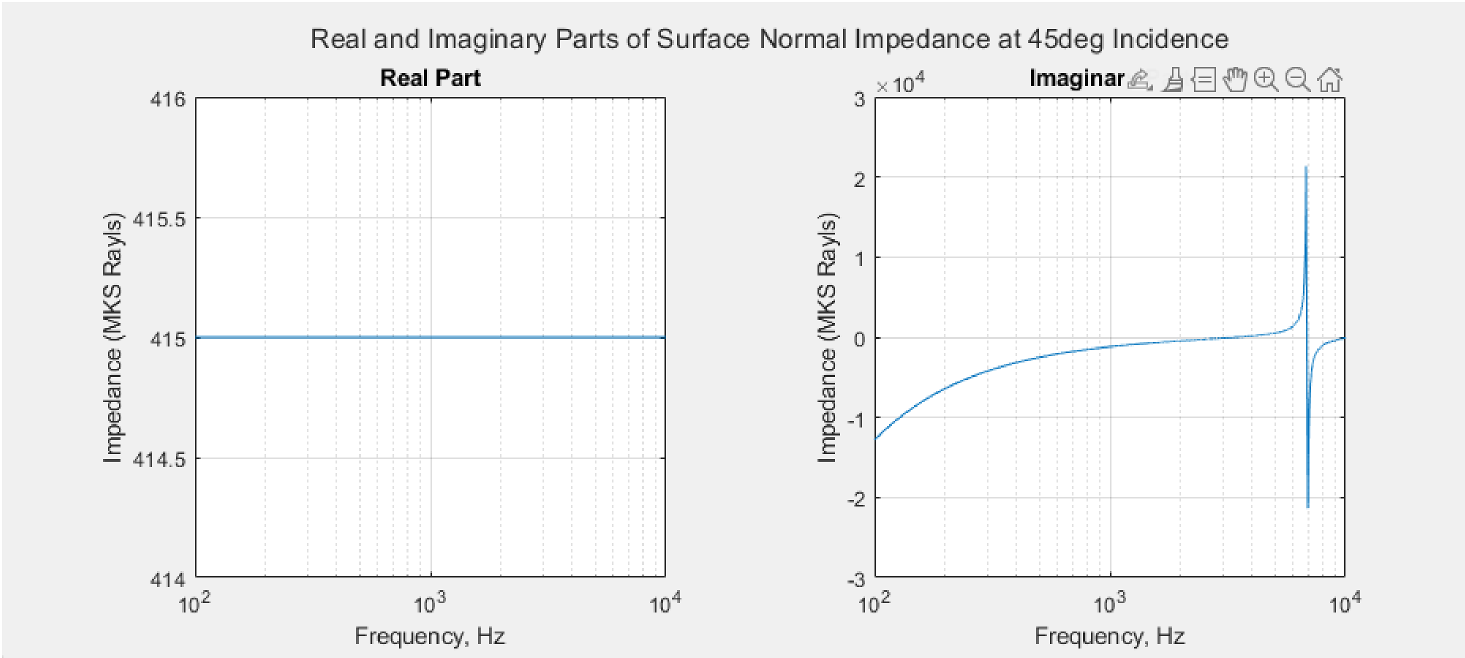


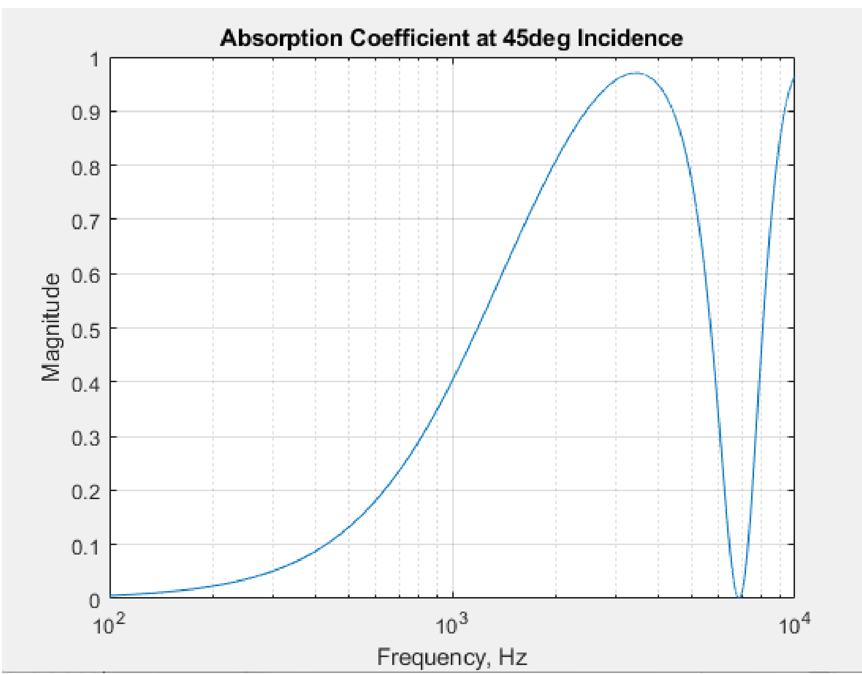


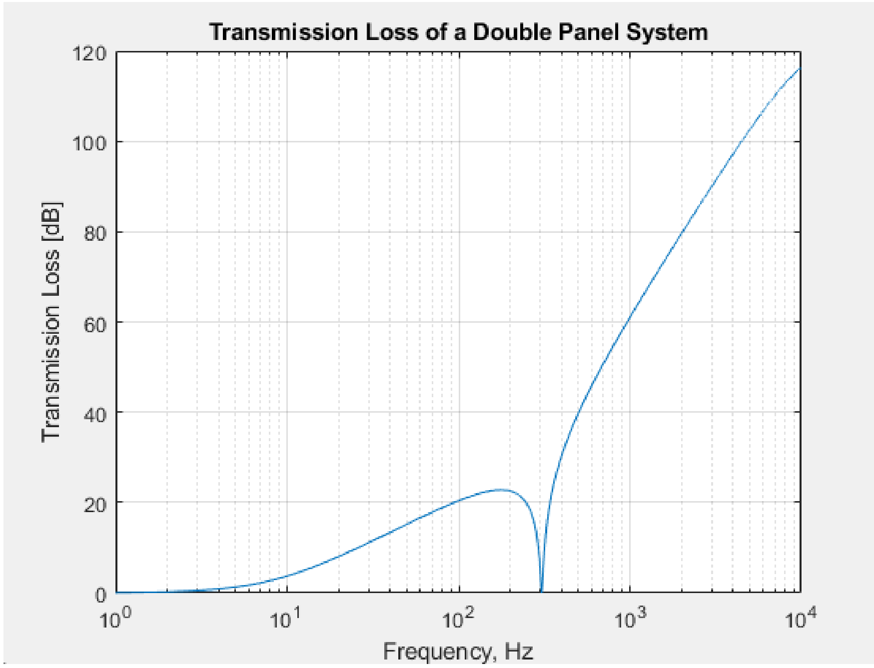
Additional Prob 1

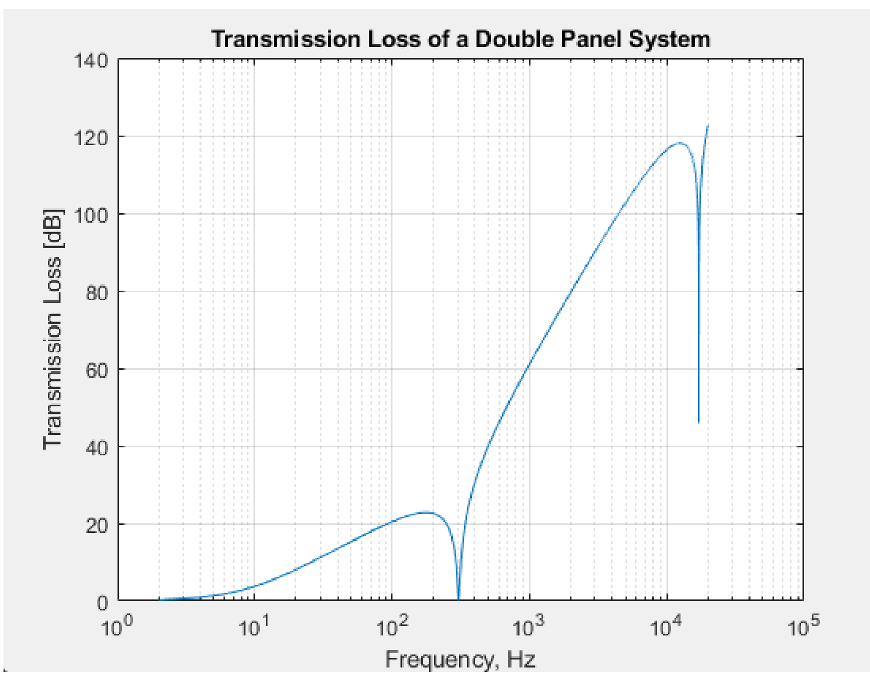

Absorption Coefficient at Normal Incidence
Magnitude
Frequency, Hz

