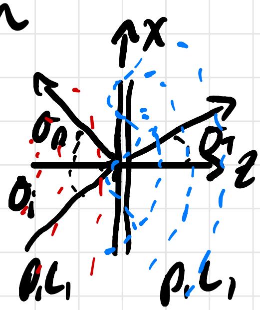
3.0 Sound Transmission through panels

3.1 introduction



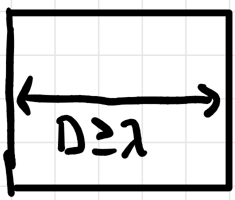
$$\theta_i = \theta_R = \theta_T$$

Sound is radiated from the back of the panel

- room-to-room transmission
- aircraft fuselages
- Submarines
- automotive dash panels

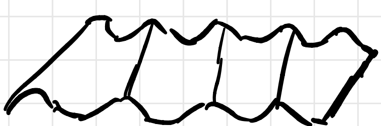
Assumptions

- Infinite panels
  - finite panels can be modeled as infinite if the panel is larger than a wavelength



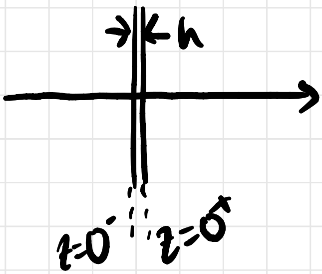
Cross sectional Dimension

- Panels are isotropic
  - same properties in all directions
  - excludes panels with different flexural stiffness in different directions



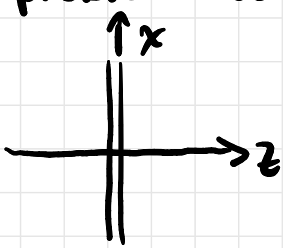
exclude corrugated panels

- Panel thickness is negligible when it comes to applying boundary conditions



$h \ll \lambda$

- Plane Waves
- Because of Plane Waves + infinite panels problem can be treated as 2 dimensional



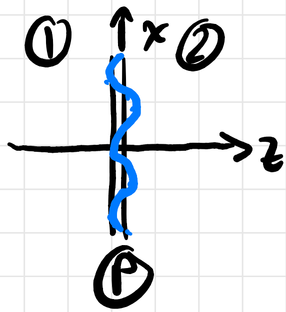
Panel Occupies xy Plane and there is no propagation in the y-direction

3.2 General Problem

Assume General Solutions for the Sound fields on both sides of the panel and for the Motion of the panel

- Identify b.c.'s
- Sub assumed soln's into b.c's to solve for R or T

- panel displaces transversely in the z direction
- propagation factors in the x-direction all must be the same in regions ①, ②, Ⓟ



Snell's Law - b.c.'s must be independent of position

if $\theta_i = \theta_r$ + if media ① + ② are the same

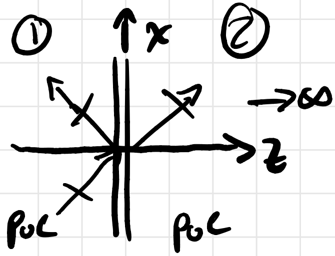
$\theta_t = \theta_i$

(otherwise $\frac{\sin\theta_t}{\sin\theta_i} = \frac{c_2}{c_1}$ when the media are different)

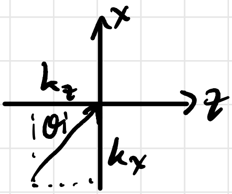
T = plane wave transmission coefficient

$TL = 10 \cdot \log_{10} \frac{1}{|T|^2}$ for same medium on both sides

Pressure field in region ①



$$P_1(x,z,t) = \left[P_i e^{-j(k_x x + k_z z)} + P_r e^{-j(k_x x - k_z z)}\right] e^{j\omega t}$$



$k_z = k\cos\theta_i$

$k_x = k\sin\theta_i$

$k = \frac{\omega}{c} = |\bar{k}|$

$$P_2(x,z,t) = P_t e^{-j(k_x x + k_z z)} e^{j\omega t}$$

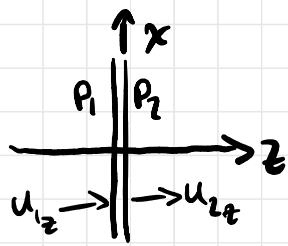
panel displacement in the z-direction:

$$w(x,t) = W(x)\cdot e^{j\omega t} \quad \text{panel displacement}$$
$$= \tilde{W}_p e^{-jKx} e^{j\omega t}$$
$$= \tilde{W}_p e^{-jk_x x} e^{j\omega t}$$

$K = k_x$ since the panel is forced by the incident sound field)

Unknowns: P_i, P_r, P_t, W_p

3 unknowns → 3 b.c.'s



normal particle velocity at the surface in regions 1 & 2 must equal the transverse velocity of the panel itself

(panel is assumed to be incompressible)

- necessary to maintain contact between the fluid and panel
- no condition applies in the x-direction

- fluid can slip in the x-direction
- "ideal" acoustics world
  - no viscosity
  - adiabatic compression

Pressure difference

$P_1 - P_2$ = net force per unit area acting on the panel

= EOM on the panel

Normal Velocities

① $$U_{z_1} = \frac{\cos\theta}{\rho_0 c}\left[P_i e^{-j(k_x x + k_z z)} - P_r e^{-j(k_x x - k_z z)}\right] e^{j\omega t}$$

$$U_{z_2} = \frac{\cos\theta}{\rho_0 c}\left[P_t e^{-j(k_x x + k_z z)}\right] e^{j\omega t}$$

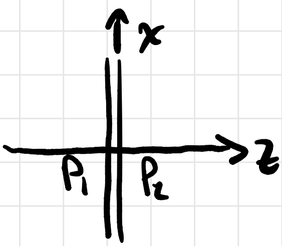
Boundary Conditions:

① $$U_{z_1}\Big|_{z=0^-} = \underbrace{\frac{\partial W}{\partial t}}_{\text{transverse velocity of the panel}} = U_{z_2}\Big|_{z=0^+}$$ ②

(only 2 independent Conditions)

Pressures

$$P_1\Big|_{z=0^-} - P_2\Big|_{z=0^+} = \text{EOM of panel}$$



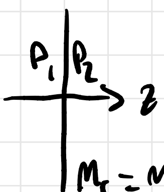
forcing the panel into motion...

depends on the panel

3.3 The Limp Panel (membrane)

- flexural stiffness is assumed negligible
  - applies to flexurally stiff panels below their critical freq
  - assuming no tension applied
- no stiffness → no free wave propagation is possible
- surface of local reaction - transverse displacement of a panel at a point is proportional to the net applied force at that point

$F = m \cdot a$



M_s = mass/area (basis weight)

$$(P_1 - P_2)_{z=0} = M_s a = M_s \frac{\partial^2 W}{\partial t^2}$$

$$W(x,t) = W_p e^{-jk_x x} e^{j\omega t}$$

$$\frac{\partial W}{\partial t} = j\omega W_p e^{-jk_x x} e^{j\omega t}$$

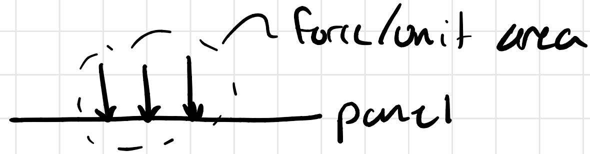
$$\frac{\partial^2 W}{\partial t^2} = -\omega^2 W_p e^{-jk_x x} e^{j\omega t}$$

$P_i + P_r - P_t = -M_s \omega^2 W_p$ .... propagation factors cancel out

$1 + R - T = -M_s \omega^2 W_p'$ ... $W_p' = \frac{W_p}{P_i}$ = Panel displacement coefficient

3

Convenient to express b.c. 3 in terms of a $\underbrace{\text{specific}}_{\text{per unit area}}$ $\underbrace{\text{in-vacuo}}_{\text{in a vacuum}}$ mechanical impedance



$$[\text{force/unit area}] = M_s a = j\omega M_s \cdot V = -\omega^2 M_s W_p$$

$$\frac{\text{Force/unit area}}{V} = j\omega M_s \quad \text{z pure mass-like impedance}$$

$$z_M = \frac{\text{Force/unit area}}{V} \ldots \text{(lower case z)}$$

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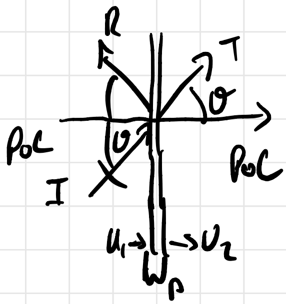


Limp panel
- no flexural stiffness
- no tension

Surface of local reaction

$$(P_1 - P_2)\big|_{z=0} = M_s \cdot a \qquad z_m = j\omega M_s \qquad W_p' = \frac{W_p}{P_I}$$

b.c. (iii) $1 + R - T = j\omega (j\omega M_s) W_p' \Rightarrow j\omega W_p' = \underbrace{V_p'}_{\text{panel velocity coefficient}}$

