Ratios, Rates, and Proportional Relationships

Understanding Rates and Proportionality

Unit Rates

Name: Calculate unit rates with whole numbers

Description: This skill focuses on understanding and calculating the unit rate when given a ratio involving whole-number quantities. Students will learn that a unit rate expresses how many units of one quantity there are for one unit of the second quantity. This is a foundational skill for understanding proportionality and serves as a direct precursor to working with fractional rates.

Core example questions:

- A car travels 240 miles in 4 hours. What is its speed in miles per hour?
- If 5 oranges cost $3, how much does 1 orange cost?
  Difficulty: Introductory
  Common pitfalls:
- Incorrectly dividing the quantities (e.g., dividing hours by miles instead of miles by hours).
- Not understanding what the "unit" refers to in the context of the problem.
- Confusing the order of the quantities in the ratio.
  Introduced definitions: Ratio, Rate, Unit Rate, Per
- Concept / Introduction / Definition: 4
- Practice: 15
- Common Pitfall avoidance: 5
- Application/word problem: 3
- Challenging: 2
- Total number of questions for each goal: 29

Name: Compute unit rates with fractions

Description: This skill extends the concept of unit rates to include scenarios where one or both quantities are fractions, including complex fractions. Students will need to apply their knowledge of fraction division (multiplying by the reciprocal) to accurately determine the unit rate. This builds directly on the previous skill but increases the computational complexity.

Core example questions:

- A baker uses 43 cup of flour to make 21 batch of cookies. How much flour is needed for one full batch?

- If a snail crawls 85 of a mile in 41 of an hour, what is its speed in miles per hour?
  Difficulty: Core
  Common pitfalls:

- Errors in dividing fractions (e.g., not multiplying by the reciprocal, or reciprocating the incorrect fraction).

- Misinterpreting the complex fraction setup and which quantity is being "per" the other.

- Not simplifying the resulting fractional unit rate.
  Introduced definitions: Complex Fraction

- Concept / Introduction / Definition: 3

- Practice: 12

- Common Pitfall avoidance: 6

- Application/word problem: 4

- Challenging: 3

- Total number of questions for each goal: 28

Name: Solve real-world problems involving unit rates with fractions

Description: This skill requires students to apply their understanding of unit rates with fractions to practical, real-world contexts. Problems will involve various units and scenarios (e.g., lengths, areas, prices, speeds), requiring students to set up the appropriate ratio and calculate the unit rate to answer a specific question. This moves from pure computation to problem-solving.

Core example questions:

- A recipe calls for 32 cup of sugar for every 21 pound of butter. If you have 1 pound of butter, how much sugar do you need?

- Sarah can paint 51 of a room in 43 of an hour. At this rate, how long will it take her to paint the entire room?
  Difficulty: Application
  Common pitfalls:

- Difficulty translating the word problem into a correct mathematical setup (identifying the two quantities and their relationship).

- Not identifying which quantity should be in the numerator and which in the denominator for the unit rate.

- Calculation errors with fractions in a word problem context.
  Introduced definitions: (No new specific definitions, but reinforces prior ones in application scenarios)
- Concept / Introduction / Definition: 2
- Practice: 7
- Common Pitfall avoidance: 5
- Application/word problem: 8
- Challenging: 4
- Total number of questions for each goal: 26

Identifying Proportional Relationships

Name: Identify proportional relationships from tables

Description: Students will analyze data presented in tables to determine if a relationship between two quantities is proportional. This involves checking for a constant ratio between corresponding values (y/x) for all pairs (x,y) in the table, assuming x=0. Understanding this concept is fundamental for recognizing proportionality across different representations.

Core example questions:

- Determine if the following table represents a proportional relationship. Explain your reasoning.

x	y
2	8
3	12
5	20

- Which of the following tables shows a proportional relationship?

| Table A | x | y | | Table B | x | y |

|---|---|---|---|---|---|---|

| | 1 | 3 | | | 1 | 2 |

| | 2 | 6 | | | 2 | 5 |

| | 3 | 9 | | | 3 | 8 |

Difficulty: Core

Common pitfalls:

- Not consistently checking the ratio for every pair of values, or only checking the first pair.
- Confusing additive relationships (e.g., y=x+c) with multiplicative (proportional) relationships (y=kx).
- Calculation errors when finding ratios, especially if values are not simple.
  Introduced definitions: Proportional Relationship, Constant Ratio
- Concept / Introduction / Definition: 4
- Practice: 10
- Common Pitfall avoidance: 5
- Application/word problem: 3
- Challenging: 3
- Total number of questions for each goal: 25

Name: Identify proportional relationships from graphs

Description: This skill focuses on recognizing proportional relationships when they are represented graphically. Students will learn that a proportional relationship is visually characterized by a straight line that passes through the origin (0,0). This provides a strong visual intuition for proportionality.

Core example questions:

- Which of the following graphs represents a proportional relationship? Justify your answer. (Provide several graphs, some proportional, some not).
- Explain why a graph showing the cost of apples versus the number of apples purchased must pass through the origin if the relationship is proportional.
  Difficulty: Core
  Common pitfalls:
- Mistaking any straight line for a proportional relationship (forgetting or ignoring the origin requirement).
- Difficulty interpreting the meaning of the origin (0,0) in the context of a given graph.
- Confusing increasing non-linear graphs with proportional ones.
  Introduced definitions: Origin
- Concept / Introduction / Definition: 3

- Practice: 9
- Common Pitfall avoidance: 5
- Application/word problem: 3
- Challenging: 3
- Total number of questions for each goal: 23

Name: Identify proportional relationships from equations

Description: Students will learn to recognize proportional relationships when they are expressed as algebraic equations. They will identify that equations of the form y=kx (where k is a non-zero constant) represent proportional relationships, and understand why other forms of equations do not.

Core example questions:

- Is the equation y=5x a proportional relationship? Why or why not?
- Which of the following equations represents a proportional relationship: y=x+2, y=3x, y=x2, y=21x?
  Difficulty: Core
  Common pitfalls:
- Confusing y=kx+b (linear, but not necessarily proportional unless b=0) with y=kx.
- Not understanding that k must be a constant, not a variable.
- Difficulty with equations where the constant of proportionality is a fraction or decimal.
  Introduced definitions: (No new definitions, reinforces prior ones)
- Concept / Introduction / Definition: 3
- Practice: 10
- Common Pitfall avoidance: 4
- Application/word problem: 3
- Challenging: 2
- Total number of questions for each goal: 22

Name: Represent proportional relationships with multiple representations

Description: This skill requires students to translate and represent proportional relationships between different forms: tables, graphs, equations, and verbal descriptions. This demonstrates a deeper, more integrated understanding of the concept by requiring students to connect and move fluidly between these representations.

Core example questions:

- A car travels at a constant speed of 60 miles per hour.
    - Write an equation that represents this relationship.
    - Create a table showing the distance traveled after 1, 2, and 3 hours.
    - Sketch a graph of this relationship.
- Given the equation y=2.5x, create a verbal description, a table, and a graph that represent this proportional relationship.
  Difficulty: Synthesis
  Common pitfalls:

- Inconsistency between representations (e.g., an equation not matching the table values, or a graph not reflecting the equation).

- Difficulty in creating an appropriate scale for graphs, especially if values are large or small.

- Struggling to articulate clear and accurate verbal descriptions that capture the proportionality.
  Introduced definitions: (No new definitions, synthesizes prior ones)

- Concept / Introduction / Definition: 2

- Practice: 6

- Common Pitfall avoidance: 4

- Application/word problem: 7

- Challenging: 4

- Total number of questions for each goal: 23

Name: Interpret points (x, y) on the graph of a proportional relationship

Description: This skill focuses on understanding the meaning of specific points on the graph of a proportional relationship within the context of the problem. Special attention is given to the significance of the origin (0,0) (meaning zero of one quantity results in zero of the other) and the point (1, r), where 'r' represents the unit rate (the value of y when x is 1).

Core example questions:

- The graph shows the cost of gasoline based on the number of gallons purchased. Explain the meaning of the point (0,0) and the point (1, 3.50) on this graph.

- If a graph shows the relationship between hours worked and money earned, what does the point (5, 75) represent?
  Difficulty: Conceptual
  Common pitfalls:

- Not connecting the coordinates (x,y) to the specific quantities they represent in the problem's context.

- Difficulty explaining the "why" behind the significance of (0,0) and (1,r) in terms of the situation.

- Only stating what the numbers are, rather than interpreting their meaning.
  Introduced definitions: (No new definitions, but deepens the conceptual understanding of proportionality and graphical representation)

- Concept / Introduction / Definition: 4

- Practice: 6

- Common Pitfall avoidance: 4

- Application/word problem: 6

- Challenging: 3

- Total number of questions for each goal: 23

Constant of Proportionality

Name: Find the constant of proportionality from tables

Description: This skill involves calculating the constant of proportionality (which is numerically equivalent to the unit rate) directly from a table that represents a proportional relationship. Students will consistently divide the 'y' value by the corresponding 'x' value for any given pair in the table. This is a crucial procedural step to identify the constant.

Core example questions:

- Find the constant of proportionality from the following table:

x	y
2	10
4	20
7	35

- If a table shows the number of hours worked (x) and the amount earned (y), and you earn $15 for 1 hour, $30 for 2 hours, what is the constant of proportionality?
  Difficulty: Procedural
  Common pitfalls:

- Dividing x by y instead of y by x.

- Errors in calculation, especially with larger numbers, decimals, or fractions within the table.
- Not recognizing that if the ratio is not constant, there is no single constant of proportionality.
  Introduced definitions: Constant of Proportionality (re-emphasized as 'k' in the context of y=kx)
- Concept / Introduction / Definition: 3
- Practice: 15
- Common Pitfall avoidance: 6
- Application/word problem: 4
- Challenging: 3
- Total number of questions for each goal: 31

Name: Find the constant of proportionality from graphs

Description: Students will determine the constant of proportionality from the graph of a proportional relationship. This often involves selecting a convenient point on the straight line (excluding the origin) and calculating the ratio of the y-coordinate to the x-coordinate (y/x). This reinforces the connection between the visual representation and the numerical constant.

Core example questions:

- A graph shows the relationship between the number of gallons of paint and the area it covers. If the line passes through (2, 80), what is the constant of proportionality?
- Identify the constant of proportionality from the given graph. (Provide a clear graph of a proportional relationship).
  Difficulty: Procedural
  Common pitfalls:
- Misreading coordinates from the graph, especially if the grid lines are not clearly marked.
- Not selecting a point that is easy to read accurately (e.g., picking a point that doesn't fall exactly on grid intersections).
- Dividing x by y instead of y by x.
  Introduced definitions: (No new definitions, but solidifies the graphical interpretation of the constant)
- Concept / Introduction / Definition: 3
- Practice: 12
- Common Pitfall avoidance: 5
- Application/word problem: 4
- Challenging: 3

- Total number of questions for each goal: 27

Name: Find the constant of proportionality from equations

Description: This skill focuses on directly identifying the constant of proportionality in an equation of a proportional relationship, which is always in the form y=kx. Students will recognize 'k' as the constant multiplier that relates 'x' to 'y'. This is the most straightforward method for finding the constant once the equation is in the correct form.

Core example questions:

- What is the constant of proportionality in the equation y=7x?
- For the equation C=2.5G (where C is cost and G is gallons), what is the constant of proportionality?
  Difficulty: Introductory
  Common pitfalls:
- Confusing the 'x' or 'y' with the constant 'k'.
- If the equation is not explicitly in y=kx form (e.g., 2y=6x), not rearranging it first (dividing by the coefficient of y).
- Mistaking a constant added or subtracted in an equation for the constant of proportionality (e.g., in y=3x+5).
  Introduced definitions: (No new definitions, reinforces the algebraic representation of the constant)
- Concept / Introduction / Definition: 3
- Practice: 10
- Common Pitfall avoidance: 4
- Application/word problem: 2
- Challenging: 2
- Total number of questions for each goal: 21

Name: Interpret the constant of proportionality in context

Description: This skill moves beyond just calculating the constant of proportionality to understanding what it actually means in a real-world scenario. Students will articulate the constant's meaning, often as a unit rate, clearly relating the two quantities involved in the problem's context. This is crucial for applying mathematical concepts to real-world situations.

Core example questions:

- If the constant of proportionality for a relationship between distance traveled (miles) and time (hours) is 55, what does this number represent in the context of the problem?

- A constant of proportionality of 0.75 describes the relationship between the amount of sugar (cups) and the amount of flour (cups) in a recipe. Explain what 0.75 means in this situation.
  Difficulty: Conceptual
  Common pitfalls:

- Stating the value without explaining its meaning in terms of the units involved (e.g., "55" instead of "55 miles per hour").

- Not clearly defining the relationship between the two quantities (e.g., "it's how much you get" instead of "it's the cost per item").

- Difficulty in articulating a clear, concise, and complete interpretation.
  Introduced definitions: (No new definitions, but deepens understanding and application of existing ones)

- Concept / Introduction / Definition: 4

- Practice: 7

- Common Pitfall avoidance: 4

- Application/word problem: 9

- Challenging: 4

- Total number of questions for each goal: 28

Name: Compare constants of proportionality

Description: This advanced skill requires students to compare and contrast different proportional relationships by first determining their respective constants of proportionality from various representations (tables, graphs, equations, verbal descriptions) and then comparing those values to draw conclusions. This synthesizes multiple previous skills and promotes higher-order thinking.

Core example questions:

- Company A charges $2.50 per square foot for painting. Company B's pricing is shown in the table below:

Area (sq ft)	Cost
10	$22.00
20	$44.00

Which company offers a better deal? Justify your answer.

- Graph 1 shows the relationship between distance and time for a car. Equation 2 is y=65x for another car. Which car is traveling faster? Explain.
  Difficulty: Application / Synthesis
  Common pitfalls:

- Not correctly identifying the constant of proportionality from each given representation.

- Making comparison errors (e.g., confusing a smaller constant with a better deal when it might imply less of a desired outcome, or vice-versa).

- Difficulty in clearly articulating the comparison and justifying the conclusion.

- Calculation errors when determining constants.
  Introduced definitions: (No new definitions, synthesizes prior concepts and terms)

- Concept / Introduction / Definition: 2

- Practice: 5

- Common Pitfall avoidance: 4

- Application/word problem: 10

- Challenging: 6

- Total number of questions for each goal: 27

Equations for Proportional Relationships

Name: Write equations for proportional relationships from tables

Description: This skill requires students to analyze a table of values that represents a proportional relationship and then express that relationship as an algebraic equation in the form y=kx. This involves identifying the constant of proportionality (k) from the table and then constructing the equation. This solidifies the connection between tabular data and algebraic representation.

Core example questions:

- Write an equation that represents the proportional relationship shown in the table:

x	y
3	12
5	20

8	32

- A table shows that 2 pounds of apples cost $4, and 4 pounds cost $8. Write an equation to represent the cost (C) for any number of pounds (P) of apples.
  Difficulty: Procedural
  Common pitfalls:

- Incorrectly calculating the constant of proportionality from the table (e.g., x/y instead of y/x).

- Not writing the equation in the correct y=kx form.

- Assuming a relationship is proportional without verifying the constant ratio for all pairs.
  Introduced definitions: (No new definitions, but applies previous ones to equation writing)

- Concept / Introduction / Definition: 3

- Practice: 12

- Common Pitfall avoidance: 5

- Application/word problem: 4

- Challenging: 3

- Total number of questions for each goal: 27

Name: Write equations for proportional relationships from graphs

Description: Given a graph that displays a proportional relationship, students will write its corresponding algebraic equation in the form y=kx. This involves identifying a point on the graph (other than the origin), using its coordinates to determine the constant of proportionality (k), and then forming the equation. This reinforces the visual to algebraic translation.

Core example questions:

- Write an equation for the proportional relationship shown in the graph below. (Provide a clear graph of a proportional relationship passing through, e.g., (2,6)).

- The graph shows the amount of money earned (M) over time (T) in hours. Write an equation that represents this relationship.
  Difficulty: Procedural
  Common pitfalls:

- Misreading coordinates from the graph.

- Incorrectly calculating k from the coordinates (e.g., x/y).

- Not recognizing that the graph must pass through the origin for it to be a proportional relationship.
- Forgetting to write the equation in the y=kx form.
  Introduced definitions: (No new definitions, but strengthens the graphical interpretation of k)
- Concept / Introduction / Definition: 3
- Practice: 10
- Common Pitfall avoidance: 5
- Application/word problem: 4
- Challenging: 3
- Total number of questions for each goal: 25

Name: Write equations for proportional relationships from verbal descriptions

Description: This skill involves translating a verbal description of a proportional relationship into an algebraic equation of the form y=kx. Students must identify the two quantities that are proportional, determine the constant of proportionality (unit rate) from the description, and then construct the equation. This is a critical step in problem-solving.

Core example questions:

- A car travels at a constant speed of 55 miles per hour. Write an equation to represent the distance (D) traveled in any number of hours (H).
- The cost of apples is $1.50 per pound. Write an equation that shows the total cost (C) for P pounds of apples.
  Difficulty: Core
  Common pitfalls:
- Incorrectly identifying the constant of proportionality from the wording.
- Confusing which variable represents x and which represents y.
- Including additional operations (addition/subtraction) in the equation if the relationship is strictly proportional.
  Introduced definitions: (No new definitions, but integrates verbal language with algebraic form)
- Concept / Introduction / Definition: 4
- Practice: 8
- Common Pitfall avoidance: 5
- Application/word problem: 6
- Challenging: 3

- Total number of questions for each goal: 26

Name: Solve proportions using various methods

Description: Students will learn and apply different methods to solve for an unknown value in a proportion. This includes the most common method of cross-multiplication, but also other equivalent ratio strategies such as finding a scale factor or finding the unit rate. This skill is foundational for solving many real-world problems involving proportional reasoning.

Core example questions:

- Solve for x: 53=20x
- If 4 apples cost $2, how much do 10 apples cost? Use a proportion to solve.
- A1.5=103. Find A.
  Difficulty: Procedural
  Common pitfalls:
- Errors in cross-multiplication (e.g., multiplying incorrectly or setting up the equation wrong).
- Not consistently applying the same operation to both sides of the proportion when using equivalent ratios.
- Difficulty with fractional or decimal values within the proportion.
  Introduced definitions: Proportion, Cross-multiplication, Equivalent Ratio
- Concept / Introduction / Definition: 3
- Practice: 15
- Common Pitfall avoidance: 7
- Application/word problem: 5
- Challenging: 4
- Total number of questions for each goal: 34

Name: Solve word problems involving proportional relationships

Description: This is a comprehensive application skill where students must apply their understanding of proportional relationships to solve multi-step real-world problems. This involves identifying that a situation is proportional, setting up a proportion or an equation, and then solving for an unknown quantity. These problems can involve various contexts such as scaling recipes, calculating distances, or determining costs.

Core example questions:

- If a 12-ounce can of soup costs $1.80, how much would a 20-ounce can cost if the price is proportional to the size?
- A recipe calls for 3 cups of flour for 2 dozen cookies. If you want to make 5 dozen cookies, how much flour do you need?
- A map has a scale of 1 inch = 50 miles. If two cities are 3.5 inches apart on the map, what is the actual distance between them?
  Difficulty: Application
  Common pitfalls:
- Difficulty translating the word problem into a correct mathematical setup (proportion or equation).
- Incorrectly identifying the quantities that are proportional.
- Setting up the proportion incorrectly (e.g., inconsistent units in numerator/denominator).
- Calculation errors.
  Introduced definitions: (No new definitions, but applies all previous concepts)
- Concept / Introduction / Definition: 2
- Practice: 8
- Common Pitfall avoidance: 6
- Application/word problem: 12
- Challenging: 6
- Total number of questions for each goal: 34

Applying Proportional Relationships to Real-World Problems

Percent Concepts

Name: Understand the concept of percent as a rate per 100

Description: This foundational skill introduces students to percentages as a specific type of ratio, where the second term (the whole) is always 100. Students will understand that "percent" literally means "per one hundred" and can be expressed as a fraction with a denominator of 100. This conceptual understanding is vital before moving to calculations.

Core example questions:

- What does 75% mean? How can it be written as a fraction?
- If 25 out of 100 students prefer reading, what percentage of students prefer reading?
- Explain why 100% represents the whole amount.
  Difficulty: Introductory

Common pitfalls:

- Misunderstanding that percent implies "out of 100".
- Confusing percent with fractions or decimals directly without the "per 100" context.
- Not understanding that a percentage can be greater than 100 or less than 1 (e.g., 0.5%).
  Introduced definitions: Percent, Percentage, Rate per 100
- Concept / Introduction / Definition: 5
- Practice: 8
- Common Pitfall avoidance: 3
- Application/word problem: 2
- Challenging: 1
- Total number of questions for each goal: 19

Name: Convert between percents, fractions, and decimals

Description: This skill focuses on the procedural fluency of converting numbers between their three primary forms: percents, fractions, and decimals. Students will practice rules for converting each to the others (e.g., percent to decimal by dividing by 100, decimal to percent by multiplying by 100, fraction to decimal by dividing numerator by denominator, etc.). This is a highly procedural but essential skill for solving percent problems.

