ME413 HW 04

Benjamin Masters

TOTAL POINTS

98 / 100

QUESTION 1

1 P1 **10 / 10**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

QUESTION 2

2 P2 **18 / 20**

- **- 0 pts** Correct
- **- 2 Point adjustment**

QUESTION 3

3 P3 **20 / 20**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

QUESTION 4

4 P4 **20 / 20**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

QUESTION 5

5 P5 **10 / 10**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

QUESTION 6

6 P6 **20 / 20**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

1 P1 **10 / 10**

- **- 0 pts** Correct

+ 1 Point adjustment

2 P2 **18 / 20**

- **- 0 pts** Correct

- 2 Point adjustment

3 P3 **20 / 20**

- **- 0 pts** Correct

+ 1 Point adjustment

4 P4 **20 / 20**

- **- 0 pts** Correct

+ 1 Point adjustment

5 P5 **10 / 10**

- **- 0 pts** Correct

+ 1 Point adjustment

6 P6 **20 / 20**

- **- 0 pts** Correct

+ 1 Point adjustment

Question 1 (10 points)

A 250-kg table for repairing instrument is isolated from the floor by springs with stiffness 20 kN/m and dampers with the coefficient damping of 4 kN/m/s. If the floor vibrates vertically at ±2.5 mm at a frequency of 10 Hz, find the displacement of the table (magnitude and phase).

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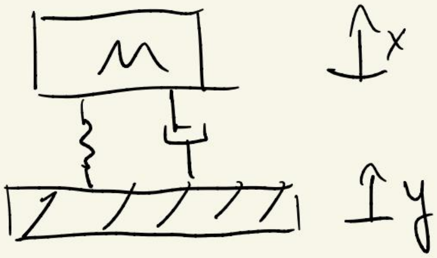
$M = 250\,kg$ $\quad K = 20\,kN/m$ $\quad C = 4\,kN/m/s$ $\quad A_y = 2.5\,mm$

$f = 10\,Hz$



$\omega = f \cdot 2\pi = 62.83\,rad/s$

$\omega_n = \sqrt{k/m} = 8.94\,rad/s$

$m\ddot{x} + k(x-y) + c(\dot{x}-\dot{y})$

$r = \omega/\omega_n = \dfrac{62.83\,rad/s}{8.94\,rad/s} = 7.03$

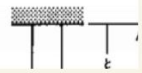
$\xi = \dfrac{c}{2\sqrt{mk}} = \dfrac{4000\,N/m/s}{2\sqrt{250\,kg \cdot 20000\,N/m}} = 0.894 = \xi$

$|TR| = \sqrt{\dfrac{1 + 4r^2\xi^2}{(1-r^2)^2 + 4r^2\xi^2}} = 0.252$

$A_x = |TR| \cdot A_y = 0.252 \cdot 2.5\,mm = \boxed{0.63\,mm = A_x}$

$\varphi = \tan^{-1}\left(\dfrac{-2\xi r^3}{(1-r^2) + (2\xi r)^2}\right) = \boxed{\varphi = -1.396\,rad}$

Question 2 (20 points)
An instrument in an aircraft is to be isolated from the engine vibrations, ranging from 1,800 to 3,600 cycles per minute. If the damping is negligible and instrument has mass of 20 kg, specify the springs for the mounting for 80 percent isolation. Use the chart and compare with the predictions using the analytical formula.



$M = 20\,Kg$ $\qquad R = 0.80 \Rightarrow TR = 0.20$ $\qquad f = 1800 \rightarrow 3600$ cyc/min

$$\xi = 0 \;\therefore\; TR = \sqrt{\frac{1 + \cancel{(2\xi r)^2}}{(1-r^2)^2 + \cancel{(2\xi r)^2}}} \Rightarrow \frac{1}{r^2 - 1} = \frac{1}{\frac{\omega^2}{\omega_n^2} - 1} = TR$$

$$\frac{1}{.20} + 1 = \frac{\omega^2}{\omega_n^2} = 6 \qquad \omega_n = \omega/\sqrt{6} = \sqrt{k/m}$$

$$\frac{\omega\sqrt{m}}{\sqrt{6}} = \sqrt{k} \Rightarrow K = \frac{\omega^2 M}{6}$$

$$\boxed{\omega_1 = \frac{1800\,\text{cyl/min}}{60\,\text{sec/min}} \cdot \frac{2\pi\,\text{rad}}{\text{cyl}} = 188.5\,\frac{\text{rad}}{\text{s}}}$$

$$\omega_2 = \frac{3600\,\text{cyc/min}}{60\,\text{sec/min}} \cdot \frac{2\pi\,\text{rad}}{\text{cyc}} = 377\,\text{rad/s}$$

Using lower freq...