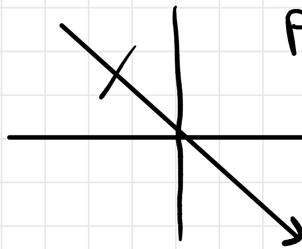
$$1 + R - T = z_m \cdot V_p'$$

i) $u_{1z}\big|_{z=0} = \frac{dW}{dt} \Rightarrow \frac{\cos\theta}{\rho_0 c}[P_I - P_R] e^{-jk_x x} e^{j\omega t} = j\omega W_p' e^{-jk_x x} e^{j\omega t}$

$$1 - R = \frac{\rho_0 c}{\cos\theta} j\omega W_p' = \frac{\rho_0 c}{\cos\theta} V_p' \quad \text{b.c. (I)}$$

ii) $v_{z_t}|_{z=0} = \frac{dW}{dt} \Rightarrow T = \frac{\rho_0 c}{\cos\theta} V_p' \quad$ b.c.(II)

what is $\frac{\rho_0 c}{\cos\theta}$?



$$p = e^{-jk_x x} e^{-jk_z z}$$

$$v_z = \frac{\cos\theta}{\rho_0 c} e^{-jk_x x} e^{-jk_z z}$$

$\frac{p}{v_z} = \frac{\rho_0 c}{\cos\theta}$ ... kind of like a characteristic impedance

* characteristic impedance referred to the θ direction

$Z_{cn} = \frac{Z_c}{\cos\theta}$ ~ characteristic impedance

normal fluid loading

B.C. summary

i) $1 - R = Z_m V_p'$

ii) $T = Z_{cn} V_p'$

iii)
$$\begin{bmatrix} Z_m V_p' & -R & +T & = 1 \\ Z_{cn} V_p' & +R & & = 1 \\ Z_{cn} V_p' & & -T & = 0 \end{bmatrix}$$

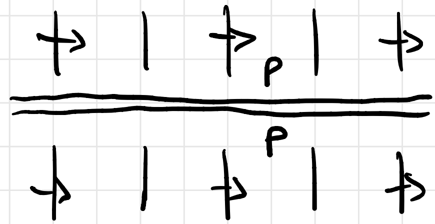
$$\begin{bmatrix} Z_m & -1 & +1 \\ Z_{cn} & 1 & 0 \\ Z_{cn} & 0 & -1 \end{bmatrix} \begin{bmatrix} V_p' \\ R \\ T \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Solve for T ...

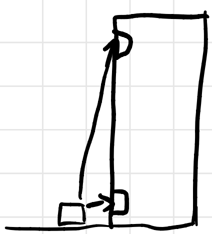
$$T = \frac{2}{2 + \xi_m} \qquad \xi_m = \left(\frac{Z_m}{Z_{cn}}\right)$$

single parameter $\xi_m = \frac{j\omega M_s}{\rho_0 c} \cdot \cos\theta$

$T \to 1$ as $\omega \to 0$ & $M_s \to 0$ & $\theta \to \pi/2$



when $\theta \to \pi/2$



anomalous transmission near grazing

interior traffic noise is independent of height

$$T = \frac{2}{2 + \xi_m} \qquad T \to 0 \text{ as } \xi_m \gg 2$$

$$\simeq T = \frac{2}{\xi_m} = \frac{2\left(\frac{\rho_0 c}{\cos\theta}\right)}{j\omega M_s} \qquad \underbrace{|T| \propto \frac{1}{\omega} \propto \frac{1}{M_s}}_{\text{mass law}}$$

$\xi_m \gg 1$ ... mass controlled region

Same medium on both sides

$$TL \simeq 20 \cdot \log_{10}\left[\frac{\omega M_s}{2(\rho_0 c/\cos\theta)}\right]$$

6 dB increase per doubling of mass/frequency

Examples

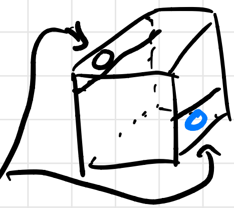
i) Light material in air   Mylar … 0.1 kg/m²

$|T|$ at 1 kHz at 45°

$$z_m = \frac{j\omega M_s}{\rho_0 c}\cos\theta = j\frac{(2\pi)(1000)(0.1)}{415}(.717)$$

$$T = \frac{2}{2+j1.07} \Rightarrow |T| = 0.88 \Rightarrow TL = 1.09\text{ dB}$$

fan noise measurement device
… mylar sheets cover the sides
except where fan is mounted
and control opening



INCE Planum

ii) Heavy Material in air

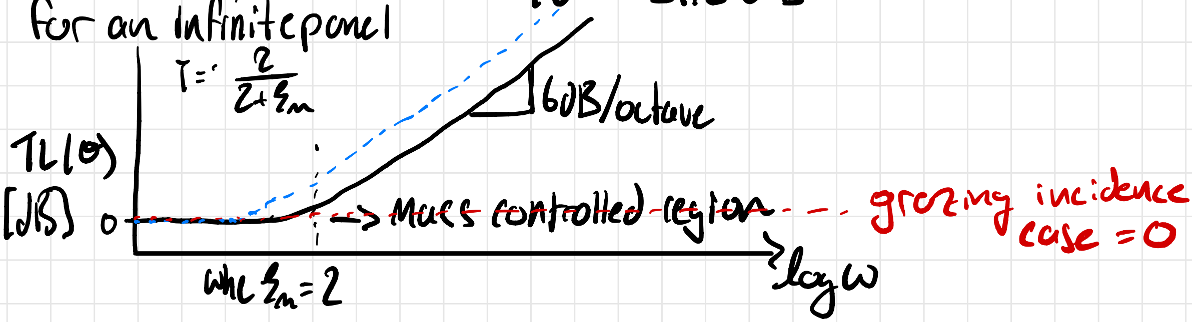
$\frac{1}{4}''$ lead $\quad M_s = 70.6 \, kg/m^2$

ξ_m at $45°$ at $1kHz$

$\xi_m = 756 \gg 1 \qquad |T| \to 0.00265$

$\qquad\qquad\qquad\qquad TL \to 51.5\,dB$

for an infinite panel



increasing mass shifts the frequency of entering the mass-controlled region lower

decreasing theta has the same effect

Highest transmission loss happens at normal incidence
$(\theta = 0)$

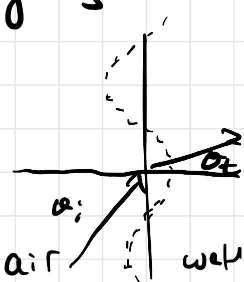
Feb 28th

air - air

$$T = \frac{2}{2+\xi_m} = \frac{2}{2 + \frac{j\omega M_s}{\rho_0 c}\cos\theta} = \frac{2\rho_0 c \sec\theta}{2\rho_0 c \sec\theta + j\omega M_s}$$

air - water

$$T = \frac{\rho_w c_w \sec\theta_t + \rho_0 c \sec\theta_i}{\rho_w c_w \sec\theta_t + \rho_0 c \sec\theta_i + j\omega M_s}$$

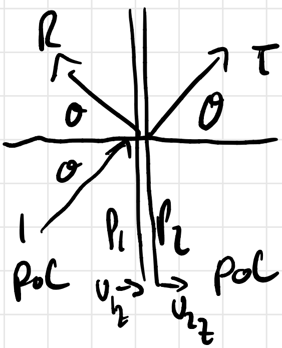


$\rho_w c_w \gg \rho_0 c_0$

Barrier performance depends on barrier properties and on properties of the medium

may be effective in air and transparent in water

3.4 Flexurally Stiff panel (Euler-Bernoulli Plate)



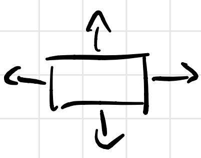
$$\underbrace{D}_{\text{flexural stiffness per unit width (into the page)}}\frac{\partial^4 w}{\partial x^2} + m_s\frac{\partial^2 w}{\partial t^2} = \text{Force / unit area} = (P_1 - P_2)_{z=0}$$

$$D = \frac{E h^3}{12(1-\nu^2)}$$

E = Young's Modulus
h = panel thickness
ν = poissons ratio

$0 \le \nu \le 0.5$
incompressible

auxetic Materials – negative poissons ratio



pull in one-direction, also expands in the other

Assume harmonic motion of the panel

$$w(x,t) = W_p e^{-jk_x x} e^{j\omega t} \qquad k_x = k\sin\theta$$

Sub into b.c. iii

$$(D k_x^4 - M_s\omega^2) W_p e^{-jk_x x} e^{j\omega t} = (P_i + P_r - P_t) e^{-jk_x x} e^{j\omega t}$$