Core example questions:

- Convert 45% to a decimal and a fraction in simplest form.
- Convert the fraction 43 to a decimal and a percentage.
- Convert 0.125 to a fraction and a percentage.
  Difficulty: Procedural
  Common pitfalls:
- Incorrectly moving the decimal point (e.g., multiplying by 10 instead of 100).
- Errors in simplifying fractions.
- Difficulty with repeating decimals or converting fractions to decimals.
- Forgetting to include the percent symbol when converting to percent.
  Introduced definitions: (No new definitions, focuses on conversion procedures)
- Concept / Introduction / Definition: 2

- Practice: 18
- Common Pitfall avoidance: 7
- Application/word problem: 3
- Challenging: 3
- Total number of questions for each goal: 33

Solving Percent Problems

Name: Solve percent problems (finding the part, whole, or percent)

Description: This fundamental skill in solving percent problems involves understanding the relationship between the part, the whole, and the percentage. Students will learn to use various strategies, such as setting up a proportion (wholepart=100percent) or using the percent equation (Part = Percent × Whole), to find the unknown quantity when the other two are given. This is a core procedural skill for all subsequent percent applications.

Core example questions:

- What is 25% of 80?
- 15 is what percent of 60?
- 30 is 40% of what number?
- If 7 out of 28 students scored an A on a test, what percentage of students scored an A?
  Difficulty: Core
  Common pitfalls:
- Misidentifying the part, whole, or percent in a given problem.
- Incorrectly setting up the proportion or equation.
- Errors in decimal placement when converting percentages.
- Forgetting to convert the percentage to a decimal or fraction before multiplying.
  Introduced definitions: Part, Whole, Percent
- Concept / Introduction / Definition: 4
- Practice: 15
- Common Pitfall avoidance: 7
- Application/word problem: 5
- Challenging: 3

- Total number of questions for each goal: 34

Name: Solve multistep ratio problems

Description: This skill focuses on solving complex problems involving ratios that require multiple steps or a combination of different ratio concepts. Students will need to break down problems into smaller, manageable parts, often using proportions or unit rates in sequence to arrive at a solution. These problems often bridge between general ratio understanding and real-world application.

Core example questions:

- A recipe calls for a ratio of 2 parts flour to 1 part sugar. If you use 6 cups of flour, how much sugar do you need? If the entire mixture weighs 18 cups, how much flour and how much sugar are there?
- Sarah runs 3 miles in 30 minutes. At this rate, how far will she run in 2 hours?
- The ratio of boys to girls in a class is 3:2. If there are 15 boys, how many students are there in total?
  Difficulty: Application
  Common pitfalls:
- Not identifying all the steps required to solve the problem.
- Errors in setting up intermediate proportions or unit rates.
- Confusing parts of a ratio with the total.
- Not checking if the final answer makes sense in the context of the problem.
  Introduced definitions: (No new definitions, but synthesizes existing ratio and proportion concepts)
- Concept / Introduction / Definition: 2
- Practice: 8
- Common Pitfall avoidance: 5
- Application/word problem: 10
- Challenging: 6
- Total number of questions for each goal: 31

Name: Calculate percent increase and decrease

Description: This skill teaches students how to determine the percentage change (either an increase or a decrease) of a quantity relative to its original amount. The formula for percent change is Original AmountChange×100%. Students will differentiate between finding the amount of change and finding the percentage of that change.

Core example questions:

- A shirt's price increased from $20 to $25. What is the percent increase?
- The number of students in a class decreased from 30 to 24. What is the percent decrease?
- If a stock's value goes from $50 to $40, calculate the percent decrease.
  Difficulty: Procedural
  Common pitfalls:
- Using the new amount instead of the original amount in the denominator.
- Incorrectly calculating the "change" (e.g., just using the new amount).
- Forgetting to multiply by 100 to convert the decimal to a percentage.
- Not distinguishing between increase and decrease in the final answer.
  Introduced definitions: Percent Increase, Percent Decrease, Original Amount, Change (in amount)
- Concept / Introduction / Definition: 3
- Practice: 10
- Common Pitfall avoidance: 6
- Application/word problem: 4
- Challenging: 3
- Total number of questions for each goal: 26

Name: Solve word problems involving percent increase and decrease

Description: This skill applies the calculation of percent increase and decrease to various real-world scenarios. Students will need to identify the original amount, the new amount, and whether the change is an increase or decrease to correctly set up and solve the problem. Contexts include population changes, sales figures, or growth rates.

Core example questions:

- Last year, a town's population was 10,000. This year, it's 10,500. What was the percent increase in population?
- A jacket originally cost $60 and is now on sale for $45. What is the percent decrease in price?
- The average temperature in July was 75 degrees Fahrenheit. In August, it was 81 degrees Fahrenheit. What was the percent increase in temperature?
  Difficulty: Application
  Common pitfalls:
- Misidentifying the original amount (denominator) in the problem.
- Calculation errors when determining the amount of change.

- Not articulating whether it's an increase or decrease in their answer.
  Introduced definitions: (No new definitions, applies previous ones in context)

- Concept / Introduction / Definition: 2

- Practice: 7

- Common Pitfall avoidance: 5

- Application/word problem: 10

- Challenging: 5

- Total number of questions for each goal: 29

Name: Solve word problems involving discounts, markups, and commissions

Description: This skill focuses on practical applications of percentages in financial contexts. Students will calculate and apply discounts (reduction in price), markups (increase in price by a business), and commissions (percentage of sales earned). These are common real-world scenarios that demonstrate the utility of percent calculations.

Core example questions:

- A store is offering a 20% discount on a $75 pair of shoes. How much is the discount? What is the new price?

- A salesperson earns a 15% commission on all sales. If they sell a car for $25,000, how much commission do they earn?

- A bicycle costs the store $120. They mark it up by 40%. What is the selling price of the bicycle?
  Difficulty: Application
  Common pitfalls:

- Confusing whether to add or subtract the calculated percent amount (e.g., adding a discount).

- Calculating the percentage of the wrong amount (e.g., calculating commission on the cost instead of the selling price).

- Errors in multi-step calculations.
  Introduced definitions: Discount, Markup, Commission, Original Price, Sale Price, Selling Price

- Concept / Introduction / Definition: 3

- Practice: 8

- Common Pitfall avoidance: 6

- Application/word problem: 12

- Challenging: 6

- Total number of questions for each goal: 35

Name: Solve percent error problems

Description: This skill introduces students to the concept of percent error, which measures the accuracy of a measurement or estimation compared to an actual or accepted value. The formula for percent error is $\left|\frac{\text{Actual Value} - \text{Measured Value}}{\text{Actual Value}}\right| \times 100\%$. This helps students understand the precision and limitations of data.

Core example questions:

- John estimated the length of a desk to be 70 inches. The actual length was 72 inches. What is the percent error in his estimation?
- A scientist measures the mass of a substance to be 5.2 grams. The known mass is 5.0 grams. Calculate the percent error.
- If a recipe calls for 2 cups of sugar, but you accidentally use 2.1 cups, what is your percent error?
  Difficulty: Application
  Common pitfalls:
- Forgetting to use the absolute value for the difference.
- Using the measured value instead of the actual value in the denominator.
- Errors in decimal or fraction arithmetic.
- Forgetting to multiply by 100 to express as a percentage.
  Introduced definitions: Percent Error, Actual Value, Measured Value, Estimated Value
- Concept / Introduction / Definition: 3
- Practice: 6
- Common Pitfall avoidance: 4
- Application/word problem: 8
- Challenging: 4
- Total number of questions for each goal: 25

Name: Solve multistep percent problems

Description: This is the most complex application skill within the percent unit, requiring students to integrate various percent concepts to solve real-world problems that involve multiple steps. These problems often combine discounts, sales tax, tips, percent increase/decrease, or other scenarios, challenging students to analyze the problem, determine the correct order of operations, and apply multiple percentage calculations.

Core example questions:

- A jacket costs $80. It is on sale for 25% off. After the discount, a 5% sales tax is added. What is the final price of the jacket?

- Sarah earned $300 last week. This week, her earnings increased by 10%. If she spent 20% of her earnings this week on groceries, how much did she spend on groceries?

- A restaurant bill is $45. You want to leave a 15% tip. If there is also a 6% sales tax applied to the original bill before the tip, what is the total amount you will pay?
  Difficulty: Synthesis
  Common pitfalls:

- Incorrect order of operations (e.g., adding tax before applying a discount).

- Calculating percentages on the wrong base amount at each step.

- Missing a step or misinterpreting the problem's requirements.

- Errors in complex arithmetic involving decimals.
  Introduced definitions: (No new definitions, synthesizes all prior percent concepts)

- Concept / Introduction / Definition: 2

- Practice: 7

- Common Pitfall avoidance: 5

- Application/word problem: 12

- Challenging: 7

- Total number of questions for each goal: 33

Name: Solve simple interest problems

Description: This skill introduces students to calculating simple interest, which is a common application of percentages in finance. Students will use the formula $I=Prt$ (Interest = Principal × rate × time) to find the interest earned or paid, or to solve for one of the other variables when the others are given. Understanding simple interest is a foundational concept for personal finance.

Core example questions:

- Calculate the simple interest earned on a principal of $500 at an annual interest rate of 4% for 3 years.

- If you borrow $1000 at a simple interest rate of 6% per year, how much interest will you owe after 6 months?

- If you earn $100 in simple interest on a principal of $2000 at a rate of 5% per year, for how many years was the money invested?
  Difficulty: Application

Common pitfalls:

- Not converting the percentage rate to a decimal before calculation.
- Not matching the time unit to the rate unit (e.g., using months for time when the rate is annual).
- Incorrectly performing the multiplication or division in the formula.
- Confusing interest with the total amount (principal + interest).
  Introduced definitions: Simple Interest, Principal, Interest Rate (Annual Rate), Time (in years), Total Amount
- Concept / Introduction / Definition: 3
- Practice: 10
- Common Pitfall avoidance: 5
- Application/word problem: 9
- Challenging: 5
- Total number of questions for each goal: 32

Scale Drawings

Name: Identify scale copies

Description: This skill focuses on helping students visually differentiate between figures that are true scale copies of an original figure and those that are not. Students will learn that a scale copy maintains all angle measures and all corresponding side lengths are in proportion. This is a foundational visual recognition skill.

Core example questions:

- Given a set of figures, identify which ones are scale copies of Figure A. Explain why the others are not.
- True or False: If you stretch a photograph horizontally, it is still a scale copy. Justify your answer.
  Difficulty: Introductory
  Common pitfalls:
- Confusing stretched or skewed figures with true scale copies.
- Focusing only on one dimension (e.g., length) without considering all dimensions or angles.
- Not understanding that angles must remain the same.
  Introduced definitions: Scale Copy, Corresponding Sides, Corresponding Angles, Original Figure
- Concept / Introduction / Definition: 5

- Practice: 10
- Common Pitfall avoidance: 5
- Application/word problem: 2
- Challenging: 2
- Total number of questions for each goal: 24

Name: Explore scale copies

Description: This skill deepens the conceptual understanding of scale copies. Students will investigate the properties of scale copies, recognizing that while side lengths change proportionally, angle measures remain invariant. This involves hands-on exploration or visual analysis of scaled figures.

Core example questions:

- Draw a simple figure (e.g., a rectangle). Then draw a larger version of it that is a scale copy and a smaller version that is a scale copy.
- If you double the length of all sides of a triangle, what happens to its angles? Explain.
- Describe what happens to the shape of a figure if you only multiply its width by a factor but not its height. Is it a scale copy?
  Difficulty: Conceptual
  Common pitfalls:
- Incorrectly assuming angle measures also scale.
- Difficulty in drawing accurate scale copies without a grid or explicit dimensions.
- Not precisely describing the observed properties.
  Introduced definitions: (Reinforces Scale Copy, Corresponding Sides, Corresponding Angles)
- Concept / Introduction / Definition: 4
- Practice: 7
- Common Pitfall avoidance: 3
- Application/word problem: 4
- Challenging: 3
- Total number of questions for each goal: 21

Name: Corresponding sides and points in scale drawings

Description: Students will learn to accurately identify corresponding sides, angles, and points between an original figure and its scale copy. This foundational skill is crucial for setting up proportions and understanding scale factors, ensuring that comparisons are made between the correct parts of the figures.

Core example questions:

- Given Figure A and its scale copy, Figure B, identify the side in Figure B that corresponds to side XY in Figure A.
- Point P is a vertex in the original figure. What is its corresponding point in the scale copy?
- If angle C in Figure 1 measures 90 degrees, what is the measure of its corresponding angle in a scale copy, Figure 2?
  Difficulty: Introductory
  Common pitfalls:
- Mismatching corresponding parts due to rotation or reflection of the scale copy.
- Confusing sides with angles, or points with sides.
- Not understanding that corresponding angles have equal measures.
  Introduced definitions: Corresponding Sides, Corresponding Angles, Corresponding Points, Vertex
- Concept / Introduction / Definition: 4
- Practice: 10
- Common Pitfall avoidance: 5
- Application/word problem: 2
- Challenging: 1
- Total number of questions for each goal: 22

Name: Identify scale factor in scale drawings

Description: This skill involves determining the scale factor (the constant ratio by which all corresponding lengths in a figure are multiplied to create a scale copy) between an original figure and its scaled copy. Students will use the ratio of a length in the copy to the corresponding length in the original.

Core example questions:

- A rectangle measures 3 cm by 5 cm. Its scale copy measures 9 cm by 15 cm. What is the scale factor?
- If a side in the original drawing is 4 inches and its corresponding side in the scale drawing is 1 inch, what is the scale factor?
  Difficulty: Procedural
  Common pitfalls:
- Incorrectly setting up the ratio (e.g., original/copy instead of copy/original).

- Not using corresponding sides.
- Calculation errors, especially with fractions or decimals.
- Confusing a scale factor greater than 1 for a reduction, or less than 1 for an enlargement.
  Introduced definitions: Scale Factor, Enlargement, Reduction
- Concept / Introduction / Definition: 3
- Practice: 12
- Common Pitfall avoidance: 6
- Application/word problem: 4
- Challenging: 3
- Total number of questions for each goal: 28

Name: Interpret scale factor in scale drawings

Description: Students will move beyond just calculating the scale factor to understanding what it means in the context of scaling. They will explain how scale factors affect the size of a scaled copy (e.g., a scale factor > 1 means enlargement, < 1 means reduction) and how scaling can be reversed using the reciprocal of the scale factor.

Core example questions:

- If a figure is scaled by a factor of 3, how does its new size compare to the original? How would you get back to the original size?
- Explain the difference between a scale factor of 0.5 and a scale factor of 2.
- A map uses a scale factor of 1/1000. What does this tell you about the map compared to the actual area?
  Difficulty: Conceptual
  Common pitfalls:
- Difficulty articulating the relationship between the scale factor and the size change.
- Not understanding that multiplying by the reciprocal reverses the scaling.
- Confusing fractions and decimals in terms of enlargement/reduction.
  Introduced definitions: (No new definitions, focuses on conceptual understanding of Scale Factor)
- Concept / Introduction / Definition: 4
- Practice: 6
- Common Pitfall avoidance: 3

- Application/word problem: 5
- Challenging: 3
- Total number of questions for each goal: 21

Name: Construct scale drawings given a scale factor

Description: This skill requires students to physically or graphically draw a scaled copy of a given figure using a specified scale factor. This could involve using graph paper, rulers, or digital tools. This is a hands-on application of the definition of a scale factor and proportional reasoning.

Core example questions:

- Draw a rectangle with dimensions 4 cm by 6 cm. Then draw a scale copy of this rectangle using a scale factor of 1.5.
- On graph paper, draw a simple polygon. Then draw a scale copy of it using a scale factor of 1/2.
  Difficulty: Procedural
  Common pitfalls:
- Calculation errors when multiplying dimensions by the scale factor.
- Inaccurate drawing or measurement.
- Not scaling all dimensions proportionally.
- Difficulty with fractional or decimal scale factors.
  Introduced definitions: (No new definitions, applies previous ones in construction)
- Concept / Introduction / Definition: 2
- Practice: 8
- Common Pitfall avoidance: 4
- Application/word problem: 3
- Challenging: 3
- Total number of questions for each goal: 20

Name: Solve problems involving scale drawings: finding actual lengths

Description: Students will use a given scale drawing (e.g., a map, blueprint) and its stated scale (e.g., 1 inch = 10 feet, 1:200) to calculate actual distances or lengths of objects in the real world. This typically involves setting up and solving proportions.

Core example questions:

- On a map, 1 cm represents 50 km. If two cities are 3.5 cm apart on the map, what is the actual distance between them?

- A blueprint has a scale of 1 inch = 4 feet. A room is 5 inches long on the blueprint. What is the actual length of the room?
  Difficulty: Application
  Common pitfalls:

- Incorrectly setting up the proportion (e.g., inverting the scale ratio).

- Errors in unit conversion (e.g., forgetting to convert cm to km).

- Calculation errors, especially with decimals or fractions.

- Not clearly identifying the drawing length and the actual length.
  Introduced definitions: Scale (of a drawing), Actual Length, Drawing Length

- Concept / Introduction / Definition: 2

- Practice: 8

- Common Pitfall avoidance: 5

- Application/word problem: 10

- Challenging: 5

- Total number of questions for each goal: 30

Name: Determine the scale of a drawing

Description: This skill reverses the previous one. Students will be given actual object dimensions and their corresponding dimensions on a scale drawing and will need to determine the scale that was used for the drawing. This involves finding the ratio of the drawing length to the actual length and simplifying it to a unit scale (e.g., 1 inch : X feet).

Core example questions:

- A building is 100 feet tall. On a model, it is 10 inches tall. What is the scale of the model? (Express as "1 inch = X feet").

- A cell is 0.05 mm long. Its diagram in a science book is 5 cm long. What is the scale of the diagram?
  Difficulty: Procedural
  Common pitfalls:

- Not converting units to be consistent before setting up the ratio.

- Incorrectly simplifying the ratio or expressing the scale.

- Confusing which value goes in the numerator and which in the denominator.
  Introduced definitions: (No new definitions, applies existing ones in a new context)

- Concept / Introduction / Definition: 2
- Practice: 9
- Common Pitfall avoidance: 5
- Application/word problem: 6
- Challenging: 4
- Total number of questions for each goal: 26

Name: Reproduce a scale drawing at a different scale

Description: This advanced skill requires students to take an existing scale drawing (with its implied or stated scale) and create a new scale drawing of the same object but at a different, specified scale. This involves a two-step process: first determining the actual dimensions from the original drawing, then using those actual dimensions to create the new drawing with the new scale.

Core example questions:

- A map has a scale of 1 cm = 10 km. Redraw a section of this map, using a new scale of 1 cm = 5 km. (Provide a simple map section).
- A blueprint of a house has a scale of 1 inch = 10 feet. Create a new drawing of one room from the blueprint using a scale of 1 inch = 5 feet.
  Difficulty: Synthesis
  Common pitfalls:
- Errors in the intermediate step of finding actual dimensions.
- Incorrectly applying the new scale.
- Calculation mistakes with multiple conversions.
- Inaccurate drawing.
  Introduced definitions: (No new definitions, synthesizes all prior scale drawing concepts)
- Concept / Introduction / Definition: 2
- Practice: 5
- Common Pitfall avoidance: 4
- Application/word problem: 7
- Challenging: 5
- Total number of questions for each goal: 23

Name: Relate scale drawings to area

Description: This skill focuses on understanding how the area of a scaled copy is related to the original area and the scale factor. Students will discover that if linear dimensions are scaled by a factor 'k', then the area is scaled by a factor of k2. This is a conceptual leap from linear scaling to two-dimensional scaling.

Core example questions:

- If a photograph is enlarged by a scale factor of 2, how much larger is its area compared to the original?
- A rectangular garden has an area of 50 square feet. If you create a scale drawing of the garden using a scale factor of 1/10, what will be the area of the garden in the drawing?
- Explain why doubling the sides of a square makes its area four times larger.
  Difficulty: Conceptual
  Common pitfalls:
- Incorrectly assuming area scales by 'k' instead of k2.
- Difficulty in applying the k2 rule, especially with fractional scale factors.
- Not understanding the conceptual reason behind the k2 relationship.
  Introduced definitions: (No new definitions, but introduces the relationship between linear scale factor and area scale factor)
- Concept / Introduction / Definition: 4
- Practice: 6
- Common Pitfall avoidance: 4
- Application/word problem: 7
- Challenging: 4
- Total number of questions for each goal: 25

Name: Solve problems involving scale drawings: word problems with scale factors

Description: This skill focuses on solving real-world word problems that specifically require applying the concept of scale factor to find unknown lengths or dimensions. These problems often do not explicitly state a traditional "scale" but imply a scale factor through given dimensions of original and scaled objects.

