ME413 HW 06

Benjamin Masters

TOTAL POINTS

95 / 100

QUESTION 1

1 Q1 **20 / 20**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

QUESTION 2

2 Q2 **20 / 20**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

QUESTION 3

3 Q3 **15 / 20**

- **- 0 pts** Correct
- **- 5 Point adjustment**

QUESTION 4

4 Q4 **20 / 20**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

QUESTION 5

5 Q5 **20 / 20**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

1 Q1 **20 / 20**

- 0 pts Correct

+ 1 Point adjustment

2 Q2 **20 / 20**

- **- 0 pts** Correct

+ 1 Point adjustment

3 Q3 **15 / 20**

- **- 0 pts** Correct

- 5 Point adjustment

4 Q4 **20 / 20**

- 0 pts Correct

+ 1 Point adjustment

5 Q5 **20 / 20**

- **- 0 pts** Correct

+ 1 Point adjustment

Question 1 (20 points)

The Krakatoa volcanic eruption: Not only did it cause serious damage to the island, the eruption of Krakatoa (located in Indonesia) in 1883 created the loudest sound ever reported at 180 dB. It was so loud it was heard 3,000 miles (5,000 km) away.

(a) Estimate the acoustic pressure, acoustic density, particle velocity and particle displacement closed to source. You may assume the transmission of the eruption sound can be modeled a one-dimensional propagation with no reflections. Take the speed of sound as 340 m/s, ambient pressure as 101.3 kPa and ambient density as 1.21 kg/m^3.

(b) Would it be reasonable to analyze this event with a model of linear acoustics/continuum mechanics? Why or why not?

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$20\,\mu Pa = P_{re}$

$$p = 20\,\mu Pa \cdot 10^{180\,dB/20} = \boxed{20\,kPa = p}$$

$$u = P/\rho_0 c_0 = 20\,kPa/(1.21\,kg/m^3 \cdot 340\,m/s)$$

$$\boxed{u = 48.6\,m/s}$$

$$\rho = P/c_0^2 = \frac{20\,kPa}{(340\,m/s)^2} = \boxed{0.173\,kg/m^3 = \rho}$$

$$\xi = \frac{u}{2\pi \cdot f} \qquad \text{Assume } f = 100\,Hz$$

$$\xi = \frac{48.6\,m/s}{2\pi \cdot 100\,Hz} = \boxed{.0773\,m = \xi}$$

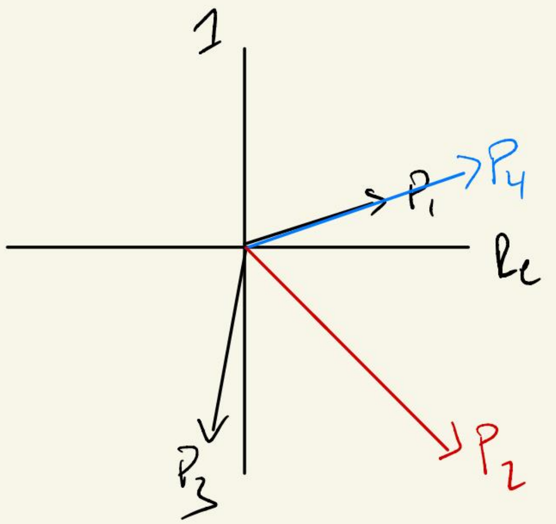
b) yes linear acoustics can be used to analyze because the length of 5000 km is much larger than the mean free path of the system.

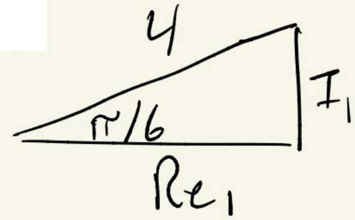
Question 2 (20 points)
The sound field at a point has four components, p_1, p_2, p_3 and p_4 where

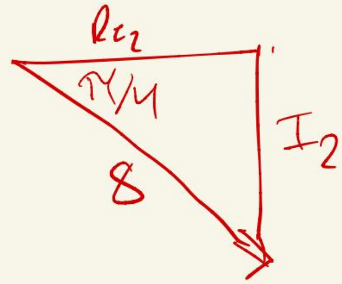
$$p_1 = 4\cos(\omega t + \pi/6)\ ,\ p_2 = 8\cos(\omega t - \pi/4)\ ,$$
and $$p_3 = 6\sin(\omega t - \pi/8)\ .\ p_4 = 5\sin(\omega t + 2\pi/3)$$

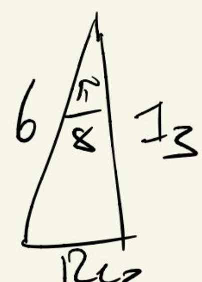
Sketch a phasor diagram to represent these three components.