$/P_i$ $\quad 1 + R - T = (D k_x^4 - M_s\omega^2) W_p' \qquad W_p' = W_p / P_i$

$$= j\omega\left(j\omega M_s - j\frac{D}{\omega}k_x^4\right) W_p' \Rightarrow j\omega W_p' = V_p'$$

$$1 + R - T = \left[j\omega M_s - j\frac{D}{\omega}k_x^4\right] V_p'$$

$$\frac{\text{force/unit area}}{\text{transverse velocity}} = j\left[\omega M_s - \frac{D}{\omega}\right] k_x^4 = Z_m$$

two terms can cancel
- spatial resonance

Stiff panel, free waves are possible

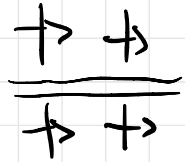
b.c. iii $\quad 1 + R - T = Z_m V_p'$

$Z_m V_p' - R + T = 1$ ... Same as Limp panel

b.c. i and ii are unchanged

$$T = \frac{2}{2 + \zeta_m} \qquad \zeta_m = \frac{Z_m}{Z_{cn}} = j\frac{\cos\theta}{\rho_0 c}\left[\underbrace{\omega M_s}_{\text{low freq}} - \underbrace{\frac{D}{\omega}k_x^4}_{\text{high freq}}\right]$$

as $\theta \to \pi/2 \quad T \to 1$



Stiff panel exhibits the same behaviour

Stiff panel discussion - Euler-Bernoulli Plate

$$z_m = j\left(\underbrace{\omega m_s - \frac{D}{\omega} k_x^4}_{\text{Spatial Resonance}}\right) \qquad k_x = k\sin\theta_i$$

$$T = \frac{2}{2+\zeta_m} \qquad \zeta_m = \frac{z_m}{z_{cn}} = j\frac{\cos\theta}{\rho_0 c}\left(\omega m_s - \frac{D}{\omega}k_x^4\right)$$

Mass-controlled at low frequencies
Stiffness controlled at high frequencies

Damping - introduce dissipation due to flexure by using a complex stiffness

$$D = D_m(1 + j\eta) \qquad \begin{aligned} &D_m \ldots \text{ real, static flexural stiffness} \\ &\eta \ldots \text{ loss factor} \end{aligned}$$

Referred to as hysteretic damping

for metals $\eta \simeq 0.001$ $\quad \eta \ll 1$

much higher in polymers and viscoelastic materials $\eta \simeq 0.25$

η - fraction of the vibrational energy dissipated per cycle

$$\zeta_m = \frac{\cos\theta}{\rho_0 c}\left[\eta\frac{D_m}{\omega}k_x^4 + j\left(\omega m_s - \frac{D_m}{\omega}k_x^4\right)\right]$$

$$T = \frac{2}{2+\zeta_m} = \frac{2\rho_0 c \sec\theta}{\left[\underbrace{2\rho_0 c\sec\theta}_{\text{acoustic loading}} + \underbrace{\eta\frac{D_m}{\omega}k_x^4}_{\text{mechanical damping}}\right] + j\left[\underbrace{\omega m_s}_{\text{inertia}} - \underbrace{\frac{D_m}{\omega}k_x^4}_{\text{stiffness}}\right]}$$

Three frequency regions

1) mass-controlled

$$\omega M_s \gg \frac{D_m}{\omega} k_x^4 \Rightarrow \omega \ll \frac{c^2}{\sin^2\theta}\sqrt{\frac{M_s}{D_m}}$$ critical freq

$$\omega \ll \frac{\omega_c}{\sin^2\theta}$$ } coincidence frequency $\omega_{cn} \geq \omega_c$

The critical frequency is the lowest coincidence frequency

as $\theta \to 0$ $\omega_{cn} \to \infty$

no pressure gradient if sound is normally incident

in the mass-controlled region

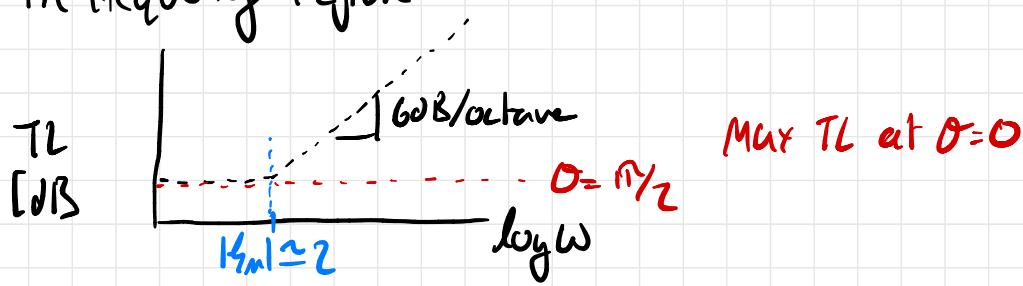
$$T = \frac{2\rho_0 c \sec\theta}{2\rho_0 c \sec\theta + j\omega M_s}$$ ... Same as limp panel

given that $\frac{\eta D_m}{\omega} k_x^4$ is negligible ...

the addition of damping does not increase TL of the panel in this region

only effect is to add mass

* true in frequency region below coincidence



2) Coincidence Region - damping controls the performance

$$\mathrm{Im}\{ z_m \} \simeq 0 \qquad \text{mass + stiffness terms cancel}$$

$$\omega M_s \simeq \frac{D_m}{\omega} k_x^4 \qquad \omega \simeq \omega_{cn}$$

$$T = \frac{2\rho_0 c \sec\theta}{2\rho_0 c \sec\theta + \eta \frac{D_m}{\omega} k_x^4} \qquad k_x = k \sin\theta_i$$

damping prevents $T \to 1$

in the absence of damping $T \to 1$ ... panel is transparent

TL plunges

damping is very important in this region

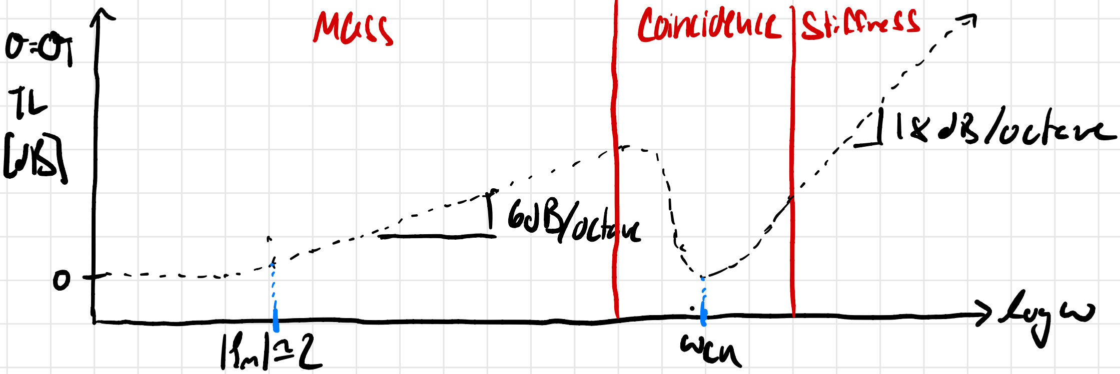
3) Stiffness controlled region - high frequency behavior

$$\omega M_s \ll k_x^4 \Rightarrow \omega \gg \omega_{cn}$$

$$T = \frac{2\rho_0 c \sec\theta}{-j\left(\frac{D_m}{\omega}\right)(1 + j\eta) k_x^4}$$

$$= \frac{2\rho_0 c \sec\theta}{-j\, D_m (1 + j\eta) \frac{\omega^3}{c^4} \sin^4\theta}$$

$$|T| \propto \frac{1}{\omega^3} \propto \frac{1}{D_m}$$



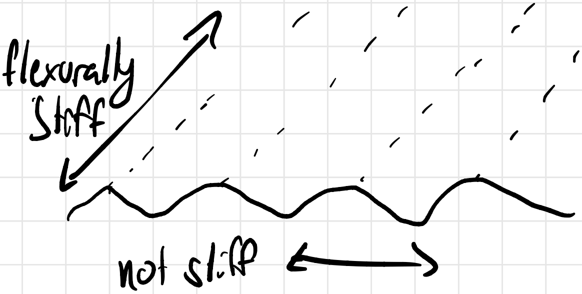
Safe to design for mass controlled region

$\omega_c = c^2 \sqrt{M_s / D_m}$ $\quad \omega_{cn} = \dfrac{\omega_c}{\sin^2\theta}$ $\quad$ as $\theta \to 0$ $\omega_{cn} \to \infty$

- D_m has no impact at normal incidence $\theta = 0$
  $\omega_{cn} \to \infty$ ... mass line continues forever
- different ω_{cn} for each angle of incidence
- at any freq $> \omega_c$, always an angle where $T \to 1$
  (prevented only by damping)

- Coincidence freq is different for each angle of incidence
  - coincidence region broadens for random incidence sound
  - generally try to ensure that the coincidence freq is above the frequency range of interest

