# ME413 HW 05

Benjamin Masters

TOTAL POINTS

**120 / 120**

QUESTION 1

1 Q1 **20 / 20**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

QUESTION 2

2 Q2 **20 / 20**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

QUESTION 3

3 Q3 **20 / 20**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

QUESTION 4

4 Q4 **20 / 20**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

QUESTION 5

5 Q5 **20 / 20**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

QUESTION 6

6 Q6 **20 / 20**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

1 Q1 **20 / 20**

- **- 0 pts** Correct

**+ 1 Point adjustment**

2 Q2 **20 / 20**

- **- 0 pts** Correct

**+ 1 Point adjustment**

3 Q3 **20 / 20**

- **- 0 pts** Correct

**+ 1 Point adjustment**

4 Q4 **20 / 20**

- **- 0 pts** Correct

**+ 1 Point adjustment**

5 Q5 **20 / 20**

- **- 0 pts** Correct

**+ 1 Point adjustment**

6 Q6 **20 / 20**

- **- 0 pts** Correct

**+ 1 Point adjustment**

**Question 1 (20 points)**
Describe, with the aid of a sketch or relevant equations when necessary, each of the following:
(a) spring force, excitation, natural frequency and resonance of a vibratory system.
(b) Kinetic energy and pressure energy of a one-dimensional planar waves.
(c) Amplitude and phase of a pressure.

Benjamin Masters

a) spring force: a restoring force by a spring, force always acts in the direction opposite of the displacement.

Excitation: applying a force that initiates motion of a system.

Natural Frequency: The frequency at which a system tends to vibrate at naturally.

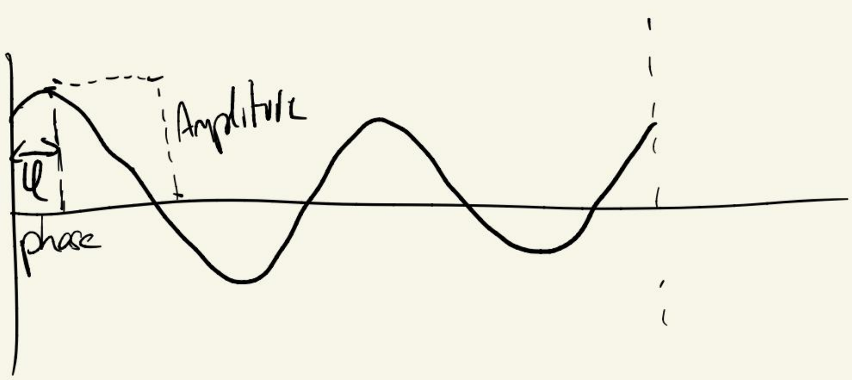
Resonance: The increase in amplitude when a system is driven at or near it's natural frequency

b) Kinetic Energy of 1-D planer wave: Energy resulting from movement of a fluid. represented by: $E_K(t) = \frac{1}{2} mu^2 \approx \frac{1}{2} \rho_0 V_0 u^2$

Pressure Energy: Potential energy stored by compression of a fluid represented by: $\frac{1}{2} \frac{p(t)^2}{\rho_0 c_0^2} = e_p(t)$

c) Amplitude: the magnitude of the pressure

phase: the angular difference between a pressure wave and another reference wave.



**Question 2 (20 points)**
A train traveling at 100 mph (you need to convert it to S.I. Unit) enters a long tunnel with the same cross-sectional area as the train.
(a) Determine the strength of the pressure wave generated in the tunnel.
(b) What is the <u>peak</u> intensity of the sound pressure?
(d) If the cross-sectional area of the train is 5 m$^2$, what is the total (peak) sound power of generated by the pressure wave?
You may take the density of air as 1.21 kg m$^{-3}$ and sound speed in air as 340 m s$^{-1}$.

$$u' = 100\,\text{mph} = \frac{100\,\text{mi}}{\text{hr}} \cdot \frac{1\,\text{hr}}{3600\,\text{s}} \cdot \frac{1609.3\,\text{m}}{1\,\text{mi}} = 44.7\,\text{m/s} = u'$$

$$p' = \rho_0 c_0 u' \qquad \rho_0 = 1.21\,\text{kg/m}^3 \qquad c_0 = 340\,\text{m/s}$$

$$p' = (1.21\,\text{kg/m}^3)(340\,\text{m/s})(44.7\,\text{m/s}) = \boxed{18.39\,\text{kPa} = p'}$$

$$b)\ I = p'u' = 18.39\,\text{kPa} \cdot 44.7\,\text{m/s} = \boxed{822\,\text{kW/m}^2 = I_{pk}}$$

$$c)\ W_{pk} = I_{pk} S = (822\,\text{kW/m}^2)(5\,\text{m}^2) = \boxed{4110\,\text{kW} = W_{pk}}$$

**Question 3 (20 points)**

(a) Convert the following rms pressure into dB

(i) 20 μPa; (ii) 150 μPa; (iii) 1 kPa, and (iv) 50 kPa.