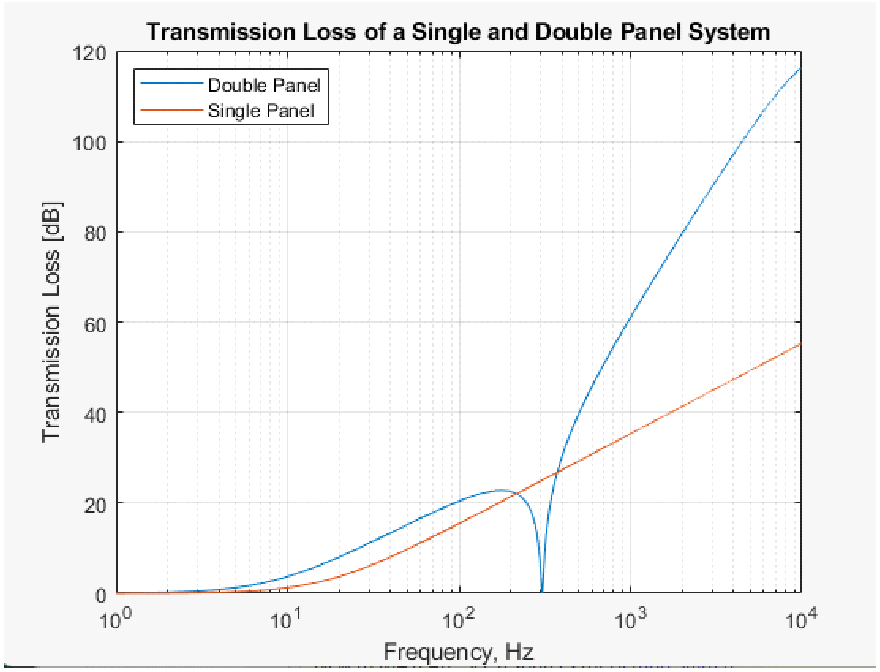

Real and Imaginary Parts of Surface Normal Impedance at Normal Incidence
Real Part
Imaginary Part
Impedance (MKS Rayls)
Frequency, Hz


Real and Imaginary Parts of Surface Normal Impedance at 45deg Incidence
Real Part
Imaginar
Impedance (MKS Rayls)
Frequency, Hz
416
415.5
415
414.5
414
10^2
10^3
10^4
$\times 10^4$
3
2
1
0
-1
-2
-3


Absorption Coefficient at 45deg Incidence
Magnitude
Frequency, Hz
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10^2
10^3
10^4