Core example questions:

- A toy car is a scale model of a real car. If the toy car is 10 inches long and the real car is 180 inches long, what is the scale factor of the toy car to the real car?
- A designer wants to enlarge a logo. The original logo is 5 cm wide. If the enlarged logo needs to be 20 cm wide, what scale factor should the designer use?

- A photocopy machine can reduce images by a scale factor of 0.8. If an original document is 15 inches long, how long will it be after being reduced by the machine?
  Difficulty: Application
  Common pitfalls:

- Misidentifying the original and scaled dimensions in the problem.

- Incorrectly calculating the scale factor (e.g., original/copy).

- Errors when applying the scale factor to find an unknown.

- Difficulty with wording that implies a scale factor rather than stating a direct scale.
  Introduced definitions: (No new definitions, applies previous scale factor concepts)

- Concept / Introduction / Definition: 2

- Practice: 7

- Common Pitfall avoidance: 5

- Application/word problem: 10

- Challenging: 6

- Total number of questions for each goal: 30

Name: Use different scales to describe drawings

Description: Students will understand and apply different types of scales used to describe and work with scale drawings, including ratio scales (e.g., 1:100), fractional scales (e.g., 1/100), and verbal/unit scales (e.g., 1 cm = 10 km). This skill ensures flexibility in interpreting and using diverse scale notations common in real-world applications.

Core example questions:

- A map states its scale is 1:25,000. Explain what this means in terms of actual distances.

- Convert the verbal scale "1 inch = 2 feet" into a ratio scale and a fractional scale.

- If a drawing has a fractional scale of 1/50, what does this imply about the size of the drawing compared to the actual object?
  Difficulty: Conceptual / Procedural
  Common pitfalls:

- Misinterpreting ratio scales, especially without units (e.g., 1:100 means 1 unit on drawing to 100 same units actual).

- Errors in unit conversions when converting between different types of scales.

- Difficulty in expressing scales clearly and accurately.
  Introduced definitions: Ratio Scale, Fractional Scale, Verbal Scale (Unit Scale)

- Concept / Introduction / Definition: 4
- Practice: 8
- Common Pitfall avoidance: 4
- Application/word problem: 5
- Challenging: 3
- Total number of questions for each goal: 24

The Number System Exercise Curriculum

Operations with Rational Numbers

Understanding Rational Numbers

Name: Interpreting Negative Numbers

Description: This skill introduces students to the meaning and application of negative numbers within real-world contexts. Students will interpret signed numbers (positive and negative) to represent quantities such as temperatures below zero, depths below sea level, debts, or movements in opposite directions, understanding their magnitude and direction.

Core example questions:

- If the temperature is -5 degrees Celsius, what does the negative sign tell you about the temperature?
- An elevation of -200 feet means what in relation to sea level?
- What does a bank balance of -$50 indicate?
  Difficulty: Introductory
  Common pitfalls:
- Confusing the magnitude of a negative number (e.g., thinking -10 is "bigger" than -5).
- Difficulty relating negative signs to direction or position relative to a zero point.
- Not understanding that "negative" doesn't always mean "less" in terms of magnitude.
  Introduced definitions: Negative Numbers, Positive Numbers, Signed Numbers, Magnitude, Sea Level, Below Zero, Debt
- Concept / Introduction / Definition: 5
- Practice: 10
- Common Pitfall avoidance: 4
- Application/word problem: 5

- Challenging: 2
- Total number of questions for each goal: 26

Name: Changing Temperatures

Description: This skill uses the context of temperature changes to model and solve problems involving the addition of positive and negative numbers. Students will visualize changes on a number line, reinforcing the concept of adding integers with different signs.

Core example questions:

- The temperature was 5 degrees Celsius and dropped by 8 degrees. What is the new temperature? Use a number line to show your work.
- If the temperature is -3 degrees Fahrenheit and rises by 7 degrees, what is the new temperature?
- The temperature started at -10°C and ended at 2°C. What was the change in temperature?
  Difficulty: Core
  Common pitfalls:
- Incorrectly performing integer addition, especially when signs are different.
- Difficulty visualizing movement on a number line.
- Confusing "rise" with "decrease" or "fall" with "increase."
  Introduced definitions: Number Line, Temperature Change, Drop, Rise
- Concept / Introduction / Definition: 3
- Practice: 12
- Common Pitfall avoidance: 6
- Application/word problem: 7
- Challenging: 3
- Total number of questions for each goal: 31

Name: Changing Elevation

Description: This skill builds on the understanding of interpreting negative numbers by applying addition of positive and negative numbers within the context of changing elevation. Students will model movements up and down from a reference point (like sea level) and calculate final positions. This reinforces integer addition rules in a spatial context.

Core example questions:

- A submarine is at an elevation of -250 feet. It ascends 100 feet. What is its new elevation?
- A hiker starts at 500 feet above sea level and descends 700 feet. What is their new elevation?
- If a bird flies from an elevation of -10 meters to an elevation of 5 meters, what is the total change in elevation?
  Difficulty: Core
  Common pitfalls:
- Errors in applying integer addition rules.
- Misinterpreting "ascends" as negative or "descends" as positive.
- Difficulty setting up the initial number (e.g., starting point) and the change.
  Introduced definitions: Elevation, Ascend, Descend, Above Sea Level, Below Sea Level
- Concept / Introduction / Definition: 3
- Practice: 10
- Common Pitfall avoidance: 5
- Application/word problem: 8
- Challenging: 4
- Total number of questions for each goal: 30

Name: Money and Debts

Description: This skill applies the concept of positive and negative numbers to financial situations, specifically focusing on money and debts. Students will learn to represent money owed as negative values and money gained or available as positive values, using addition of rational numbers to calculate account balances or net financial positions.

Core example questions:

- You have $75 in your bank account. You write a check for $100. What is your new balance?
- If you owe your friend $20 and then pay them back $15, how much do you still owe?
- A business starts with a debt of $500. They make a profit of $700. What is their new financial standing?
  Difficulty: Core
  Common pitfalls:
- Difficulty in consistently applying signs to financial transactions (e.g., withdrawals as positive).
- Errors in integer or rational number addition with mixed signs.

- Not distinguishing between the amount of a transaction and the resulting balance.
  Introduced definitions: Debt, Credit, Debit, Balance, Account Balance, Profit, Loss

- Concept / Introduction / Definition: 3

- Practice: 10

- Common Pitfall avoidance: 5

- Application/word problem: 8

- Challenging: 4

- Total number of questions for each goal: 30

Name: Understand absolute value

Description: This skill introduces the concept of absolute value as the distance of a number from zero on a number line. Students will learn that absolute value is always non-negative, regardless of whether the original number is positive or negative. This is a foundational concept for later understanding of distance between numbers.

Core example questions:

- What is the absolute value of -7?

- Explain what $|-12|$ means.

- Find the value of $|9|$.

- Which number is farther from zero: -5 or 3? Explain using absolute value.
  Difficulty: Introductory
  Common pitfalls:

- Confusing absolute value with simply changing the sign of a number (e.g., thinking $|-x|=x$ always, even if x is negative).

- Not understanding the "distance from zero" definition.

- Miscalculating absolute values of fractions or decimals.
  Introduced definitions: Absolute Value, Distance from Zero

- Concept / Introduction / Definition: 4

- Practice: 12

- Common Pitfall avoidance: 6

- Application/word problem: 3

- Challenging: 2
- Total number of questions for each goal: 27

Name: Absolute value and opposite integers

Description: This skill builds on the understanding of absolute value by exploring its relationship with opposite integers. Students will learn that opposite integers (e.g., 5 and -5) have the same absolute value because they are equidistant from zero on the number line.

Core example questions:

- What is the opposite of -8? What is the absolute value of both -8 and its opposite?
- Are the absolute values of two opposite integers always the same? Explain why or why not.
- Give an example of two numbers that have the same absolute value but are not the same number.
  Difficulty: Conceptual
  Common pitfalls:
- Confusing "opposite" with "absolute value" (e.g., thinking the opposite of -5 is |-5|).
- Not clearly understanding the concept of "equidistant from zero".
- Difficulty articulating the relationship verbally.
  Introduced definitions: Opposite Integers
- Concept / Introduction / Definition: 4
- Practice: 8
- Common Pitfall avoidance: 4
- Application/word problem: 3
- Challenging: 2
- Total number of questions for each goal: 21

Name: Converting fractions to decimals

Description: This skill focuses on the procedural fluency of converting fractions or mixed numbers into their decimal form using long division. Students will also learn to recognize and distinguish between terminating decimals (those that end) and repeating decimals (those with a pattern of digits that repeats infinitely).

Core example questions:

- Convert 83 to a decimal.

- Convert 32 to a decimal. Is it terminating or repeating?
- Convert 241 to a decimal.
- Express 0.45 as a fraction in simplest form.
  Difficulty: Procedural
  Common pitfalls:
- Errors in long division.
- Not recognizing the repeating pattern in a decimal.
- Incorrectly handling mixed numbers (e.g., only converting the fraction part and forgetting the whole number).
- Difficulty converting repeating decimals back to fractions.
  Introduced definitions: Terminating Decimal, Repeating Decimal, Long Division
- Concept / Introduction / Definition: 3
- Practice: 15
- Common Pitfall avoidance: 7
- Application/word problem: 2
- Challenging: 3
- Total number of questions for each goal: 30

Name: Order rational numbers

Description: This skill requires students to order a set of rational numbers presented in various forms (fractions, decimals, integers, and possibly percents). Students will need to convert numbers to a common form (usually decimals) to facilitate comparison and then arrange them in ascending or descending order.

Core example questions:

- Order the following numbers from least to greatest: 21, 0.4, -1, 43, -0.75.
- Arrange in descending order: 2.3, 37, 2.05, 221.
- Place the following numbers on a number line: -2.5, 25, 0, –43, 1.8.
  Difficulty: Procedural
  Common pitfalls:
- Errors in converting all numbers to a common format.
- Incorrectly comparing negative numbers (e.g., thinking -3 is greater than -1).

- Difficulty comparing fractions with different denominators without conversion.
- Misplacing numbers on a number line.
  Introduced definitions: Rational Numbers, Ascending Order, Descending Order
- Concept / Introduction / Definition: 3
- Practice: 12
- Common Pitfall avoidance: 6
- Application/word problem: 4
- Challenging: 4
- Total number of questions for each goal: 29

Adding and Subtracting Rational Numbers

Name: Add with integer chips

Description: This skill uses a concrete manipulative, integer chips (e.g., red for negative, yellow for positive), to model and understand the addition of integers. Students will visually represent integers and explore how "zero pairs" (one positive and one negative chip) cancel each other out, leading to the sum. This hands-on approach builds conceptual understanding.

Core example questions:

- Use integer chips to model 3+(–5). Draw your chip representation.
- How many zero pairs can you form with 4 positive chips and 2 negative chips? What is the sum?
- Model –6+2 using integer chips and explain your result.
  Difficulty: Introductory
  Common pitfalls:
- Incorrectly representing negative or positive numbers with chips.
- Not understanding the concept of a "zero pair" and how it simplifies the sum.
- Difficulty in drawing clear chip models.
  Introduced definitions: Integer Chips, Zero Pair
- Concept / Introduction / Definition: 5
- Practice: 10
- Common Pitfall avoidance: 5

- Application/word problem: 2
- Challenging: 1
- Total number of questions for each goal: 23

Name: Adding negative numbers on the number line

Description: This skill focuses on representing the addition of negative numbers visually on a number line. Students will start at a given number and move in the appropriate direction (right for positive, left for negative) by the specified distance, reinforcing the idea of direction and magnitude in integer addition.

Core example questions:

- Use a number line to show how to calculate $4+(-6)$.
- Start at -2 on the number line. If you add -3, where do you land? Draw your movement.
- Model the expression $-5+7$ on a number line.
  Difficulty: Core
  Common pitfalls:
- Moving in the wrong direction for negative numbers.
- Miscounting units on the number line.
- Starting at the wrong initial point.
- Difficulty with larger numbers or non-integer values on a number line.
  Introduced definitions: (No new definitions, but reinforces Number Line)
- Concept / Introduction / Definition: 4
- Practice: 12
- Common Pitfall avoidance: 6
- Application/word problem: 3
- Challenging: 2
- Total number of questions for each goal: 27

Name: Number equations & number lines

Description: This skill connects addition and subtraction equations involving rational numbers to movements on a number line. Students will demonstrate their understanding by representing equations visually and deriving equations from given number line models. This helps bridge the gap between abstract equations and concrete visual representations.

Core example questions:

- Write an equation that represents the following movement on a number line: Start at 3, move left 7 units.
- Model the equation $2.5+(-4)=-1.5$ on a number line.
- Explain why subtracting a negative number is equivalent to adding a positive number, using a number line as an example.

Difficulty: Conceptual
Common pitfalls:

  - Incorrectly translating movement direction/distance to positive/negative numbers in equations.
  - Difficulty showing subtraction on a number line, especially subtraction of negative numbers.
  - Not accurately placing rational numbers (decimals/fractions) on the number line.
    Introduced definitions: (No new definitions, connects equations to number lines)

- Concept / Introduction / Definition: 4
- Practice: 8
- Common Pitfall avoidance: 5
- Application/word problem: 4
- Challenging: 3
- Total number of questions for each goal: 24

Name: Add integers using counters

Description: Similar to integer chips, this skill uses generic counters (e.g., colored circles) to model integer addition. It reinforces the concept of positive and negative values combining and canceling out, providing another concrete visual aid for understanding integer operations.

Core example questions:

- Use counters to show the sum of -4 and 6. Draw your model.
- If you have 7 red counters (negative) and 3 blue counters (positive), what is the sum?
- Model $5+(-5)$ using counters. What do you notice?
  Difficulty: Introductory
  Common pitfalls:

- Similar to integer chips, not forming zero pairs correctly.
- Inconsistent assignment of colors to positive/negative.
- Difficulty translating the physical model to the abstract sum.
  Introduced definitions: (No new definitions, reinforces chip/counter modeling)
- Concept / Introduction / Definition: 4
- Practice: 9
- Common Pitfall avoidance: 4
- Application/word problem: 2
- Challenging: 1
- Total number of questions for each goal: 20

Name: Add integers using number lines

Description: This skill provides focused practice on modeling and solving integer addition problems exclusively using a number line. It helps students internalize the directional movements associated with positive and negative numbers on a linear scale, leading to a strong mental model for integer addition.

Core example questions:

- Use a number line to find the sum of -3 and -4.
- Show how to solve 8+(–5) using a number line.
- The temperature started at -1 degree and rose by 6 degrees. Use a number line to find the new temperature.
  Difficulty: Procedural
  Common pitfalls:
- Errors in direction and magnitude of jumps on the number line.
- Miscounting units or landing on the wrong integer.
- Not accurately drawing or labeling the number line for clarity.
  Introduced definitions: (No new definitions, reinforces number line modeling)
- Concept / Introduction / Definition: 3
- Practice: 12
- Common Pitfall avoidance: 6
- Application/word problem: 4

- Challenging: 2
- Total number of questions for each goal: 27

Name: Integer addition rules

Description: This skill transitions students from concrete models to abstract rules for adding integers. Students will learn and apply the rules for adding integers with the same sign (add magnitudes, keep the sign) and different signs (subtract magnitudes, keep the sign of the number with the larger absolute value). This is critical for efficient computation.

Core example questions:

- Explain the rule for adding two negative integers. Provide an example.
- What is the rule for adding a positive and a negative integer? Give an example where the sum is positive, and one where it's negative.
- Solve: $-15+(-7)$ and $20+(-12)$.
  Difficulty: Core
  Common pitfalls:
- Confusing rules for different signs vs. same signs.
- Errors in determining the sign of the sum when signs are different.
- Subtracting the larger magnitude from the smaller, or vice versa, incorrectly.
- Forgetting to take the absolute value when determining the dominant sign.
  Introduced definitions: (No new definitions, focuses on procedural rules)
- Concept / Introduction / Definition: 5
- Practice: 15
- Common Pitfall avoidance: 8
- Application/word problem: 3
- Challenging: 3
- Total number of questions for each goal: 34

Name: Add integers

Description: This skill focuses on developing fluency in adding integers without relying on models. Students will apply the learned integer addition rules to quickly and accurately calculate sums of two integers, fostering computational proficiency.

Core example questions:

- Calculate: $-8+12$
- Solve: $-10+(-5)$
- Find the sum of 18 and -25.
- $-3+(-7)+10$
  Difficulty: Procedural
  Common pitfalls:
- Residual errors from confusing addition rules.
- Careless computational mistakes.
- Lack of immediate recall for basic integer sums.
  Introduced definitions: (No new definitions, focuses on fluency)
- Concept / Introduction / Definition: 1
- Practice: 20
- Common Pitfall avoidance: 8
- Application/word problem: 4
- Challenging: 3
- Total number of questions for each goal: 36

Name: Add three or more integers

Description: This skill extends integer addition to include sums of three or more integers. Students will apply the rules for integer addition iteratively or by grouping positive and negative numbers before combining, building efficiency and accuracy with longer expressions.

Core example questions:

- Find the sum: $-2+5+(-8)$
- Calculate: $15+(-6)+3+(-10)$
- The temperature was -5°F, then rose 12°, then fell 7°. What is the final temperature?
  Difficulty: Procedural
  Common pitfalls:
- Losing track of signs in multi-step calculations.
- Calculation errors from combining too many numbers at once.

- Not simplifying expressions methodically.
  Introduced definitions: (No new definitions, extends integer addition)

- Concept / Introduction / Definition: 2

- Practice: 12

- Common Pitfall avoidance: 6

- Application/word problem: 5

- Challenging: 4

- Total number of questions for each goal: 29

Name: Quantities that combine to zero: word problems

Description: This skill requires students to describe and identify real-world situations where two opposite quantities combine to make zero. This reinforces the concept of additive inverses and the meaning of zero in various contexts (e.g., gains and losses, movements in opposite directions).

Core example questions:

- Describe a situation where taking 5 steps forward and then 5 steps backward results in a net movement of zero.

- Give an example of a financial transaction that would result in a balance of $0 if you start with $0.

- If a balloon rises 100 feet and then its elevation changes by -100 feet, what is its final change in elevation?
  Difficulty: Conceptual
  Common pitfalls:

- Difficulty articulating real-world scenarios.

- Not understanding that the quantities must be "opposite" and equal in magnitude.

- Confusing a final state of zero with simply doing nothing.
  Introduced definitions: Additive Inverse, Net Change, Zero Result

- Concept / Introduction / Definition: 5

- Practice: 6

- Common Pitfall avoidance: 3

- Application/word problem: 7

- Challenging: 3

- Total number of questions for each goal: 24

Name: Signs of sums

Description: This skill focuses on developing the ability to determine the sign of a sum involving positive and negative numbers before calculating the exact value. Students will use their understanding of integer addition rules, especially the concept of absolute value, to predict whether the sum will be positive, negative, or zero.

Core example questions:

- Without calculating, will the sum of $-15+10$ be positive or negative? Explain why.
- If you add a positive number and a negative number, when will the sum be zero?
- What is the sign of $-20+(-5)$?
- If a number is added to a smaller negative number, will the sum always be positive? Give an example.
  Difficulty: Conceptual
  Common pitfalls:
- Incorrectly comparing absolute values to determine the dominant sign.
- Confusing the sign of the number with the sign of the sum.
- Rushing to calculate instead of first considering the signs.
  Introduced definitions: (No new definitions, reinforces sign rules conceptually)
- Concept / Introduction / Definition: 4
- Practice: 10
- Common Pitfall avoidance: 6
- Application/word problem: 4
- Challenging: 3
- Total number of questions for each goal: 27

Name: Absolute value to find distance

Description: This skill introduces the application of absolute value to calculate the distance between any two rational numbers on a number line. Students will learn that the distance between two numbers 'a' and 'b' is given by $|a-b|$ or $|b-a|$, which is always a positive value.

Core example questions:

- What is the distance between -3 and 5 on a number line? Use absolute value.