(b) Convert the following harmonic waves expressed in dB into rms and peak pressures

(i) 20 dB; (ii) 60 dB; (iii) 90 dB, and (iv) 130 dB

(c) You are asked to conduct a measurement of noise level of a machine $L$ in a workshop. It is known that the workshop has a background noise level of $L_b$. Show that no correction for background noise level is needed if $L - L_b > 10$ dB.

$P_{ref} = 20\mu Pa$

a)

i) $L_p = 10 \cdot \log\left(20\mu Pa / 20\mu Pa\right)^2 = 0 dB$

ii) $L_p = 10\log\left(150\mu Pa / 20\mu Pa\right)^2 = 17.5 dB$

iii) $L_p = 10\log\left(1kPa / 20\mu Pa\right)^2 = 153.98 dB$

iv) $L_p = 10\log\left(50kPa / 20\mu Pa\right)^2 = 187.96 dB$

b) i) $P_{rms} = 20\mu Pa \cdot 10^{20/20} = 200\mu Pa$

$P_{pk} = \sqrt{2} \cdot P_{rms} = 282.8\mu Pa$

ii) $P_{rms} = 20\mu Pa \cdot 10^{60dB/20} = 20000\ \mu Pa = .02 Pa$

$P_{pk} = \sqrt{2} \cdot .02 = 0.0283 Pa$

iii) $P_{rms} = 20\mu Pa \cdot 10^{90/20} = 0.632 Pa$

$P_{pk} = \sqrt{2} \cdot 0.632 Pa = 0.894 Pa$

iv) $P_{rms} = 20\mu Pa \cdot 10^{130/20} = 63.25 Pa$

$P_{pk} = \sqrt{2} \cdot 63.25 Pa = 89.44 Pa$

c) $L - L_B > 10 dB$ $\quad L_t = L \oplus L_B$

knowing $\Delta L = 10 dB$ $\quad C_t(\Delta L) = 10\log\left(1 + 10^{-\Delta L/10}\right)$

$C_t = 10\log(1.1) = 0.41 dB$

Based on Assumption that change of less than .5 dB is negligible, the calculated difference of .4 is negligible and can be ignored, along with any dB difference greater than 10 dB.

**Question 4 (20 points)**

(a) What will be the increase in sound pressure level if pressure is doubled?
(b) Calculate the sound power level (re $10^{-12}$ W) of a noise source whose power is 0.25 W.
(c) Calculate the sound power, in watts, corresponding to a sound power level of 97 dB re$10^{-12}$ W.
(d) Measurements of SPL are conducted in a large room at six different locations. The measured SPLs are 64 dB, 60 dB, 55 dB, 67 dB, 61.5 dB and 66.2 dB respectively. Calculate the average SPL in the room.

a) $P_2^2 = (2P_1)^2 \Rightarrow L_{P_2} - L_{P_1} = 10\log\frac{(2P_1)^2}{P_1^2} = 20\log 2 \approx 6\,dB$

if pressue is doubled SPL increases by 6dB.

b) $L_w = 10\log\left(0.25\,W/10^{-12}\,W\right) = 104\,dB = L_w$

c) $\frac{97\,dB}{10} = \frac{10\log(W/10^{-12})}{10} \Rightarrow 9.7 = \log(W/10^{-12}) \Rightarrow 10^{9.7} = W/10^{-12}$

$W = 10^{9.7} \cdot 10^{-12} = .005\,W$

d) $\overline{L_p} = 10\log\left[\frac{1}{6}\left(10^{64/10} + 10^{60/10} + 10^{55/10} + 10^{67/10} + 10^{61.5/10} + 10^{66.2/10}\right)\right]$

$\underline{L_p = 63.81\,dB}$

**Question 5 (20 points)**

Two coherent sound sources (with the angular frequency of $\omega$) operate together that give a total pressure field of

$$p_t = \mathrm{Re}\left\{4e^{i\pi/3}e^{i\omega t}\right\}$$

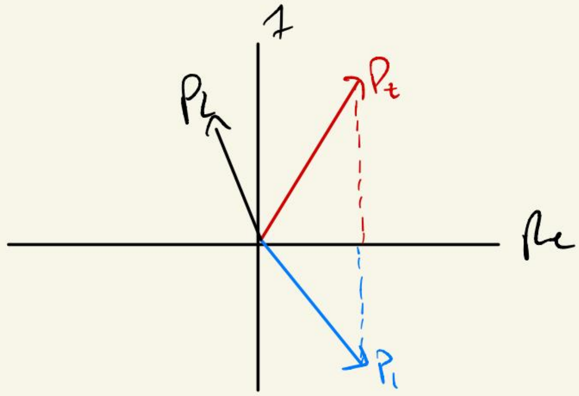
(a) If the first source produces a sound field of $p_1 = 3\sin(\omega t + \pi/4)$, find the magnitude and phase of the second source, $p_2$. Express your solution in a complex form.