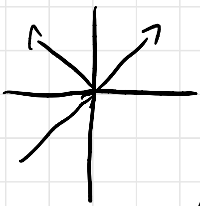
- uniform panels $\quad \omega_c = c^2 \sqrt{M_s / D_m}$



corrugated
- orthotropic

- can have very low critical frequencies in the direction parallel to the peaks of the corrugations

03/09/22



$$T = \frac{2}{2 + \xi_n} \qquad \xi_n = \frac{z_n}{z_{cn}}$$

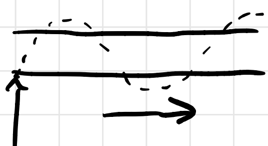
(flexurally stiff)

$$T = \frac{2\rho_0 c \sec\theta}{2\rho_0 c \sec\theta + \underbrace{\eta \frac{D_m}{\omega} k_x^4}_{\text{loss factor}} + j\left[\omega \cancel{m_s} - \cancel{\frac{D_m}{\omega}} k_x^4\right]}$$

coincidence

$\omega \simeq \omega_{cn}$ coincidence region

Notes on Coincidence



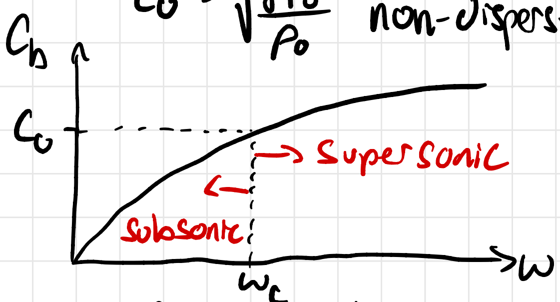
coincidence occurs when the free flexural wave speed in the panel is equal to the ambient sound speed

Speed of free flexural waves

$$C_b = \omega^{1/2} \left[\sqrt[4]{\frac{D_m}{M_s}} \right].$$

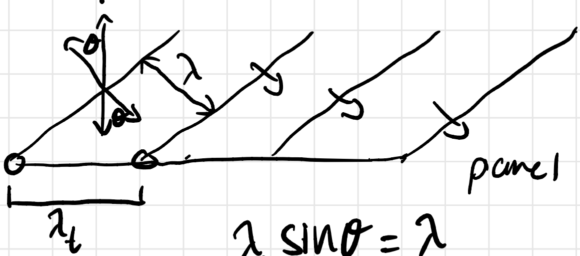
Dispersive wave... propagation wave speed is frequency dependent

$$C_0 = \sqrt{\frac{\gamma P_0}{\rho_0}}$$ non-dispersive



(Critical Freq)

Trace wave speed



$$\lambda_t \sin\theta = \lambda$$

$$\lambda_t = \frac{\lambda}{\sin\theta} \qquad \lambda_t \geq \lambda$$

$\lambda_t \to \infty$ as $\theta \to 0$

$\lambda_t \to \lambda$ as $\theta \to \pi/2$ (Grazing Incidence)

Pressure Pattern moves λ_t across the panel in one period, $T = 1/f$

$$\lambda_t = C_t T \qquad C_t = \frac{\lambda_t}{T} = \lambda_t f = \frac{\lambda f}{\sin\theta} = \frac{c}{\sin\theta} = C_t$$

$$C_t \geq C$$

C_t is the speed at which points of constant phase in the soundfield move across the surface

$\theta = \pi/2$

as θ decreases from $\pi/2$, C_t increases

$\theta \to 0 \quad C_t \to \infty$

Coincidence Occurs when

free flexural wave speed $C_b = C_t$ trace wave speed imposed by incident sound field

- Same freq, same speed, same wavelength
- perfect spatial matching of the incident sound field + the free response of the panel

"spatial resonance"

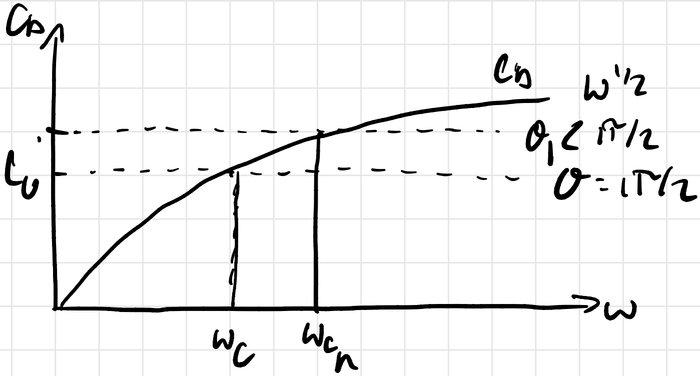
in the absence of damping $T \to 1$

at ω_{cn} $\quad C_b = C_t$

$$\omega_{cn}^{1/2} \sqrt[4]{\frac{D_m}{m_s}} = \frac{c}{\sin\theta} \Rightarrow \omega_{cn} = \frac{c^2}{\sin^2\theta} \sqrt{\frac{m_s}{D_m}}$$

ω_c

ω_c = lowest coincidence frequency, critical freq



Since in the absence of damping $TL \to 0$
at coincidence

- from a barrier performance point of view
  - like to stay below ω_c
  - make ω_c as large as possible

$$\omega_c = c^2 \sqrt{\frac{m_s}{D_m}}$$

high mass and
low stiffness

barrier should have low flexural stiffness
+ large mass/unit area

perfect barrier is lead

worst barrier is carbon fiber composites

for a homogeneous panel

$$\omega_c = \frac{c^2 \, m_s^{1/2}}{\left[Eh^3/(12(1-\nu^2))\right]^{1/2}}$$

c = ambient sound speed
m_s = mass/unit area
$= \rho_s h$
density, thickness

$$\omega_{ch} = \frac{c^2 \rho_s^{1/2}}{\left[E/(12(1-\nu^2))\right]^{1/2}}$$

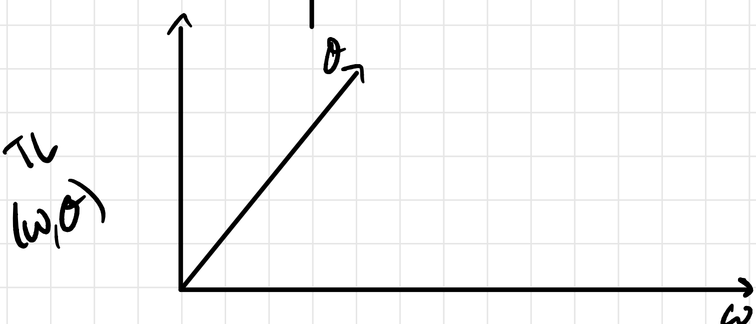
Material property only

For air at 20°C

Material	hf_c (ms^{-1})
Steel	12.4
Aluminium	12.0
Glass	12.7
Plexiglass	27.7
Plywood	20.0
Concrete	19-34

for 1mm thick...
corresponds to kHz

1mm Al → 12kHz



In reverberent conditions
- Sound arrives from many angles at once

Random Incidence power transmission coefficient
- assumes equal energy from all angles

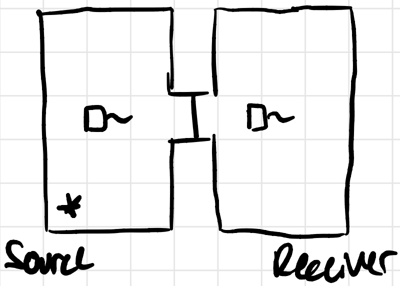
Diffuse / Random Incidence transmission coeff

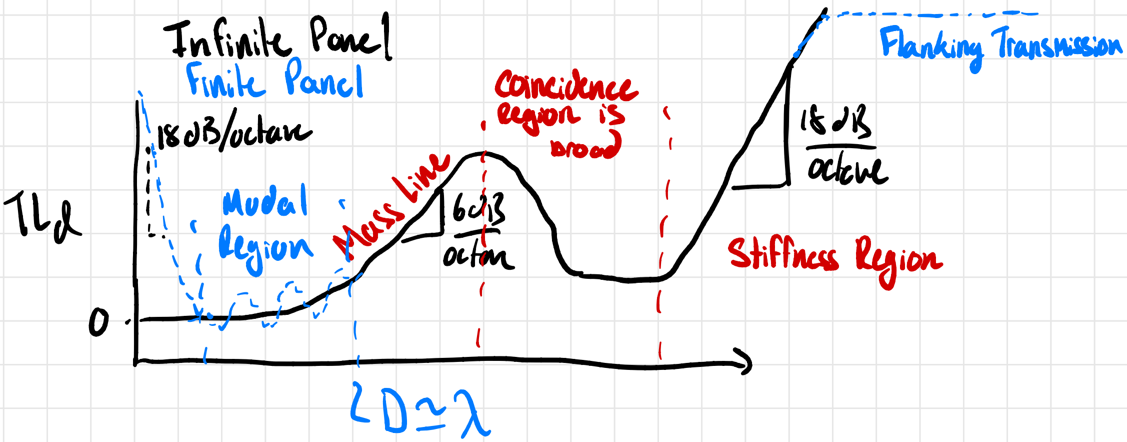
$$\tau_d(\omega) = \int_0^{\pi/2} |T(\omega,\theta)|^2 \sin(2\theta)\, d\theta$$

$$TL_d = 10 \log_{10}(1/\tau_d)$$

if corrugated...