(a) Determine the total sound pressure, p_t (real and imaginary parts) at the point.
(b) What are the peak pressure and the phase of p_t.
(c) What are mean square and root mean square values for p_1, p_2, p_3, and p_t.



$P_1 \Rightarrow$


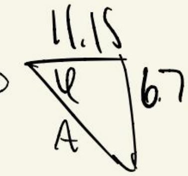
$P_2 \Rightarrow$


$P_3 \Rightarrow$


$P_4 \Rightarrow$


$\Sigma Re = Re_1 + Re_2 - Re_3 + Re_4$
$\Sigma I = I_1 - I_2 - I_3 + I_4$

$\Sigma Re = 4\cos\pi/6 + 8\cos\pi/4 - 6\sin\pi/8 + 5\cos\pi/6$
$Re = 11.15$

$P_t \Rightarrow$


$\Sigma I = 4\sin\pi/6 - 8\sin\pi/4 - 6\cos\pi/8 + 5\sin\pi/6$
$I = -6.70$

b) $A_{pk} = \sqrt{11.15^2 + 6.70^2} = \boxed{13.01 = A_{pk}}$ $\quad \varphi = \tan^{-1}\left(\frac{6.70}{11.15}\right) = \boxed{-0.54\,rad}$

c) $P^2_{1MS} = \frac{|P_1|^2}{2} = 4^2/2 = 8$ $\quad P_{1rms} = \sqrt{8}$; $P^2_{2MS} = \frac{|P_2|^2}{2} = 32$ $\quad P_{2rms} = \sqrt{32}$

$P^2_{3MS} = \frac{|P_3|^2}{2} = 6^2/2 = 18$ $\quad P_{3rms} = \sqrt{18}$; $P^2_{tMS} = \frac{13.01^2}{2} = 84.6 = P^2_{tMS}$

$P_{trms} = 9.20$

Question 3 (20 points)
An underwater sonar beam of diameter 0.75 m carries 50 watts of acoustic power in a plane wave of frequency 25 kHz. Determine
(a) the wavelength,
(b) the sound pressure level in dB, (you should take the reference pressure for water as 1×10^{-6} Pa.)
(c) the maximum particle velocity in the beam of sound and the maximum particle displacement.

(You may take the density and sound speed of water as 1000 kg/m^3 and 1480 m/s respectively.)

$f = 25\,kHz \quad \rho = 1000\,kg/m^3 \quad c_0 = 1480\,m/s \quad d = .75\,m \quad W = 50\,W$

$P_{ref} = 1\times10^{-6}\,Pa$

$$A = \frac{\pi d^2}{4} = \frac{\pi(.75\,m)^2}{4} = 0.442\,m^2$$

b) $Z = \rho c_0 = 1000\,kg/m^3 \cdot 1480\,m/s = 1480\,kN\cdot s/m^3$

$$W = \frac{p^2 A}{Z} \quad p^2 = \frac{ZW}{A} \Rightarrow p = \sqrt{\frac{1480\,kN\cdot s/m^3 \cdot 50\,W}{.442\,m^2}}$$

$p = 12.94\,kPa$

$L_p = 2\cdot10\log(12.94\,kPa/1\,\mu Pa) = \boxed{202.2\,dB = L_p}$

a) $\lambda = \frac{c_0}{f} = 1480\,m/s \,/\, 25\,kHz = \boxed{\lambda = .059\,m}$

c) $P/Z = u = \frac{12.94\,kPa}{1000\,kg/m^3 \cdot 1480\,m/s} = \boxed{.0087\,m/s = u}$

$$\xi = \frac{u}{\omega} = \frac{u}{2\pi f} = \frac{.0087\,m/s}{2\pi\cdot 25\,kHz} = \boxed{5.54\times10^{-8}\,m = \xi}$$

Question 4 (20 points)

(a) What is the increase in SPL at a certain point if the intensity at that point is doubled?

(b) Calculate the sound pressure level that corresponds to a sound pressure of 0.04 Pa.

(c) Calculate the sound pressure in Pa, corresponding to an SPL of 87 dB.

