ME413 HW 03

Benjamin Masters

TOTAL POINTS

93 / 100

QUESTION 1

1 Q1 **20 / 20**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

QUESTION 2

2 Q2 **18 / 20**

- **- 0 pts** Correct
- **- 2 Point adjustment**

QUESTION 3

3 Q3 **20 / 20**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

QUESTION 4

4 Q4 **20 / 20**

- **- 0 pts** Correct
- **+ 1 Point adjustment**

QUESTION 5

5 Q5 **15 / 20**

- **- 0 pts** Correct
- **- 5 Point adjustment**

1 Q1 **20 / 20**

- **- 0 pts** Correct

+ 1 Point adjustment

2 Q2 **18 / 20**

- **- 0 pts** Correct

- 2 Point adjustment

3 Q3 **20** / **20**

- **- 0 pts** Correct

+ 1 Point adjustment

4 Q4 **20 / 20**

- 0 pts Correct

+ 1 Point adjustment

5 Q5 **15 / 20**

- **- 0 pts** Correct

- 5 Point adjustment

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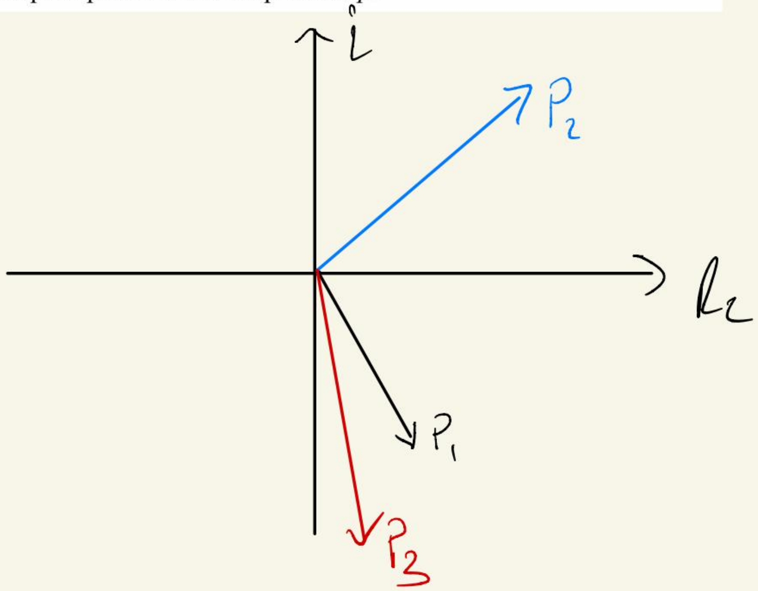
Question 1 (20 points)
The sound field at a point has three components, p_1, p_2 and p_3 where

$$p_1 = 5\cos(\omega t - \pi/3)\ ,\ p_2 = 8\cos(\omega t + \pi/4)\ ,$$

and $$p_3 = 6\sin(\omega t + \pi/10)\ .$$

Sketch a phasor diagram to represent these three components.

(i) determine the total sound pressure, p_t (real and imaginary parts) at the point.
(ii) What are the peak pressure and the phase of p_t.



$P_t = \sum P_1, P_2, P_3$

$Re: 5\cos(\pi/3) + 8\cos(\pi/4) + 6\sin(\pi/10)$
$= 10.01$

$i: 8\sin(\pi/4) - 5\sin(\pi/3) - 6\cos(\pi/10)$
$= -4.38$

i) $\boxed{10.01 - 4.38i}$

ii) peak pressure $= A = \sqrt{10.01^2 + 4.38^2} = \boxed{10.93 = A}$
phase $= \varphi = \tan^{-1}(-4.38/10.01) = \boxed{-.412\text{ rad} = \varphi}$

$10.93e^{-.412i}$

Question 2 (20 points)

Consider a spring-dashpot system that has the following parameters:

$W = 40$ N, $C = 0$, and $k = 0.39$ N/cm.

(i) If the mass is initially displaced and released at rest, how will the system response? (Hint: Find the natural frequency of the system.)

For the same spring-dashpot system but the coefficient of damping C has now been increased to 0.10 N/cm/s. If the initial displacement of the mass is 2.5 cm and the initial velocity is zero,

(ii) determine the damping frequency and logarithmic decrement of the oscillation.

(iii) What is the time when the mass reach its first peak?

