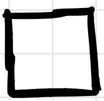
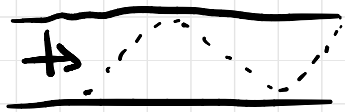
2.0 Pipes, Resonators, and Filters

Kinsler & Frey Ch. 10

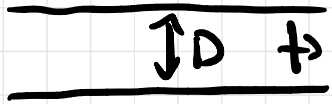
- Series Connected Duct Elements
  - automotive muffler
  - acoustic filters
- Plane Wave Approach - Low frequencies

 $L < \frac{\lambda}{2}$



- Lumped elements
  - small compared to a wavelength
  - geometrical details don't matter
- Impedances
- Duct area changes & brenches
- Acoustic filters
  - low pass
  - high pass
  - bandstop/band pass

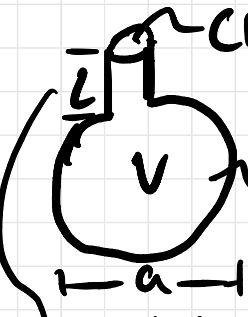
In all cases:
low freq approach.



$\rightarrow D < \frac{\lambda}{2}$

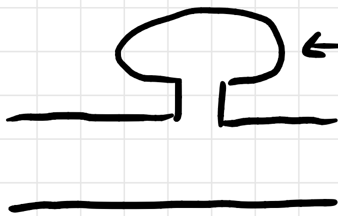
Plane wave propagation

2.1 Helmholtz Resonators



cross-sectional area of neck

volume



Helmholtz Resonater

neck Length, L - Fluid in neck acts like a solid body moving back n forth

$ka \ll 1$
Lumped elements

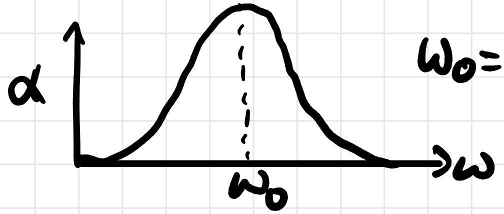
Pressure is Assumed to be instentaneously constant

- Shape is not important
- incompressible in neck
- fluid in neck acts as inertial element
- fluid trapped in reservoir acts like a stiffness

Frictional losses due to viscosity as air moves back and forth in the neck
- Thermoviscous boundary layers form in the neck

- Radiation from the mouth to infinity (radiative loss)

SDOF



$\omega_0 = \sqrt{S_e/m}$

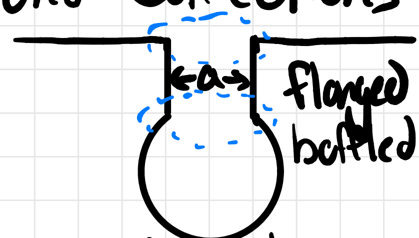
2.2 Mass Modeling

$m = \rho_0 S_a L'$ ~ effective length

$L' = L$ + end corrections

~ end correction

End Corrections

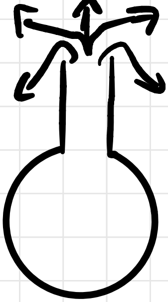


flanged case
baffled case

$E.C. \simeq \frac{8}{3\pi} a \simeq 0.85a$

Added to both inner and outer ends

total end correction $L' = L + 1.7a$



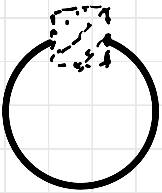
unflanged Case:

external: 0.6a - outer end-correction

.85a - inner end-correction

Total $= 0.6a + 0.85a = 1.45a$

$L' = L + 1.45a$



... 0 neck length...

still have end-corrections

$L' = 0.6a + 0.85a$

True even when $L = 0$

2.2.2 Stiffness

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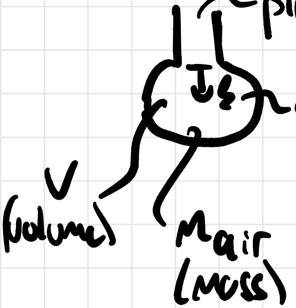


plug of fluid

ξ displacement of the piston (plug of fluid) ... positive into the reservoir

$$p = B s$$

B bulk modulus of the fluid

s condensation

$$s = \frac{\rho - \rho_0}{\rho} = \frac{\Delta\rho}{\rho_0}$$

ρ instantaneous density

Initial Density

$$\rho_0 = \frac{M_{air}}{V}$$

Final Density

$$\rho = \frac{M_{air}}{V + \Delta V}$$

$$s = \frac{\rho - \rho_0}{\rho_0} = \frac{\dfrac{\cancel{M_{air}}}{V+\Delta V} - \dfrac{\cancel{M_{air}}}{V}}{\dfrac{\cancel{M}_{air}}{V}} = \frac{V}{V + \Delta V} - 1$$

$$s = \frac{-\Delta V}{V(1 + \cancel{\Delta V / V})} \rightarrow 0$$

due to small changes and linear acoustics assumption

$$s \simeq \frac{-\Delta V}{V} \simeq + \frac{S_a \xi}{V}$$

S_a cross-sectional neck area

ξ displacement of fluid in neck

$p = Bs \qquad B = \rho_0 c^2 = \gamma P_0$ ~ ambient pressure

$\simeq 1.4 \times 10^5$ Pa

$c = \dfrac{\gamma P_0}{\rho_0} \simeq \dfrac{\sqrt{\rho_0 c^2}}{\rho_0}$ ~ stiffness

$\rho_0 c$ - impedence

$\rho_0 c^2$ - stiffness

$p = Bs = \rho_0 c^2 \cdot \dfrac{S_a}{V} \cdot \xi$

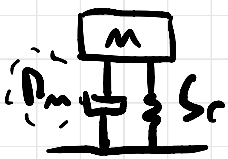
pressure inside reservoir

- force per unit area resisting the inward motion of the piston

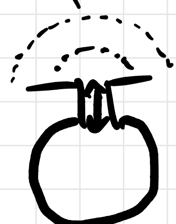
$f = S_a p = \left(\dfrac{\rho_0 c^2 S_a^2}{V} \right) \xi$

force = ? · displacement $\Rightarrow \dfrac{\text{force}}{\text{displacement}}$ = Mechanical Stiffness of air trapped in reservoir

$s_r = \dfrac{\rho_0 c^2 S_a^2}{V}$



Sound Radiation



2.2.3 Resistace

- visco thermal boundary layer forms in the neck
  - shearing of the fluid → energy loss
- heat transfer from compressed fluid in the res. to the walls

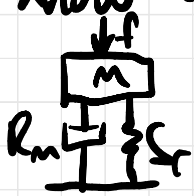
- heat transfer back and forth from fluid to walls
  - irreversible → energy loss
- Sound Radiation Loss

$$R_m = \underbrace{R_w}_{\text{viscous and thermal losses}} + \underbrace{R_r}_{\text{radiation to infinity}}$$

<u>R_m is usually small</u>

Formulas - K + F Ch. 10

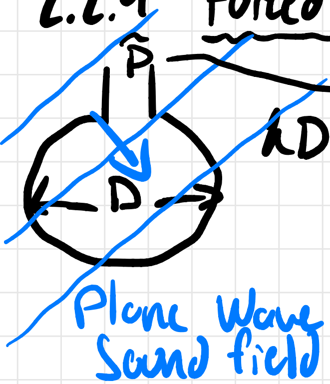
Now know all 3...