$$K_1 = \frac{\omega_1^2 M}{6} = \frac{(188.5\,\text{rad/s})^2 \cdot 20\,\text{kg}}{6} = \boxed{118.4\,\text{kN/m} = K}$$

$$\delta_{st} = \frac{Mg}{K} = \frac{20\,\text{kg} \cdot 9.81\,\text{m/s}^2}{118440\,\text{N/m}} = 1.6\,\text{mm}$$

Comparing with chart: $\omega_n = \sqrt{g/\delta_{st}}$

Using 30 Hz: δ_{st} from chart $\approx 2\,\text{mm} = \delta_{st,\,chart}$

$$\omega_{n,c} = \sqrt{9.81\,\text{m/s}^2 / .002\,\text{m}} = 70.04\,\text{rad/s} = \omega_{n,c}$$

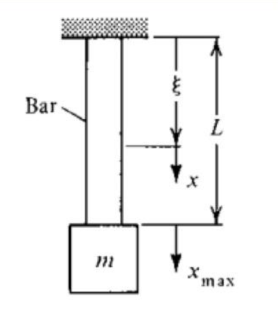
$$K_c = \omega_{n,c}^2 m = (70.04\,\text{rad/s})^2 \cdot 20\,\text{kg} = \boxed{98.11\,\text{kN/m} = K_{chart}}$$

Chart is hard to read as no 80% isolation line, 2mm is an approximation, probably δ_{st} should be read as less from the chart, giving closer answer. However, the springs are within 20% with this rough approximation.

δ_{st} from calculation is found to be 1.6mm vs. 2mm read from chart.

Question 3 (20 points)

A uniform bar of density ρ mass/length with an attached mass is shown in the diagram. Assume the elongation of the bar is linear, i.e. $x/x_{\text{mass}} = \xi/L$. Find the resonant frequency for the longitudinal vibration of the bar.



$\omega_n = \sqrt{k/m_t}$ $\quad k(\text{extension}) = m_t g \quad m_t = m + \rho L$

$\omega_n = \sqrt{g/\text{extension}}$

$\text{extension} = \delta_{st} = L - \xi$

$$\boxed{\omega_n = \sqrt{g/(L-\xi)}}$$

Question 4 (20 points)

(a) What would the speed of sound in ambient air be if the compression and the expansion of the gas were isothermal instead of adiabatic? Calculate the difference (in %) relative to the usual value for an adiabatic process.
[*Note: The isothermal assumption is valid for sound in very small (microscopic) pores.*]

(b) What is the speed of sound in helium (in gaseous state at atmospheric pressure) (i) at room temperature, and, (ii) at 250 °C? (The properties of helium are in your notes).

a) $C_o(\text{adiabatic}) = 343 \text{ m/s}$

isothermal: $P = \rho RT \Rightarrow \frac{\partial P}{\partial \rho} = RT \Rightarrow C_o^2 = RT$

Assuming 20°C, $R = 287$, $T = 293K$ so $C_o = \sqrt{287 \cdot 293} = \boxed{C_o = 290 \text{ m/s}}$

% difference $= \left(1 - \frac{290}{340}\right) \cdot 100\% = \boxed{14.7\% \text{ difference}}$

b) $\gamma_{He} = 1.667$ $R_{He} = 2.077 \text{ kJ/kgK}$ $T = 20°c, 250°c$

i) $c = \sqrt{\gamma RT} = \sqrt{1.667 \cdot 2077 \text{ J/kgK} \cdot 293K} = \boxed{1007 \text{ m/s} = C_{He}}$ 20°C

ii) $c = \sqrt{1.667 \cdot 2077 \text{ J/kgK} \cdot 523K} = \boxed{1346 \text{ m/s} = C_{He}}$ 250°C

Question 5 (10 points)

(a) What are the two necessary components for a system to vibrate and support the wave motion?

(b) There is an office in the floor level of a factory. In the floor level, there is a heavy machinery generating significant noise and vibration causing disturbances in the office. Suggest two possible methods that you will do to reduce the noise levels in the office.

(c) Name the different types of vibration isolators. Suggest which types of vibration isolators you will use if (i) the machine runs at a steady speed, and (ii) the machine will be operated near the resonant frequency of the system for an extended period of time.

(d) A heavy machine producing low frequency vibration in a room of a concrete building with low internal damping. What are the possible remedies if these vibrations cause noise issues?

a) mass and stiffness? $W = \sqrt{k/m}$ or continuity and compressible

b) 1) isolate the machinery from the floor using springs.
2) isolate the offices from the factory floor.

c) Elastometric, Fiberglass and cork, Springs, pneumatic
i) Spring isolators, as the resonance can be arranged optimally
ii) neoprene laminate due to high internal damping to reduce amplification at resonance.

d) reinforcing the floor to increase stiffness or mounting the machine on pillars driven into the ground beneath the floor.