(iv) What is the initial acceleration of the mass?

i) $\omega_n = \sqrt{k/m}$ $\quad m = \frac{W}{g} = 40N/9.81 m/s^2 = 4.077 kg$

$\omega_n = \sqrt{\frac{39 N/m}{4.077 kg}} = 3.09$ rad/s $\quad C=0 \therefore \xi = 0$

X_0 = initial displacement $\Rightarrow \boxed{x(t) = X_0 \cos(3.09t)}$

ii) $C = .10$ N/cm/s $= 10$ N-s/m $\quad X_0 = 2.5$ cm $\quad \dot{x}_0 = 0$

$\xi = \frac{C}{2\omega_n M} = \frac{10 N\text{-}s/m}{2 \cdot 3.09 rad/s \cdot 4.077 kg} = 0.397 = \xi$

$\omega_d = \omega_n\sqrt{1-\xi^2} = 3.09 rad/s \cdot \sqrt{1-.397^2} = \boxed{\omega_d = 2.84 rad/s}$

$\Delta = 2\pi\xi/\sqrt{1-\xi^2} = 2\pi(.397)/\sqrt{1-.397^2} = \boxed{\Delta = 2.72}$

iii) $T_d = 2\pi/\omega_d = 2.21 s$ first peak $= T_d/2 = \boxed{T_1 = 1.11 s \text{ or } 0s?}$

iv) $x(t) = 2.5cm\, e^{-.397 \cdot 3.09 rad/s \cdot t} \cdot \cos(3.09\sqrt{1-.397^2} \cdot t)$

$\dot{x}(t) = 2.5cm(-.397 \cdot 3.09)e^{\cdots} \cdot \cos(\cdots) - 2.5 \cdot 3.09 \cdot \sqrt{1-.397^2}\, e^{\cdots}\cos(\cdots)$

$\ddot{x}(t) = 2.5cm\left[(-.397 \cdot 3.09)^2 + 2(.397 \cdot 3.09^2 \cdot \sqrt{1-.397^2}) + (3.09\sqrt{1-.397^2})^2\right]$

$\boxed{\ddot{x}(0) = 41.27 cm/s^2}$

Question 3 (20 points)

A spring-mass-dashpot system is mounted on a solid foundation. The system has a quality factor Q of 5, a mass of 15 kg and a static deflection of 2 mm.

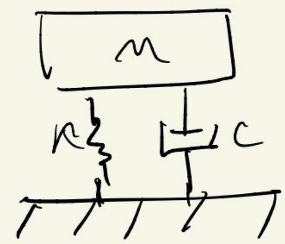
(i) What are the bandwidth, resonant frequency (give the answer in Hz), spring stiffness, coefficient of damping of the system?

(ii) If the mass is subjected to a sinusoidal force which has a magnitude of 100 N and a frequency of 25 Hz, what is the magnification factor? What is the maximum displacement of the vibration?

(iii) What are the magnitude and phase of transmissibility (TR) if the same load as (ii) is applied to the system? Hint: Show that the magnitude of TR and its phase are given by:

$$|\mathrm{TR}| = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}} \quad \text{and} \quad \tan\psi = \frac{-2\xi r^3}{(1-r^2)^2+(2\xi r)^2}$$

(iv) What is the minimum frequency of the force that the system will provide an effective vibration isolation?



$Q = 5 \quad m = 15\,kg \quad \delta_{st} = 2\,mm \qquad K = m \cdot g / \delta_{st}$

$Q = \dfrac{1}{2\xi} \quad \xi = \dfrac{1}{2Q} = \dfrac{1}{10} = 0.1 = \xi$

bandwidth $= 2\xi = \boxed{0.2 = B.W.}$

$K = 15\,kg \cdot 9.81\,m/s^2 / 2\,mm = \boxed{73.575\ N/mm = K}$

$\omega_n = \sqrt{K/m} = \sqrt{g/\delta_{st}} = \sqrt{9.81\,m/s^2 / .002\,m} = 70.04\,rad/s$

$f_n = \dfrac{\omega_n}{2\pi} = \dfrac{70.04\,rad/s}{2\pi} = \boxed{11.15\,Hz = f_n}$

$C = 2\xi \omega_n m = 2(.1)(70.04\,rad/s)(15\,kg) = \boxed{C = 0.934\ N\text{-}s/m}$

ii) $|F_0| = 100\,N \quad f_{F_0} = 25\,Hz \qquad M.F. = \dfrac{1}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$