Mechanical Impedance

$$Z_m = \frac{f}{u} \sim \dot{z}$$

2.2.4 Forced Response



$kD \ll 1 \ \ldots \ D \ll \lambda$

$\hat{P}_{ext}$ at resonator mouth is unaffected by the presence of the resonator

External Driving Force:

$$\hat{f} = S_a \hat{p}_x e^{j\omega t}$$

EOM

$$M\frac{\partial^2 \xi}{\partial t^2} + R_m \frac{\partial \xi}{\partial t} + S_r \xi = S_a \tilde{P} e^{j\omega t}$$

Convenient to Solve for $j\omega\tilde{\xi}$ = complex velocity

Mechanical impedance...

= ratio of driving force to velocity

$= S_a \tilde{P} / j\omega \xi$

$$Z_M = R_m + j\omega\left(M - \frac{S_r}{\omega}\right)$$

Acoustic Impedence = Ratio of Pressure to Volume Velocity

$$Z_a = \frac{Z_m}{S_a^2}$$

2.2.5 Natural Frequency

$$Im\{Z_m\} = 0$$

$$\omega_0 M - \frac{S_r}{\omega_0} = 0$$

$$\boxed{\omega_0 = \sqrt{\frac{S_r}{M}} = c\sqrt{\frac{S_a}{VL'}}}$$

w_0 decreases as neck length increases

w_0 decreases as volume increases

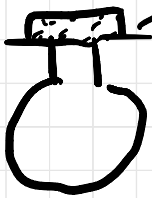
w_0 increases as neck area increases

Resonance occurrs when driven at the natural Freq

- large velocity in the neck
  - dissipate significant amounts of energy due to viscous boundary layer in neck

- Acoustic Absorption

May want to increase R_m

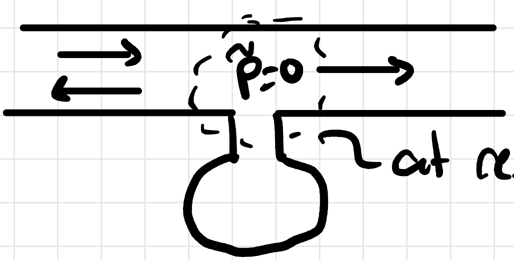


fabric layer to increase resistance

Helmholtz Resonators

(i) Used to absorb energy at the natural frequency

(ii) Used to reflect energy in duct systems due to the input impedance at resonance



at resonance

reflected at pressure release with $R=-1$

2.2.6 Pressure Amplifier

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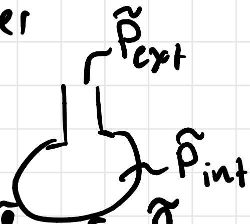

EOM in harmonic form

$$-\omega^2 M\tilde{\xi} + j\omega R_m\tilde{\xi} + S_R\tilde{\xi} = S_a\tilde{P}_{ext}$$

$$\tilde{\xi} = \frac{S_a\tilde{P}_{ext}}{-\omega^2 M + j\omega R_m + S_R}$$

$$\tilde{S} = \frac{S_a}{V}\tilde{\xi}$$

$$\tilde{P}_{int} = B\tilde{S}$$

$$\tilde{P}_{int} = \left(\frac{S_a^2 B}{j\omega V\left(j\omega M - \frac{jS_R}{\omega} + R_m\right)}\right)\tilde{P}_{ext}$$

if you drive at ω_0

$$\frac{\tilde{P}_{int}}{\tilde{P}_{ext}} = \frac{S_a^2 B}{j\omega V(R_m)}$$

Can perform freq analysis using a set of helmholtz resonators

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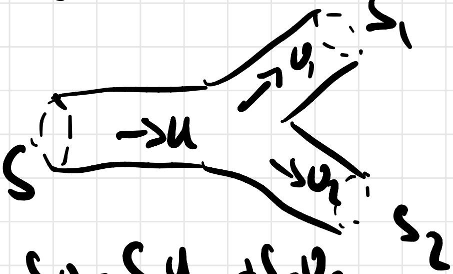
2.3 Acoustic Impedances

Acoustic impedance is analogous to electrical impedance

Voltage ⟺ Pressure
Current ⟺ Volume Velocity
Inductance ⟺ Mass
Capacitance ⟺ Stiffness
Resistance ⟺ Resistance

At a junction:



$Su = S_1 u_1 + S_2 u_2$

Acoustic Impedance

$$Z_a = \frac{p}{U} = \frac{p}{Su}$$

p ~ sound pressure

U ~ volume velocity

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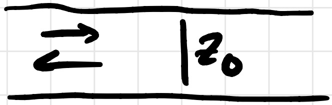
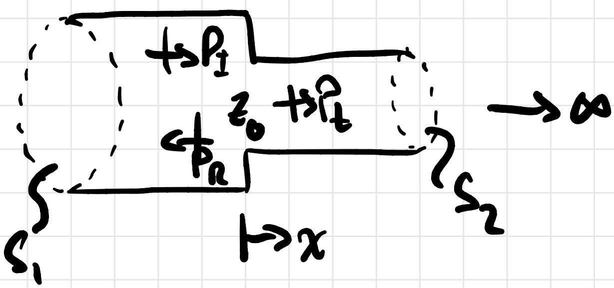
Acoustic Impedance $= \dfrac{p}{U}$ ~ volume velocity



$$R = \frac{Z_0 - \frac{\rho_0 c}{S}}{Z_0 + \frac{\rho_0 c}{S}}$$

$$R_\Pi = |R|^2 \qquad T_\Pi = 1 - R_\Pi = \alpha$$

2.4.2 Area transition



Z_0 = acoustic impedance "looking into" the downstream section

$$p_t = C e^{-jkx}$$

$$u_t = \frac{C}{\rho_0 c} e^{-jkx} \quad \text{... particle velocity}$$

$$U_t = \frac{S_2 \cdot C}{\rho_0 c} e^{-jkx} = S_2 \cdot u_t$$

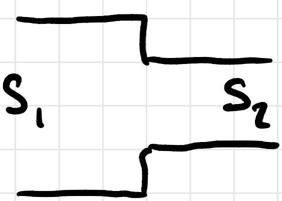
$$Z_0 = \frac{P_t}{U_t}\bigg|_{x=0} = \frac{c}{S_2 c} \cdot \rho_0 c = \frac{\rho_0 c}{S_2} = Z_0$$

$$R = \frac{Z_0 - \frac{\rho_0 c}{S_1}}{Z_0 + \frac{\rho_0 c}{S_1}} = \frac{\frac{\rho_0 c}{S_2} - \frac{\rho_0 c}{S_1}}{\frac{\rho_0 c}{S_2} + \frac{\rho_0 c}{S_1}} = \frac{\frac{1}{S_2} - \frac{1}{S_1}}{\frac{1}{S_2} + \frac{1}{S_1}}$$

$$R = \frac{1 - \left(\frac{S_2}{S_1}\right)}{1 + \left(\frac{S_2}{S_1}\right)}$$

$$R_\pi = |R|^2$$

$$T_\pi = \frac{4\left(\frac{S_2}{S_1}\right)}{\left(1 + \left(S_2/S_1\right)\right)^2}$$



