Dinh, Bryan
CSCI 2114: Tashfeen's Data Structures
December 5, 2025

Homework 6

Question 1. Head over to the https://www.gutenberg.org and find an interesting book that is available in the text format. Perform word frequency analysis on the text of the book using the Java's built-in hash map. Before you start any counting, make sure you clean your text of any characters that are not lower-case English alphabet or an invisible space character, visibly rendered as "␣". Print out the top ten words with their respective frequencies in descending order. The words should be *at least six letters*. For example, a sample output for the George Orwell's novel *1984* is given bellow. Place the code for this question in a file called `Frequency.java` and copy any output in your PDF to answer this question.

```
|   winston | 282|
|   thought | 158|
|    seemed | 133|
|    moment | 131|
|    always | 122|
|    obrien | 119|
|    almost | 106|
|   another | 105|
|    though | 104|
|    before | 103|
```

The Strange Case of Dr. Jekyll and Mr. Hyde

```
utterson: 127
project:  88
gutenberg:  87
jekyll:  86
lawyer:  67
before:  48
thought:  39
seemed:  30
should:  30
moment:  28
myself:  28
```

Question 2. Let the 27 letter alphabet consist of a space and 26 lower case English letters: $\{␣, \text{a}, \text{b}, \text{c}, \ldots, \text{z}\}$. We may further order the alphabet with $f = \{(␣, 0), (\text{a}, 1), (\text{b}, 2), (\text{c}, 3), (\text{d}, 4), \ldots, (\text{z}, 26)\}$. A *hash* function h takes any arbitrary string s of the alphabet's characters to a non-negative integer. Let s be of length n where s_i is the i^{th} character of s, $h(s)$ is given as,

$$h(s) \equiv \sum_{i=0}^{n-1} (f(s_i) \times 27^i) \mod m.$$

As trivial examples, for a sufficiently large value of m,

$$h(␣) \equiv 0 \equiv (0 \times 27^0), \quad h(\text{a}) \equiv 1 \equiv (1 \times 27^0), \quad h(\text{b}) \equiv 2 \equiv (2 \times 27^0) \mod m.$$

And, for a non-trivial example,

$$\begin{aligned} h(\text{bobby}) &\equiv (f(\text{b}) \times 27^4) + (f(\text{o}) \times 27^3) + (f(\text{b}) \times 27^2) + (f(\text{b}) \times 27^1) + (f(\text{y}) \times 27^0) \mod m \\ &\equiv (2 \times 27^4) + (15 \times 27^3) + (2 \times 27^2) + (2 \times 27^1) + (25 \times 27^0) \mod m \\ &\equiv 1062882 + 295245 + 1458 + 54 + 25 \mod m \\ &\equiv 1359664 \mod m \end{aligned}$$

Write a Java program that takes $h(s)$ and prints out s, e. g., `java Inverse 1359664` prints bobby. Use this program to figure out the string s for the following $h(s)$,

`1446443617214273139220623316754031238673439502425204756407075972832616437749257375015894090763500398 14`

Yes, you have to use `BigInteger`; you'll be okay. if a machine is expected to be infallible it cannot also be intelligent

Question 3. Download the professor's implementation of the above hash function in Java from the linked file `HashBrown.java`. Also download the list of 370,105 unique English words. Write a program that finds all the hash collisions among these words for each m_i given bellow.

$$m_1 = 2^{31} - 1 = \texttt{Integer.MAX\_VALUE}$$
$$m_2 = 2^{63} - 1 = \texttt{Long.MAX\_VALUE}$$