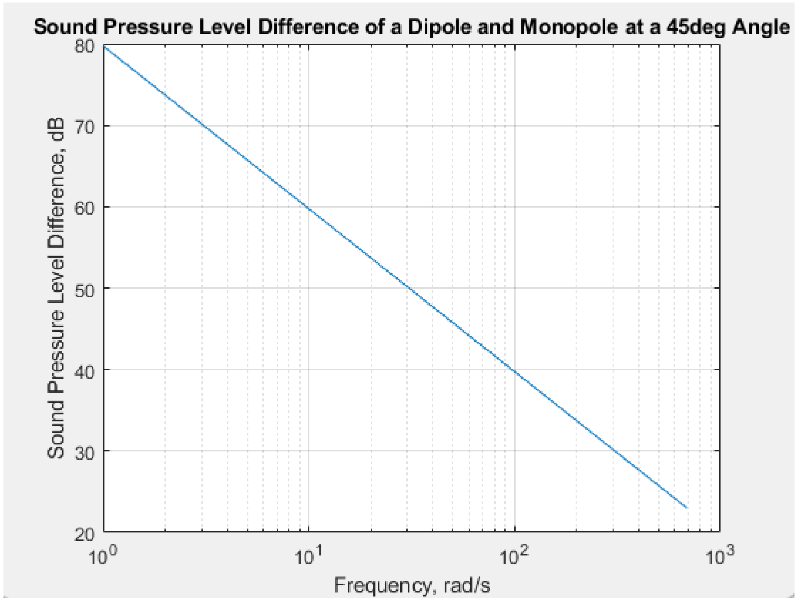

Transmission Loss of a Double Panel System
Transmission Loss [dB]
Frequency, Hz

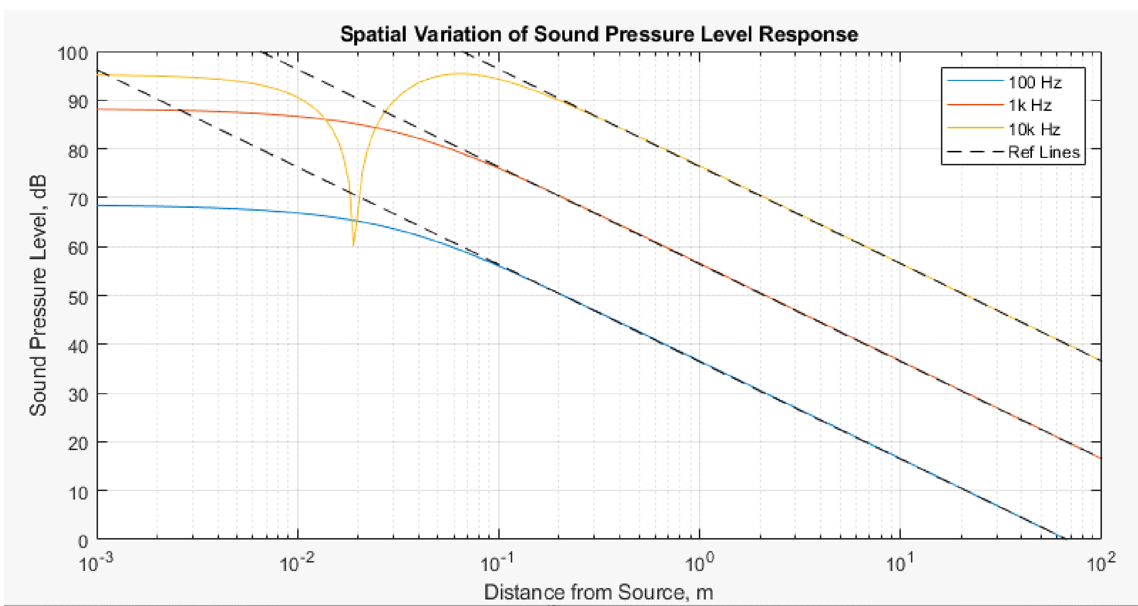

Transmission Loss of a Double Panel System
Transmission Loss [dB]
Frequency, Hz

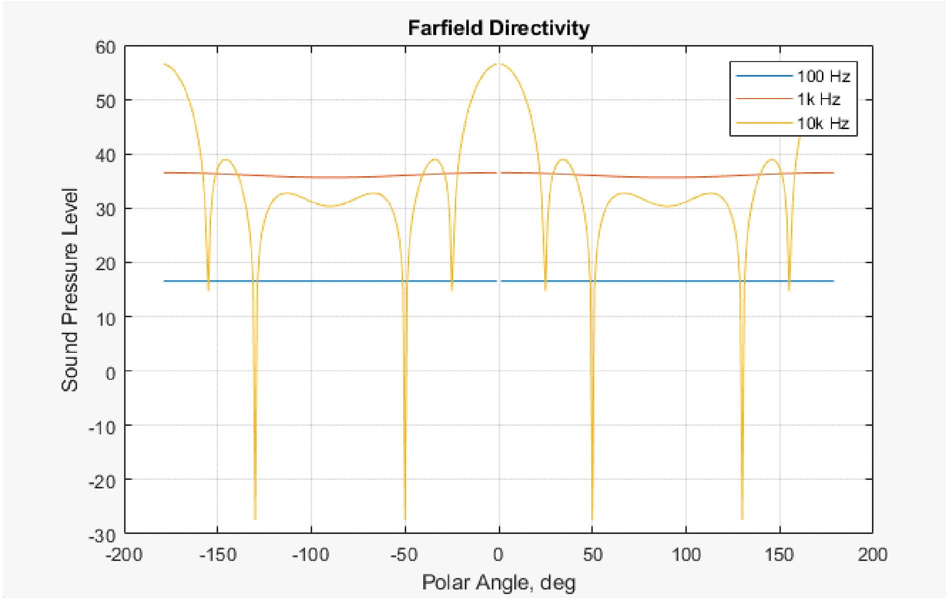

Transmission Loss of a Single and Double Panel System
Double Panel
Single Panel
Transmission Loss [dB]
120
100
80
60
40
20
0
10^0
10^1