$r = \dfrac{\omega}{\omega_n}$ or $\dfrac{f}{f_n} = \dfrac{25\,Hz}{11.15\,Hz} = 2.24, \ \xi = 0.1$

$M.F = \dfrac{1}{\sqrt{(1-2.24^2)^2+(2(.1)(2.24))^2}} = \boxed{M.F. = 0.243}$

$X_{max} = \dfrac{MF \cdot F_0}{K} = \dfrac{0.243 \cdot 100\,N}{73.575 \times 10^3\,N/m} = 3.303 \times 10^{-4}\,m = \boxed{.3303\,mm = X_{max}}$

iii) $|TR| = \sqrt{\frac{1+(2\xi r)^2}{(1-r^2)^2+(2\xi r)^2}} = \sqrt{\frac{1+(2\cdot .1\cdot 2.24)^2}{(1-2.24^2)+(2\cdot .1\cdot 2.24)^2}}$

$\boxed{|TR| = 0.266}$

$\varphi_{TR} = \tan^{-1}\left(\frac{-2\xi r^3}{(1-r^2)+(2\xi r)^2}\right) = \boxed{\varphi_{TR} = 0.532\,\text{rad}}$

iv) $\omega_{eff} = \sqrt{2}\cdot\omega_n = \sqrt{2}\cdot 70.04\,\text{rad/s} = \boxed{99.05\,\text{rad/s} = \omega_{eff}}$

or $f_{eff} = \sqrt{2}\cdot 11.15\,\text{Hz} = \boxed{15.77\,\text{Hz} = f_{eff}}$

Question 4 (20 points)

An air compressor of 450 kg operates at a constant speed of 1750 rpm. The rotating parts are well balanced. The reciprocating parts are of 10 kg. The crank radius is 100 mm. If the damper for the mounting introduces a damping ratio of 0.15,

(i) specify the springs for the mounting such that only 20 percent of the unbalanced force is transmitted to the foundation, and

(ii) estimate the amplitude of the transmitted force and the force reduction.

$M_t = 450\,\text{Kg}$ $\quad \omega = 1750\text{ rpm} = \frac{1750\text{ rot}}{1\text{ min}} \cdot \frac{1\text{ min}}{60\text{ sec}} \cdot \frac{2\pi\text{ rad}}{1\text{ rot}}$

$m = 10\,\text{Kg}$ $\quad \omega = 183.26\text{ rad/s}$

$e = 100\text{mm} = .1\text{m}$ $\quad \xi = 0.15$

i) $TR = 0.20$ $\quad F_{eq} = me\omega^2 = 10\text{Kg} \cdot .1\text{m} \cdot (183.26\text{rad/s})^2$

$F_{eq} = 33,584.23\text{N}$

$$|TR|^2 = \frac{1 + 4r^2\xi^2}{(1-r^2)^2 + 4r^2\xi^2} \Rightarrow |TR|^2(1-r^2)^2 + |TR|^2 \cdot 4r^2\xi^2 = 1 + 4r^2\xi^2$$

$$4r^2\xi^2(1-|TR|^2) + 1 = TR^2(1-2r^2+r^4)$$

$$.09r^2 - .0036r^2 + 1 = .04 - .08r^2 + .04r^4$$

$$.04r^4 - .1664r^2 - .96 = 0$$

$$r^4 - 4.16r^2 - 24 = 0 \Rightarrow r^2 = 7.402$$

$$r = \sqrt{7.402} = 2.721 = r = \omega/\omega_n \quad \omega_n = \frac{\omega}{r} = \frac{183.26\text{rad/s}}{2.721}$$

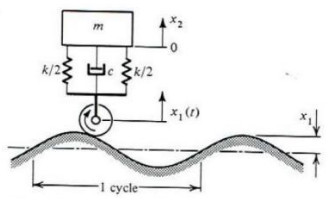
$\omega_n = 67.35\text{ rad/s}$ $\quad \omega_n = \sqrt{K/M_t}$ $\quad K = \omega_n^2 \cdot M_t$

$$K = (67.35\text{rad/s})^2 \cdot 450\text{Kg} = \boxed{2.04\times10^6\text{ N/M} = K}$$

ii) $F_{TR} = TR \cdot F_{eq} = 0.20 \cdot 33.584\text{KN} = \boxed{6.717\text{KN} = |F_{TR}|}$

$\Delta F = F_{eq} - F_{TR} = \boxed{\Delta F = 26.867\text{ KN (force reduction)}}$

Question 5 (20 points)
A vehicle is a complex system with many degrees of freedom. Consider a first approximation of the problem: the following figure may be considered as a vehicle driven on a rough road.