$T(\theta, \phi, \omega)$
ϕ azimuth

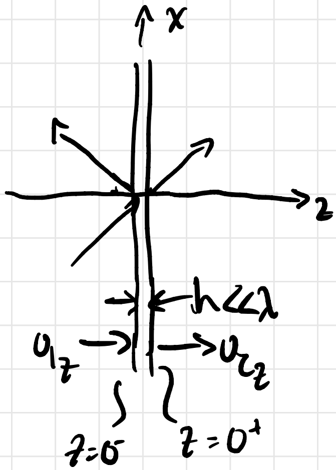




High Frequency TL is limited in practice by flanking transmission

3.5 Rigid + Flexible Resistive Screen

Rigid - material matrix does not move but the material is porous
- Fluid moves back and forth through the material



$$(P_1 - P_2)_{z=0} = R_{fd} \cdot v_{1z}\big|_{z=0}$$
$$= R_{fd} \cdot v_{2z}\big|_{z=0}$$

Assuming $v_{1z}\big|_{z=0} = v_{2z}\big|_{z=0}$

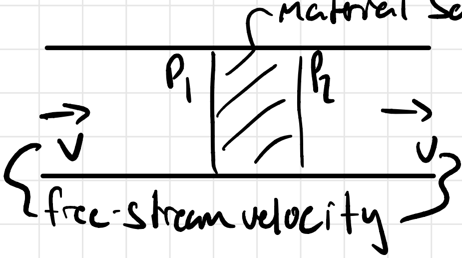
Implies Incompressible flow within the layer

Also assume inertial effects within the layer are negligible

- viscous effects are much larger then inertial effects within the layer

- velocity is oscillitory

R_{fd} = dynamic specific macroscopic flow resistivity



$R_{fs} = \dfrac{P_1 - P_2}{v}$ Static Flow Resistance

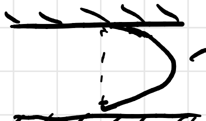
$R_{fs} = \sigma h$ ~ thickness
~ flow resistivity

Resistivity ... resistance/unit thickness

Units of flow resistance = $Pa/m/s$ = [Rayls]

usually assumed that $R_{f_d} \simeq R_{f_s}$

true at "sufficiently low" freq's



~ Parabolic Velocity Profile

Small channel + viscous flow

Poiseuille flow

pore dimension should be on the order of 0.1 mm

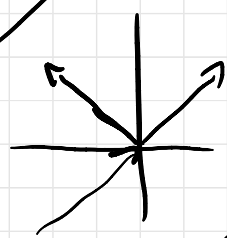
$$(P_1 - P_2)_{z=0} = R_f \, v_{1_z}\big|_{z=0} = R_f \, v_{2_z}\big|_{z=0}$$

Assuming incompressible flow

Specific in-vacuo mechanical impedance

$$\frac{(P_1 - P_2)\big|_{z=0}}{v_{1_z}\big|_{z=0}} = R_f = Z_m$$

Assume negligible inertial effects



(i) $(P_1 - P_2)_{z=0} = R_F \, v_{1_z}\big|_{z=0}$

$v_{1_z}\big|_{z=0} = v_{2_z}\big|_{z=0}$

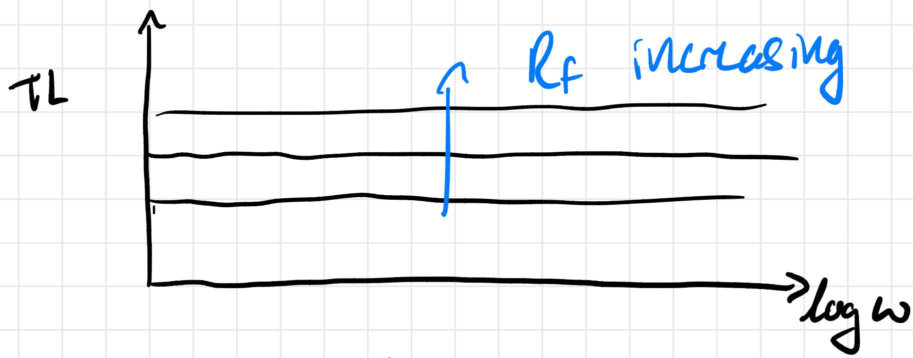
Assume incompressible flow

$$\zeta_m = \frac{R_F}{\rho_0 c} \cdot \cos\theta$$

$$T = \frac{2}{2 + \zeta_m} = \frac{2}{2 + R_f \frac{\cos\theta}{\rho_0 c}}$$

$$T = \frac{2\rho_0 c \sec\theta}{2\rho_0 c \sec\theta + \underbrace{R_f}_{\text{positive real number}}} < 1$$

T is real

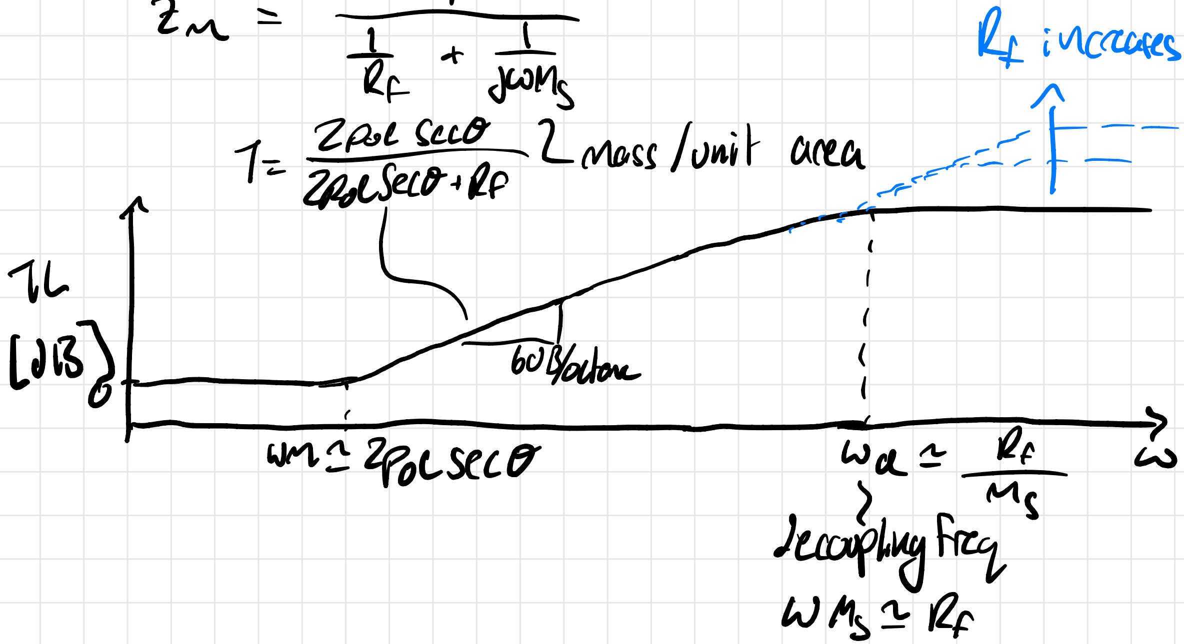


- if R_f is too large, motion of the solid matrix becomes significant and limits to the limp or stiff panel

- Impedances of the solid + fluid components add in parallel

Limp, Resistive Screen

$$Z_m = \frac{1}{\frac{1}{R_f} + \frac{1}{j\omega M_s}}$$



$\omega << \frac{R_f}{M_s}$ acts like a limp panel
- fluid + solid are very closely coupled

$\omega >> \frac{R_f}{M_s}$ acts like a rigid resistive screen

Assumptions

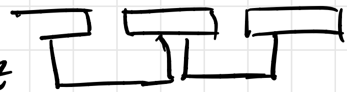
1. Flow in the layer is incompressible so that $u_{1z}|_{z=0} = u_{2z}|_{z=0} = 0$
   - Thin layers << wavelength
2. Fluid inertial effects are negligible in the layer
   - usually true in thin resistive layers

- Thin resistive layers → scrims

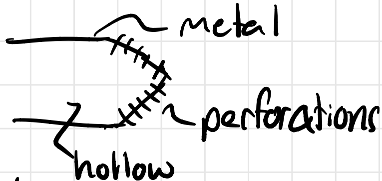
viscous drag through the holes are much larger than inertial effects