Special Cases

$\frac{S_2}{S_1} \to 1$ $R \to 0$ $T \to 1$ ... no area change

$\frac{S_2}{S_1} \to 0$ $R \to 1$ $T \to 0$ ... rigid termination

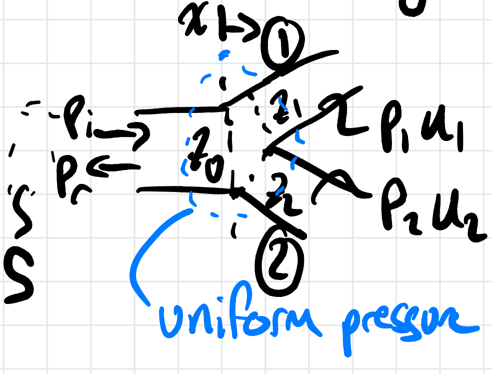
$\frac{S_2}{S_1} \to \infty$ $R \to -1$ ... Pressure Release Case

Magnitude of R

$$\frac{S_2}{S_1} = 10 \text{ or } \frac{1}{10} \ldots \ |R| \text{ is the same}$$

if $\frac{S_2}{S_1} < 1$ ... contraction so R is positive
~inphase reflection

2.4.5 Splitting duct Case



Assume junction is small compared to a wavelength

- point junction

at $x=0$ $\quad P_i + P_r = P_1 = P_2 \quad (1)$

$$U_i + U_r = U_1 + U_2 \quad (2)$$

Divide (2) by (1)

$$\frac{U_i + U_r}{P_i + P_r} = \frac{U_1 + U_2}{P_1} = \frac{U_1 + U_2}{P_2}$$

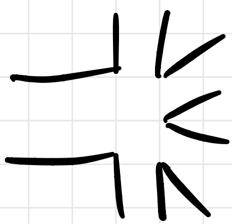
$$\left(\frac{1}{Z_0}\right)_{x=0} = \frac{1}{Z_1}\bigg|_{x=0} + \frac{1}{Z_2}\bigg|_{x=0}$$

parallel addition of impedances

- Smallest impedance is the most important

$$Z_0 = \frac{Z_1 Z_2}{Z_1 + Z_2} \Rightarrow R = \frac{Z_0 - \frac{\rho_0 c}{S}}{Z_0 + \frac{\rho_0 c}{S}}$$

Easy to extend to many branches

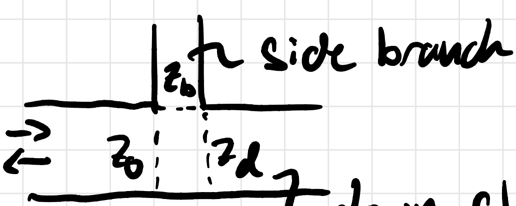


$$\frac{1}{Z_0} = \frac{1}{Z_1} + \frac{1}{Z_2} \cdots + \frac{1}{Z_n}$$

n = num of branches

Junction must be small compared to a wavelength

Side branches can be treated as junctions



$$\frac{1}{Z_0} = \frac{1}{Z_d} + \frac{1}{Z_b}$$

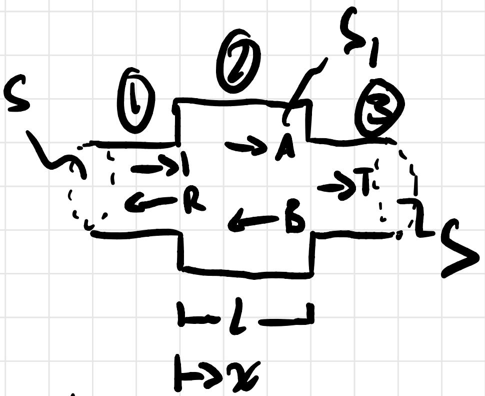
* made no assumptions about the impedances

2.5 Acoustic Filters

- used in Automotive mufflers, HVAC systems, compressors, musical instruments
- low pass, band pass, band stops, high pass

· Combine expansions, contractions, orifices, side branches
· All dimensions, $D < \lambda/2$

2.5.1 - Low Pass



4 unknowns, apply 2 b.c.'s at each junction

- pressure
- volume velocity

① $P_1 = e^{-jkx} + Re^{jkx}$

$U_1 = \frac{S}{\rho_0 c}\left[e^{-jkx} - Re^{jkx}\right]$

② $P_2 = Ae^{-jkx} + Be^{jkx}$

$U_2 = \frac{S_1}{\rho_0 c}\left[Ae^{-jkx} - Be^{jkx}\right]$

③ $P_3 = Te^{-jkx}$

$V_3 = \frac{S}{\rho_0 c} T e^{-jkx}$

interested in R & T

at $x=0$ $\quad 1 + R = A + B$

$$\frac{S}{\rho_0 c} - \frac{SR}{\rho_0 c} = \frac{S_1 A}{\rho_0 c} - \frac{S_1 B}{\rho_0 c}$$

at $x = L$

$$Ae^{-jkL} + Be^{jkL} = Te^{-jkL}$$

$$\frac{S_1}{\rho_0 c} Ae^{-jkL} - \frac{S_1}{\rho_0 c} Be^{jkL} = \frac{S}{\rho_0 c} Te^{-jkL}$$

$$\begin{bmatrix} & & & \\ & 4\times 4 & & \\ & & & \end{bmatrix} \begin{bmatrix} A \\ B \\ R \\ T \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

Solve for R and T

Power Transmission Coefficient

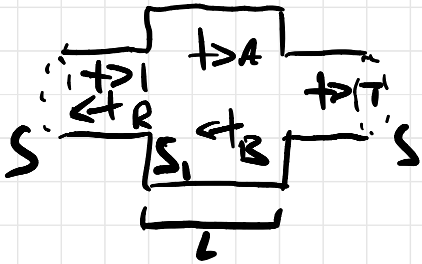
$T_{\pi} = |T|^2$ (since the upstream and downstream cross-sectional areas are the same)

$$T_{\pi} = \frac{4}{4\cos^2 kL + \left(\frac{S_1}{S} + \frac{S}{S_1}\right)^2 \sin^2 kL}$$