It is assumed that
(a) the vehicle is constrained to one degree of freedom in the vertical direction,
(b) the spring constant of the tires is infinite , i.e. the road roughness is transmitted directly to the suspension system of the vehicle, and
(c) the tires do not leave the road surface.

Assume a trailer has 1000 kg mass fully loaded and 250 kg empty. The spring of the suspension is of 350 kN/m. The damping ratio is 0.5 when the trailer is fully loaded. The speed of the trailer is 100 kph (kilometer per hour). The road varies sinusoidally with 5.0 m/cycle.
(i) Determine the amplitude ratio when the trailer is fully loaded or empty.
(ii) What is the ratio of the maximum force transmitted to the trailer through the suspension when it is fully loaded to that when it is empty?

$M_l = 1000 kg \quad M_e = 250 kg \quad K = 350 kN/m \quad \xi = 0.5$ (loaded)

$|V_t| = 100 kph \quad T = 5.0 m/cycle$

$C = 2\xi\sqrt{km} = 2(0.5)\sqrt{350 kN/m \cdot 1000 kg} = 18.708 \, kN\text{-}s/m$

$\omega_n = \sqrt{K/m} = \sqrt{350 kN/m / 1000} = \omega_n = 18.71 \, rad/s$

$V_t = 100 kph \cdot \frac{1 hr}{3600 sec} \cdot \frac{1000 m}{1 km} = 27.78 \, m/s$

$\omega = V_t \cdot \frac{2\pi}{5m} = 27.78 m/s \cdot \frac{2\pi \, rad}{5m} = 34.91 \, rad/s = \omega$

$r = \frac{\omega}{\omega_n} = 34.91 \, rad/s / 18.71 \, rad/s = 1.87 = r$

$$TR = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} = \frac{\sqrt{1 + (2 \cdot .5 \cdot 1.87)^2}}{\sqrt{(1-1.87^2)^2 + (2 \cdot .5 \cdot 1.87)^2}} = 0.68$$

$\boxed{TR = \text{amplitude ratio} = 0.68 \text{ when loaded}}$

ii) $$\left|\frac{F_{max}}{\hat{X}_1}\right| = m\left|\frac{\hat{a}}{\hat{X}_1}\right| = m\omega^2\left|\frac{\hat{X}_2}{\hat{X}_1}\right| = \frac{m\omega^2\sqrt{1 + 4r^2\xi^2}}{\sqrt{(1-r^2)^2 + 4r^2\xi^2}} = \left|\frac{F_{max}}{\hat{X}_1}\right|$$

$$\left|\frac{F_{max}}{\hat{x}_1}\right|_{loaded} = \frac{1000\,kg\,(34.91\,rad/s)^2\sqrt{1+4(1.87^2)(0.5)^2}}{\sqrt{(1-1.87^2)^2+4\cdot 1.87^2\cdot .5^2}} = 828.5\,kN$$

$$\omega_{n,empty} = \sqrt{350\,kN/m / 250\,kg} = 37.42\,rad/s$$

$$\zeta_{empty} = \frac{C}{2\sqrt{KM}} = \frac{18.708\,kN\text{-}s/m}{2\sqrt{350\,kN/m\cdot 250}}$$

$$r_{empty} = 34.91\,rad/s / 37.42\,rad/s = 0.933$$

$$\zeta_{empty} = 1$$

$$\left|\frac{F_{max}}{\hat{x}_1}\right|_{empty} = \frac{250\,kg\,(34.91\,rad/s)^2\sqrt{1+4(.933^2)(1)^2}}{\sqrt{(1-.933^2)^2+4\cdot .933^2\cdot (1)^2}}$$

$$= 344.84\,kN$$

$$\frac{F_{max,load}}{F_{max,empty}} = \frac{828.5\,kN}{344.84\,kN} = \boxed{2.40 = \frac{F_{max,l}}{F_{max,e}}}$$