- Find the distance between 2.5 and -4.5.
- The temperature changed from 10°F to -8°F. What is the total change in temperature (distance)?
  Difficulty: Procedural
  Common pitfalls:
- Forgetting to use absolute value, resulting in a negative distance.
- Errors in subtraction before taking the absolute value.
- Difficulty with fractional or decimal numbers.
- Not understanding that distance is always non-negative.
  Introduced definitions: Distance (on a number line)
- Concept / Introduction / Definition: 3
- Practice: 10
- Common Pitfall avoidance: 5
- Application/word problem: 6
- Challenging: 4
- Total number of questions for each goal: 28

Name: Absolute value to find distance challenge

Description: This skill involves solving more complex problems where finding the distance between rational numbers using absolute value is a key step, potentially integrated into multi-step scenarios or problems requiring deeper interpretation.

Core example questions:

- A diver is at -15 feet relative to the water surface. A bird is flying at 20 feet above the water surface. What is the vertical distance between the diver and the bird?
- Point A is at -21 and Point B is at 341 on a number line. What is the distance between them?
- If the distance between two numbers is 8, and one number is -3, what are the possible values for the other number?
  Difficulty: Challenging
  Common pitfalls:
- Misinterpreting the problem setup.
- Errors in multi-step calculations.
- Forgetting absolute value or performing subtraction incorrectly.

- Difficulty with non-integer rational numbers.
  Introduced definitions: (No new definitions, applies previous ones in complex problems)
- Concept / Introduction / Definition: 2
- Practice: 5
- Common Pitfall avoidance: 3
- Application/word problem: 8
- Challenging: 6
- Total number of questions for each goal: 24

Name: Adding & subtracting negative numbers

Description: This skill integrates both addition and subtraction operations involving negative numbers, including integers and other rational numbers (fractions and decimals). Students will master the rule "subtracting a negative is the same as adding a positive" and fluently apply all integer operation rules.

Core example questions:

- Calculate: $-5-(-8)$
- Solve: $12+(-7)-3$
- Find the value of $-4.5-2.1+(-1.3)$.
- The temperature dropped from 10°C to -2°C. What was the total change in temperature? (Emphasize subtraction as finding change).
  Difficulty: Procedural
  Common pitfalls:
- Confusing subtraction of a negative with addition of a negative.
- Errors in converting subtraction to addition (e.g., $a-b=a+(-b)$).
- Calculation errors with fractions and decimals.
- Not performing operations from left to right in multi-operation expressions.
  Introduced definitions: (No new definitions, reinforces integer addition/subtraction rules)
- Concept / Introduction / Definition: 2
- Practice: 15
- Common Pitfall avoidance: 7
- Application/word problem: 5

- Challenging: 4
- Total number of questions for each goal: 33

Name: Adding & subtracting rational numbers

Description: This is a culminating skill for addition and subtraction, requiring students to perform these operations with any rational numbers: positive and negative integers, fractions, and decimals. This demands fluency in converting forms if necessary, applying rules for signs, and accurate computation.

Core example questions:

- Calculate: $-321+45$
- Solve: $7.2-(-3.8)-1.5$
- Find the value of $-52+21-(-103)$.
- A stock's price started at $50.25, dropped by $7.50, then rose by $3.75. What is the final price?
  Difficulty: Procedural
  Common pitfalls:
- Errors in finding common denominators for fractions.
- Decimal alignment errors in addition/subtraction.
- Sign errors, especially with multiple operations.
- Difficulty converting between fractions and decimals if needed.
  Introduced definitions: (No new definitions, synthesizes all prior addition/subtraction skills)
- Concept / Introduction / Definition: 2
- Practice: 18
- Common Pitfall avoidance: 8
- Application/word problem: 7
- Challenging: 5
- Total number of questions for each goal: 40

Name: Addition & subtraction: find the missing value

Description: This skill involves solving for a missing value (variable) in addition and subtraction equations involving rational numbers. Students will use inverse operations to isolate the unknown, applying their knowledge of rational number arithmetic.

Core example questions:

- Solve for x: $x+(-7)=15$
- Find the missing value: $12.3-y=-5.8$
- Solve for z: $-31+z=65$
- If the temperature went from T to -4°C after a drop of 9°C, what was the initial temperature T?
  Difficulty: Core
  Common pitfalls:
- Incorrectly applying inverse operations (e.g., adding instead of subtracting).
- Sign errors when moving terms across the equals sign.
- Computational errors with rational numbers.
- Difficulty with equations where the variable is being subtracted (e.g., $5-x=2$).
  Introduced definitions: (No new definitions, applies inverse operations to rational numbers)
- Concept / Introduction / Definition: 2
- Practice: 10
- Common Pitfall avoidance: 6
- Application/word problem: 5
- Challenging: 4
- Total number of questions for each goal: 27

Name: Equivalent expressions with negative numbers

Description: This skill focuses on understanding and creating equivalent expressions involving negative numbers, primarily through the application of the properties of addition and subtraction. Students will learn that subtraction can be rewritten as adding the opposite ($a-b=a+(-b)$) and identify other equivalent forms, deepening their algebraic reasoning with rational numbers.

Core example questions:

- Write an equivalent expression for $5-(-3)$.
- Which of the following expressions is equivalent to $-7+2$? (Options: $2-7$, $7-2$, $-7-2$)
- Show that $10-4$ is equivalent to $10+(-4)$.
  Difficulty: Conceptual
  Common pitfalls:

- Sign errors when converting subtraction to addition (e.g., a−(−b) incorrectly becoming a−b).
- Not recognizing that the order of terms changes with subtraction if not using additive inverse (e.g., a−b=b−a).
- Difficulty with expressions involving three or more terms.
  Introduced definitions: Equivalent Expressions
- Concept / Introduction / Definition: 4
- Practice: 8
- Common Pitfall avoidance: 5
- Application/word problem: 2
- Challenging: 3
- Total number of questions for each goal: 22

Name: Interpret negative number addition and subtraction expressions

Description: This skill challenges students to understand and explain the meaning of abstract mathematical expressions involving addition and subtraction of negative numbers within a real-world context. For example, interpreting −5+(−3) as a debt increasing or a temperature dropping further. This connects the abstract symbols to concrete situations.

Core example questions:

- Describe a real-world scenario that can be represented by the expression −10+7.
- What does the expression 20−(−5) mean in terms of elevation?
- Explain the meaning of the operation in −3−4 in a financial context.
  Difficulty: Conceptual
  Common pitfalls:
- Difficulty translating abstract operations into coherent real-world narratives.
- Misinterpreting the meaning of signs in context (e.g., a negative result meaning "more" of something undesirable).
- Not distinguishing between the operation and the result.
  Introduced definitions: (No new definitions, but reinforces conceptual understanding of previous ones)
- Concept / Introduction / Definition: 5
- Practice: 7

- Common Pitfall avoidance: 4
- Application/word problem: 6
- Challenging: 3
- Total number of questions for each goal: 25

Name: Interpreting negative number statements

Description: Building on interpretation, this skill focuses on analyzing and making sense of statements or scenarios that involve negative numbers and their relationships through addition and subtraction. Students will evaluate the accuracy or implications of given statements.

Core example questions:

- A statement says: "If your bank account is at -$50 and you deposit $20, your balance will be -$70." Is this statement correct? Explain why or why not.
- True or False: "A temperature drop of -8 degrees means the temperature increased by 8 degrees." Justify your answer.
- If a submarine is at -100 meters and descends another 50 meters, the statement "its new depth is -50 meters" is incorrect. Explain the correct depth.
  Difficulty: Conceptual
  Common pitfalls:
- Misinterpreting the effect of operations with negative numbers.
- Errors in applying integer addition/subtraction rules mentally while interpreting.
- Not fully explaining the reasoning behind their interpretation.
  Introduced definitions: (No new definitions, reinforces critical thinking with negative numbers)
- Concept / Introduction / Definition: 4
- Practice: 6
- Common Pitfall avoidance: 5
- Application/word problem: 7
- Challenging: 4
- Total number of questions for each goal: 26

Name: Negative number addition and subtraction: word problems

Description: This skill focuses on solving real-world word problems that explicitly require the addition and subtraction of negative numbers. These problems will integrate various contexts such as finance, temperature, elevation, and sports scores, demanding accurate setup and calculation.

Core example questions:

- The highest temperature recorded in a city was 35°C, and the lowest was -12°C. What is the difference between the highest and lowest temperatures?
- A submarine is at a depth of -300 feet. It then rises 120 feet and later descends another 50 feet. What is its final depth?
- Sarah had -$25 in her account. She deposited $40, and then withdrew $15. What is her final account balance?
  Difficulty: Application
  Common pitfalls:
- Incorrectly translating word cues (e.g., "drop," "rise," "debt") into positive or negative numbers.
- Setting up the equation incorrectly (e.g., subtracting when adding is appropriate).
- Sign errors during calculation.
- Not considering multiple steps in the problem.
  Introduced definitions: (No new definitions, applies previous concepts in problem solving)
- Concept / Introduction / Definition: 2
- Practice: 8
- Common Pitfall avoidance: 6
- Application/word problem: 12
- Challenging: 6
- Total number of questions for each goal: 34

Name: One-step equations with negatives (add & subtract)

Description: This skill introduces solving one-step linear equations where the operations are addition or subtraction, and involve negative numbers. Students will apply inverse operations to isolate the variable, demonstrating their understanding of how to undo operations with signed rational numbers.

Core example questions:

- Solve for x: $x+(-5)=10$
- Find the value of y: $y-8=-3$
- Solve: $z-(-4)=-9$

- The temperature T dropped by 7 degrees to reach -2 degrees. Write and solve an equation to find the original temperature T.
  Difficulty: Procedural
  Common pitfalls:

- Incorrectly performing the inverse operation (e.g., adding 7 to both sides when it should be subtracting 7).

- Sign errors when applying inverse operations with negative numbers.

- Forgetting to account for the negative sign in front of the variable if it occurs (e.g., $-x=5$).
  Introduced definitions: (No new definitions, applies inverse operations to equations)

- Concept / Introduction / Definition: 2

- Practice: 12

- Common Pitfall avoidance: 6

- Application/word problem: 4

- Challenging: 3

- Total number of questions for each goal: 27

Name: Order of operations with negative numbers

Description: This skill requires students to apply the order of operations (PEMDAS/GEMDAS) to expressions that include all four operations: addition, subtraction, multiplication, and division, specifically involving negative numbers. This ensures students can handle complex numerical expressions accurately.

Core example questions:

- Evaluate: $-3\times(5-8)+10\div(-2)$

- Calculate: $12-(-6)\div3+(-4)$

- Solve: $(-2)2+5\times(-1)-7$
  Difficulty: Procedural
  Common pitfalls:

- Incorrectly applying the order of operations (e.g., adding/subtracting before multiplying/dividing).

- Sign errors during multiplication and division of negative numbers.

- Errors in applying integer addition/subtraction rules after intermediate calculations.

- Forgetting the rules for exponents with negative bases (e.g., $(-2)2=4$ but $-22=-4$).
  Introduced definitions: Order of Operations (PEMDAS/GEMDAS)

- Concept / Introduction / Definition: 2
- Practice: 12
- Common Pitfall avoidance: 7
- Application/word problem: 4
- Challenging: 5
- Total number of questions for each goal: 30

Name: Ordering negative number expressions

Description: This skill combines evaluation of expressions with the ability to order rational numbers. Students will first simplify expressions involving addition and subtraction of negative numbers and then arrange the resulting values from least to greatest or greatest to least. This reinforces both computation and number sense.

Core example questions:

- Evaluate each expression and then order them from least to greatest:
    - A: $-5+12$
    - B: $3-(-4)$
    - C: $-10-2$
- Arrange the results of the following calculations in descending order:
    - Expression 1: $1.5+(-3.2)$
    - Expression 2: $-0.8-(-1.5)$ Difficulty: Synthesis Common pitfalls:
- Calculation errors when evaluating the expressions.
- Incorrectly ordering negative numbers on the number line.
- Forgetting to evaluate all expressions before attempting to order them.
  Introduced definitions: (No new definitions, synthesizes evaluation and ordering skills)
- Concept / Introduction / Definition: 2
- Practice: 8
- Common Pitfall avoidance: 5
- Application/word problem: 4
- Challenging: 4
- Total number of questions for each goal: 23

Name: Substitution with negative numbers

Description: This skill focuses on correctly substituting negative numbers into algebraic expressions and then evaluating them using the rules for operations with rational numbers. This builds foundational algebraic skills and reinforces accurate computation with signed numbers.

Core example questions:

- Evaluate $3x+7$ when $x=-4$.
- If $a=-2$ and $b=-5$, find the value of $a-b$.
- Calculate $y2-2y$ when $y=-3$.
  Difficulty: Procedural
  Common pitfalls:
- Sign errors when substituting (e.g., $3(-4)$ vs $3-4$).
- Errors in applying order of operations after substitution, especially with exponents or multiplication by a negative.
- Careless computational mistakes.
  Introduced definitions: Substitute, Evaluate (revisit)
- Concept / Introduction / Definition: 2
- Practice: 10
- Common Pitfall avoidance: 5
- Application/word problem: 3
- Challenging: 3
- Total number of questions for each goal: 23

Name: Subtracting negative numbers

Description: This skill specifically targets the common challenge of subtracting negative numbers. Students will develop fluency in applying the rule that subtracting a negative number is equivalent to adding its positive counterpart ($a-(-b)=a+b$). This is a crucial procedural rule for rational number operations.

Core example questions:

- Calculate: $5-(-3)$
- Solve: $-10-(-2)$
- Find the value of $7.5-(-1.5)$
- $-341-(-21)$
  Difficulty: Procedural

Common pitfalls:

- Forgetting to change both the operation AND the sign of the second number.
- Only changing the sign of the second number, or only changing the operation.
- Calculation errors after the conversion.
  Introduced definitions: (No new definitions, focuses on the specific rule)
- Concept / Introduction / Definition: 3
- Practice: 15
- Common Pitfall avoidance: 8
- Application/word problem: 3
- Challenging: 3
- Total number of questions for each goal: 32

Name: Understand subtraction as adding the opposite

Description: This skill provides the formal conceptual basis for all subtraction of rational numbers: the rule that subtracting any number is equivalent to adding its additive inverse (its opposite). Students will explain why $a-b=a+(-b)$ and apply this understanding consistently. This unifies addition and subtraction operations.

Core example questions:

- Explain why $8-5$ gives the same result as $8+(-5)$.
- Rewrite $-10-(-3)$ as an equivalent addition problem and solve it.
- If you owe $10 and then your debt is reduced by $5, explain how this can be seen as adding the opposite.
  Difficulty: Conceptual
  Common pitfalls:
- Difficulty in articulating the "why" behind the rule.
- Not consistently applying the rule to all subtraction problems.
- Misidentifying the additive inverse (opposite) of a number.
  Introduced definitions: Additive Inverse (formal revisit)
- Concept / Introduction / Definition: 5
- Practice: 8

- Common Pitfall avoidance: 4
- Application/word problem: 4
- Challenging: 3
- Total number of questions for each goal: 24

Name: Representing Subtraction

Description: This skill focuses on using a number line to model and solve subtraction problems involving both positive and negative numbers. Students will explore subtraction as finding the distance between two numbers or as starting at the first number and moving in the opposite direction of the second number (which is equivalent to adding the opposite).

Core example questions:

- Use a number line to model 4−7.
- Show −2−3 on a number line.
- Model the expression −5−(−2) on a number line and explain the result.
  Difficulty: Core
  Common pitfalls:
- Incorrectly depicting "taking away" or "moving in the opposite direction."
- Sign errors when moving on the number line.
- Difficulty with problems involving subtracting a negative number.
  Introduced definitions: (No new definitions, focuses on number line modeling for subtraction)
- Concept / Introduction / Definition: 3
- Practice: 10
- Common Pitfall avoidance: 5
- Application/word problem: 4
- Challenging: 2
- Total number of questions for each goal: 24

Name: Subtracting Rational Numbers

Description: This skill aims for fluency in solving subtraction expressions involving any rational numbers, including positive and negative integers, fractions, and decimals. Students will consistently apply the rule of adding the opposite to transform subtraction into addition, ensuring accurate and efficient computation.

Core example questions:

- Calculate: 43−(−21)
- Solve: −8.2−3.5
- Find the value of −231−132.
- What is the difference between an elevation of 50.5 meters and a depth of -10.2 meters?
  Difficulty: Procedural
  Common pitfalls:
- Errors in converting subtraction to addition of the opposite.
- Computational errors with fractions (common denominators) or decimals (alignment).
- Sign errors after the conversion.
  Introduced definitions: (No new definitions, focuses on fluency with all rational number subtraction)
- Concept / Introduction / Definition: 2
- Practice: 15
- Common Pitfall avoidance: 7
- Application/word problem: 5
- Challenging: 4
- Total number of questions for each goal: 33

Name: Subtract integers using counters

Description: This skill employs visual counters (like integer chips) to model and solve integer subtraction problems. It specifically addresses how to handle situations where there aren't enough of the "right" type of counters to take away, requiring the introduction of zero pairs to facilitate the subtraction.

Core example questions:

- Use counters to model 3−5. Show how you add zero pairs to solve.
- Model −4−(−2) using counters.
- Explain how using zero pairs helps you solve 2−(−3) with counters.
  Difficulty: Introductory
  Common pitfalls:
- Difficulty understanding when and how to add zero pairs.
- Confusing the "take away" action with addition.

- Incorrectly representing negative numbers with counters.
  Introduced definitions: (Reinforces Zero Pair, Counter Modeling for Subtraction)
- Concept / Introduction / Definition: 4
- Practice: 9
- Common Pitfall avoidance: 5
- Application/word problem: 2
- Challenging: 1
- Total number of questions for each goal: 21

Name: Subtract integers using number lines

Description: This skill provides dedicated practice for modeling and solving integer subtraction problems using a number line. Students will visualize subtraction by considering it as adding the opposite or by interpreting a−b as the distance from b to a. This solidifies the number line as a versatile tool for signed number operations.

Core example questions:

- Show how to solve 6−9 on a number line.
- Model −3−(−5) on a number line.
- If you start at -7 and want to reach -1, what subtraction problem does this represent on the number line?
  Difficulty: Core
  Common pitfalls:
- Incorrect direction of movement for subtraction.
- Difficulty translating "subtracting a negative" into movement on the number line.
- Miscounting units or landing on the wrong integer.
  Introduced definitions: (No new definitions, reinforces number line modeling for subtraction)
- Concept / Introduction / Definition: 3
- Practice: 12
- Common Pitfall avoidance: 6
- Application/word problem: 4
- Challenging: 3

- Total number of questions for each goal: 28

Name: Integer subtraction rules

Description: This skill formalizes and provides focused practice on applying the rules for subtracting integers. The primary rule emphasized is converting subtraction to addition of the opposite ($a-b=a+(-b)$). Students will apply this rule to efficiently and accurately solve various integer subtraction problems.

Core example questions:

- State the rule for subtracting integers. Give an example.
- Rewrite $-12-(-5)$ as an addition problem and solve it.
- Calculate: $20-(-7)$
- Solve: $-15-8$
  Difficulty: Procedural
  Common pitfalls:
- Not consistently applying the "add the opposite" rule.
- Sign errors after converting to an addition problem.
- Confusing rules with integer addition.
  Introduced definitions: (No new definitions, formalizes integer subtraction rules)
- Concept / Introduction / Definition: 3
- Practice: 15
- Common Pitfall avoidance: 7
- Application/word problem: 3
- Challenging: 3
- Total number of questions for each goal: 31

Here is a curriculum design for the specified middle school math skills, following Common Core standards and incorporating your question distribution guidelines.

Skill 31. Subtract integers

Name: Subtract integers

Description: Students will learn to fluently subtract integers, understanding that subtracting an integer is equivalent to adding its opposite. This skill builds on the understanding of integer addition and the concept of absolute value.

* Core Example Question: Calculate $7-(-3)$.

* Core Example Question: What is $-5-2$?

* Core Example Question: Find the value of $-10-(-6)$.

Difficulty: Procedural

Common pitfalls: Forgetting to change the operation to addition when subtracting a negative, incorrectly determining the sign of the result after converting to addition, misinterpreting the order of operations in expressions involving multiple subtractions.

Introduced definitions:

- Opposite (Additive Inverse): The number that when added to a given number results in a sum of zero. For example, the opposite of 5 is -5, and the opposite of -3 is 3.
- Subtraction of Integers: The operation of finding the difference between two integers. Defined as $a-b=a+(-b)$.

Concept / Introduction / Definition (no of questions): 3

Practice no of questions: 10

Common Pitfall avoidance no of questions: 5

Application/word problem no of questions: 2

Challenging number of questions: 2

Total number of questions for each goal: 22

Skill 32. Add and subtract integers using counters

Name: Add and subtract integers using counters

Description: Students will model and solve addition and subtraction problems using integer counters (or similar manipulatives like number lines), developing a concrete understanding of integer operations before moving to abstract rules.

