ME413 HW 01

Benjamin Masters

TOTAL POINTS

99 / 100

QUESTION 1

1 10 / 10

- 0 pts Correct

+ 1 Point adjustment

QUESTION 2

2 9 / 10

- 0 pts Correct

- 1 Point adjustment

QUESTION 3

3 40 / 40

- 0 pts Correct

+ 1 Point adjustment

QUESTION 4

4 20 / 20

- 0 pts Correct

+ 1 Point adjustment

QUESTION 5

5 20 / 20

- 0 pts Correct

+ 1 Point adjustment

HW1
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Question 1 (10 Points)
A harmonic displacement is given by

$$x(t) = 5\times10^{-4}\sin(30t + 2\pi/3) \quad \text{m}$$

Find
(a) the frequency and the period of the motion
(b) the maximum displacement, velocity, and acceleration,
(c) the displacement, velocity, and acceleration at $t = 0$ s.

a) $\omega = 30$ rad/s $\quad f = \frac{\omega}{2\pi} = \frac{30 \text{ rad/s}}{2\pi} = 4.77 \text{ Hz} = f$

$T = \frac{1}{f} = \frac{1}{4.77 \text{ Hz}} = 0.209 \text{ s} = T$

b) $x_{max} = A = 5 \times 10^{-4} \text{ m} = x_{max}$

$\dot{x}(t) = (5\times10^{-4})(30)\cos(30t + 2\pi/3)$ m/s

$\dot{x}_{max} = \omega \cdot A = 5\times10^{-4} \text{ m} \cdot 30 \text{ rad/s} = 0.015 \text{ m/s} = \dot{x}_{max}$

$\ddot{x}(t) = -5\times10^{-4}(30)^2 \sin(30t + 2\pi/3)$ m/s^2

$\ddot{x}_{max} = \omega^2 A = (30 \text{ rad/s})^2 \cdot 5\times10^{-4} \text{ m} = 0.45 \text{ m/s}^2 = \ddot{x}_{max}$

c) $x(0) = 5\times10^{-4}\sin(30(0) + 2\pi/3) \text{ m} = 5\times10^{-4}\sin(2\pi/3)$

$x(0) = 4.33\times10^{-4}$ m

$\dot{x}(0) = (5\times10^{-4})(30)\cos(30(0) + 2\pi/3)$ m/s $= 0.015\cos(2\pi/3)$ m/s

$\dot{x}(0) = -.0075$ m/s

$\ddot{x}(0) = -(5\times10^{-4})(30)^2 \sin(30(0) + 2\pi/3)$ m/s^2

$\ddot{x}(0) = -0.390$ m/s^2

1 **10 / 10**

- **0 pts** Correct

+ **1** **Point adjustment**

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Question 2 (10 points)
For a harmonic, the velocity is given in **m/s** by

$$\dot{x}(t) = 0.15\cos(4t - \pi/6)$$

Find
(a) the frequency and the period of the motion
(b) the maximum displacement, velocity, and acceleration,
(c) the displacement, velocity, and acceleration at $t = 0.3$ s.

a) $\omega = 4\ rad/s \quad f = \frac{\omega}{2\pi} = \frac{4\ rad/s}{2\cdot\pi} = 0.637\ Hz = f$

$T = \frac{1}{f} = \frac{1}{0.637\ Hz} = 1.57\ s = T$

b) $\dot{x}(t) = A\omega\cos(4t - \pi/6) \quad x_{max} = A \quad \omega = 4\ rad/s$

$A\omega = 0.15\ m/s \quad A = \frac{0.15\ m/s}{4\ rad/s} = x_{max} = 0.0375\ m$

$\dot{x}_{max} = A\omega = 0.15\ m/s = \dot{x}_{max}$

$\ddot{x}_{max} = A\omega^2 = 0.15\ m/s \cdot 4\ rad/s = 0.60\ m/s^2 = \ddot{x}_{max}$

c) $\dot{x}(t) = 0.15\cos(4t - \pi/6) \quad x(t) = 0.0375\sin(4t - \pi/6)$

$\ddot{x}(t) = -0.60\sin(4t - \pi/6)$

$x(0) = 0.0375\sin(4(0) - \pi/6) \Rightarrow 0.0375\sin(-\pi/6)$

$x(0) = -.01875\ m$

$\dot{x}(0) = 0.15\cos(4(0) - \pi/6) \quad \dot{x}(0) = 0.15\cos(-\pi/6)$

$\dot{x}(0) = 0.130\ m/s$

$\ddot{x}(0) = -0.60\sin(4(0) - \pi/6) \quad \ddot{x}(0) = -0.60\sin(-\pi/6)$

$\ddot{x}(0) = 0.3\ m/s^2$

2 9 / 10

- **0 pts** Correct

- **1 Point adjustment**

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Question 3 (40 points)

(a) An accelerometer indicates that the acceleration of a body is sinusoidal at a frequency of 40 Hz. If the maximum acceleration is 100 m s^{-2}, find the amplitudes of the displacement and the velocity.