3.6 Perforated Rigid Screen

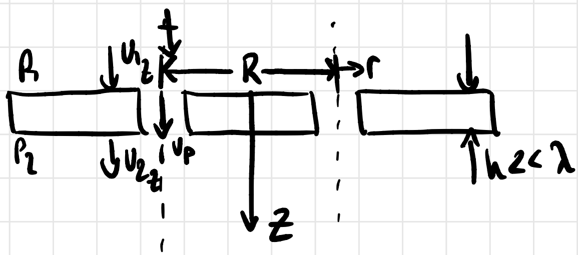
- jet engine liners
- duct systems (HVAC - protect fibrous layers)
- room acoustics
  - array of helmholtz resonators



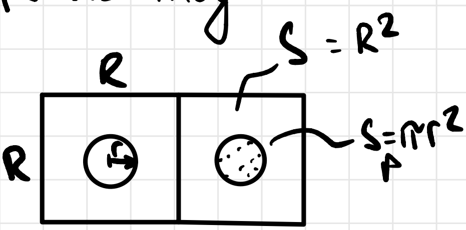
Metadyne Wedges
- used in anechoic chambers where testing might be dirty



Side View



Square periodic Array



Continuity of Volume Velocity (Assume locally incompressible flow)

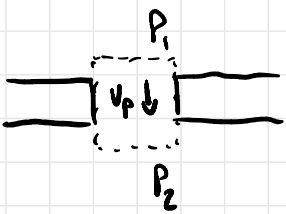
at $z=0^-$ $\quad S \cdot U_{1z} = S_p V_p$

at $z=0^+$ $\quad S \cdot U_{2z} = S_p V_p$

$\therefore \; U_{1z} = U_{2z} = \frac{S_p}{S} V_p$

$\frac{S_p}{S} = \Omega_s$ (surface porosity)

Equation of motion of fluid in the perforation



$$\underbrace{S_p(P_1 - P_2)}_{\text{Force}} = \underbrace{\rho_0 \cdot S_p (h + \Delta)}_{\text{Mass}} \underbrace{\frac{\partial V_p}{\partial t}}_{\text{Acceleration}}$$

Δ ... end correction

$\Delta = (1.6)r$ ... accounting for inner + outer end-correction

★ true if $r/R < 0.2$ ... holes are independent

$$S_p(P_1 - P_2) = \rho_0 S_p h' \frac{\partial V_p}{\partial t}$$

$$V_p = \frac{S}{S_p} U_{1z}\Big|_{z=0}$$

$$V_p = \frac{1}{\Omega_s} U_{1z}\Big|_{z=0}$$

$$(P_1 - P_2)\Big|_{z=0} = \frac{\rho_0 h'}{\Omega_s} \frac{\partial U_{1z}}{\partial t}\Big|_{z=0}$$

$$= \frac{j\omega \rho_0 h'}{\Omega_s} U_{1z}\Big|_{z=0}$$

$$\left.\frac{P_1-P_2}{v_{1z}}\right|_{z=0} = \frac{j\omega\rho_0 h'}{\Omega_s} = Z_m = j\omega\underbrace{\boxed{\left(\frac{\rho_0 h'}{\Omega_s}\right)}}_{M_{se}}$$

M_{se} ⋯ effective mass

$$Z_m = j\omega\cdot M_{se}$$

$$T = \frac{2}{2+\zeta_m} \quad \cdots \quad \zeta_m = \frac{Z_m}{Z_{cn}} = \frac{j\omega\cancel{\rho_0}h'}{\cancel{\rho_0}c\cdot\Omega_s}\cdot\cos\theta$$

$$\zeta_m = \frac{jkh'}{\Omega_s}\cdot\cos\theta$$

$$\left.M_{se} \propto \frac{1}{\Omega_s}\right]$$ represents the inertial effect of fluid acceleration in + out of the holes

$$M_{se} = \frac{h'\rho_0}{\Omega_s}$$

$$\left.\begin{aligned}(P_1-P_2)\big|_{z=0} &= Z_m v_{1z}\big|_{z=0}\\ v_{1z}\big|_{z=0} &= v_{2z}\big|_{z=0}\end{aligned}\right]\ 2\text{ b.c.'s}$$

$$T = \frac{2}{2+\zeta_m}$$

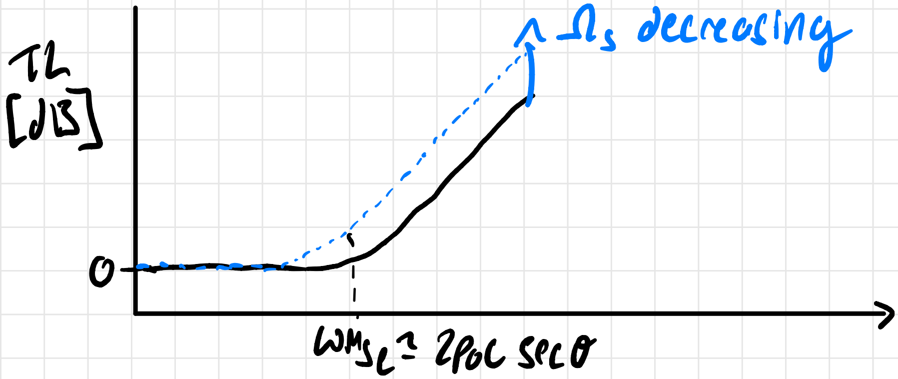
$$T = \frac{2\rho_0 c\cdot\sec\theta}{2\rho_0 c\cdot\sec\theta + j\omega M_{se}}$$ (like a limp panel w/ M_{se})

in high freq (mass controlled) region

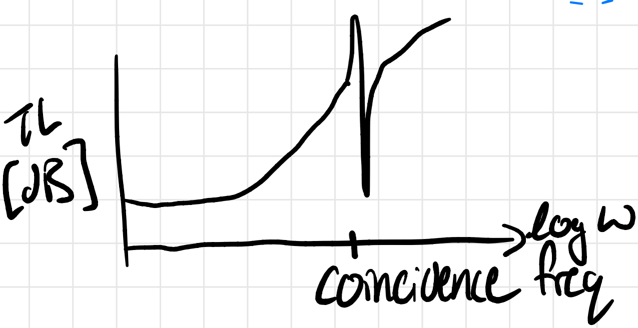
$\zeta_m \gg 1$ $\quad T \to \dfrac{-2j\Omega_s\sec\theta}{(kh')}$ ← non-dimensional thickness

As $\Omega_s \to 0$, $T \to 0$, TL increases

... hole shape (within reason) does not matter
... so long as hole cross section is very small compared to a wavelength

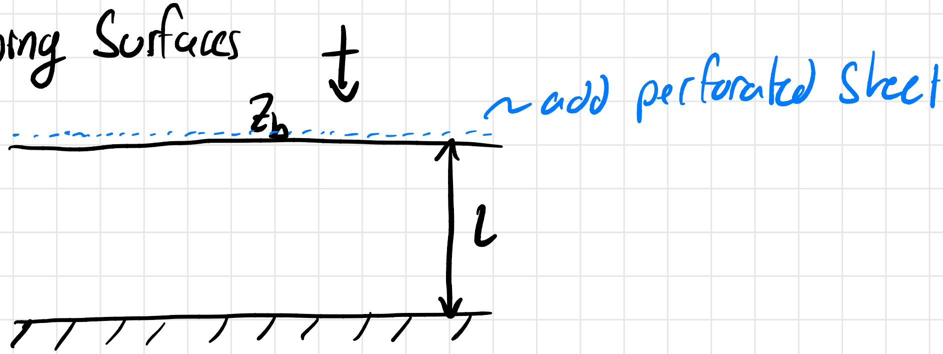


Ω_s decrease corresponds to M_{se} increase



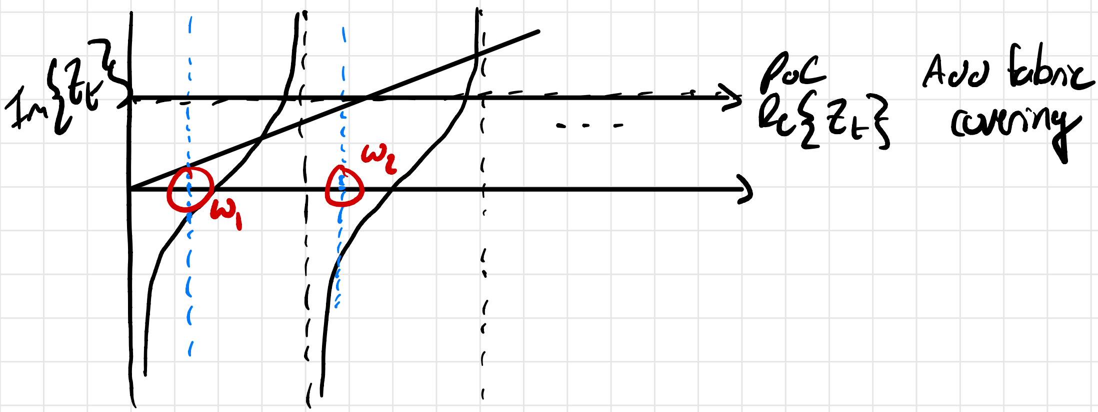
Book: Lectures on The Theory of Sound
Rschevkin - practical info related to perforated treatments