2.5 Acoustic Filters

2/09/22

2.5.1 Low pass Filter

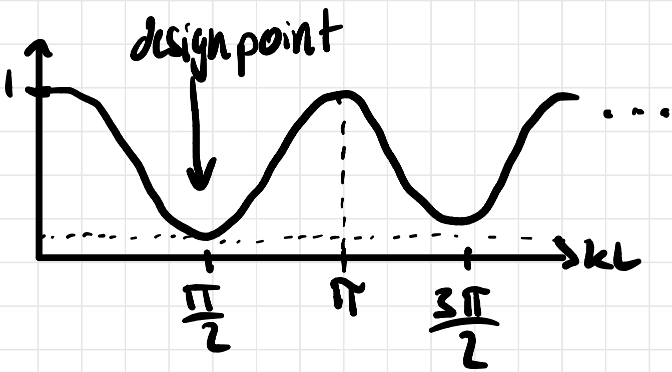


$$T_\pi = |T|^2$$

$$T_\pi = \frac{4}{4\cos^2 kL + \left(\frac{S_1}{S} + \frac{S}{S_1}\right)\sin^2 kL}$$

$$kL \ll 1 \Rightarrow T_\pi \simeq 1$$

Low freq's pass without attenuation



$$\frac{2\pi L}{\lambda} = \frac{\pi}{2} \Rightarrow \frac{L}{\lambda} = \frac{1}{4}$$ ... for max attenuation

if $kL = \frac{\pi}{2}$ $\cos \to 0$

$$T_\pi = \frac{4}{\frac{S_1}{S} + \frac{S}{S_1}} \Rightarrow T_{\pi_{min}} = \left[\frac{2\left(\frac{S}{S_1}\right)^2}{1 + \left(\frac{S}{S_1}\right)^2}\right]^2$$

depth of minimum is controlled by the expansion ratio

if expansion is large

$\frac{S}{S_1} << 1$ $T_{\pi_{min}} = 4\left(\frac{S}{S_1}\right)^2$

Subject to the constraint that we should be operating in the plane wave region

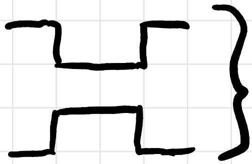
Notes:

i) "Attenuation" produced by reflection (reactive systems - no energy dissipation)

ii) S_1 can be larger or smaller and create the same effect

$\frac{S_1}{S} = 10$ or $\frac{S_1}{S} = \frac{1}{10}$ T_π is the same in both cases (expansion or contraction)

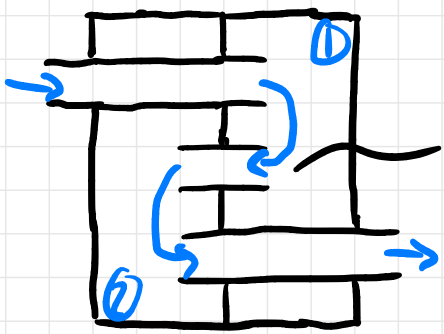
lower back pressure
(less flow resistance)



limited space

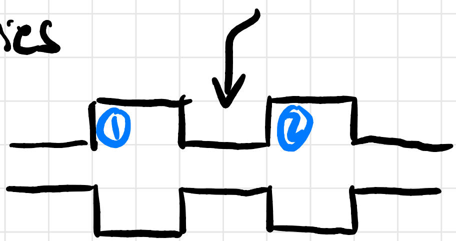
Real automotive mufflers are more complicated
- greater attenuation over range of freq's

Triple Pass -



Possibly Perforated elements to add energy dissipation

Series

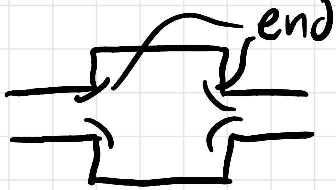


if all dimensions are small compared to a wavelength, only plane waves propagate so plane wave analysis works and the details of geometry are not important

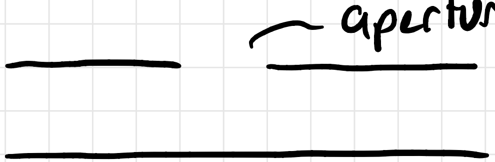
Higher Frequencies

- Finite Element Software
- Boundary Elements
- Modal expansion

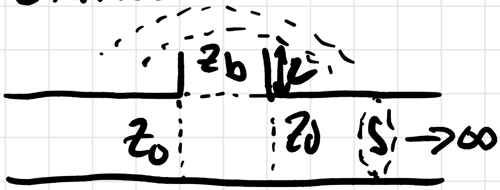
Flow - also complicates
- sound speed in up and downstream sections is different



2.5.2 High Pass Filter



- Allows only HF components to propagate into downstream section past the orifice



$$\frac{1}{Z_0} = \frac{1}{Z_b} + \frac{1}{Z_d}$$

$$Z_d = \frac{\rho_0 c}{S}$$

Z_b = radiation acoustic impedance of the aperture
~ model as an unflanged piston

$Im\{Z_b\}$ = mass reactance of fluid in the neck + mass reactance of an unflanged piston (exterior end correction) + + internal end correction (flanged opening)

$\underbrace{Re\{Z_b\}}_{\text{very small at low freq's}}$ = resistance = radiation away to infinity + viscothermal losses in the neck

$$Z_b \simeq \frac{\rho_0 c k^2}{4\pi} + j\left(\frac{\rho_0 L' \omega}{\pi \underbrace{a}_{\text{aperture radius}}^2}\right)$$

~ Mass-like impedance

L' - effective neck length

$$L' = L + \underbrace{1.5a}_{\text{end corrections}}$$ ~ inner flanged, outer unflanged

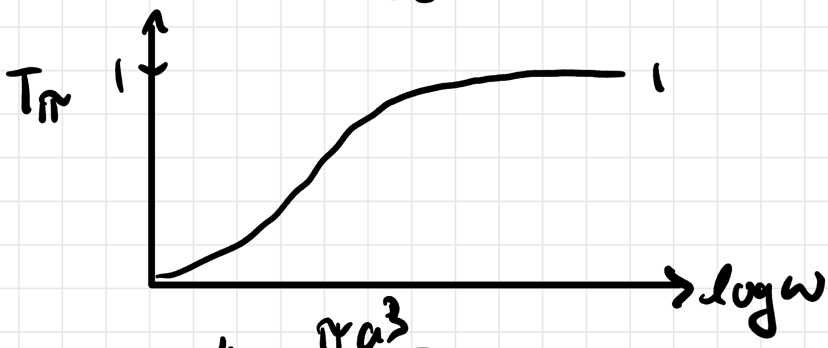
at low freq's...