(b) A table has a vertical sinusoidal motion with constant frequency. What is the largest **displacement** amplitude that the table can have, if an object on the table is to remain in contact?

a) $\omega = 40Hz$ $\quad \ddot{x}_{max} = 100 m/s^2$

$$x_{max} = A, \quad \dot{x}_{max} = A\omega, \quad \ddot{x}_{max} = A\omega^2$$

$$100 m/s^2 = A\omega^2 = A(40Hz)^2 \Rightarrow A = \frac{100 m/s^2}{(40Hz)^2} = 0.625$$

$$\boxed{A_x = A = 0.625 m} \qquad A\dot{x} = A\omega = 0.625 m \cdot 40Hz$$

$$\boxed{A_{\dot{x}} = 25 m/s}$$

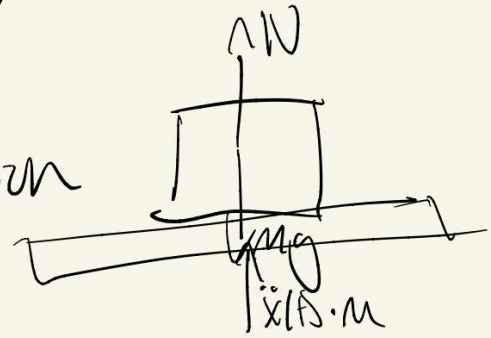
b) $\omega = constant$ $\quad x(t) = A\sin(\omega t)$

$$\dot{x}(t) = A\omega\cos(\omega t)$$

$$\ddot{x}(t) = -A\omega^2\sin(\omega t)$$

$\ddot{x}(t)m = mg$

For max condition



$$F = ma = m\ddot{x} = mg$$

$$\ddot{x}(t) = g \Rightarrow A\omega^2\sin(\omega t) = g$$

$$\boxed{|A_{max}| = \frac{g}{\omega^2}}$$

3 40 / 40

- **0 pts** Correct

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Question 4 (20 points)
Given that

$$x(t) = X\cos(3t+\psi) = A\cos 3t + B\sin 3t \ ,$$

find A, B, X and ψ for each set of the following conditions:

(a) $x(0.1)$ = -7.5 mm, and $x(0.15)$ = 10.0 mm
(b) $x(0)$ = -7.5 mm, and $\dot{x}(0)$ = 400 mm s^{-1}
(c) $x(-0.1)$ = -7.5 mm, and $\ddot{x}(0.1)$ = - 8×10^4 mm s^{-2}

a) $-7.5mm = A\cos(3(.1)) + B\sin(3(.1))$

$-7.5 = 0.955A + 0.296B$

$10mm = A\cos(3(.15)) + B\sin(3(.15))$

$10 = 0.90A + 0.435B \Rightarrow A = 11.1 - .483B$

$-7.5 = 0.955A + 0.296B$

$-7.5 = 0.955(11.1 - .483B) + 0.296B$

$-7.5 = 10.60 - .461B + .296B$

$18.1 = .165B$ $\quad B = 109.7mm \quad A = -41.88mm$

$-7.5mm = X\cos(3(.1) + \varphi) \ldots 10 = X\cos(3(.15) + \varphi)$

$A = X\cos\varphi \quad B = -X\sin\varphi \Rightarrow -109.7 = X\sin\varphi$

$-41.88 = X\cos\varphi$

$X = \frac{-41.88}{\cos\varphi} \quad -109.7 = -41.88 \cdot \frac{\sin\varphi}{\cos\varphi}$

$\tan\varphi = \frac{109.7}{41.88} = 2.619 \Rightarrow \tan^{-1}(2.619) = \varphi = 1.206\ Rad$

$X = \frac{-41.88}{\cos(1.206)} = 117.4mm = X$

b)

(b) $x(0) = -7.5$ mm, and $\dot{x}(0) = 400$ mm s^{-1}

$x(t) = A\cos(3t) + B\sin(3t)$

$\dot{x}(t) = -3A\sin(3t) + 3B\cos(3t)$

$-7.5\text{ mm} = A\cos(0) + B\sin(0)$ $\quad A = -7.5$ mm

$400\text{ mm/s} = -3A\sin(0) + 3B\cos(0)$ $\quad 3B = 400$ m/s $\quad B = 133.3$ mm