Absorbing Surfaces



$Z_b = -j \rho_0 c \cdot \cot(kL)$ at normal incidence

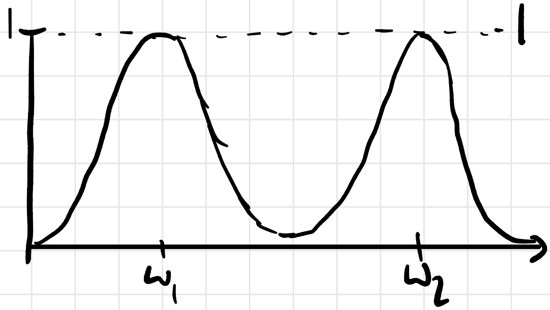
$$Z_t = Z_m + Z_b = j\omega M_{sc} - j\rho_0 c \cdot \cot kL$$



Combination of perforated Screen and a resistive layer of $R_f = \rho_0 c$

- creates perfect absorption at $\omega_1, \omega_2 \cdots$

normal incidence
α
(abs coeff)



* combination of perforated sheet & thin resistive layer

3.7 General Approach to Sound Transmission Through Panels

We have observed:

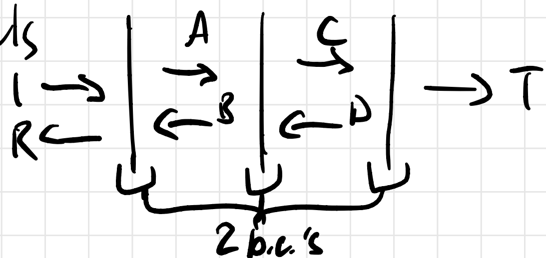
- limp membrane
- Stiff Panel
- Resistive Screen
- Perforated Panel

Can be used in combination

Z_m: $(P_1 - P_2)_{z=0} = Z_m U_{1z}\big|_{z=0}$

$$U_{1z}\big|_{z=0} = U_{2z}\big|_{z=0}$$

Multiple panels



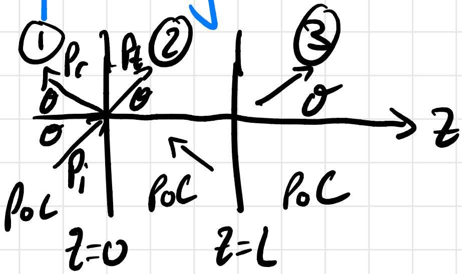
solve as 6x6 Matrix

4 panels: 8x8

$$\begin{bmatrix} P_1 \\ V_1 \end{bmatrix}_{z=0} = [T] \begin{bmatrix} P_2 \\ V_2 \end{bmatrix}_{z=0}$$

$$\begin{bmatrix} 1 & z_M \\ 0 & 1 \end{bmatrix}$$

3.8 Multi panel System



Each Panel is defined by Z_m

Each Panel
2 b.C.'s

Double Panel
$$\underset{4\times4}{[A]} \begin{bmatrix} R \\ T \\ R' \\ T' \end{bmatrix} = \underset{4\times4}{[B]}$$

$$\begin{bmatrix} P_2 \\ v_{2_z} \end{bmatrix} = \begin{bmatrix} \cos k_z l & \frac{j\rho_0 c}{\cos\theta}\sin k_z l \\ j\frac{\cos\theta}{\rho_0 c}\sin k_z l & \cos k_z l \end{bmatrix} \begin{bmatrix} P_3 \\ v_{3_z} \end{bmatrix} \qquad k_z = k\cos\theta$$

$[T_A]$ depends only on l (space between two elements, not on absolute position)

$$\begin{array}{c|cc|c} & 1 & 2 & \\ P_1 & P_2 & P_3 & P_4 \\ v_{1_z} & v_{2_z} & v_{3_z} & v_{4_z} \end{array}$$

$$\begin{bmatrix} P_3 \\ v_{3_z} \end{bmatrix} = [T_2] \begin{bmatrix} P_4 \\ v_{4_z} \end{bmatrix}$$ T_2 - T-matrix for panel 2

$$\begin{bmatrix} P_2 \\ v_{2_z} \end{bmatrix} = [T_A] \begin{bmatrix} P_3 \\ v_{3_z} \end{bmatrix}$$ T_A - T-matrix for air space

$$= [T_A][T_2] \begin{bmatrix} P_4 \\ v_{4_z} \end{bmatrix}$$

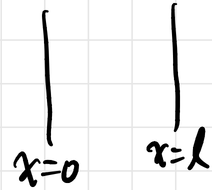
$$\begin{bmatrix} P_1 \\ v_{1_z} \end{bmatrix} = [T_1][T_A][T_2] \begin{bmatrix} P_4 \\ v_{4_z} \end{bmatrix}$$

$$= [T_t] \begin{bmatrix} P_4 \\ v_{4_z} \end{bmatrix}$$ T_t - 2x2 transfer matrix

2 eqn's ... 2 unknowns

$$T = \frac{2 e^{j k_z l}}{T_{t_{11}} + \frac{\cos\theta}{\rho_0 c} T_{t_{12}} + \frac{\rho_0 c}{\cos\theta} T_{t_{21}} + T_{22}}$$

Double Panel $\quad [T_t] = [T_1][T_A][T_2]$



To model a Single Panel...

let $T_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, let $l \to 0$... $T_A \to \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$[T_1] = \begin{bmatrix} 1 & Z_M \\ 0 & 1 \end{bmatrix} \qquad [T_t] = [T_1]$$

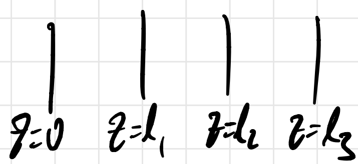
$$T = \frac{2}{1 + \frac{\cos\theta}{\rho_0 c} Z_{M_1} + 0 + 1} = \frac{2\rho_0 c}{2\rho_0 c + \cos\theta Z_{M_1}}$$

$$T = \frac{2\rho_0 c \sec\theta}{2\rho_0 c \sec\theta + Z_{M_1}}$$

exactly the same as the single panel case we've worked out before...

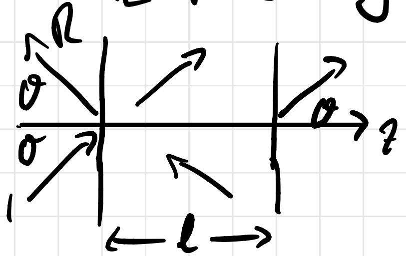
General Approach to solve for T

1. Find the total transfer matrix from back to front
2. Use general expression for T



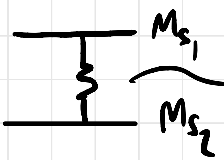
$$T = \frac{2e^{jk_z l_s}}{\cdots}$$

3.9 Double Panel System – 2 homogeneous elastic panels separated by finite depth air space

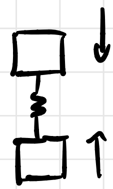


Mass-air-mass resonance (typically at low frequencies)

when $kl \ll 1$ ... air space is shallow



Stiffness of air between panels



when $kl \ll 1$ (at normal incidence)

$$\omega_0 = \left[\frac{\rho_0 c^2}{l}\left(\frac{M_{s_1} + M_{s_2}}{M_{s_1} \cdot M_{s_2}}\right)\right]^{1/2}$$

...mass-air-mass resonance frequency...