* Core Example Question: Use integer counters to show $4+(-6)$.

* Core Example Question: Model $2-5$ using integer counters.

* Core Example Question: How would you represent $-3-(-1)$ with counters?

Difficulty: Conceptual

Common pitfalls: Incorrectly pairing positive and negative counters (zero pairs), difficulty representing subtraction, especially subtracting a negative number, not understanding that adding zero pairs doesn't change the value.

Introduced definitions:

- Integer Counters: Physical or virtual manipulatives representing positive and negative integer values, often depicted as different colored chips (e.g., yellow for positive, red for negative).
- Zero Pair: A positive counter and a negative counter that, when combined, have a sum of zero.

Concept / Introduction / Definition (no of questions): 4

Practice no of questions: 8

Common Pitfall avoidance no of questions: 4

Application/word problem no of questions: 0

Challenging number of questions: 1

Total number of questions for each goal: 17

Skill 33. Adding & subtracting negative fractions

Name: Adding & subtracting negative fractions

Description: Students will add and subtract negative fractions, applying properties of operations, including finding common denominators and correctly handling signs.

* Core Example Question: Calculate $-21+43$.

* Core Example Question: What is $32-(-61)$?

* Core Example Question: Find the value of $-53-101$.

Difficulty: Procedural

Common pitfalls: Errors in finding common denominators, misapplying integer rules to the numerators, forgetting to simplify fractions to their lowest terms, sign errors when combining.

Introduced definitions:

- Negative Fraction: A fraction where the numerator or the fraction itself is negative. For example, $-\frac{4}{3}$ or $\frac{4}{-3}$.

Concept / Introduction / Definition (no of questions): 2

Practice no of questions: 10

Common Pitfall avoidance no of questions: 5

Application/word problem no of questions: 2

Challenging number of questions: 2

Total number of questions for each goal: 21

Skill 34. Commutative and associative properties of addition with integers

Name: Commutative and associative properties of addition with integers

Description: Students will apply the commutative and associative properties of addition to simplify expressions involving integers, understanding how these properties allow for flexible grouping and ordering of numbers.

* Core Example Question: Rewrite $5+(-8)+3$ using the commutative property to make it easier to add.

* Core Example Question: Simplify $(2+(-7))+4$ using the associative property.

* Core Example Question: Explain how the commutative and associative properties can help you solve $-9+4+(-1)$.

Difficulty: Conceptual

Common pitfalls: Confusing the commutative and associative properties, incorrectly applying the properties to subtraction, not recognizing when these properties can simplify calculations.

Introduced definitions:

- Commutative Property of Addition: For any numbers a and b, $a+b=b+a$. The order of the addends does not affect the sum.
- Associative Property of Addition: For any numbers a, b, and c, $(a+b)+c=a+(b+c)$. The way addends are grouped does not affect the sum.

Concept / Introduction / Definition (no of questions): 3

Practice no of questions: 7

Common Pitfall avoidance no of questions: 3

Application/word problem no of questions: 0

Challenging number of questions: 1

Total number of questions for each goal: 14

Skill 35. Adding and Subtracting to Solve Problems

Name: Adding and Subtracting to Solve Problems

Description: Students will represent gains and losses using signed numbers and apply rational number addition and subtraction to solve real-world problems. This skill focuses on translating word problems into mathematical expressions and interpreting the results in context.

* Core Example Question: A submarine is at -250 feet relative to sea level. If it ascends 120 feet, what is its new depth?

* Core Example Question: The temperature dropped from 15°F to -8°F. What was the change in temperature?

* Core Example Question: Sarah had $45.75 in her bank account. She wrote a check for $60.50. What is her new balance?

Difficulty: Application

Common pitfalls: Incorrectly assigning positive or negative signs to values from the problem, misinterpreting the operation (addition vs. subtraction) required, not accurately calculating with rational numbers, failing to label answers with appropriate units.

Introduced definitions:

- Real-World Problems: Problems that involve practical situations and contexts, often requiring the application of mathematical concepts to solve.
- Signed Numbers: Numbers with a positive or negative sign, used to represent quantities that have both magnitude and direction (e.g., profit/loss, above/below sea level, temperature changes).

Concept / Introduction / Definition (no of questions): 2

Practice no of questions: 5

Common Pitfall avoidance no of questions: 3

Application/word problem no of questions: 8

Challenging number of questions: 3

Total number of questions for each goal: 21

Skill 36. Integer addition and subtraction rules

Name: Integer addition and subtraction rules

Description: Students will apply combined rules for integer addition and subtraction, synthesizing their understanding of different cases (same signs, different signs, subtracting negatives) into a coherent set of operational rules. This skill focuses on the procedural fluency of applying the rules.

* Core Example Question: Simplify $12+(-5)-3$.

* Core Example Question: Calculate $-8-(-4)+6$.

* Core Example Question: Evaluate $-2+7-(-9)$.

Difficulty: Procedural

Common pitfalls: Mixing up addition and subtraction rules, particularly when signs are different, making errors in multi-step problems, incorrectly converting subtraction to addition of the opposite.

Introduced definitions:

- Rules for Adding Integers (Same Signs): Add the absolute values and keep the common sign.
- Rules for Adding Integers (Different Signs): Subtract the smaller absolute value from the larger absolute value, and use the sign of the number with the larger absolute value.
- Rules for Subtracting Integers: To subtract an integer, add its opposite.

Concept / Introduction / Definition (no of questions): 2

Practice no of questions: 12

Common Pitfall avoidance no of questions: 6

Application/word problem no of questions: 1

Challenging number of questions: 2

Total number of questions for each goal: 23

Skill 37. Add and subtract integers

Name: Add and subtract integers

Description: Students will fluently add and subtract integers using a combination of mental math, rules, and strategies. This skill emphasizes mastery and efficiency in performing these fundamental operations.

* Core Example Question: Calculate −15+7.

* Core Example Question: What is 9−(−4)?

* Core Example Question: Find the value of −6−11.

Difficulty: Core

Common pitfalls: Slowness in recalling rules, frequent sign errors, difficulty with larger numbers, lacking confidence in results.

Introduced definitions: (No new definitions; this skill reinforces previously introduced concepts.)

Concept / Introduction / Definition (no of questions): 0 (This skill builds on previous concepts; focus is on fluency)

Practice no of questions: 15

Common Pitfall avoidance no of questions: 7

Application/word problem no of questions: 2

Challenging number of questions: 3

Total number of questions for each goal: 27

Skill 38. Complete addition and subtraction equations with integers

Name: Complete addition and subtraction equations with integers

Description: Students will fill in missing values in addition and subtraction equations involving integers, requiring them to use inverse operations and their understanding of integer properties to solve for unknowns.

* Core Example Question: Complete the equation: $5 + \text{___} = -2$.

* Core Example Question: Find the missing number: $-8 - \text{___} = -3$.

* Core Example Question: If $\text{___} + (-6) = 1$, what is the missing integer?

Difficulty: Procedural

Common pitfalls: Incorrectly applying inverse operations, sign errors when isolating the unknown, difficulty with equations where the unknown is the subtrahend.

Introduced definitions:

- Inverse Operations: Operations that undo each other (e.g., addition and subtraction are inverse operations).

Concept / Introduction / Definition (no of questions): 2

Practice no of questions: 8

Common Pitfall avoidance no of questions: 4

Application/word problem no of questions: 0

Challenging number of questions: 2

Total number of questions for each goal: 16

Skill 39. Add and subtract positive and negative decimals

Name: Add and subtract positive and negative decimals

Description: Students will add and subtract decimals, including positive and negative values, by aligning decimal points and applying integer addition and subtraction rules.

* Core Example Question: Calculate $3.5+(-1.2)$.

* Core Example Question: What is $-0.75-2.1$?

* Core Example Question: Find the value of $-4.05-(-1.3)$.

Difficulty: Procedural

Common pitfalls: Misaligning decimal points, errors in borrowing/regrouping, applying integer rules incorrectly to decimal numbers, sign errors.

Introduced definitions: (No new definitions; this skill extends integer operations to decimals.)

Concept / Introduction / Definition (no of questions): 2

Practice no of questions: 10

Common Pitfall avoidance no of questions: 5

Application/word problem no of questions: 2

Challenging number of questions: 2

Total number of questions for each goal: 21

Skill 40. Add and subtract positive and negative fractions

Name: Add and subtract positive and negative fractions

Description: Students will add and subtract fractions, including positive and negative values, by finding common denominators and applying rules for operations with signed numbers. (This is a refinement and slightly different phrasing of skill 33, focusing on fluency across all types of fractions).

* Core Example Question: Calculate $41+(-83)$.

* Core Example Question: What is $-65-(-31)$?

* Core Example Question: Find the value of $-221+143$.

Difficulty: Procedural

Common pitfalls: Errors in finding common denominators, improper conversion of mixed numbers to improper fractions, incorrect application of integer rules to numerators, not simplifying answers.

Introduced definitions: (No new definitions; reinforces and applies prior fraction and integer concepts.)

Concept / Introduction / Definition (no of questions): 1

Practice no of questions: 10

Common Pitfall avoidance no of questions: 5

Application/word problem no of questions: 2

Challenging number of questions: 2

Total number of questions for each goal: 20

Skill 41. Add and subtract rational numbers

Name: Add and subtract rational numbers

Description: Students will fluently add and subtract rational numbers (integers, fractions, and decimals), demonstrating a comprehensive mastery of operations across all forms of rational numbers. This skill serves as a capstone, requiring students to identify the number type and apply appropriate strategies.

* Core Example Question: Calculate $-3+4.5-21$.

* Core Example Question: What is $0.25-(-43)+(-1)$?

* Core Example Question: Find the value of $-251+0.8-3$.

Difficulty: Synthesis

Common pitfalls: Difficulty converting between different forms of rational numbers (e.g., decimals to fractions), procedural errors in combining different types of rational numbers, making multiple sign errors in multi-step problems, struggling to choose the most efficient method for calculation.

Introduced definitions:

- Rational Number: Any number that can be expressed as a fraction $\frac{p}{q}$ where p and q are integers and $q=0$. This includes integers (e.g., $\frac{5}{1}=5$), fractions (e.g., $\frac{1}{2}$), and terminating or repeating decimals (e.g., $\frac{3}{4}=0.75$).

Concept / Introduction / Definition (no of questions): 2

Practice no of questions: 10

Common Pitfall avoidance no of questions: 5

Application/word problem no of questions: 4

Challenging number of questions: 3

Total number of questions for each goal: 24

Multiplying and Dividing Rational Numbers

1. Exponents with negative fractional bases

Description: Students will learn to evaluate expressions where a negative fractional base is raised to a whole-number exponent. This involves understanding how the sign of the result is determined by the exponent's parity (even or odd) and correctly applying the exponent to both the numerator and the denominator of the fraction.

* Core Example Question: Evaluate $(-21)3$.

* Core Example Question: Calculate $(-32)2$.

* Core Example Question: What is the value of $(-43)1$?

Difficulty: Procedural

Common pitfalls: Incorrectly applying the exponent to the sign (e.g., $(-x)2=-x2$), forgetting to raise both the numerator and denominator to the power, sign errors, especially with odd exponents.

Introduced definitions:

- Base: In an exponential expression b^n, b is the base, the number being multiplied.
- Exponent (Power): In an exponential expression b^n, n is the exponent, indicating how many times the base is multiplied by itself.
- Whole-Number Exponent: An exponent that is a non-negative integer $(0,1,2,3,\ldots)$.

Concept / Introduction / Definition (no of questions): 3

Practice no of questions: 8

Common Pitfall avoidance no of questions: 4

Application/word problem no of questions: 0

Challenging number of questions: 1

Total number of questions for each goal: 16

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2. Position, Speed, and Direction

Description: Students will develop an understanding of how the multiplication of signed numbers applies to real-world contexts such as position, speed, and direction. This involves interpreting positive and negative values for movement and rate, and using multiplication to determine final positions or changes in position over time.

* Core Example Question: If a car travels at a speed of 60 mph (miles per hour) and its position is represented by positive numbers for east and negative for west, what is its position after 2 hours if it starts at 0 and travels east?

* Core Example Question: A submarine descends at a rate of -50 feet per minute. What is its change in depth after 3 minutes? (Assume positive is above sea level, negative is below).

* Core Example Question: If an object is moving in the negative direction at a speed of 10 units/second, where will it be relative to its starting point after 4 seconds?

Difficulty: Conceptual

Common pitfalls: Misinterpreting positive/negative signs in context (e.g., negative speed meaning slowing down instead of opposite direction), difficulty connecting the abstract multiplication rules to concrete scenarios, sign errors in the final calculation.

Introduced definitions:

- Position: A location relative to a reference point, often represented by signed numbers (e.g., 5 miles north, −3 meters below sea level).
- Speed: The rate at which an object is moving.
- Direction: The course along which someone or something is moving or pointing. In this context, often represented by the sign of the speed or velocity (e.g., positive for one direction, negative for the opposite).
- Signed Numbers: Numbers that represent both magnitude and direction.

Concept / Introduction / Definition (no of questions): 3

Practice no of questions: 5

Common Pitfall avoidance no of questions: 3

Application/word problem no of questions: 5

Challenging number of questions: 2

Total number of questions for each goal: 18

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3. Multiplying Rational Numbers

Description: Students will interpret signed numbers for speed and direction and, crucially, understand the rationale behind why multiplying two negative numbers results in a positive product. This involves exploring conceptual models (like patterns, number lines, or real-world scenarios of undoing actions) to justify the multiplication rules, rather than just memorizing them.

* Core Example Question: Explain why (−2)×(−3) equals 6 using a pattern or a real-world example.

* Core Example Question: If "moving backward" is represented by a negative sign and "doing it for a negative amount of time" (i.e., undoing an action) is also negative, how does this help understand (−negative)×(−negative)=positive?

* Core Example Question: Consider the pattern: 3×2=6, 3×1=3, 3×0=0, $3 \times (-1) = \text{___}$. What does this pattern suggest about multiplying a positive and a negative? Extend the pattern to explain (−3)×(−2).

Difficulty: Conceptual

Common pitfalls: Simply memorizing the rules without conceptual understanding, struggling to articulate the "why" behind multiplying two negative numbers, difficulty connecting abstract rules to concrete models.

Introduced definitions:

- Rational Numbers: Any number that can be expressed as a fraction qp where p and q are integers and q=0. This encompasses integers, fractions, and terminating/repeating decimals.
- Product: The result of a multiplication operation.

Concept / Introduction / Definition (no of questions): 4

Practice no of questions: 6

Common Pitfall avoidance no of questions: 3

Application/word problem no of questions: 2

Challenging number of questions: 2

Total number of questions for each goal: 17

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4. Multiply!

Description: Students will fluently multiply rational numbers, including expressions with multiple factors. This skill focuses on the procedural application of multiplication rules for integers, fractions, and decimals, ensuring accuracy and efficiency.

* Core Example Question: Calculate -2.5×4.

* Core Example Question: What is $(-43) \times (52)$?

* Core Example Question: Find the product of $-1 \times (-2) \times (-3)$.

Difficulty: Procedural

Common pitfalls: Sign errors, errors in multiplying fractions (e.g., not simplifying before multiplying), errors in multiplying decimals (e.g., incorrect decimal placement), difficulty managing multiple negative signs.

Introduced definitions: (No new definitions; this skill reinforces previously introduced concepts.)

Concept / Introduction / Definition (no of questions): 0

Practice no of questions: 12

Common Pitfall avoidance no of questions: 6

Application/word problem no of questions: 2

Challenging number of questions: 2

Total number of questions for each goal: 22

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5. Understand multiplying by a negative integer using a number line

Description: Students will visualize and understand multiplication involving negative integers using a number line model. This provides a visual and intuitive way to grasp the concept of multiplying by a negative number as repeated movement in the opposite direction.

* Core Example Question: Use a number line to show $3 \times (-2)$. Describe your steps.

* Core Example Question: How would you model $(-2) \times 4$ on a number line?

* Core Example Question: Explain how the number line helps visualize why multiplying a positive and a negative results in a negative product.

Difficulty: Conceptual

Common pitfalls: Difficulty visualizing "negative groups" or "groups of negatives," misinterpreting movements on the number line, struggling to use the model for all types of integer multiplication.

Introduced definitions:

- Number Line: A line on which every point corresponds to a real number.
- Repeated Addition: A way to conceptualize multiplication, where a×b means adding a to itself b times.

Concept / Introduction / Definition (no of questions): 3

Practice no of questions: 7

Common Pitfall avoidance no of questions: 3

Application/word problem no of questions: 0

Challenging number of questions: 1

Total number of questions for each goal: 14

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6. Integer multiplication rules

Description: Students will apply rules for multiplying integers with different signs and same signs. This skill focuses on the precise and consistent application of the sign rules for integer multiplication.

* Core Example Question: State the rule for multiplying two integers with the same sign. Provide an example.

* Core Example Question: State the rule for multiplying two integers with different signs. Provide an example.

* Core Example Question: Determine the sign of the product: (−5)×8.

Difficulty: Procedural

Common pitfalls: Forgetting the rules, confusing multiplication rules with addition rules, making sign errors when applying the rules.

Introduced definitions:

- Rule for Multiplying Integers (Same Signs): The product of two integers with the same sign (both positive or both negative) is positive.
- Rule for Multiplying Integers (Different Signs): The product of two integers with different signs (one positive and one negative) is negative.

Concept / Introduction / Definition (no of questions): 3

Practice no of questions: 10

Common Pitfall avoidance no of questions: 5

Application/word problem no of questions: 1

Challenging number of questions: 1

Total number of questions for each goal: 20

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7. Multiply integers

Description: Students will fluently multiply integers, demonstrating accuracy and efficiency in applying the multiplication rules. This skill focuses on direct computation.

* Core Example Question: Calculate $(-7)\times(-4)$.

* Core Example Question: What is $6\times(-9)$?

* Core Example Question: Find the product of -12×3.

Difficulty: Core

Common pitfalls: Slowness in recalling rules, frequent sign errors, difficulty with larger numbers, lacking confidence in results.

Introduced definitions: (No new definitions; this skill reinforces previously introduced concepts.)

Concept / Introduction / Definition (no of questions): 0

Practice no of questions: 15

Common Pitfall avoidance no of questions: 7

Application/word problem no of questions: 2

Challenging number of questions: 3

Total number of questions for each goal: 27

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8. Dividing by zero

Description: Students will understand why division by zero is undefined. This involves exploring the concept through inverse operations (multiplication) and considering what happens if one tries to divide by zero.

* Core Example Question: Explain why 05 is undefined by thinking about what number times 0 would equal 5.

* Core Example Question: What is the result of 00? Why is this also considered undefined or indeterminate?

* Core Example Question: Can you ever divide something into zero equal parts? Discuss why or why not.

Difficulty: Conceptual

Common pitfalls: Simply stating "undefined" without understanding the reason, confusing division by zero with zero divided by a non-zero number, difficulty articulating the mathematical justification.

Introduced definitions:

- Undefined: A mathematical expression that does not have a meaningful value.
- Division: The operation of finding how many times one number is contained in another. Defined as $a\div b=c$ if and only if $b\times c=a$.

Concept / Introduction / Definition (no of questions): 3

Practice no of questions: 4

Common Pitfall avoidance no of questions: 2

Application/word problem no of questions: 0

Challenging number of questions: 1

Total number of questions for each goal: 10

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9. Dividing Rational Numbers

Description: Students will use multiplication and division relationships to determine division rules for rational numbers. This includes understanding that division by a number is equivalent to multiplication by its reciprocal, and extending integer division rules to fractions and decimals.

* Core Example Question: Explain how knowing that $3\times(-4)=-12$ helps you understand why $-12\div3=-4$.

* Core Example Question: How can you rewrite $21\div(-43)$ as a multiplication problem?

* Core Example Question: Describe the steps to divide a negative decimal by a positive decimal.

Difficulty: Conceptual

Common pitfalls: Not understanding the reciprocal concept, struggling to apply the sign rules consistently across different rational number forms, difficulty connecting division to multiplication.

Introduced definitions:

- Reciprocal (Multiplicative Inverse): For a number a (where $a=0$), its reciprocal is $a1$, such that $a\times a1=1$.
- Division of Rational Numbers: To divide by a rational number, multiply by its reciprocal. For example, $ba\div dc=ba\times cd$.

Concept / Introduction / Definition (no of questions): 3

Practice no of questions: 7

Common Pitfall avoidance no of questions: 3

Application/word problem no of questions: 1

Challenging number of questions: 2

Total number of questions for each goal: 16

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10. Negative Rates

Description: Students will multiply and divide rational numbers in constant rate problems involving negative values. This extends their understanding of signed numbers in real-world contexts to include rates of change, such as temperature drops, financial depreciation, or depletion of resources.