For every hash collision corresponding to each m_i give the hash value and the collided words. E. g., there are 30 hash collisions when $m = 2^{31} - 1$ with one of them being $h(\text{decoy}) = 2226796 = h(\text{eternish})$. Use the main method of `HashBrown.java` for this question.

```
881:[aeq, grillades]
13747:[elocutionists, rwd]
2115518164:[elkslip, vapourization]
491318560:[nonoptimistic, ploughgang]
1403560615:[pluvialis, primeness]
1183410379:[paleophytology, piscators]
395425281:[ketoketene, weenong]
2226796:[decoy, eternish]
1929526801:[paumari, uplinks]
1541549160:[cappelenite, postorbital]
1828421126:[cosinesses, outfits]
1147362968:[calumniative, deveined]
743636998:[submaniacally, trigeminus]
1862481874:[amblygonite, jillion]
1339762618:[homemaking, marattia]
513542510:[disroost, gymnastically]
276890:[decerning, nave]
52720974:[cremor, epimanikion]
1399643975:[encroachments, identic]
966567163:[myriologist, shiftable]
1520219554:[reseason, uniformal]
391988318:[orography, xanthide]
911563435:[assibilating, bingeys]
158695826:[araliophyllum, kapote]
194905270:[motets, verrucano]
1054238897:[birthbed, cumbrously]
2008476663:[counteragitate, discourteousness]
164046042:[rebinding, viviparously]
287790385:[decertified, tanged]
25127934:[blandishingly, praecordial]
5731686:[indistinctiveness, unmanipulatable]
```

31 total collisions.

Question 4. Read Cormen, et al. *Introduction to Algorithms* (3^{rd} Edition) pages 647–652 and pages 658–659. You'll be implementing the Dijkstra's and the Bellman-Ford[1] algorithms. Download the files containing the adjacency matrices,

1) dijkstra1.txt
2) dijkstra2.txt
3) bellmanford1.txt
4) bellmanford2.txt

The above matrices define edge weights between vertices that should be named by taking as many labels as the rows/columns in the list: a, b, c, . . . , z.

Write a Java class `Graph.java` that implements both the Dijkstra's[2] and the Bellman-Ford algorithms. Use your Dijkstra's implementation to solve the files `dijkstra1.txt` and `dijkstra2.txt`. Similarly, use your Bellman-Ford algorithm to solve the files `bellmanford1.txt` and `bellmanford2.txt`. For each of the graphs and the algorithm, report the minimum distance with which each vertex can be reached and their respective parent vertex in your answers PDF. E. g, the output of Dijkstra's algorithm on `dijkstra1.txt` should look like,

```
a: 0 via a
b: 8 via d
c: 9 via b
d: 5 via a
e: 7 via d
```

You may verify the above by manually drawing the `dijkstra1.txt` graph on a piece of paper and running Dijkstra on it by hand.

Question 5. Can you run the Bellman-Ford algorithm on `dijkstra1.txt` and `dijkstra2.txt` to obtain the same result as you did by running the Dijkstra's algorithm? Similarly, can you run Dijkstra's algorithm on `bellmanford1.txt` and `bellmanford2.txt` to obtain the same results as you did by running Bellman-Ford algorithm?

```
Dijkstra 1 results:
a: 0.0 via a
b: 8.0 via d
c: 9.0 via b
d: 5.0 via a
e: 7.0 via d
Dijkstra 2 results:
a: 0.0 via a
b: 243.0 via t
c: 186.0 via p
d: 237.0 via y
e: 356.0 via b
f: 173.0 via a
g: 191.0 via p
h: 196.0 via p
i: 244.0 via k
j: 188.0 via u
k: 3.0 via a
l: 295.0 via a
m: 204.0 via t
n: 212.0 via f
o: 188.0 via z
p: 115.0 via a
```

[1] Also known as the "distance vector routing" algorithm in the networks field.

[2] Note that Java's class PriorityQueue is not dynamic and breaks ties arbitrarily.

```
q: 293.0 via t
r: 233.0 via u
s: 253.0 via v
t: 202.0 via k
u: 183.0 via f
v: 216.0 via y
w: 184.0 via k
x: 271.0 via d
y: 202.0 via j
z: 175.0 via p
Bellman Ford 1 results:
a: 0.0 via a
b: 2.0 via c
c: 4.0 via d
d: 7.0 via a
e: -2.0 via b
Bellman Ford 2 results:
a: 0.0 via a
b: 24.0 via z
c: -2.0 via a
d: 38.0 via s
e: 56.0 via p
f: 31.0 via d
g: -8.0 via a
h: -2.0 via l
i: 75.0 via u
j: 7.0 via c
k: 110.0 via l
l: -11.0 via g
m: 23.0 via p
n: -1.0 via u
o: -5.0 via h
p: 16.0 via v
q: 29.0 via b
r: 4.0 via g
s: 33.0 via x
t: 83.0 via v
u: -5.0 via r
v: 24.0 via j
w: 15.0 via c
x: 10.0 via c
y: -3.0 via l
z: 11.0 via u
```

Question 6. On a graph of exclusively positive edge weights, which algorithm is a better choice?
Dijkstra's algorithm

Question 7. Watch A* (A-Star) Pathfinding Algorithm Visualization on a Real Map on YouTube to see how a path finding algorithm known as the A* looks in execution on a real world graph. Note that A* is essentially the Dijkstra's algorithm with the addition of an "heuristic" function.