10^2
10^3
10^4
Frequency, Hz

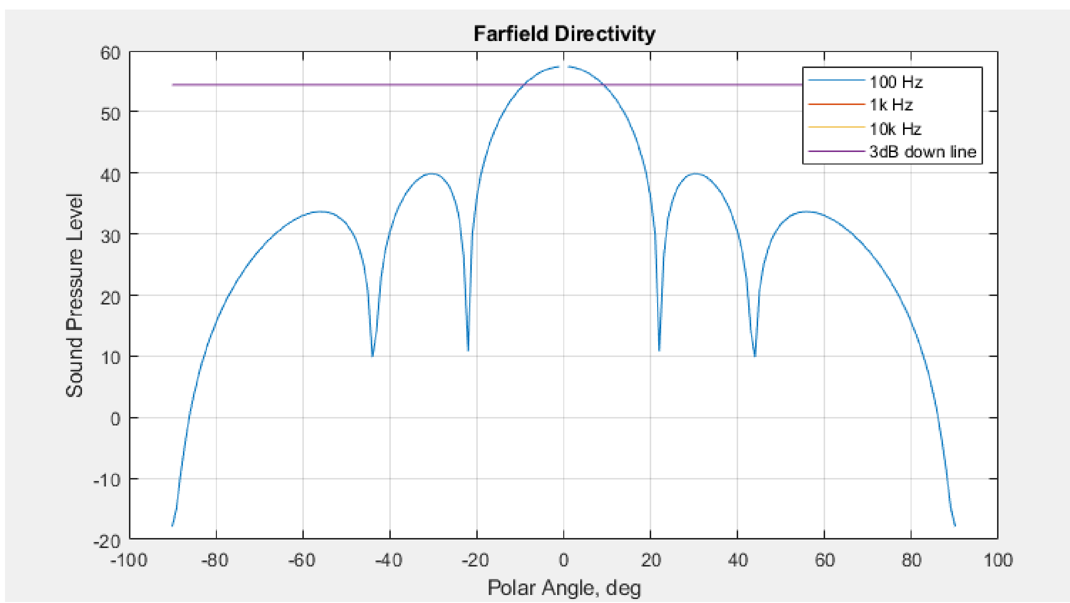
Ps5

Thursday, December 2, 2021 3:18 PM






Farfield Directivity
Sound Pressure Level
Polar Angle, deg
100 Hz
1k Hz
10k Hz
60
50
40
30
20
10
0
-10
-20
-30
-200
-150
-100
-50
0
50
100
150
200


Farfield Directivity
Sound Pressure Level
Polar Angle, deg
100 Hz
1k Hz
10k Hz
3dB down line
60
50
40
30
20
10
0
-10
-20
-100
-80
-60
-40
-20
0
20
40
60
80
100