* Core Example Question: A submersible descends at an average rate of −2.5 meters per second. How deep will it be after 10 seconds if it starts at the surface?

* Core Example Question: The temperature in a freezer is decreasing at a rate of −3◦F per hour. If the temperature started at 5◦F, how long will it take to reach −10◦F?

* Core Example Question: A bank account balance is decreasing by $15.50 per day. If the balance changed by −$124, how many days passed?

Difficulty: Application

Common pitfalls: Incorrectly setting up the equation, misinterpreting negative rates (e.g., negative rate of change for temperature meaning it's getting warmer), sign errors in calculations, not properly labeling units.

Introduced definitions:

- Rate: A ratio that compares two quantities with different units (e.g., miles per hour, dollars per day).
- Constant Rate: A rate that does not change over time.

Concept / Introduction / Definition (no of questions): 2

Practice no of questions: 5

Common Pitfall avoidance no of questions: 3

Application/word problem no of questions: 8

Challenging number of questions: 3

Total number of questions for each goal: 21

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11. Integer division rules

Description: Students will apply rules for dividing integers with different signs and same signs. This skill focuses on the precise and consistent application of the sign rules for integer division, recognizing the relationship between division and multiplication rules.

* Core Example Question: State the rule for dividing two integers with the same sign. Provide an example.

* Core Example Question: State the rule for dividing two integers with different signs. Provide an example.

* Core Example Question: Determine the sign of the quotient: 48÷(−6).

Difficulty: Procedural

Common pitfalls: Forgetting the rules, confusing division rules with addition/subtraction rules, making sign errors.

Introduced definitions:

- Rule for Dividing Integers (Same Signs): The quotient of two integers with the same sign (both positive or both negative) is positive.

- Rule for Dividing Integers (Different Signs): The quotient of two integers with different signs (one positive and one negative) is negative.
- Quotient: The result of a division operation.

Concept / Introduction / Definition (no of questions): 3

Practice no of questions: 10

Common Pitfall avoidance no of questions: 5

Application/word problem no of questions: 1

Challenging number of questions: 1

Total number of questions for each goal: 20

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12. Equal quotients of integers

Description: Students will identify equal quotients involving negative integers. This skill helps reinforce the understanding of equivalent division expressions and the properties of signed numbers in division.

* Core Example Question: Which of the following expressions is equivalent to $5-10$: -510, -510, or $-5-10$?

* Core Example Question: Is $-12\div(-3)$ equal to $12\div3$? Explain why or why not.

* Core Example Question: Create two different division expressions that have the same quotient as -618.

Difficulty: Conceptual

Common pitfalls: Misinterpreting the placement of the negative sign in fractions, incorrectly simplifying expressions before comparing, not fully understanding the rules for determining the sign of a quotient.

Introduced definitions:

- Equivalent Quotients: Division expressions that have the same numerical value.

Concept / Introduction / Definition (no of questions): 2

Practice no of questions: 6

Common Pitfall avoidance no of questions: 3

Application/word problem no of questions: 0

Challenging number of questions: 2

Total number of questions for each goal: 13

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13. Divide integers

Description: Students will fluently divide integers, demonstrating accuracy and efficiency in applying the division rules. This skill focuses on direct computation.

* Core Example Question: Calculate $-24 \div 6$.

* Core Example Question: What is $-45 \div (-9)$?

* Core Example Question: Find the value of $36 \div (-4)$.

Difficulty: Core

Common pitfalls: Slowness in recalling rules, frequent sign errors, difficulty with larger numbers, lacking confidence in results.

Introduced definitions: (No new definitions; this skill reinforces previously introduced concepts.)

Concept / Introduction / Definition (no of questions): 0

Practice no of questions: 15

Common Pitfall avoidance no of questions: 7

Application/word problem no of questions: 2

Challenging number of questions: 3

Total number of questions for each goal: 27

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14. Dividing mixed numbers with negatives

Description: Students will divide mixed numbers that include negative values. This requires converting mixed numbers to improper fractions, applying the rule for dividing fractions (multiplying by the reciprocal), and correctly handling the signs.

* Core Example Question: Calculate $-121 \div 43$.

* Core Example Question: What is $231 \div (-67)$?

* Core Example Question: Find the value of $-351 \div (-153)$.

Difficulty: Procedural

Common pitfalls: Errors in converting mixed numbers to improper fractions, forgetting to flip the second fraction (reciprocal), sign errors, not simplifying the final answer.

Introduced definitions:

- Mixed Number: A number consisting of a whole number and a fraction.
- Improper Fraction: A fraction in which the absolute value of the numerator is greater than 1 or equal to the absolute value of the denominator.2

Concept / Introduction / Definition (no of questions): 2

Practice no of questions: 8

Common Pitfall avoidance no of questions: 4

Application/word problem no of questions: 2

Challenging number of questions: 2

Total number of questions for each goal: 18

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15. Dividing negative numbers

Description: Students will perform division operations involving negative numbers across integers, fractions, and decimals, reinforcing the consistency of sign rules for division across all rational number forms. This skill serves as a broader category for division of negative rational numbers.

* Core Example Question: Calculate $-0.5 \div 2.5$.

* Core Example Question: What is $-32 \div (-94)$?

* Core Example Question: Find the result of $18 \div (-0.6)$.

Difficulty: Reinforcement

Common pitfalls: Inconsistent application of sign rules, errors when combining different rational number forms in division, general computational errors.

Introduced definitions: (No new definitions; this skill reinforces previously introduced concepts.)

Concept / Introduction / Definition (no of questions): 0

Practice no of questions: 10

Common Pitfall avoidance no of questions: 5

Application/word problem no of questions: 2

Challenging number of questions: 2

Total number of questions for each goal: 19

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16. Dividing positive and negative fractions

Description: Students will learn to divide fractions, including those with positive and negative values. This involves understanding the concept of multiplying by the reciprocal and applying the rules for dividing signed numbers to fractions.

* Core Example Question: Calculate $43 \div (-21)$.

* Core Example Question: What is $-65 \div 32$?

* Core Example Question: Find the value of $(-87) \div (-41)$.

Difficulty: Procedural

Common pitfalls: Forgetting to flip the second fraction (reciprocal), sign errors in the quotient, not simplifying the resulting fraction, issues with complex fractions.

Introduced definitions:

- Reciprocal: For a fraction $\frac{a}{b}$ (where $a,b=0$), its reciprocal is $\frac{b}{a}$.

- Division of Fractions: To divide by a fraction, multiply by its reciprocal.

Concept / Introduction / Definition (no of questions): 2

Practice no of questions: 10

Common Pitfall avoidance no of questions: 5

Application/word problem no of questions: 1

Challenging number of questions: 2

Total number of questions for each goal: 20

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17. Equivalent expressions with negative numbers (multiplication and division)

Description: Students will identify or create equivalent expressions involving multiplication and division of negative numbers. This reinforces the understanding of how negative signs behave in operations and how different forms can represent the same value.

* Core Example Question: Which expression is equivalent to -3×4: $3\times(-4)$, $(-3)\times(-4)$, or $-(3\times4)$?

* Core Example Question: Create an expression equivalent to $3-15$ using a positive dividend.

* Core Example Question: Is $(-2)\times(-5)\div(-1)$ equivalent to $2\times5\div1$? Explain.

Difficulty: Conceptual

Common pitfalls: Misinterpreting the placement of negative signs, especially with fractions or multiple operations, incorrectly applying rules for signs, confusing equivalence with equality for expressions.

Introduced definitions:

- Equivalent Expressions: Expressions that have the same value for all possible values of their variables. (In this context, specific numerical values.)

Concept / Introduction / Definition (no of questions): 3

Practice no of questions: 7

Common Pitfall avoidance no of questions: 4

Application/word problem no of questions: 0

Challenging number of questions: 2

Total number of questions for each goal: 16

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18. Exponents with integer bases

Description: Students will evaluate expressions with integer bases raised to whole-number exponents. This builds upon prior knowledge of exponents and extends it to include negative integer bases, emphasizing the impact of the exponent's parity on the sign of the result.

* Core Example Question: Evaluate $(-3)2$.

* Core Example Question: Calculate $(-2)^3$.

* Core Example Question: What is the value of $(-5)^0$?

Difficulty: Procedural

Common pitfalls: Incorrectly applying the exponent to the sign (e.g., -3^2 vs. $(-3)^2$), forgetting that anything to the power of zero is one, making errors with larger bases or exponents.

Introduced definitions:

- Integer Base: The base of an exponential expression that is an integer (positive, negative, or zero).
- Whole-Number Exponent: An exponent that is a non-negative integer $(0,1,2,3,\ldots)$.

Concept / Introduction / Definition (no of questions): 2

Practice no of questions: 8

Common Pitfall avoidance no of questions: 4

Application/word problem no of questions: 0

Challenging number of questions: 2

Total number of questions for each goal: 16

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19. Multiplying & dividing negative numbers word problems

Description: Students will solve real-world word problems involving multiplication and division of negative numbers. This involves translating contextual situations into mathematical expressions, performing the operations, and interpreting the signed results in the context of the problem.

* Core Example Question: The temperature is dropping at a rate of $-2^\circ C$ per hour. If it continues at this rate for 3 hours, what will be the total change in temperature?

* Core Example Question: A submersible is at a depth of -300 feet. If it ascends by covering -50 feet for each minute (i.e., it is rising at 50 feet per minute, expressed as a negative change in depth for a given time), how many minutes will it take to reach -100 feet?

* Core Example Question: A stock price decreased by an average of $-\$1.50$ per day over 5 days. What was the total change in the stock price?

Difficulty: Application

Common pitfalls: Incorrectly setting up the problem, misinterpreting the meaning of negative values in context, choosing the wrong operation, making sign errors in the final answer, failing to label units.

Introduced definitions: (No new definitions; applies previously introduced concepts to problem-solving.)

Concept / Introduction / Definition (no of questions): 2

Practice no of questions: 5

Common Pitfall avoidance no of questions: 3

Application/word problem no of questions: 8

Challenging number of questions: 3

Total number of questions for each goal: 21

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20. Multiplying negative numbers

Description: Students will perform multiplication operations involving negative numbers across integers, fractions, and decimals, focusing on the consistent application of sign rules. This skill serves as a broader category for multiplication of negative rational numbers.

* Core Example Question: Calculate $-1.5\times(-0.4)$.

* Core Example Question: What is $(-31)\times6$?

* Core Example Question: Find the product of $-10\times(-0.02)$.

Difficulty: Reinforcement

Common pitfalls: Inconsistent application of sign rules, errors when combining different rational number forms in multiplication, general computational errors.

Introduced definitions: (No new definitions; this skill reinforces previously introduced concepts.)

Concept / Introduction / Definition (no of questions): 0

Practice no of questions: 10

Common Pitfall avoidance no of questions: 5

Application/word problem no of questions: 2

Challenging number of questions: 2

Total number of questions for each goal: 19

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21. Multiplying positive and negative fractions

Description: Students will multiply fractions, including those with positive and negative values. This involves applying the rules for multiplying fractions (multiplying numerators and denominators) and the rules for signs of products.

* Core Example Question: Calculate $32\times(-41)$.

* Core Example Question: What is $(-75)\times(-103)$?

* Core Example Question: Find the product of -121×52.

Difficulty: Procedural

Common pitfalls: Sign errors, not simplifying before or after multiplying, making errors with mixed numbers (not converting to improper fractions first).

Introduced definitions: (No new definitions; reinforces fraction and integer multiplication concepts.)

Concept / Introduction / Definition (no of questions): 2

Practice no of questions: 10

Common Pitfall avoidance no of questions: 5

Application/word problem no of questions: 1

Challenging number of questions: 2

Total number of questions for each goal: 20

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22. Negative signs in fractions

Description: Students will understand the placement and meaning of negative signs in fractions, recognizing that $-\frac{a}{b}=\frac{-a}{b}=\frac{a}{-b}$. This skill clarifies potential ambiguities and helps students correctly interpret and manipulate signed fractions.

* Core Example Question: Show that $-\frac{5}{3}$ is equivalent to $\frac{5}{-3}$ and $\frac{-5}{3}$.

* Core Example Question: Explain why $\frac{-7}{-2}$ is a positive fraction.

* Core Example Question: Which of these fractions is NOT equivalent to $-\frac{9}{4}$: $\frac{9}{-4}$, $\frac{-9}{4}$, or $-\frac{9}{-4}$?

Difficulty: Conceptual

Common pitfalls: Incorrectly associating the negative sign with only the numerator or denominator, struggling to recognize equivalent forms, making sign errors when operating with fractions where the negative sign is in an unusual position.

Introduced definitions:

- Negative Fraction: A fraction representing a negative rational number. The negative sign can be placed in front of the fraction, with the numerator, or with the denominator (e.g., $-\frac{a}{b}$, $\frac{b}{-a}$, or $\frac{-a}{b}$ are all equivalent).

Concept / Introduction / Definition (no of questions): 3

Practice no of questions: 6

Common Pitfall avoidance no of questions: 3

Application/word problem no of questions: 0

Challenging number of questions: 1

Total number of questions for each goal: 13

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23. One-step equations with negatives (multiply & divide)

Description: Students will solve one-step linear equations involving multiplication and division of negative numbers. This requires applying inverse operations and correctly handling the signs of numbers in the equations.

* Core Example Question: Solve for x: $-4x=20$.

* Core Example Question: Solve for y: $-3y=-7$.

* Core Example Question: If $0.5z=-2.5$, what is z?

Difficulty: Procedural

Common pitfalls: Incorrectly applying inverse operations (e.g., adding instead of dividing), making sign errors when dividing or multiplying by negative numbers, not simplifying the solution.

Introduced definitions:

- One-Step Equation: An algebraic equation that can be solved in a single step using an inverse operation.
- Inverse Operations: Operations that undo each other (e.g., multiplication and division are inverse operations).

Concept / Introduction / Definition (no of questions): 2

Practice no of questions: 10

Common Pitfall avoidance no of questions: 5

Application/word problem no of questions: 1

Challenging number of questions: 2

Total number of questions for each goal: 20

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24. Order of operations with negative numbers

Description: Students will apply the order of operations (PEMDAS/BODMAS) to expressions involving multiplication and division of negative numbers, as well as addition, subtraction, and exponents. This integrates all learned operations with rational numbers and sign rules.

* Core Example Question: Evaluate $-5+(-2)\times3$.

* Core Example Question: Calculate $10\div(-2)-(-4)$.

* Core Example Question: Simplify $6-(4\div(-2))2$.

Difficulty: Procedural

Common pitfalls: Incorrectly applying the order of operations, making sign errors at each step, struggling with parentheses or exponents involving negative numbers, not paying attention to detail in multi-step problems.

Introduced definitions:

- Order of Operations: A set of rules that dictate the sequence in which mathematical operations should be performed to ensure a consistent result (commonly remembered as PEMDAS/BODMAS: Parentheses/Brackets, Exponents/Orders, Multiplication and Division (from left to right), Addition and Subtraction (from left to right)).

Concept / Introduction / Definition (no of questions): 2

Practice no of questions: 10

Common Pitfall avoidance no of questions: 6

Application/word problem no of questions: 2

Challenging number of questions: 3

Total number of questions for each goal: 23

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25. Signs of expressions

Description: Students will determine the sign of the result of expressions involving multiplication and division of rational numbers without necessarily calculating the exact value. This focuses on understanding the rules for signs and how they combine.

* Core Example Question: Will the expression $(-5)\times2\times(-3)$ result in a positive or negative number?

* Core Example Question: Determine the sign of $-6-12\times4$.

* Core Example Question: What is the sign of the product $(-0.1)\times(-0.2)\times(-0.3)\times(-0.4)$?

Difficulty: Conceptual

Common pitfalls: Miscounting negative signs, incorrectly applying rules for odd/even numbers of negative factors, errors in determining the overall sign after multiple operations.

Introduced definitions: (No new definitions; applies rules for signs in multiplication and division.)

Concept / Introduction / Definition (no of questions): 3

Practice no of questions: 8

Common Pitfall avoidance no of questions: 4

Application/word problem no of questions: 0

Challenging number of questions: 2

Total number of questions for each goal: 17

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26. Signs of expressions challenge

Description: Students will solve challenging problems involving determining the signs of complex expressions with multiplication and division of rational numbers. These problems may include more factors, mixed operations, or require more careful analysis of signs.

* Core Example Question: Determine the sign of $(-1)5\times(-2)\div(-3)\times(-4)\times(-5)$.

* Core Example Question: If $a>0$ and $b<0$ and $c<0$, what is the sign of $ca\times b$?

* Core Example Question: For which values of n (positive integer) will $(-1)n\times(-1)n+1$ be positive?

Difficulty: Synthesis

Common pitfalls: Overlooking a negative sign, incorrectly applying exponent rules to signs, struggling with variable expressions for signs, errors in combining signs across many factors.

Introduced definitions: (No new definitions; this skill synthesizes previous understanding of signs.)

Concept / Introduction / Definition (no of questions): 1

Practice no of questions: 5

Common Pitfall avoidance no of questions: 3

Application/word problem no of questions: 0

Challenging number of questions: 4

Total number of questions for each goal: 13

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27. Simplify complex fractions

Description: Students will simplify complex fractions involving rational numbers. This requires understanding that a complex fraction represents division and applying the rules for dividing fractions, including those with signed numbers.

* Core Example Question: Simplify $\frac{-4}{3}$$\frac{2}{1}$.

* Core Example Question: Simplify $\frac{9}{7}$$\frac{-2}{3}$$\frac{1}{}$.

* Core Example Question: Simplify $\frac{5}{-3}$$\frac{0.6}{}$.

Difficulty: Procedural

Common pitfalls: Incorrectly identifying the main division bar, procedural errors in dividing fractions (e.g., not multiplying by the reciprocal), sign errors, not simplifying the final result.

Introduced definitions:

- Complex Fraction: A fraction in which the numerator, denominator, or both contain fractions.

Concept / Introduction / Definition (no of questions): 2

Practice no of questions: 8

Common Pitfall avoidance no of questions: 4

Application/word problem no of questions: 0

Challenging number of questions: 2

Total number of questions for each goal: 16

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28. Integer multiplication and division rules

Description: Students will apply combined rules for integer multiplication and division. This skill consolidates the separate rules for multiplication and division of integers into a unified understanding.

* Core Example Question: State the combined rules for multiplying and dividing integers (same signs and different signs).

* Core Example Question: Will $-12\times(-3)$ have the same sign as $12\div(-3)$?

* Core Example Question: Calculate $(-5)\times4\div(-2)$.

Difficulty: Procedural

Common pitfalls: Confusing multiplication and division rules, particularly for signs, making errors in multi-step problems that mix operations.

Introduced definitions: (No new definitions; this skill reinforces previously introduced rules.)

Concept / Introduction / Definition (no of questions): 2

Practice no of questions: 10

Common Pitfall avoidance no of questions: 5

Application/word problem no of questions: 1

Challenging number of questions: 2

Total number of questions for each goal: 20

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29. Multiply and divide integers

Description: Students will fluently multiply and divide integers, demonstrating accuracy and efficiency in performing these fundamental operations. This skill emphasizes overall mastery of integer multiplication and division.

* Core Example Question: Calculate $-63 \div 9$.

* Core Example Question: What is $-8 \times (-7)$?

* Core Example Question: Find the value of $100 \div (-25)$.

Difficulty: Core

Common pitfalls: Slowness in recalling rules, frequent sign errors, difficulty with larger numbers, lacking confidence in results.

Introduced definitions: (No new definitions; this skill reinforces previously introduced concepts.)

Concept / Introduction / Definition (no of questions): 0

Practice no of questions: 15

Common Pitfall avoidance no of questions: 7

Application/word problem no of questions: 2

Challenging number of questions: 3

Total number of questions for each goal: 27

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30. Complete multiplication and division equations with integers

Description: Students will fill in missing values in multiplication and division equations with integers. This requires using inverse operations and their understanding of integer properties to solve for unknowns in various positions within the equation.

* Core Example Question: Complete the equation: $-3 \times \text{___} = 18$.

* Core Example Question: Find the missing number: $\text{___} \div (-5) = 6$.

* Core Example Question: If $40 \div \text{___} = -8$, what is the missing integer?

Difficulty: Procedural

Common pitfalls: Incorrectly applying inverse operations, sign errors when isolating the unknown, difficulty with equations where the unknown is the divisor or multiplicand.

Introduced definitions: (No new definitions; applies inverse operations and integer rules to equations.)

Concept / Introduction / Definition (no of questions): 2

Practice no of questions: 8

Common Pitfall avoidance no of questions: 4

Application/word problem no of questions: 0

Challenging number of questions: 2

Total number of questions for each goal: 16

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31. Multiply and divide positive and negative decimals

Description: Students will multiply and divide decimals, including positive and negative values, by correctly handling decimal placement and applying the rules for signs of products and quotients.