$\rho_0 c^2$ indicates stiffness

at non-normal angles, the m-a-m frequency increases at ω_0, TL is a minimum

when $M_{s_1} = M_{s_2}$ $TL \to 0$ at ω_0

and ω_0 is a minimum for a given $M_{s_1} + M_{s_2}$

Performance of a double panel system is at its worst when the panels are identical

$$\boxed{\text{AVOID IDENTICAL PANELS}}$$

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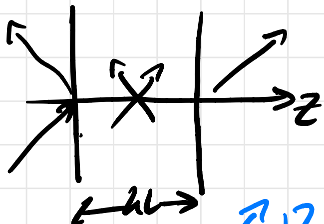
Double Panels

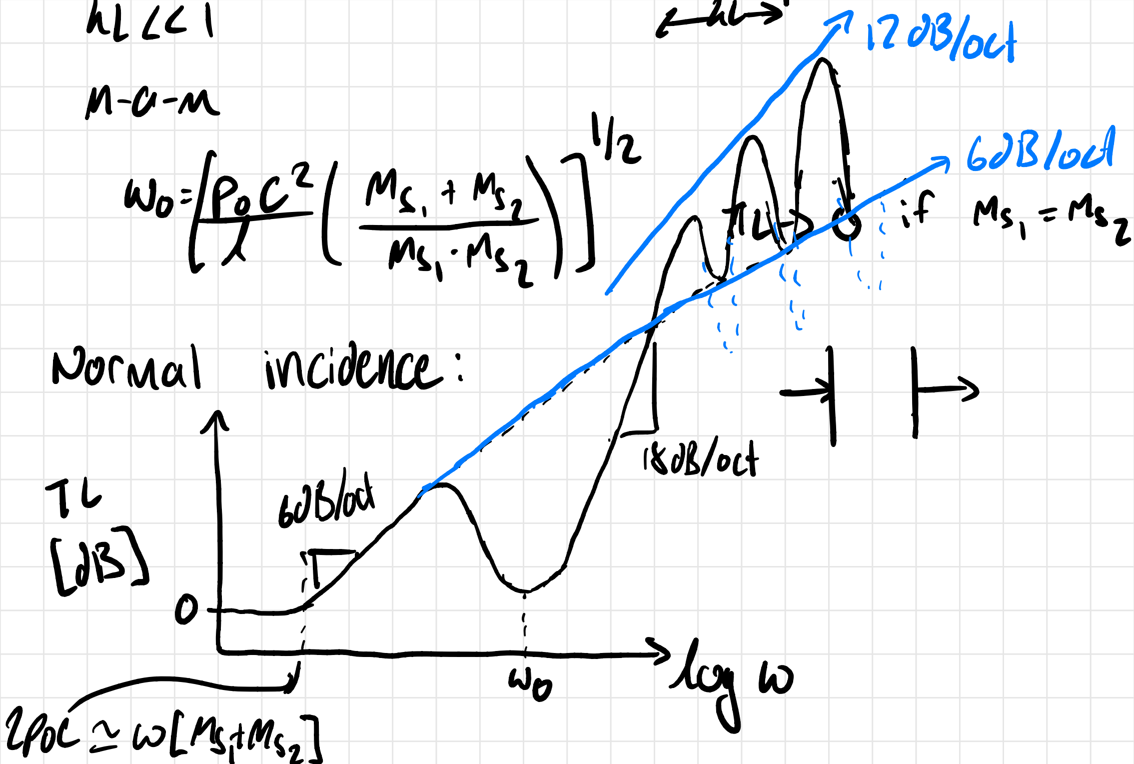


$kL \ll 1$

m-a-m

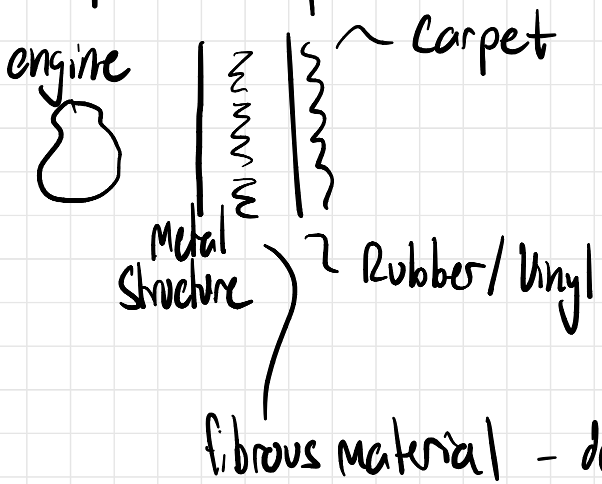
$$\omega_0 = \left[\frac{\rho_0 c^2}{L}\left(\frac{M_{s_1} + M_{s_2}}{M_{s_1} \cdot M_{s_2}}\right)\right]^{1/2}$$

Normal incidence:



Generally the space between the panels should be filled with fibrous Matls.

Automotive dash panels



- Sound level should decay ~40 dB in 2 trips across the space between the panels

Given sufficient absorbent

above m-a-m resonance

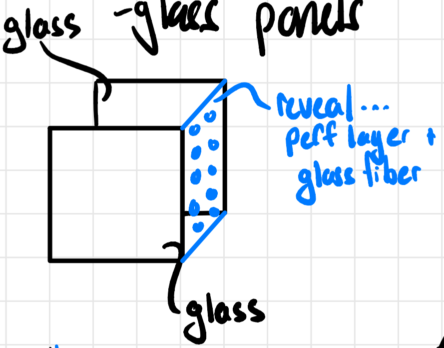
$$\underbrace{TL_{total} = TL_1 + TL_2 + 6\,dB}_{\text{if panels are decoupled}}$$

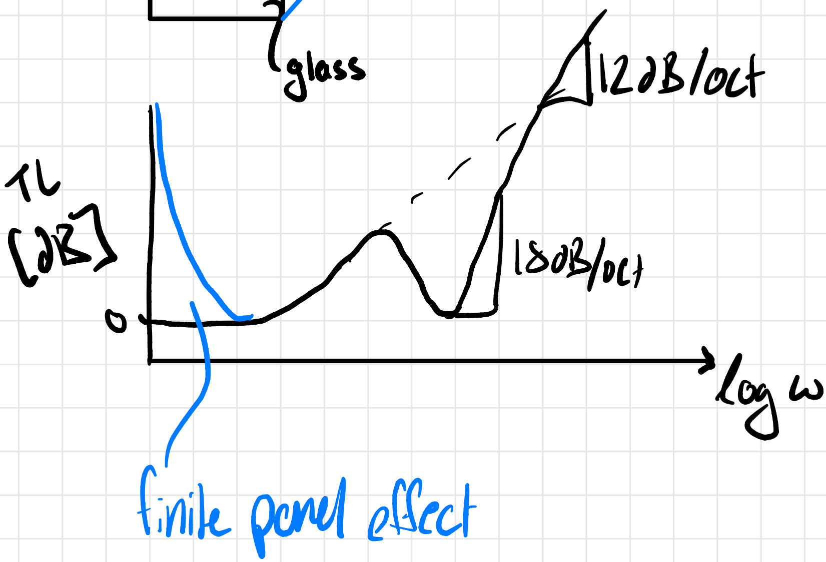
$TL_{double} \gg TL_{single}$ having same M_s

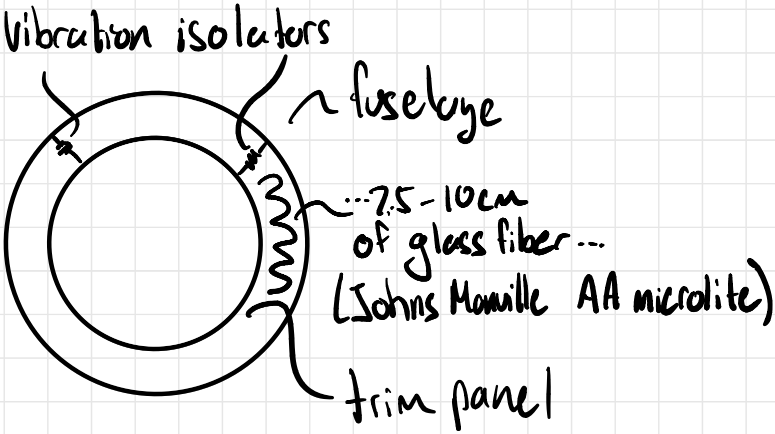
* Supernal * Joby

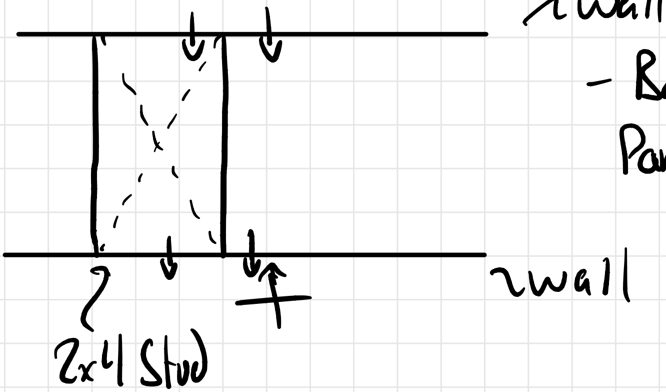
Triple panel - further improvement

glass - glass panels





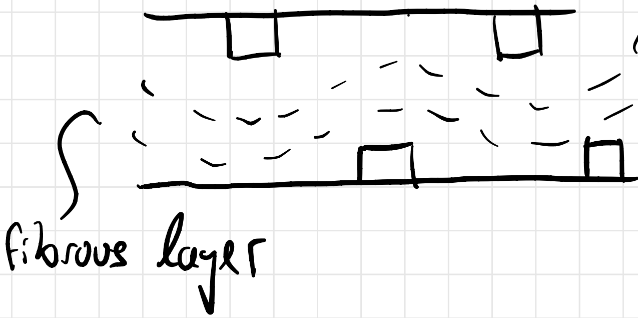




- Bad Wall Construction
Panels are directly connected by studs

Air space is short circuited by structural vibration path

Make connection resilient



~Staggered Studs to provide independent support

- always include absorbing layer