$A = X\cos\varphi \quad B = -X\sin\varphi \Rightarrow -7.5 = X\cos\varphi \ldots 133.3 = X\sin\varphi$

$X = \frac{-7.5}{\cos\varphi} \qquad 133.3 = +7.5\frac{\sin\varphi}{\cos\varphi} \Rightarrow \tan\varphi = 17.77$

$\varphi = \tan^{-1}(17.77) = 1.515\text{ Rad} = \varphi$

$X = \frac{-7.5}{\cos(1.515)} = 133.5\text{ mm} = X$

(c) $x(-0.1) = -7.5$ mm, and $\ddot{x}(0.1) = -8\times10^4$ mm s^{-2}

$x(t) = A\cos 3t + B\sin 3t \qquad \dot{x}(t) = -3A\sin 3t + 3B\cos 3t$

$\ddot{x}(t) = -9A\cos(3t) - 9B\sin 3t$

$x(-.1) = -7.5 = A\cos(-.3) + B\sin(-.3)$

$\ddot{x}(.1) = \frac{-8\times10^4}{-9} = -9A\cos.3 - 9B\sin.3 \quad (-9)\ (-9)$

$A\cos.3 + B\sin.3 = 8888.9$ and $A\cos(-.3) + B\sin(-.3) = -7.5$

$A = \frac{8888.9 - B\sin.3}{\cos.3} \Rightarrow \frac{\cos(-.3)(8888.9 - B\sin.3) + B\sin(-.3)}{\cos.3}$

$8888.9 - B\sin.3 + B\sin(-.3) = -7.5 \Rightarrow .591B = 8896.4$

$B = 15053$ mm $\qquad A = \frac{8888.9 - 15053\sin.3}{\cos.3} = 4649\text{ mm} = A$

$x(t) = 4649\cos(\omega t) + 1505\sin(\omega t)$

$= \sqrt{4649^2 + 1505^2}\cos(\omega t + \phi)$

$\boxed{X = 1575\,mm}$

$\tan\phi = \frac{1505}{4649} \Rightarrow \tan^{-1}\left(\frac{1505}{4649}\right) = \boxed{\phi = 1.271\,Rad}$

4 20 / 20

- **0 pts** Correct

+ **1 Point adjustment**

Question 5 (20 points)

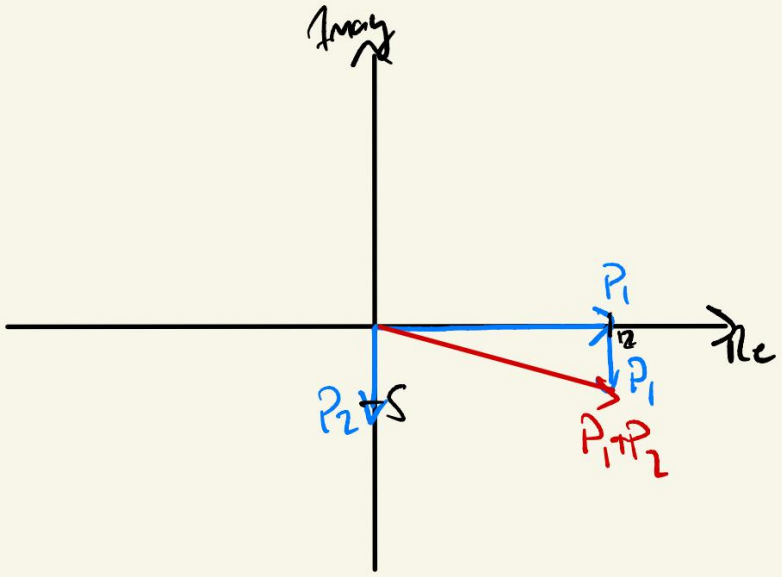
The total acoustic pressure is given as $p = p_1 + p_2$ where

$$p_1 = 12\cos(\omega t) \quad \text{and}$$

$$p_2 = 5\sin(\omega t)$$

(i) Draw p_1 and p_2 in a phasor diagram.
(ii) Sketch the solution for p in the same phasor diagram.
(iii) Determine the magnitdue and phase of p.
(iv) Hence, or otherwise, express the solution for p in terms of a complex number.

(i) (ii)



iii) $\varphi = -\tan^{-1}(5/12) = -0.395 \text{ rad} = \varphi$

$|p| = \sqrt{5^2 + 12^2} = 13 = |p|$

iv) $p = 13\cos(\omega t - 0.395) = 13e^{i(\omega t - 0.395)} = p$

5 20 / 20

- **0 pts** Correct

+ **1 Point adjustment**