* Core Example Question: Calculate $0.7 \times (-1.2)$.

* Core Example Question: What is $-4.8 \div 0.6$?

* Core Example Question: Find the value of $-2.5 \div (-0.05)$.

Difficulty: Procedural

Common pitfalls: Incorrect decimal placement during multiplication or division, sign errors, computational errors with decimal arithmetic.

Introduced definitions: (No new definitions; extends decimal operations to include negative values.)

Concept / Introduction / Definition (no of questions): 2

Practice no of questions: 10

Common Pitfall avoidance no of questions: 5

Application/word problem no of questions: 2

Challenging number of questions: 2

Total number of questions for each goal: 21

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32. Multiply and divide positive and negative fractions

Description: Students will multiply and divide fractions, including positive and negative values, by applying appropriate rules for fraction operations and signs. This skill serves as a broader category for operations with signed fractions.

* Core Example Question: Calculate $(-41)\times232$.

* Core Example Question: What is $-109\div(-53)$?

* Core Example Question: Find the result of $85\times(-154)$.

Difficulty: Procedural

Common pitfalls: Procedural errors in fraction arithmetic, sign errors, not simplifying answers, issues with mixed numbers.

Introduced definitions: (No new definitions; reinforces previously introduced fraction and sign concepts.)

Concept / Introduction / Definition (no of questions): 1

Practice no of questions: 10

Common Pitfall avoidance no of questions: 5

Application/word problem no of questions: 2

Challenging number of questions: 2

Total number of questions for each goal: 20

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33. Multiply and divide rational numbers

Description: Students will fluently multiply and divide rational numbers (integers, fractions, and decimals). This skill serves as a capstone for the unit, requiring students to identify the number type, select the appropriate strategy, and apply the correct sign rules for multiplication and division across all forms of rational numbers.

* Core Example Question: Calculate $-2.5\times(-21)$.

* Core Example Question: What is $43\div(-0.25)$?

* Core Example Question: Find the value of $(-6)\times31\div(-0.5)$.

Difficulty: Synthesis

Common pitfalls: Difficulty converting between different forms of rational numbers, procedural errors in combining different types of rational numbers, making multiple sign errors in multi-step problems, struggling to choose the most efficient method for calculation.

Introduced definitions: (No new definitions; this skill synthesizes understanding of rational numbers and their operations.)

Concept / Introduction / Definition (no of questions): 2

Practice no of questions: 10

Common Pitfall avoidance no of questions: 5

Application/word problem no of questions: 4

Challenging number of questions: 3

Total number of questions for each goal: 24

Problem Solving with Rational Numbers

1. Expressions with Rational Numbers

    - Name: Expressions with Rational Numbers
    - Description: Students will learn to evaluate numerical expressions that involve various rational numbers (integers, fractions, and decimals) and all four basic arithmetic operations (addition, subtraction, multiplication, and division). This skill reinforces the understanding of order of operations (PEMDAS/BODMAS) and the properties of rational numbers.
        - Core Example Questions:
            - Evaluate: $-21+43\times(-32)$
            - Calculate: $5.25\div(-0.5)-3.1$
            - Simplify: $(65-31)\times(-109)+2$
    - Difficulty: Procedural
    - Common pitfalls:
        - Incorrect application of order of operations.
        - Errors with signs when multiplying or dividing negative numbers.
        - Difficulty with fraction arithmetic (finding common denominators, multiplying/dividing fractions).
        - Mistakes in decimal alignment during addition/subtraction, or place value errors in multiplication/division.
    - Introduced definitions: Rational Number, Expression, Evaluate, Order of Operations (PEMDAS/BODMAS), Integer, Fraction, Decimal, Numerator, Denominator, Mixed Number, Product, Quotient, Sum, Difference.
    - Concept / Introduction / Definition (no of questions): 3
    - Practice no of questions: 10
    - Common Pitfall avoidance no of questions: 5
    - Application/word problem no of questions: 2
    - Challenging number of questions: 2
    - Total number of questions for each goal: 22
2. Solving Problems with Rational Numbers

    - Name: Solving Problems with Rational Numbers
    - Description: This skill focuses on applying the understanding of rational numbers and all four operations to solve practical, real-world scenarios. Students will translate verbal descriptions into mathematical expressions or equations and then solve them, emphasizing the relevance of rational numbers in everyday contexts.
        - Core Example Questions:
            - A submarine is at an elevation of -250 feet. It ascends 120 feet and then descends 75 feet. What is its new elevation?
            - Sarah has $\frac{4}{3}$ of a pizza. She eats $\frac{3}{1}$ of what she has. How much of the whole pizza did she eat?
            - The temperature dropped from $8.5^\circ C$ to $-2.3^\circ C$. What was the total change in temperature?
    - Difficulty: Application
    - Common pitfalls:
        - Misinterpreting the language of the problem (e.g., confusing "increase" with "decrease").
        - Setting up the incorrect operation for a given situation.

        - Not paying attention to units or context in the answer.
        - Performing calculations correctly but failing to answer the specific question asked.
    - Introduced definitions: Real-World Problem, Context, Elevation, Temperature Change, Net Change.
    - Concept / Introduction / Definition (no of questions): 2
    - Practice no of questions: 8
    - Common Pitfall avoidance no of questions: 4
    - Application/word problem no of questions: 6
    - Challenging number of questions: 3
    - Total number of questions for each goal: 23
3. Solving Equations With Rational Numbers

    - Name: Solving Equations With Rational Numbers
    - Description: Students will learn to solve one-step and multi-step linear equations where the coefficients and constants are rational numbers, including negative numbers. This involves isolating the variable using inverse operations and maintaining equality on both sides of the equation.
        - Core Example Questions:
            - Solve for x: $x+52=-101$
            - Solve for y: $-3.5y=14.7$
            - Solve for z: $4z-1=-5$
    - Difficulty: Procedural
    - Common pitfalls:
        - Incorrectly applying inverse operations (e.g., adding when should subtract).
        - Errors with signs when moving terms across the equals sign.
        - Mistakes in performing rational number arithmetic during the solution process.
        - Not simplifying answers to their simplest form.
    - Introduced definitions: Equation, Variable, Coefficient, Constant, Inverse Operations, Solution, Isolate, Equality.
    - Concept / Introduction / Definition (no of questions): 3
    - Practice no of questions: 12
    - Common Pitfall avoidance no of questions: 6
    - Application/word problem no of questions: 3
    - Challenging number of questions: 2
    - Total number of questions for each goal: 26
4. Representing Contexts with Equations

    - Name: Representing Contexts with Equations
    - Description: This skill focuses on the critical step of translating real-world scenarios into mathematical equations, specifically involving negative numbers and all four operations. Students will identify the unknown quantity, define a variable, and then construct an equation that accurately models the given situation.
        - Core Example Questions:
            - Write an equation to represent the following: "The temperature dropped by 15 degrees Fahrenheit to reach -8 degrees Fahrenheit. What was the initial temperature?"
            - A submarine is at -120 feet. It then moves to a new depth that is 3 times its current depth. Write an equation to find the new depth.
            - You have $ $25 inyourbankaccount.Afterwritingacheck,yourbalanceis -$15 $. Write an equation to find the amount of the check.
    - Difficulty: Application

- Common pitfalls:
  - Misidentifying the variable or the unknown quantity.
  - Using the wrong operation to represent a given action (e.g., using addition for a decrease).
  - Confusing the initial value with the final value.
  - Not correctly incorporating negative numbers into the equation.
- Introduced definitions: Model, Translate, Unknown Quantity, Initial Value, Final Value, Balance.
- Concept / Introduction / Definition (no of questions): 3
- Practice no of questions: 7
- Common Pitfall avoidance no of questions: 4
- Application/word problem no of questions: 8
- Challenging number of questions: 3
- Total number of questions for each goal: 25

5. The Stock Market

   - Name: The Stock Market
   - Description: Students will explore the application of positive and negative rational numbers in the context of the stock market. This includes understanding concepts like stock price changes, gains, and losses, and applying rational number operations to calculate overall changes in portfolio value or individual stock performance.
     - Core Example Questions:
       - A stock opens at $ $50.25 $. During the day, it drops $ $3.50 $ and then gains $ $1.75 $. What is the closing price?
       - You own 100 shares of a company. If the stock price decreases by $ $1.25 $ per share, what is the total loss in your portfolio?
       - Over three days, a stock's changes were $ + $0.75 $, $ - $1.50 $, and $ + $0.30 $. What is the average daily change?
   - Difficulty: Application
   - Common pitfalls:
     - Confusing a decrease with a positive change.
     - Errors in calculating total change when multiple transactions occur.
     - Misinterpreting "per share" versus "total portfolio" changes.
     - Difficulty with multi-step calculations involving decimals.
   - Introduced definitions: Stock, Share, Stock Price, Gain, Loss, Portfolio, Directed Change, Opening Price, Closing Price, Average Change.
   - Concept / Introduction / Definition (no of questions): 3
   - Practice no of questions: 6
   - Common Pitfall avoidance no of questions: 3
   - Application/word problem no of questions: 7
   - Challenging number of questions: 3
   - Total number of questions for each goal: 22

6. Add and subtract integers: word problems

   - Name: Add and subtract integers: word problems
   - Description: This skill focuses on solving word problems that specifically involve the addition and subtraction of integers. Students will identify integer quantities and translate the problem into an integer addition or subtraction expression to find the solution.
     - Core Example Questions:
       - A hiker starts at an elevation of 150 feet above sea level and descends 200 feet. What is her new elevation?
       - The temperature was $-5^\circ C$ in the morning and rose by $12^\circ C$ by noon. What was the temperature at noon?

            - A submarine is at a depth of −300 feet. It rises 100 feet. What is its new depth?
    - Difficulty: Application
    - Common pitfalls:
        - Incorrectly setting up the operation (e.g., subtracting when should add a negative).
        - Errors in integer arithmetic, especially with negative numbers.
        - Misinterpreting terms like "above," "below," "rise," "fall."
        - Not understanding that "subtracting a negative" is equivalent to adding a positive.
    - Introduced definitions: Integer, Elevation, Sea Level, Depth, Temperature.
    - Concept / Introduction / Definition (no of questions): 2
    - Practice no of questions: 5
    - Common Pitfall avoidance no of questions: 3
    - Application/word problem no of questions: 7
    - Challenging number of questions: 2
    - Total number of questions for each goal: 19
7. Add, subtract, multiply, and divide integers

    - Name: Add, subtract, multiply, and divide integers
    - Description: Students will master performing all four basic arithmetic operations (addition, subtraction, multiplication, and division) with integers, paying close attention to the rules of signs for each operation. This is a foundational skill for working with all rational numbers.
        - Core Example Questions:
            - Calculate: −8+5
            - Evaluate: −12−(−7)
            - Find the product: −6×9
            - Determine the quotient: 48÷(−6)
    - Difficulty: Core
    - Common pitfalls:
        - Confusing rules for adding/subtracting integers with rules for multiplying/dividing integers regarding signs.
        - Errors when subtracting a negative number.
        - Forgetting that dividing by zero is undefined.
        - Simple arithmetic errors with larger numbers.
    - Introduced definitions: Positive Integer, Negative Integer, Zero, Absolute Value.
    - Concept / Introduction / Definition (no of questions): 3
    - Practice no of questions: 15
    - Common Pitfall avoidance no of questions: 7
    - Application/word problem no of questions: 2
    - Challenging number of questions: 3
    - Total number of questions for each goal: 30
8. Add, subtract, multiply, or divide two decimals

    - Name: Add, subtract, multiply, or divide two decimals
    - Description: Students will perform any of the four arithmetic operations on two decimal numbers. This skill requires careful attention to decimal point placement, especially during multiplication and division.
        - Core Example Questions:
            - Calculate: 3.75+12.8
            - Find the difference: 9.2−4.55
            - Multiply: 0.6×2.3

        - Divide: 15.3÷0.3
    - Difficulty: Procedural
    - Common pitfalls:
        - Misaligning decimal points during addition and subtraction.
        - Incorrectly counting decimal places in multiplication.
        - Errors in moving the decimal point in the divisor and dividend during division.
        - Forgetting to add trailing zeros when needed for division.
    - Introduced definitions: Decimal Point, Place Value, Dividend, Divisor, Product (of decimals), Quotient (of decimals).
    - Concept / Introduction / Definition (no of questions): 2
    - Practice no of questions: 12
    - Common Pitfall avoidance no of questions: 6
    - Application/word problem no of questions: 2
    - Challenging number of questions: 2
    - Total number of questions for each goal: 24
9. Add, subtract, multiply, and divide decimals: word problems

    - Name: Add, subtract, multiply, and divide decimals: word problems
    - Description: This skill focuses on solving multi-step real-world problems that necessitate the use of all four operations with decimal numbers. Students will interpret the problem, set up the appropriate decimal operations, and solve to find a practical solution.
        - Core Example Questions:
            - Sarah buys 3.5 pounds of apples at $ $1.80 $ per pound and 2.25 pounds of grapes at $ $2.40 $ per pound. How much did she spend in total?
            - A recipe calls for 0.75 cups of sugar. If you want to make 2.5 times the recipe, how much sugar do you need?
            - A long jump record is 8.95 meters. If a new jumper jumps 0.7 meters less than the record, what is their jump distance?
    - Difficulty: Application
    - Common pitfalls:
        - Misidentifying the operation needed for each step.
        - Errors in decimal arithmetic from the previous skill.
        - Failing to perform all necessary steps in a multi-step problem.
        - Not checking for reasonableness of the answer in context.
    - Introduced definitions: Unit Price, Total Cost, Recipe Scaling, Difference (in context).
    - Concept / Introduction / Definition (no of questions): 2
    - Practice no of questions: 6
    - Common Pitfall avoidance no of questions: 3
    - Application/word problem no of questions: 8
    - Challenging number of questions: 3
    - Total number of questions for each goal: 22
10. Add, subtract, multiply, and divide fractions and mixed numbers: word problems

    - Name: Add, subtract, multiply, and divide fractions and mixed numbers: word problems
    - Description: Students will solve word problems that involve all four operations with fractions and mixed numbers. This requires understanding how to convert between mixed numbers and improper fractions, finding common denominators, and correctly applying fraction operation rules in real-world contexts.
        - Core Example Questions:
            - A recipe calls for 221 cups of flour. If you only have 143 cups, how much more flour do you need?

            - You have 65 of a pizza left. If you eat 21 of the remaining pizza, how much of the *whole* pizza did you eat?
            - A plank of wood is 841 feet long. If you cut it into 3 equal pieces, how long is each piece?
    - Difficulty: Application
    - Common pitfalls:
        - Difficulty with fraction arithmetic in a problem-solving context.
        - Not converting mixed numbers to improper fractions when appropriate (especially for multiplication/division).
        - Misinterpreting "of" as multiplication or "left" as subtraction.
        - Failing to simplify fractions to their lowest terms or convert improper fractions back to mixed numbers for the answer if needed.
    - Introduced definitions: Improper Fraction, Proper Fraction, Lowest Terms, Simplest Form, Common Denominator.
    - Concept / Introduction / Definition (no of questions): 2
    - Practice no of questions: 6
    - Common Pitfall avoidance no of questions: 4
    - Application/word problem no of questions: 8
    - Challenging number of questions: 3
    - Total number of questions for each goal: 23
11. Multi-step word problems with positive rational numbers

    - Name: Multi-step word problems with positive rational numbers
    - Description: This skill integrates the ability to solve multi-step real-world problems using all four operations, specifically with positive rational numbers (including positive integers, fractions, and decimals). It emphasizes problem decomposition, strategic planning, and accurate execution of calculations.
        - Core Example Questions:
            - John earns $ $15.50 $ per hour. He worked 7.5 hours on Monday and 6 hours on Tuesday. If he spent 41 of his total earnings on groceries, how much money does he have left?
            - A rectangular garden is 1021 feet long and 6 feet wide. If fencing costs $ $2.50 $ per foot, what is the total cost to fence the garden?
            - A bakery uses 3.25 cups of sugar for a batch of cookies. If they want to make 4 batches and have 10 cups of sugar already, how much more sugar do they need?
    - Difficulty: Synthesis
    - Common pitfalls:
        - Not breaking down the problem into manageable steps.
        - Incorrectly identifying the sequence of operations.
        - Errors in arithmetic with fractions and decimals.
        - Failing to answer all parts of a multi-step question.
        - Overlooking information or using irrelevant information.
    - Introduced definitions: Multi-step Problem, Problem Decomposition, Strategy, Perimeter, Area, Total Earnings, Remaining Amount.
    - Concept / Introduction / Definition (no of questions): 2
    - Practice no of questions: 5
    - Common Pitfall avoidance no of questions: 4
    - Application/word problem no of questions: 10
    - Challenging number of questions: 4
    - Total number of questions for each goal: 25

The Number System

Solving Real-World Problems with Rational Numbers

Problem Solving with Rational Numbers

1. Expressions with Rational Numbers

    - Name: Expressions with Rational Numbers
    - Description: Students will learn to evaluate numerical expressions that involve various rational numbers (integers, fractions, and decimals) and all four basic arithmetic operations (addition, subtraction, multiplication, and division). This skill reinforces the understanding of order of operations (PEMDAS/BODMAS) and the properties of rational numbers.
        - Core Example Questions:
            - Evaluate: $-21+43\times(-32)$
            - Calculate: $5.25\div(-0.5)-3.1$
            - Simplify: $(65-31)\times(-109)+2$
    - Difficulty: Procedural
    - Common pitfalls:
        - Incorrect application of order of operations.
        - Errors with signs when multiplying or dividing negative numbers.
        - Difficulty with fraction arithmetic (finding common denominators, multiplying/dividing fractions).
        - Mistakes in decimal alignment during addition/subtraction, or place value errors in multiplication/division.
    - Introduced definitions: Rational Number, Expression, Evaluate, Order of Operations (PEMDAS/BODMAS), Integer, Fraction, Decimal, Numerator, Denominator, Mixed Number, Product, Quotient, Sum, Difference.
    - Concept / Introduction / Definition (no of questions): 3
    - Practice no of questions: 10
    - Common Pitfall avoidance no of questions: 5
    - Application/word problem no of questions: 2
    - Challenging number of questions: 2
    - Total number of questions for each goal: 22
2. Solving Problems with Rational Numbers

    - Name: Solving Problems with Rational Numbers
    - Description: This skill focuses on applying the understanding of rational numbers and all four operations to solve practical, real-world scenarios. Students will translate verbal descriptions into mathematical expressions or equations and then solve them, emphasizing the relevance of rational numbers in everyday contexts.
        - Core Example Questions:
            - A submarine is at an elevation of -250 feet. It ascends 120 feet and then descends 75 feet. What is its new elevation?
            - Sarah has 43 of a pizza. She eats 31 of what she has. How much of the whole pizza did she eat?
            - The temperature dropped from $8.5^\circ C$ to $-2.3^\circ C$. What was the total change in temperature?
    - Difficulty: Application
    - Common pitfalls:
        - Misinterpreting the language of the problem (e.g., confusing "increase" with "decrease").
        - Setting up the incorrect operation for a given situation.
        - Not paying attention to units or context in the answer.