Question 6 (20 points)
Show that
(a) the one-dimensional equation for the linearized conservation of mass is given by

$$\frac{\partial \rho'}{\partial t} + \rho_0 \frac{\partial u'}{\partial x} = 0$$

(b) the one-dimensional linearized Euler' Equation is given by

$$\rho_0 \frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial x} = 0$$

(c) the one-dimensional wave equation can be expressed as

$$\frac{\partial^2 p'}{\partial x^2} - \frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t^2} = 0$$

(d) Show that any functions $f(\xi) = f(t - x/c_0)$ and $g(\eta) = g(t + x/c_0)$ satisfies the one-dimensional wave equation.

a) $\cancel{\rho u}\, dy dz - [\cancel{\rho u} + \frac{\partial u}{\partial x}(\rho u) dx]\, \cancel{dy dz} = \frac{\partial}{\partial t}(\rho\, dx \cancel{dy dz})$

$\frac{\partial u}{\partial x}(\rho u) dx = \frac{\partial}{\partial t}(\rho dx)$, $\rho dx = \rho' + \rho_0$

$\frac{\partial}{\partial t}(\rho_0 + \rho') + \frac{\partial}{\partial x} u'(\rho_0 + \cancel{\rho'})$ → linearize $u'\rho' = 0$

$\frac{\partial \rho_0}{\partial t} = 0$

$\frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x}\rho_0 u' = 0 \Rightarrow \boxed{\frac{\partial \rho'}{\partial t} + \rho_0 \frac{du'}{\partial x}}$

b) $\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} \Rightarrow (\rho_0 + \rho')\frac{Du'}{Dt} = \frac{-\partial(\cancel{p_0} + p')}{\partial x} = \frac{-\partial p'}{\partial x}$

$\frac{-\partial p'}{\partial x} = \rho_0\left[\frac{\partial u'}{\partial t} + u'\cancel{\frac{\partial u'}{\partial x}}\right] + \cancel{\rho'}\left[\cancel{\frac{\partial u'}{\partial t}} + u'\cancel{\frac{\partial u'}{\partial x}}\right]$

linearize $\Rightarrow \boxed{\rho_0 \frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial x} = 0}$

c) derivative of conservation of mass: $\frac{\partial}{\partial t}\left(\frac{\partial \rho'}{\partial t} + \rho_0 \frac{du'}{\partial x}\right) = 0$

com: $\frac{\partial^2 \rho'}{\partial t^2} + \rho_0 \frac{\partial^2 u'}{\partial t \partial x} = 0$

gradient of euler's: $\frac{\partial}{\partial x}\left(\frac{\partial p'}{\partial x} + \rho_0 \frac{\partial u'}{\partial t}\right) = \frac{\partial^2 p'}{\partial x^2} + \rho_0 \frac{\partial^2 u'}{\partial t \partial x} = 0$

$$-\left(\frac{\partial^2 \rho'}{\partial t^2} + \cancel{\rho_0 \frac{\partial^2 u'}{\partial t \partial x}}\right) + \left(\frac{\partial^2 p'}{\partial x^2} + \cancel{\rho_0 \frac{\partial^2 u'}{\partial t \partial x}}\right) = 0$$

$$-\frac{\partial^2 \rho'}{\partial t^2} + \frac{\partial^2 p'}{\partial x^2} = 0 \qquad p' = \left(\frac{\partial p}{\partial \rho}\right)\rho' = c_0^2 \rho' \Rightarrow \rho' = \frac{p'}{c_0^2}$$

$$\boxed{\frac{\partial^2 p'}{\partial x^2} - \frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t^2} = 0}$$

d) $\frac{\partial p}{\partial x} = \frac{\partial}{\partial x} f(t - x/c_0) = \frac{\partial f}{\partial \xi}\left(\frac{\partial}{\partial x}\xi\right) = f'(\partial\xi/\partial x) = \underline{\frac{-1}{c_0} f'(\xi)}$

$\frac{\partial^2 p}{\partial x^2} = \frac{-1}{c_0} f''(\xi)\frac{\partial \xi}{\partial x} = \underline{\frac{1}{c_0^2} f''(\xi)}$ shows that $\frac{\partial p}{\partial t} = f'(\xi)$ and $\frac{\partial^2 p}{\partial t^2} = f''(\xi)$

So $f(\xi)$ is a solution

$\frac{\partial p}{\partial x} = \frac{\partial}{\partial x} g(t + x/c_0) \qquad \eta = t + x/c_0$

$\frac{\partial p}{\partial x} = \frac{\partial g}{\partial x}\left(\frac{\partial}{\partial x}\eta\right) = g'(\partial\eta/\partial x) = \frac{1}{c_0} g'(\eta)$

$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c_0} g''(\eta)\partial\eta/\partial x = \frac{1}{c_0^2} g''(\eta)$

So $g(\eta)$ is a solution as well.