$P_{re} = 20\mu Pa$

a) $I = \frac{p^2}{Z}$ if I doubled, P is multiplied by $\sqrt{2}$

so $\Delta L_p = L_{p_2} - L_{p_1} = 20\log(\sqrt{2}p/p_{re}) - 20\log(p/p_{re})$

$\Delta L_p = 20\log\left(\frac{\sqrt{2}\cancel{p}}{\cancel{p_{re}}} \cdot \frac{\cancel{p_{re}}}{\cancel{p}}\right) = 20\log(\sqrt{2}) = \boxed{3.01\,dB = \Delta L_p}$

b) $L_p = 20\log(.04\,Pa / 20\mu Pa) = \boxed{66\,dB = L_p}$

c) $P = 20\mu Pa \cdot 10^{87dB/20dB} = \boxed{0.448\,Pa = P}$

Question 5 (20 points + extra credits of 50 points)

A pipe, which has a length of 1.2 m, is being driven at one end by a piston (negligible mass) and is open at the other end. We wish to determine the locations of node and anti-nodes along the pipe.

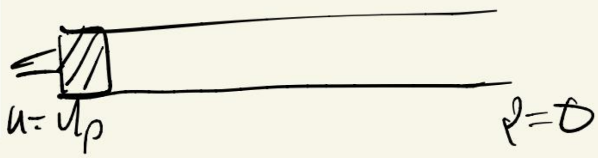
(a) What are the boundary conditions for solving this problem?

(b) Derive an expression to calculate the pressure distribution and the particle velocity distribution along the pipe. Sketch the diagrams for the pressure and velocity distributions.

(c) What are the resonant frequencies of the open pipe? What is the fundamental resonant frequency?

(To get your numerical answers, you may take any reasonable values for the air density and the sound speed in air.)

$L = 1.2\,m$



a) $u(0) = \mathrm{Re}[u_p e^{i\omega t}]$

$p(L,t) = 0$

b) $\frac{\partial^2 p}{\partial x^2} + k^2 p = 0 \Rightarrow p(x) = Ae^{-ikx} + Be^{ikx}$

$u(x) = \frac{Ae^{-ikx}}{\rho_0 c_0} - \frac{Be^{ikx}}{\rho_0 c_0}$

$u(0) = \frac{Ae^{(0)}}{\rho_0 c_0} - \frac{Be^{(0)}}{\rho_0 c_0} = \mathrm{Re}[u_p e^{i\omega t}]$

$\rho_0 c_0 (u_p) = A - B \Rightarrow A = \rho_0 c_0 u_p + B$ ①

$p(L) = Ae^{-ikL} + Be^{ikL}$ ②

$1 \to 2$ $\rho_0 c_0 u_p e^{-ikL} + Be^{-ikL} + Be^{ikL} = 0$

$\rho_0 c_0 u_p e^{-ikL} + B(e^{ikL} + e^{-ikL})$

$B = \frac{-\rho_0 c_0 u_p e^{-ikL}}{e^{ikL} + e^{-ikL}} \Rightarrow \boxed{B = \frac{-\rho_0 c_0 u_p e^{-ikL}}{2\cos kL}}$

$A = \rho_0 c_0 u_p - \frac{\rho_0 c_0 u_p e^{-ikL}}{2\cos kL} = \rho_0 c_0 u_p \left(1 - \frac{e^{-ikL}}{2\cos kL}\right)$

$\frac{\rho_0 c_0 u_p}{2\cos kL}\left(2\cos kL - e^{-ikL}\right) = \frac{\rho_0 c_0 u_p}{2\cos kL}\left(e^{ikL} + \cancel{e^{-ikL}} - \cancel{e^{-ikL}}\right)$

$\boxed{A = \frac{\rho_0 c_0 u_p e^{ikL}}{2\cos kL}}$

$$\boxed{u(x) = \frac{u_p e^{ikL}}{2\cos kL} e^{-ikx} + \frac{u_p e^{-ikL}}{2\cos kL} e^{ikx}}$$

$$\boxed{p(x) = \frac{\rho_0 c_0 u_p e^{ikL}}{2\cos kL} \cdot e^{-ikx} - \frac{\rho_0 c_0 u_p e^{-ikL}}{2\cos kL} \cdot e^{ikx}}$$

c) when $\cos kL \to 0$ $p, u \to \infty$

$$k_n = (2n+1)\frac{\pi}{2L} \Rightarrow \frac{2\cancel{\pi} f_n}{c_0} = (2n+1)\frac{\cancel{\pi}}{2L}$$

$$\boxed{f_n = \frac{(2n-1)c_0}{4L}}$$

fundamental $= f_1 = f_{n=1} = \boxed{f_1 = \frac{c_0}{4L} = \frac{340\,m/s}{4(1.2m)} = 70.8\,Hz}$

Sketch for b)