Submission Instructions

1) Turn in a PDF containing any outputs and answers from the homework.
2) Also turn in your `Frequency.java`, `Inverse.java`, `HashBrown.java` (with a main method) and finally the `Graph.java`.

Oklahoma City University, Petree College of Arts & Sciences, Computer Science

Dinh, Bryan
MATH 3503: Tashfeen's Discrete Mathematics
December 8, 2025

Homework 7

Question 1. Please read chapter 10 of Chartrand et al. and write a couple sentences about a topic/example/concept that you found difficult or interesting and why?
I found it pretty interesting how discrete math ties concepts about probability into this. I still need to get to the whole concept of counting and applying that to probability. It is somewhat difficult, but hopefully I can get a better idea of how to solve these types of problems before the final.

Question 2. During the last class period of the semester, each student in a graduate computer science class with 10 students is required to give a brief report on his or her class project. The professor randomly selects the order in which the reports are to be given. Two students have been working on similar projects and would like to give their reports consecutively. What is the probability that this will happen?
Find number of adjacent pairs (without replacement):

$$(1,2)(2,1)$$

$$(2,3)(3,2)$$

leads to ...

$$(9,10)(10,9)$$

Results with 9 adjacent pairs (without replacement)

Find the valid order positions:

$$9! \cdot 2 = 725760$$

9 for adjacent pairs, 2 for 2 possible orders for each pair, multiplied because it is implied for each pair

Find the adjacent pairs in a line of 10 students (without replacement):

$$\text{probability} = \frac{\text{favorable}}{\text{total}}$$

725760 is the number of favorable outcomes

$$= \frac{725760}{\text{total}}$$

Arrange for 10 students in the lineup instead of choosing 2 positions

$$\frac{10!}{(10-10)!} = 10! = 3628800$$

That is the total number of outcomes

$$\frac{725760}{3628800} = \frac{1}{5} = 0.2$$

Question 3. A coin is flipped three times.

(a) What is the probability of getting three heads?

$$(\frac{1}{2})^3 = \frac{1}{8} = 0.5 = 50\%$$

$\frac{1}{2}$ for heads or tails and 3 for number of trails

(b) What is the probability of getting three heads given that the first flip came up heads?

$$(\frac{1}{2})^2 = \frac{1}{4} = 0.25 = 25\%$$

2 trails for exponent

(c) What is the probability of getting three heads given that the first two flips resulted in two heads?

$$(\frac{1}{2}) = \frac{1}{2} = 50\%$$

There is only 1 position that can be one of the, sense the first two have ot be heads.

(d) What is the probability of getting three heads given that the first three flips resulted in all heads?

$$100\%$$

All 3 trials have to result in heads.
Has to be guaranteed for this condition to happen.

(e) What is the probability of getting three heads given that at least one of the first two flips resulted in heads?
Let $E =$ number of heads (HHH) and let $A \in$ of {desired outcomes}:

$$A \in \{HH, HT, TH\}$$

Only looking at when there are there 1 or more heads that occur during the trials.
There are 3 desired outcomes and 4 possible outcomes (HH, HT, TH, TT), therefore:

$$P(A) = \frac{3}{4}$$

Calculate the probably of A occurring if E does:

$$P(E|A) = \frac{P(E \cap A)}{P(A)} = \frac{\frac{1}{8}}{\frac{3}{4}} = \frac{1}{8} \cdot \frac{4}{3} = \frac{4}{24} = \frac{1}{6} \approx 16.7\%$$