$z_0 \simeq z_b \to 0$ as $\omega \to 0$

$z_0 = \frac{p}{u} \to 0$ as $p \to 0$ ... creates pressure release b.c. at the junction

$R \to -1$ as $\omega \to 0$

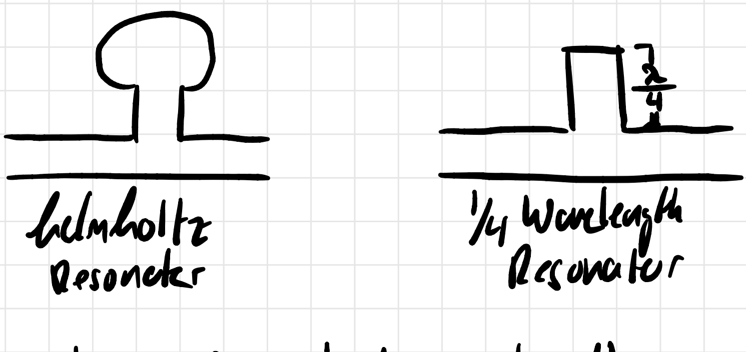
As $\omega \to HF$ $\frac{1}{Z_b} \to 0$ $Z_0 \simeq Z_0$



$k = \frac{\pi a^3}{2SL'}$

2.5.3 Bandstop filters

- attenuate a particular frequency
- side branch

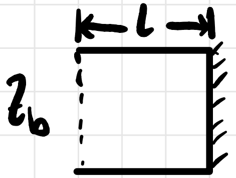


Reactive Devices

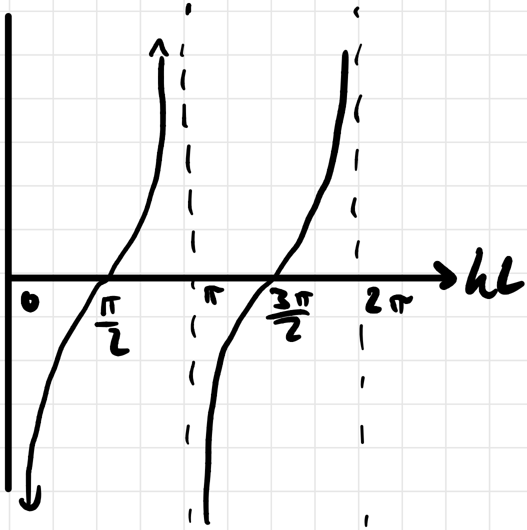
input impedance looking into the resonator $\to 0$ at tuning frequency

$Z_b \to 0$, $Z_0 \to 0$, $R \to -1$

Quarter Wave Resonator



$$Z_b = -j\frac{\rho_0 c}{S}\cot kL$$

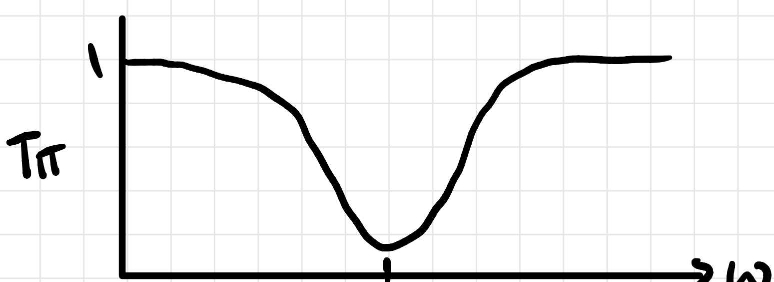


when $kL = \frac{\pi}{2}$

$\frac{2\pi}{\lambda}L = \frac{\pi}{2} \Rightarrow \frac{L}{\lambda} = \frac{1}{4}$

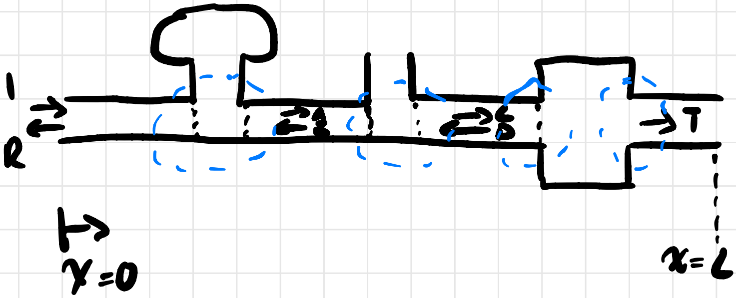
input impedance of a rigidly terminated tube $\to 0$ at freq when $L/\lambda = 1/4$

$$\frac{1}{Z_G} = \frac{1}{Z_b} + \frac{1}{Z_0} \Rightarrow R \to -1 + T_\pi \to 0$$



ω_0 ≈ natural freq of the sidebranch

2.5.4 Combinations



6 unknowns

6 boundary conditions (3x2)

Create system of equations by substituting plane wave solutions into b.c.'s

$$\begin{bmatrix} 6\times 6 \\ \text{Coeff} \end{bmatrix} \begin{bmatrix} R \\ A \\ \vdots \\ T \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

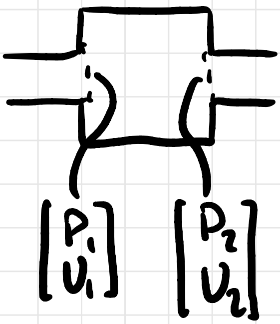
~ Forcing vector

Solve for constants as you wish

Modify system of equations each time you add a new element

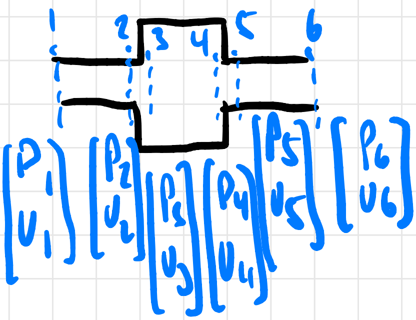
2.6 Transfer Matrix Approach

2.6.1 General Approach



- Relate p and v on 2-sides of an element by a T-Matrix [2×2]
- flexible assembly of complicated systems

$$\begin{bmatrix} p_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p_2 \\ v_2 \end{bmatrix}$$



$$\begin{bmatrix} p_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} T_1 \end{bmatrix} \begin{bmatrix} p_2 \\ v_2 \end{bmatrix}$$

$$\begin{bmatrix} p_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} T_2 \end{bmatrix} \begin{bmatrix} p_3 \\ v_3 \end{bmatrix}$$ and so on

$$\begin{bmatrix} p_1 \\ v_1 \end{bmatrix} = \underbrace{\begin{bmatrix} T_1 \end{bmatrix}}_{[2\times 2]} \underbrace{\begin{bmatrix} T_2 \end{bmatrix}}_{[2\times 2]} \underbrace{\begin{bmatrix} T_3 \end{bmatrix}}_{[2\times 2]} \underbrace{\begin{bmatrix} T_4 \end{bmatrix}}_{[2\times 2]} \underbrace{\begin{bmatrix} T_5 \end{bmatrix}}_{[2\times 2]} \begin{bmatrix} p_6 \\ v_6 \end{bmatrix} = \underbrace{\begin{bmatrix} T_t \end{bmatrix}}_{[2\times 2]} \begin{bmatrix} p_6 \\ v_6 \end{bmatrix}$$

$$P_1 = 1 + R \qquad P_6 = T$$

$$U_1 = \frac{S}{\rho_0 c}(1 - R) \qquad U_6 = \frac{S}{\rho_0 c} T$$

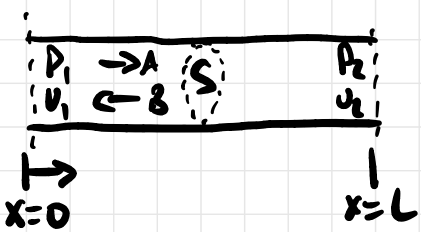
multiply out to obtain 2 equations and 2 unknowns