(b) For the same sound source $p_1$, what is the magnitude and phase of $p_2$ in order to produce a perfect 'anti-sound' to $p_1$? Give your solution in a complex form.

You should sketch a phasor diagram to show $p_1$, $p_2$ and $p_t$ in (a) and (b) respectively.

a) $P_t = \mathrm{Re}\left[4e^{i\pi/3}e^{i\omega t}\right]$ $\quad P_1 = 3\sin(\omega t + \pi/4)$ $\quad P_2 = ?$

$P_t = 4\cos(\omega t + \pi/3)$




$P_2 = P_t - P_1$

$P_{2Re} = 4\cos\pi/3 - 3\cos\pi/4$

$= -0.121$

$P_{2,I} = 4\sin\pi/3 + 3\sin\pi/4$

$= 5.59$

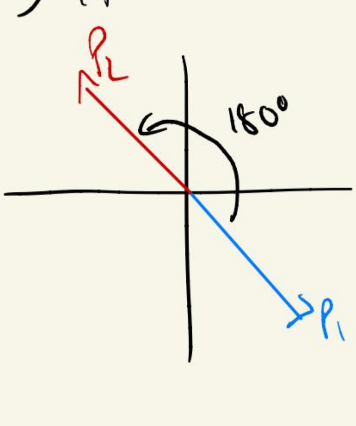
$A_{p_2} = \sqrt{.121^2 + 5.59^2} = 5.59$

$\varphi' = \tan^{-1}(5.59/.121) = 1.55$

$\varphi = \pi - \varphi' = 3.14 - 1.55 = 1.59\,(\mathrm{rad})$

$\boxed{P_2 = 5.59e^{1.59i}}$

b) for Anti-Sound, equal magnitude but 180° out of phase




So $P_2 = 3\sin(\omega t + \pi/4 \pm \pi)$

$P_2 = 3\sin(\omega t - 3\pi/4)$

$P_2 = 3\cos(\omega t + 3\pi/4)$

$\boxed{P_2 = \mathrm{Re}\left[3e^{i3\pi/4}e^{i\omega t}\right]}$

**Question 6 (20 points)**

Suppose that the transverse displacement, $y$, of a string with tension $T$ and mass per unit length $m$ satisfies the one-dimensional wave equation:

$$m\frac{\partial^2 y}{\partial t^2} - T\frac{\partial^2 y}{\partial x^2} = 0 \ .$$

If the tension $T$ of the string is 30 $N$ and mass per unit length of the string is 1.5 kg/m and a harmonic wave of frequency of 5.0 Hz is propagated along the string,

a) Express the propagation speed of the disturbances along the string in terms of $T$ and $m$. Hence or otherwise, deduce the speed in m s$^{-1}$.

b) Determine the wavelength, wave number and period of the harmonic waves.

$T = 30N \quad m = 1.5 kg/m \quad f = 5.0 Hz = \frac{5.0}{s}$

a) wave equation: $\frac{\partial^2 u}{\partial x^2} - \frac{1}{c_0^2}\frac{\partial^2 u}{\partial t^2} = 0$

converting given equation to this form:

$$\frac{M}{T}\frac{\partial^2 y}{\partial t^2} - \frac{\partial^2 y}{\partial x^2} \Rightarrow \frac{\partial^2 y}{\partial x^2} - \frac{M}{T}\frac{\partial^2 y}{\partial t^2}$$

$$\boxed{c_0 = \sqrt{T/m}}$$

$$+\frac{1}{c_0^2} = +\frac{M}{T} \Rightarrow c_0^2 = T/m$$

$$c_0 = \sqrt{30N/1.5kg/m} = c_0 = \sqrt{20 m^2/s^2} = \boxed{c_0 = 4.47 m/s}$$

b) $y(x,t) = \hat{y}(x)e^{i\omega t} \qquad \omega = 2\pi \cdot 5.0 Hz = 31.4 rad/s$

$$\frac{\partial^2 y}{\partial x^2} = \left[\frac{\partial^2 y}{\partial x^2}\right]e^{i\omega t} \qquad \frac{\partial^2 y}{\partial t^2} = -\omega^2 \hat{y}(x)e^{i\omega t}$$

$$\frac{\partial^2 y}{\partial x^2} + \frac{\omega^2}{c_0^2}\hat{y} = 0 \qquad k^2 = \frac{\omega^2}{c_0^2} \Rightarrow \sqrt{\frac{(31.4 rad/s)^2}{(4.47 m/s)^2}} = \boxed{k = 7.03 \ rad/m}$$

$$\lambda = \frac{2\pi rad}{k \ rad/m} = \frac{2\pi}{7.03} = \boxed{\lambda = 0.894 m}$$

$$T = 1/f = 1/5.0 = \boxed{0.2 s = T}$$