

Homework 7

Question 1. Please read chapter 10 of Chartrand et al. and write a couple sentences about a topic/example/concept that you found difficult or interesting and why?

I found it pretty interesting how discrete math ties concepts about probability into this. I still need to get to the whole concept of counting and applying that to probability. It is somewhat difficult, but hopefully I can get a better idea of how to solve these types of problems before the final.

Question 2. During the last class period of the semester, each student in a graduate computer science class with 10 students is required to give a brief report on his or her class project. The professor randomly selects the order in which the reports are to be given. Two students have been working on similar projects and would like to give their reports consecutively. What is the probability that this will happen?

Find number of adjacent pairs (without replacement):

$$(1, 2)(2, 1)$$

$$(2, 3)(3, 2)$$

leads to ...

$$(9, 10)(10, 9)$$

Results with 9 adjacent pairs (without replacement)

Find the valid order positions:

$$9! \cdot 2 = 725760$$

9 for adjacent pairs, 2 for 2 possible orders for each pair, multiplied because it is implied for each pair

Find the adjacent pairs in a line of 10 students (without replacement):

$$\text{probability} = \frac{\text{favorable}}{\text{total}}$$

725760 is the number of favorable outcomes

$$= \frac{725760}{\text{total}}$$

Arrange for 10 students in the lineup instead of choosing 2 positions

$$\frac{10!}{(10 - 10)!} = 10! = 3628800$$

That is the total number of outcomes

$$\frac{725760}{3628800} = \frac{1}{5} = 0.2$$

Question 3. A coin is flipped three times.

- (a) What is the probability of getting three heads?

$$\left(\frac{1}{2}\right)^3 = \frac{1}{8} = 0.5 = 50\%$$

$\frac{1}{2}$ for heads or tails and 3 for number of trials

- (b) What is the probability of getting three heads given that the first flip came up heads?

$$\left(\frac{1}{2}\right)^2 = \frac{1}{4} = 0.25 = 25\%$$

2 trials for exponent

- (c) What is the probability of getting three heads given that the first two flips resulted in two heads?

$$\left(\frac{1}{2}\right) = \frac{1}{2} = 50\%$$

There is only 1 position that can be one of the, sense the first two have to be heads.

- (d) What is the probability of getting three heads given that the first three flips resulted in all heads?

$$100\%$$

All 3 trials have to result in heads.

Has to be guaranteed for this condition to happen.

- (e) What is the probability of getting three heads given that at least one of the first two flips resulted in heads?

Let $E = \text{number of heads (HHH)}$ and let $A \in \{\text{desired outcomes}\}$:

$$A \in \{HH, HT, TH\}$$

Only looking at when there are 1 or more heads that occur during the trials.

There are 3 desired outcomes and 4 possible outcomes (HH, HT, TH, TT), therefore:

$$P(A) = \frac{3}{4}$$

Calculate the probably of A occurring if E does:

$$P(E|A) = \frac{P(E \cap A)}{P(A)} = \frac{\frac{1}{8}}{\frac{3}{4}} = \frac{1}{8} \cdot \frac{4}{3} = \frac{4}{24} = \frac{1}{6} \approx 16.7\%$$

- (f) What is the probability of getting three heads given that at most one of the first two flips resulted in heads?

Let $E = \text{number of heads (HHH)}$ and let $A \in \{\text{desired outcomes}\}$:

$$A \in \{TT, HT, TH\}$$

Only looking at when there are 0 or 1 occurrence during the trials

There are 3 desired outcomes and 4 possible outcomes (HH, HT, TH, TT), therefore:

$$P(A) = \frac{3}{4}$$

Calculate the probably of A occurring if E does:

Since $A \notin \{HH\}$, it is an empty set

Reason being because we are only looking at all the possibilities where there are either 0 or 1 head.

$$P(E|A) = \frac{P(E \cap A)}{P(A)} = \frac{0}{\frac{3}{4}} = 0$$

Question 4. Three dice are tossed. What is the probability that 1 was obtained on two of the dice given that the sum of the numbers on the three dice is 7?

List of all combinations of dice that sum up to 7:

$$(1, 1, 5)(1, 5, 1)(5, 1, 1)$$

$$(1, 2, 4)(1, 4, 2)(4, 2, 1)$$

$$(1, 3, 3)(3, 1, 3)(3, 3, 1)$$

$$(2, 1, 4)(2, 4, 1)(4, 1, 2)$$

$$(2, 3, 2)(3, 2, 2)(2, 2, 3)$$

There are a total of 15 combinations

List the combinations where there is a one on 2 dice that still sum up to 7:

$$(1, 1, 5)(1, 5, 1)(5, 1, 1)$$

Let $E \in \{(1, 1, 5), (1, 5, 1), (5, 1, 1)\}$ and $A = \text{total number of possible outcomes}:$

$$|E \cap A| = 3$$

Since the cardinality of the intersection of those conditions is 3. therefore:

$$P(E|A) = \frac{3}{15} = \frac{1}{5} = 0.2 = 20\%$$

Question 5. Early each fall, a department store manager purchases a large number of winter sweaters. He pays \$60 for each sweater. Any sweater that isn't sold by Christmas will be sold for a \$10 dollar loss. Experience says that he can sell 40% of them by Christmas if he prices the sweaters at \$100 each, he can sell 60% of them if each is priced at \$90 and he can sell 70% of them if they are priced at \$80 each. How should the manager price the sweaters?

Use Expected Value $E(X)$:

$$E = (\text{Profit of sold} \cdot P_{sold}) + (\text{Loss per unsold} \cdot P_{unsold})$$

Apply to each price:

$$E_{100} = ((100 - 60) \cdot (0.4)) + (-10 \cdot 0.6) = \$10.00$$

$$E_{90} = ((90 - 60) \cdot (0.6)) + (-10 \cdot 0.4) = \$14.00$$

$$E_{80} = ((80 - 60) \cdot (0.7)) + (-10 \cdot 0.3) = \$11.00$$

\$90 generated the most amount of profit

Question 6. A bowl contains 3 red balls, 2 white balls, and 1 blue ball

- (a) What is the expected number of white balls obtained if three balls are selected at random from the bowl?

There are 3 outcomes if 3 balls are selected from the bowl and have a white ball:
0 white, 1 white, 2 white

$$\sum_{x \in \{0,1,2\}} xP(x) = 0P(0) + 1\left(\frac{x}{\binom{6}{3}}\right) + 2\left(\frac{y}{\binom{6}{3}}\right)$$

set equal 1

$$1 = 1\left(\frac{x}{\binom{6}{3}}\right) + 2\left(\frac{y}{\binom{6}{3}}\right)$$

$$1\left(\frac{\binom{2}{1}\binom{4}{2}}{\binom{6}{3}}\right) + 2\left(\frac{\binom{2}{2}\binom{4}{1}}{\binom{6}{3}}\right) = 1$$

- (b) What is the expected number of white balls obtained if three balls are selected at random from the bowl, one at a time, where a ball is returned to the bowl after it is selected?

Define the probability of getting a white ball:

$$P(x_{white}) = \frac{2}{3+2+1} = \frac{2}{6}$$

Define expected value:

$$E(x) = E(x_1) + E(x_2) + E(x_3)$$

Since every draw resets, therefore:

$$E(x_1) = E(x_2) = E(x_3) = P(x_{White}) = \frac{2}{6}$$

Add up each probability:

$$\frac{2}{6} + \frac{2}{6} + \frac{2}{6} = \frac{6}{6} = 1$$

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