Powerful approach to modeling

- T-matrix are simple
- multiply together to obtain complete system matrix
- lends itself to automated model assembly

2.6.3 Transfer Matrices for Simple Elements

2.6.3.1 Straight Pipe Section



$$P_1 = A + B \qquad P_2 = Ae^{-jkL} + Be^{jkL}$$

$$U_1 = \frac{S}{\rho_0 c}(A - B) \qquad U_2 = \frac{S}{\rho_0 c}\left(Ae^{-jkL} - Be^{jkL}\right)$$

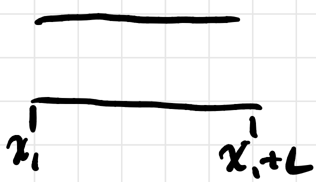
$$A = \frac{1}{2}\left(P_2 + \frac{\rho_0 c}{S} U_2\right) e^{-jkL} \qquad e^{-jkL} = \cos kL - j \sin kL$$

$$B = \frac{1}{2}\left(P_2 - \frac{\rho_0 c}{S} U_2\right) e^{jkL}$$

$$P_1 = (\cos kL) P_2 + j\left(\frac{\rho_0 c}{S} \sin kL\right) U_2$$

$$U_1 = \left(j \frac{S}{\rho_0 c} \sin kL\right) P_2 + (\cos kL) U_2$$

$$\begin{bmatrix} P_1 \\ U_1 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos kL & j\frac{\rho_0 c}{S} \sin kL \\ j\frac{S}{\rho_0 c} \sin kL & \cos kL \end{bmatrix}}_{\text{T-matrix for a pipe of length } L \quad x=0 \Rightarrow x=L} \begin{bmatrix} P_2 \\ U_2 \end{bmatrix}$$

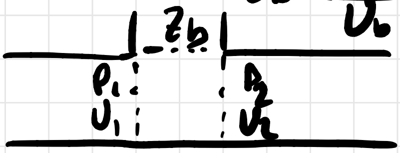


... produces same result

Transfer matrix is independent of absolute position or origin

2.6.3.2 Side branch

$$Z_b = \frac{P_b}{U_b}$$



Assumptions:
- Lumped Element
- Point Junction
- Pressure is uniform throughout the junction

$$P_1 = P_2 = P_b$$

$$P_b = Z_b U_b$$

$$P_2 = Z_b U_b \Leftarrow U_b = U_1 - U_2$$

$$P_2 = Z_b(U_1 - U_2)$$

$$U_1 Z_b = P_2 + U_2 Z_b$$

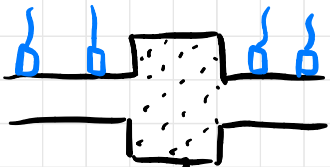
$$U_1 = \frac{P_2}{Z_b} + U_2$$

$$\begin{bmatrix} P_1 \\ U_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_b} & 1 \end{bmatrix} \begin{bmatrix} P_2 \\ U_2 \end{bmatrix}$$

transfer matrix across a side branch of impedance Z_b

Side branch: orifice/aperture, helmholtz resonator, 1/4 wave resonator

2.6.5.3 Unknown Systems



4 - microphones

2 - terminations (rigid + absorbing)

black box - unknown internal structure

$$\begin{bmatrix} p_1 \\ v_1 \end{bmatrix}_1 = [T] \begin{bmatrix} p_2 \\ v_2 \end{bmatrix}_1$$

Measurement with rigid termination

- repeat the measurement with a different termination

$$\begin{bmatrix} p_1 \\ v_1 \end{bmatrix} = [T] \begin{bmatrix} p_2 \\ v_2 \end{bmatrix}_2$$

$$\begin{bmatrix} p_{1_1} \\ v_{1_1} \\ p_{1_2} \\ v_{1_2} \end{bmatrix} = \begin{bmatrix} [T] & 0 \\ 0 & [T] \end{bmatrix} \begin{bmatrix} p_{2_1} \\ v_{2_1} \\ p_{2_2} \\ v_{2_2} \end{bmatrix}$$

4 eqn's & 4 unknowns

solve for $T_{11}, T_{12}, T_{21}, T_{22}$

Assumption: properties of unknown system don't depend on the downstream termination condition

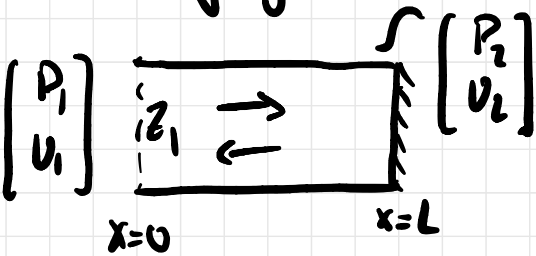
Conduct this experiment numerically - with boundary or finite element model of the system

Two-Load Method

- Munjal - Duct Acoustics
- Song + Bolton (JASA 2000)

2.6.4 Examples

2.6.4.1 Rigidly terminated tube-section



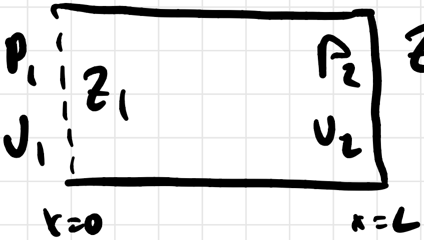
What is Z_1?

$$\begin{bmatrix} P_1 \\ U_1 \end{bmatrix} = \begin{bmatrix} \cos kL & j\frac{\rho_0 c}{S}\sin kL \\ j\frac{S}{\rho_0 c}\sin kL & \cos kL \end{bmatrix} \begin{bmatrix} P_2 \\ \cancel{U_2} \rightarrow 0 \end{bmatrix}$$

$$P_1 = \cos kL \, P_2$$

$$U_1 = j\frac{S}{\rho_0 c}\sin kL \, P_2 \Rightarrow Z_1 = \frac{\cos kL}{\sin kL} \cdot \frac{\rho_0 c}{jS}$$