(f) What is the probability of getting three heads given that at most one of the first two flips resulted in heads?
Let $E =$ number of heads (HHH) and let $A \in$ of {desired outcomes}:

$$A \in \{TT, HT, TH\}$$

Only looking at when where there are 0 or 1 occurrence during the trials

There are 3 desired outcomes and 4 possible outcomes (HH, HT, TH, TT), therefore:

$$P(A) = \frac{3}{4}$$

Calculate the probably of A occurring if E does:
Since $A \notin \{HH\}$, it is an empty set
Reason being because we are only looking at all the possibilities where there are either 0 or 1 head.

$$P(E|A) = \frac{P(E \cap A)}{P(A)} = \frac{0}{\frac{3}{4}} = 0$$

Question 4. Three dice are tossed. What is the probability that 1 was obtained on two of the dice given that the sum of the numbers on the three dice is 7?
List of all combinations of dice that sum up to 7:

$$(1,1,5)(1,5,1)(5,1,1)$$

$$(1,2,4)(1,4,2)(4,2,1)$$

$$(1,3,3)(3,1,3)(3,3,1)$$

$$(2,1,4)(2,4,1)(4,1,2)$$

$$(2,3,2)(3,2,2)(2,2,3)$$

There are a total of 15 combinations
List the combinations where there is a one on 2 dice that still sum up to 7:

$$(1,1,5)(1,5,1)(5,1,1)$$

Let $E \in \{(1,1,5),(1,5,1),(5,1,1)\}$ and $A =$ total number of possible outcomes:

$$|E \cap A| = 3$$

Since the cardinality of the intersection of those conditions in 3. therefore:

$$P(E|A) = \frac{3}{15} = \frac{1}{5} = 0.2 = 20\%$$

Question 5. Early each fall, a department store manager purchases a large number of winter sweaters. He pays \$60 for each sweater. Any sweater that isn't sold by Christmas will be sold for a \$10 dollar loss. Experience says that he can sell 40% of them by Christmas if he prices the sweaters at \$100 each, he can sell 60% of them if each is priced at \$90 and he can sell 70% of them if they are priced at \$80 each. How should the manager price the sweaters?

Use Expected Value $E(X)$:

$$E = (\text{Profit of sold} \cdot P_{sold}) + (\text{Loss per unsold} \cdot P_{unsold})$$

Apply to each price:

$$E_{100} = ((100 - 60) \cdot (0.4)) + (-10 \cdot 0.6) = \$10.00$$

$$E_{90} = ((90 - 60) \cdot (0.6)) + (-10 \cdot 0.4) = \$14.00$$

$$E_{80} = ((80 - 60) \cdot (0.7)) + (-10 \cdot 0.3) = \$11.00$$

\$90 **generated the most amount of profit**

Question 6. A bowl contains 3 red balls, 2 white balls, and 1 blue ball

(a) What is the expected number of white balls obtained if three balls are selected at random from the bowl?

There are 3 outcomes if 3 balls are selected from the bowl and have a white ball:
0 white, 1 white, 2 white

$$\sum_{x\in\{0,1,2\}} = xP(x) = 0P(0) + 1\left(\frac{x}{\binom{6}{3}}\right) + 2\left(\frac{y}{\binom{6}{3}}\right)$$

set equal 1

$$1 = 1\left(\frac{x}{\binom{6}{3}}\right) + 2\left(\frac{y}{\binom{6}{3}}\right)$$

$$1\left(\frac{\binom{2}{1}\binom{4}{2}}{\binom{6}{3}}\right) + 2\left(\frac{\binom{2}{2}\binom{4}{1}}{\binom{6}{3}}\right) = 1$$

(b) What is the expected number of white balls obtained if three balls are selected at random from the bowl, one at a time, where a ball is returned to the bowl after it is selected?

Define the probability of getting a white ball:

$$P(x_{white})\frac{2}{3+2+1} = \frac{2}{6}$$

Define expected value:

$$E(x) = E(x_1) + E(x_2) + E(x_3)$$

Since every draw resets, therefore:

$$E(x_1) = E(x_2) = E(x_3) = P(x_{White}) = \frac{2}{6}$$

Add up each probability:

$$\frac{2}{6} + \frac{2}{6} + \frac{2}{6} = \frac{6}{6} = 1$$

Computer Science, Petree College of Arts & Sciences, Oklahoma City University