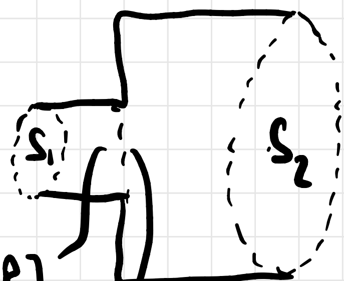
$$Z_1 = -j\frac{\rho_0 c}{S}\cot kL$$



Z_t, known but arbitrary

$$\begin{bmatrix} P_1 \\ U_1 \end{bmatrix} = \left[T, \text{ straight pipe} \right] \begin{bmatrix} P_2 \\ U_2 \end{bmatrix} \qquad U_2 = \frac{P_2}{Z_t}$$

2.6.4.2 Anechoically Terminated Area Transition



$$\begin{bmatrix} P_1 \\ U_1 \end{bmatrix} \qquad \begin{bmatrix} P_2 \\ U_2 \end{bmatrix} \Rightarrow T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdots$$ T-matrix for area transition

$(1+R)$

$$\begin{bmatrix} P_1 \\ U_1 \end{bmatrix} = \begin{bmatrix} P_2 \\ U_2 \end{bmatrix} \begin{matrix} T \\ S_2 T \end{matrix}$$

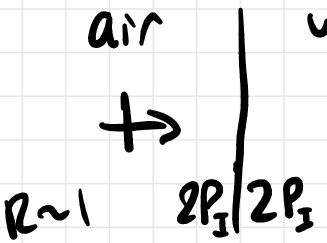
$\frac{S_1}{\rho_0 c}(1-R)$

$$T = \frac{2}{1 + \frac{S_2}{S_1}} \qquad R = \frac{1 - \frac{S_2}{S_1}}{1 + \frac{S_2}{S_1}} \qquad T_\pi + R_\pi = 1$$

if $S_2 < S_1$ ... $\frac{S_2}{S_1} < 1$ (contraction)

$T > 1 \to 2$ when $\frac{S_2}{S_1} \ll 1$

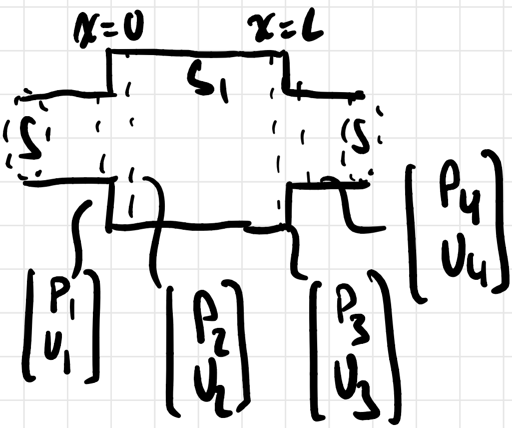
analogous to



$T>1$, but T_Π is very small

I mostly sees a rigid termination

2.6.4.3 Expansion Muffler



$$\begin{bmatrix} P_2 \\ U_2 \end{bmatrix} = [T]_{SP} \begin{bmatrix} P_3 \\ U_3 \end{bmatrix}$$

$\llcorner$ Straight pipe

$$\begin{bmatrix} P_1 \\ U_1 \end{bmatrix} = \begin{bmatrix} P_2 \\ U_2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} P_3 \\ U_3 \end{bmatrix} = \begin{bmatrix} P_4 \\ U_4 \end{bmatrix}$$

$$\begin{bmatrix} P_1 \\ U_1 \end{bmatrix} = [T]_{SP} \begin{bmatrix} P_4 \\ U_4 \end{bmatrix}$$

$$\begin{bmatrix} P_1 \\ U_1 \end{bmatrix} = \begin{bmatrix} \cos kL & j\frac{\rho_0 c}{S_1}\sin kL \\ j\frac{S_1}{\rho_0 c}\sin kL & \cos kL \end{bmatrix} \begin{bmatrix} P_4 \\ U_4 \end{bmatrix}$$

$$P_1 = 1 + R \qquad P_4 = Te^{-jkL}$$

$$U_1 = \frac{S}{\rho_0 c}(1-R) \qquad U_4 = \frac{S}{\rho_0 c} Te^{-jkL}$$

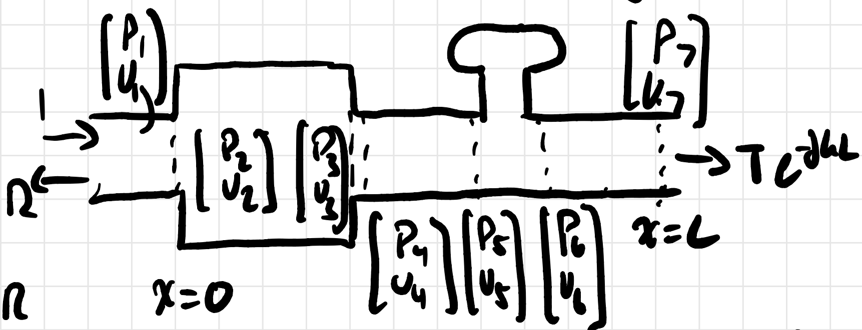
$$T = \frac{2e^{jkL}}{2\cos kL + j\left(\frac{S}{S_1} + \frac{S_1}{S}\right)\sin kL}$$

$$T_\pi = |T|^2 = \frac{4}{4\cos^2 kL + \left(\frac{S_1}{S} + \frac{S}{S_1}\right)^2 \sin^2 kL}$$

$$R = \frac{j\left(\frac{S}{S_1} - \frac{S_1}{S}\right)\sin kL}{2\cos kL + j\left(\frac{S}{S_1} + \frac{S_1}{S}\right)\sin kL} \quad \ldots \quad R_\pi = |R|^2$$

$$T_\pi + R_\pi = 1 \quad \ldots \quad \text{Energy Conservation}$$

2.6.4.4 Complicated System



$$\overbrace{\begin{bmatrix} P_1 \\ U_1 \end{bmatrix}}^{\begin{matrix}1+R \\ \frac{S}{\rho c}(1-R)\end{matrix}} = \underbrace{[T_1]}_{\text{identity}}\underbrace{[T_2]}_{\text{Straight Pipe}}\underbrace{[T_3]}_{\text{identity}}\underbrace{[T_4]}_{\text{Straight Pipe}}\underbrace{[T_5]}_{\text{Side branch}}\underbrace{[T_6]}_{\text{Straight Pipe}}\begin{bmatrix} P_7 \\ U_7 \end{bmatrix}^{T e^{-jkL}}_{\frac{S}{\rho c} T e^{-jkL}}$$

$$T_5 = \begin{bmatrix} 1 & 0 \\ 1/Z_0 & 1 \end{bmatrix}$$

Solve for R + I as the input impedance

Helmholtz Resonator

$$Z_m = j\left(\omega m - \frac{s_r}{\omega}\right)$$ ... mechanical impedance

Acoustic impedance

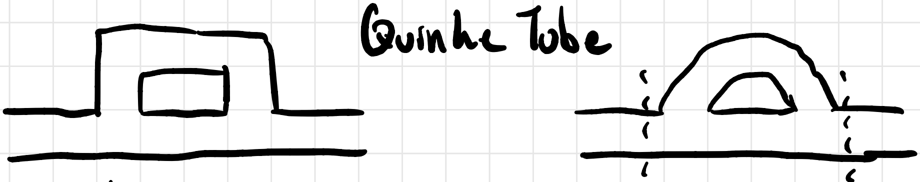
$$Z_a = \frac{Z_m}{S_n^2}$$

S_n = neck area

6.5 further Complications

T-matrix approach suits series connections

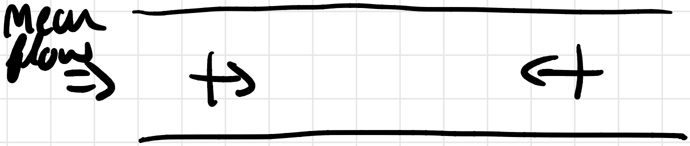
Quinke Tube



parallel connections are not so easy

Solve System separately to create a 2x2 T-matrix for the subsystems

- Flow complicates the situation
  - speed of sound is different in the up- and down-stream sections



soundfield is super-imposed on the moving flow

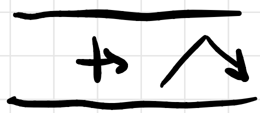
downstream $c+V = c_{eff}$ $\quad k_d = \frac{\omega}{c+V}$

upstream $c-V = c_{eff}$ $\quad k_u = \frac{\omega}{c-V}$

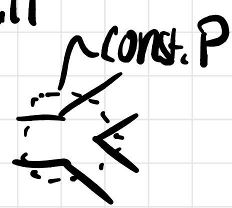
$$p = \underbrace{Ae^{-jk_d x}}_{\text{downstream}} + \underbrace{Be^{jk_u x}}_{\text{upstream}}$$

Normally Assume flow velocity is 0 if Mach number < 0.1 $\quad \frac{V}{c} < 0.1$

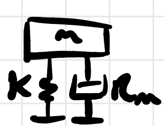
2.7 Summary



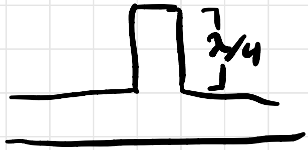
- Plane Waves
- fundamentally a low freq approach
- lumped elements, geometric features are small compared to a wavelength
- point junctions - assume pressure is constant

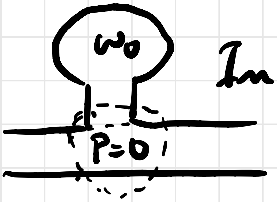


- precise geometrical details don't matter
- Helmholtz resonator



- $1/4$ wave resonator



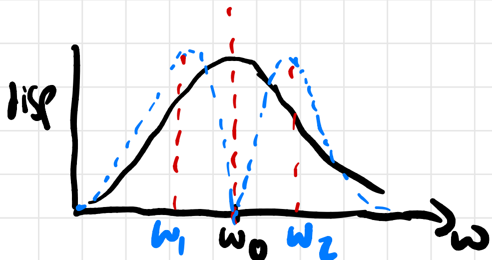


$\mathrm{Im}\{Z_m\} = 0$, at natural frequency

- Pressure release boundary condition so nothing transmits beyond point

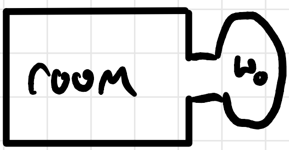
Creates Band-Stop Filter

$w_0 = \sqrt{k/m}$



Can add tuned mass-damper

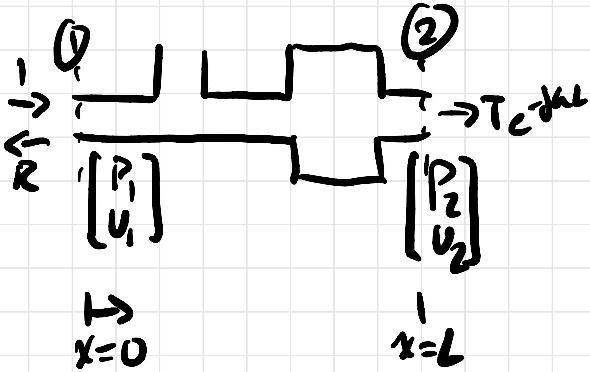
leads to above 2DOF response...



Modifies room response

How to find T, T_π...

$$\begin{bmatrix} P_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} P_2 \\ V_2 \end{bmatrix}$$



$P_1 = T_{11} P_2 + T_{12} V_2$ ①

$V_1 = T_{21} P_2 + T_{22} V_2$ ②

$$P_1 = 1 + R = T_{11}\left(Te^{-jkL}\right) + T_{12}\left(\frac{S}{\rho_0 c} Te^{-jkL}\right) \quad ①$$

$$U_1 = \frac{S}{\rho_0 c}(1-R) = T_{21}\left(Te^{-jkL}\right) + T_{22}\left(\frac{S}{\rho_0 c} Te^{-jkL}\right) \quad ②$$

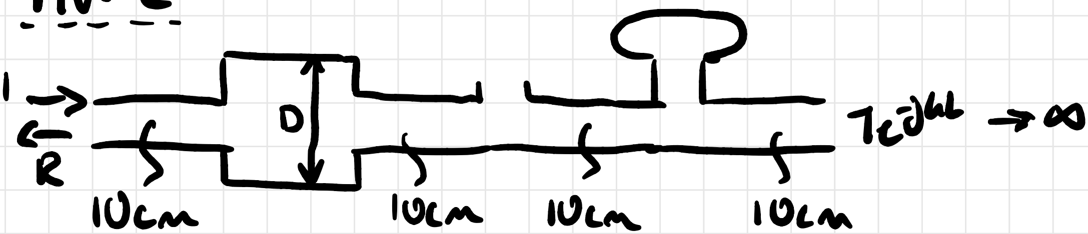
$$2 = \left[T_{11} + \left(\frac{S}{\rho_0 c}\right) T_{12} + \frac{\rho_0 c}{S} T_{21} + T_{22}\right] Te^{-jkL}$$

$$T = \frac{2e^{jkL}}{T_{11} + \frac{S}{\rho_0 c} T_{12} + \frac{\rho_0 c}{S} T_{21} + T_{22}}$$

$$TL = 10 \log \frac{1}{|T|^2}$$

Derive Similar Result for R can be derived

HW 2



$$R_\Pi + T_\Pi = 1$$

Group 4
Masters
Yu
Kim