        - Performing calculations correctly but failing to answer the specific question asked.
    - Introduced definitions: Real-World Problem, Context, Elevation, Temperature Change, Net Change.
    - Concept / Introduction / Definition (no of questions): 2
    - Practice no of questions: 8
    - Common Pitfall avoidance no of questions: 4
    - Application/word problem no of questions: 6
    - Challenging number of questions: 3
    - Total number of questions for each goal: 23
3. Solving Equations With Rational Numbers

    - Name: Solving Equations With Rational Numbers
    - Description: Students will learn to solve one-step and multi-step linear equations where the coefficients and constants are rational numbers, including negative numbers. This involves isolating the variable using inverse operations and maintaining equality on both sides of the equation.
        - Core Example Questions:
            - Solve for x: $x+52=-101$
            - Solve for y: $-3.5y=14.7$
            - Solve for z: $4z-1=-5$
    - Difficulty: Procedural
    - Common pitfalls:
        - Incorrectly applying inverse operations (e.g., adding when should subtract).
        - Errors with signs when moving terms across the equals sign.
        - Mistakes in performing rational number arithmetic during the solution process.
        - Not simplifying answers to their simplest form.
    - Introduced definitions: Equation, Variable, Coefficient, Constant, Inverse Operations, Solution, Isolate, Equality.
    - Concept / Introduction / Definition (no of questions): 3
    - Practice no of questions: 12
    - Common Pitfall avoidance no of questions: 6
    - Application/word problem no of questions: 3
    - Challenging number of questions: 2
    - Total number of questions for each goal: 26
4. Representing Contexts with Equations

    - Name: Representing Contexts with Equations
    - Description: This skill focuses on the critical step of translating real-world scenarios into mathematical equations, specifically involving negative numbers and all four operations. Students will identify the unknown quantity, define a variable, and then construct an equation that accurately models the given situation.
        - Core Example Questions:
            - Write an equation to represent the following: "The temperature dropped by 15 degrees Fahrenheit to reach −8 degrees Fahrenheit. What was the initial temperature?"
            - A submarine is at −120 feet. It then moves to a new depth that is 3 times its current depth. Write an equation to find the new depth.
            - You have $ $25 inyourbankaccount.Afterwritingacheck,yourbalanceis -$15 $. Write an equation to find the amount of the check.
    - Difficulty: Application
    - Common pitfalls:

        - Misidentifying the variable or the unknown quantity.
        - Using the wrong operation to represent a given action (e.g., using addition for a decrease).
        - Confusing the initial value with the final value.
        - Not correctly incorporating negative numbers into the equation.
    - Introduced definitions: Model, Translate, Unknown Quantity, Initial Value, Final Value, Balance.
    - Concept / Introduction / Definition (no of questions): 3
    - Practice no of questions: 7
    - Common Pitfall avoidance no of questions: 4
    - Application/word problem no of questions: 8
    - Challenging number of questions: 3
    - Total number of questions for each goal: 25
5. The Stock Market

    - Name: The Stock Market
    - Description: Students will explore the application of positive and negative rational numbers in the context of the stock market. This includes understanding concepts like stock price changes, gains, and losses, and applying rational number operations to calculate overall changes in portfolio value or individual stock performance.
        - Core Example Questions:
            - A stock opens at $ $50.25 $. During the day, it drops $ $3.50 $ and then gains $ $1.75 $. What is the closing price?
            - You own 100 shares of a company. If the stock price decreases by $ $1.25 $ per share, what is the total loss in your portfolio?
            - Over three days, a stock's changes were $ + $0.75 $, $ - $1.50 $, and $ + $0.30 $. What is the average daily change?
    - Difficulty: Application
    - Common pitfalls:
        - Confusing a decrease with a positive change.
        - Errors in calculating total change when multiple transactions occur.
        - Misinterpreting "per share" versus "total portfolio" changes.
        - Difficulty with multi-step calculations involving decimals.
    - Introduced definitions: Stock, Share, Stock Price, Gain, Loss, Portfolio, Directed Change, Opening Price, Closing Price, Average Change.
    - Concept / Introduction / Definition (no of questions): 3
    - Practice no of questions: 6
    - Common Pitfall avoidance no of questions: 3
    - Application/word problem no of questions: 7
    - Challenging number of questions: 3
    - Total number of questions for each goal: 22
6. Add and subtract integers: word problems

    - Name: Add and subtract integers: word problems
    - Description: This skill focuses on solving word problems that specifically involve the addition and subtraction of integers. Students will identify integer quantities and translate the problem into an integer addition or subtraction expression to find the solution.
        - Core Example Questions:
            - A hiker starts at an elevation of 150 feet above sea level and descends 200 feet. What is her new elevation?
            - The temperature was $-5^\circ C$ in the morning and rose by $12^\circ C$ by noon. What was the temperature at noon?

            - A submarine is at a depth of −300 feet. It rises 100 feet. What is its new depth?
    - Difficulty: Application
    - Common pitfalls:
        - Incorrectly setting up the operation (e.g., subtracting when should add a negative).
        - Errors in integer arithmetic, especially with negative numbers.
        - Misinterpreting terms like "above," "below," "rise," "fall."
        - Not understanding that "subtracting a negative" is equivalent to adding a positive.
    - Introduced definitions: Integer, Elevation, Sea Level, Depth, Temperature.
    - Concept / Introduction / Definition (no of questions): 2
    - Practice no of questions: 5
    - Common Pitfall avoidance no of questions: 3
    - Application/word problem no of questions: 7
    - Challenging number of questions: 2
    - Total number of questions for each goal: 19
7. Add, subtract, multiply, and divide integers

    - Name: Add, subtract, multiply, and divide integers
    - Description: Students will master performing all four basic arithmetic operations (addition, subtraction, multiplication, and division) with integers, paying close attention to the rules of signs for each operation. This is a foundational skill for working with all rational numbers.
        - Core Example Questions:
            - Calculate: −8+5
            - Evaluate: −12−(−7)
            - Find the product: −6×9
            - Determine the quotient: 48÷(−6)
    - Difficulty: Core
    - Common pitfalls:
        - Confusing rules for adding/subtracting integers with rules for multiplying/dividing integers regarding signs.
        - Errors when subtracting a negative number.
        - Forgetting that dividing by zero is undefined.
        - Simple arithmetic errors with larger numbers.
    - Introduced definitions: Positive Integer, Negative Integer, Zero, Absolute Value.
    - Concept / Introduction / Definition (no of questions): 3
    - Practice no of questions: 15
    - Common Pitfall avoidance no of questions: 7
    - Application/word problem no of questions: 2
    - Challenging number of questions: 3
    - Total number of questions for each goal: 30
8. Add, subtract, multiply, or divide two decimals

    - Name: Add, subtract, multiply, or divide two decimals
    - Description: Students will perform any of the four arithmetic operations on two decimal numbers. This skill requires careful attention to decimal point placement, especially during multiplication and division.
        - Core Example Questions:
            - Calculate: 3.75+12.8
            - Find the difference: 9.2−4.55
            - Multiply: 0.6×2.3

        - Divide: 15.3÷0.3
    - Difficulty: Procedural
    - Common pitfalls:
        - Misaligning decimal points during addition and subtraction.
        - Incorrectly counting decimal places in multiplication.
        - Errors in moving the decimal point in the divisor and dividend during division.
        - Forgetting to add trailing zeros when needed for division.
    - Introduced definitions: Decimal Point, Place Value, Dividend, Divisor, Product (of decimals), Quotient (of decimals).
    - Concept / Introduction / Definition (no of questions): 2
    - Practice no of questions: 12
    - Common Pitfall avoidance no of questions: 6
    - Application/word problem no of questions: 2
    - Challenging number of questions: 2
    - Total number of questions for each goal: 24
9. Add, subtract, multiply, and divide decimals: word problems

    - Name: Add, subtract, multiply, and divide decimals: word problems
    - Description: This skill focuses on solving multi-step real-world problems that necessitate the use of all four operations with decimal numbers. Students will interpret the problem, set up the appropriate decimal operations, and solve to find a practical solution.
        - Core Example Questions:
            - Sarah buys 3.5 pounds of apples at $ $1.80 $ per pound and 2.25 pounds of grapes at $ $2.40 $ per pound. How much did she spend in total?
            - A recipe calls for 0.75 cups of sugar. If you want to make 2.5 times the recipe, how much sugar do you need?
            - A long jump record is 8.95 meters. If a new jumper jumps 0.7 meters less than the record, what is their jump distance?
    - Difficulty: Application
    - Common pitfalls:
        - Misidentifying the operation needed for each step.
        - Errors in decimal arithmetic from the previous skill.
        - Failing to perform all necessary steps in a multi-step problem.
        - Not checking for reasonableness of the answer in context.
    - Introduced definitions: Unit Price, Total Cost, Recipe Scaling, Difference (in context).
    - Concept / Introduction / Definition (no of questions): 2
    - Practice no of questions: 6
    - Common Pitfall avoidance no of questions: 3
    - Application/word problem no of questions: 8
    - Challenging number of questions: 3
    - Total number of questions for each goal: 22
10. Add, subtract, multiply, and divide fractions and mixed numbers: word problems

    - Name: Add, subtract, multiply, and divide fractions and mixed numbers: word problems
    - Description: Students will solve word problems that involve all four operations with fractions and mixed numbers. This requires understanding how to convert between mixed numbers and improper fractions, finding common denominators, and correctly applying fraction operation rules in real-world contexts.
        - Core Example Questions:
            - A recipe calls for 221 cups of flour. If you only have 143 cups, how much more flour do you need?

            - You have 65 of a pizza left. If you eat 21 of the remaining pizza, how much of the *whole* pizza did you eat?
            - A plank of wood is 841 feet long. If you cut it into 3 equal pieces, how long is each piece?
    - Difficulty: Application
    - Common pitfalls:
        - Difficulty with fraction arithmetic in a problem-solving context.
        - Not converting mixed numbers to improper fractions when appropriate (especially for multiplication/division).
        - Misinterpreting "of" as multiplication or "left" as subtraction.
        - Failing to simplify fractions to their lowest terms or convert improper fractions back to mixed numbers for the answer if needed.
    - Introduced definitions: Improper Fraction, Proper Fraction, Lowest Terms, Simplest Form, Common Denominator.
    - Concept / Introduction / Definition (no of questions): 2
    - Practice no of questions: 6
    - Common Pitfall avoidance no of questions: 4
    - Application/word problem no of questions: 8
    - Challenging number of questions: 3
    - Total number of questions for each goal: 23
11. Multi-step word problems with positive rational numbers

    - Name: Multi-step word problems with positive rational numbers
    - Description: This skill integrates the ability to solve multi-step real-world problems using all four operations, specifically with positive rational numbers (including positive integers, fractions, and decimals). It emphasizes problem decomposition, strategic planning, and accurate execution of calculations.
        - Core Example Questions:
            - John earns $ $15.50 $ per hour. He worked 7.5 hours on Monday and 6 hours on Tuesday. If he spent 41 of his total earnings on groceries, how much money does he have left?
            - A rectangular garden is 1021 feet long and 6 feet wide. If fencing costs $ $2.50 $ per foot, what is the total cost to fence the garden?
            - A bakery uses 3.25 cups of sugar for a batch of cookies. If they want to make 4 batches and have 10 cups of sugar already, how much more sugar do they need?
    - Difficulty: Synthesis
    - Common pitfalls:
        - Not breaking down the problem into manageable steps.
        - Incorrectly identifying the sequence of operations.
        - Errors in arithmetic with fractions and decimals.
        - Failing to answer all parts of a multi-step question.
        - Overlooking information or using irrelevant information.
    - Introduced definitions:
    - Concept / Introduction / Definition (no of questions): 2
    - Practice no of questions: 5
    - Common Pitfall avoidance no of questions: 4
    - Application/word problem no of questions: 10
    - Challenging number of questions: 4
    - Total number of questions for each goal: 25

Equivalent Expressions and Interpretation

1. Equivalent expressions: negative numbers & distribution

- Name: Equivalent expressions: negative numbers & distribution
- Description: This skill focuses on understanding and applying the distributive property, especially when negative numbers are involved. Students will learn to simplify expressions by distributing a negative factor across terms inside parentheses. This is a foundational skill for manipulating algebraic expressions and solving equations.
    - Core Example Questions:
        - Simplify: $-3(x-5)$
        - Which expression is equivalent to $2(-4x+7)$?
        - Rewrite $-(2y+8)$ without parentheses.
- Difficulty: Core
- Common pitfalls:
    - Forgetting to distribute the negative sign to all terms inside the parentheses.
    - Incorrectly multiplying negative numbers (e.g., $-3\times-5=-15$).
    - Confusing subtraction with negative signs.
- Introduced definitions: Distributive property, equivalent expressions, terms, coefficient, constant
- Concept / Introduction / Definition: 2
- Practice: 8
- Common Pitfall avoidance: 3
- Application/word problem: 1
- Challenging: 1
- Total number of questions for each goal: 15

2. Subtraction in Equivalent Expressions

- Name: Subtraction in Equivalent Expressions
- Description: This skill builds on the distributive property by explicitly addressing subtraction within expressions. Students will learn to interpret subtraction as adding the opposite, which is crucial for correctly applying the distributive property and combining like terms.
    - Core Example Questions:
        - Rewrite $5x-(3x-4)$ as an equivalent expression.
        - Simplify: $-(x-7)$
        - Explain why $6-(2y+1)$ is equivalent to $6+(-2y-1)$.
- Difficulty: Core
- Common pitfalls:
    - Only distributing the negative sign to the first term inside the parentheses.
    - Failing to change the sign of all terms when distributing a negative.
    - Misinterpreting the difference between $a-b$ and $a+(-b)$.
- Introduced definitions: None (reinforces previous definitions)
- Concept / Introduction / Definition: 2
- Practice: 7
- Common Pitfall avoidance: 4
- Application/word problem: 1
- Challenging: 1
- Total number of questions for each goal: 15

3. Expanding and Factoring

- Name: Expanding and Factoring
- Description: This skill introduces the inverse operations of expanding (using the distributive property to multiply) and factoring (using the distributive property in reverse to find a common

factor). Students will work with expressions containing rational coefficients (fractions and decimals).
    - Core Example Questions:
        - Expand the expression: $21(4x-6)$
        - Factor the expression: $9y+12$
        - Write an expression equivalent to $2.5(x+4)$ by expanding.
- Difficulty: Procedural
- Common pitfalls:
    - Factoring out an incorrect greatest common factor.
    - Errors in multiplying or dividing with fractions or decimals.
    - Not recognizing when an expression can be factored.
- Introduced definitions: Expanding, Factoring, greatest common factor (GCF), rational coefficients
- Concept / Introduction / Definition: 3
- Practice: 8
- Common Pitfall avoidance: 3
- Application/word problem: 1
- Challenging: 2
- Total number of questions for each goal: 17

4. Write equivalent expressions using properties

- Name: Write equivalent expressions using properties
- Description: This skill focuses on applying the commutative, associative, and distributive properties to write equivalent expressions. Students will need to identify which property is being used and how it transforms an expression while maintaining its value.
    - Core Example Questions:
        - Which property is illustrated by $(3+x)+7=3+(x+7)$?
        - Rewrite $4y+5+2y$ as an equivalent expression by combining like terms.
        - Explain why $2x+3y+4x$ is equivalent to $6x+3y$.
- Difficulty: Core
- Common pitfalls:
    - Confusing the names and applications of the different properties.
    - Incorrectly combining terms that are not "like terms."
    - Making arithmetic errors when combining constants or coefficients.
- Introduced definitions: Commutative property, Associative property, Like terms
- Concept / Introduction / Definition: 3
- Practice: 7
- Common Pitfall avoidance: 2
- Application/word problem: 1
- Challenging: 1
- Total number of questions for each goal: 14

5. Interpreting linear expressions

- Name: Interpreting linear expressions
- Description: This skill requires students to understand the meaning of the components (terms, coefficients, constants) of a linear expression within a real-world context. They will explain what each part represents in a given scenario.
    - Core Example Questions:
        - In the expression $5c+10$, where c represents the number of chickens, what does the 5 represent? What does the 10 represent?
        - A rental car costs $30 per day plus $0.15 per mile. Write an expression for the total cost and explain what each part means.

            - The expression $12h-50$ represents a plumber's earnings, where h is the number of hours worked. What does the -50 likely represent in this context?
- Difficulty: Application
- Common pitfalls:
    - Confusing the variable with the coefficient.
    - Misinterpreting the constant term.
    - Struggling to connect mathematical terms to real-world quantities.
- Introduced definitions: None (applies existing definitions to context)
- Concept / Introduction / Definition: 2
- Practice: 3
- Common Pitfall avoidance: 1
- Application/word problem: 5
- Challenging: 1
- Total number of questions for each goal: 12

6. Writing expressions word problems

- Name: Writing expressions word problems
- Description: Students will translate real-world situations into linear expressions. This involves identifying the unknown quantities, defining variables, and using appropriate operations to represent the relationships described in the problem. They should also be able to recognize how different equivalent forms of an expression can highlight different aspects of the situation.
    - Core Example Questions:
        - Write an expression for the cost of p pounds of apples at $2.50 per pound and a $1.00 reusable bag fee.
        - A store offers a shirt for $20 and a 15% discount. Write an expression that represents the final cost of the shirt.
        - Sarah bought x notebooks at $3 each and y pens at $1.50 each. Write an expression for the total cost.
- Difficulty: Application
- Common pitfalls:
    - Incorrectly identifying the operation (e.g., using addition instead of multiplication).
    - Not defining variables clearly.
    - Missing parts of the expression or including extraneous information.
- Introduced definitions: None (applies existing definitions to context)
- Concept / Introduction / Definition: 1
- Practice: 4
- Common Pitfall avoidance: 1
- Application/word problem: 6
- Challenging: 2
- Total number of questions for each goal: 14

7. Identify equivalent linear expressions using algebra tiles

- Name: Identify equivalent linear expressions using algebra tiles
- Description: This skill provides a visual and concrete approach to understanding equivalent expressions. Students will use algebra tiles (representing variables and constants) to build and manipulate expressions, demonstrating equivalence through physical rearrangement and cancellation.
    - Core Example Questions:
        - Use algebra tiles to show that $2x+3+x-1$ is equivalent to $3x+2$.
        - Draw algebra tiles to represent the expression $4(x-2)$. Then simplify the expression using the tiles.

        - Which set of algebra tiles represents an expression equivalent to $x+x+5-2x$?
- Difficulty: Conceptual
- Common pitfalls:
    - Incorrectly representing negative numbers with tiles.
    - Difficulty with visualizing distribution using tiles.
    - Not understanding that zero pairs can be removed.
- Introduced definitions: Algebra tiles, zero pair
- Concept / Introduction / Definition: 4
- Practice: 6
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 13

8. Identify equivalent linear expressions I

- Name: Identify equivalent linear expressions I
- Description: This skill focuses on algebraically identifying equivalent linear expressions through simplification, combining like terms, and applying the distributive property. These will be relatively straightforward expressions.
    - Core Example Questions:
        - Is $3x+5$ equivalent to $2x+x+5$?
        - Which expression is equivalent to $7y-3+2y$?
        - Simplify: $4(x-2)+3x$
- Difficulty: Procedural
- Common pitfalls:
    - Errors in combining like terms (e.g., adding exponents).
    - Distributive property errors.
    - Sign errors when combining negative terms.
- Introduced definitions: None (reinforces previous definitions)
- Concept / Introduction / Definition: 1
- Practice: 10
- Common Pitfall avoidance: 3
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 15

9. Identify equivalent linear expressions II

- Name: Identify equivalent linear expressions II
- Description: This skill extends the previous one to include more complex linear expressions, potentially involving multiple distributive property applications, nested parentheses, or a greater variety of rational coefficients.
    - Core Example Questions:
        - Which expression is equivalent to $-2(3x-4)+5x$?
        - Simplify: $41(8y+12)-2y$
        - Is $6-(x-3)$ equivalent to $9-x$?
- Difficulty: Procedural
- Common pitfalls:
    - Order of operations errors (especially with distribution and subtraction).
    - Complex sign errors.
    - Errors in arithmetic with fractions and decimals.
- Introduced definitions: None (reinforces previous definitions)

- Concept / Introduction / Definition: 0
- Practice: 10
- Common Pitfall avoidance: 4
- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 16

10. Identify equivalent linear expressions: word problems

- Name: Identify equivalent linear expressions: word problems
- Description: Students will analyze real-world scenarios and identify different linear expressions that accurately represent the same situation. This requires them to not only translate words into expressions but also to recognize the equivalence between different algebraic forms in context.
    - Core Example Questions:
        - A t-shirt costs $15. If you buy 3 t-shirts and get a $5 discount on the total purchase, which of the following expressions represents the total cost? (A) $15\times3-5$ (B) $3\times(15-5)$ (C) $15+15+15-5$
        - A plumber charges a $40 service fee plus $60 per hour. Which expression is equivalent to the total cost if h is the number of hours? (A) $40+60h$ (B) $20(2+3h)$
        - Describe a real-world scenario that could be represented by both $3(x+2)$ and $3x+6$.
- Difficulty: Application
- Common pitfalls:
    - Failing to correctly set up the initial expression from the word problem.
    - Not recognizing the algebraic equivalence between different forms of the expression.
    - Misinterpreting the meaning of terms in context.
- Introduced definitions: None (applies existing definitions to context)
- Concept / Introduction / Definition: 1
- Practice: 3
- Common Pitfall avoidance: 1
- Application/word problem: 6
- Challenging: 2
- Total number of questions for each goal: 13

Solving Equations and Inequalities

Solving Equations

1. Rational number word problems

- Name: Rational number word problems
- Description: This skill focuses on solving multi-step word problems that involve all forms of rational numbers (positive and negative integers, fractions, and decimals). Students will need to choose appropriate strategies and tools to solve these problems, demonstrating their understanding of operations with rational numbers in real-world contexts.
    - Core Example Questions:
        - A recipe calls for $2\frac{1}{4}$ cups of flour. If you only want to make half of the recipe, how much flour do you need?
        - The temperature dropped from $15.3^\circ C$ to $-2.7^\circ C$. What was the total temperature change?
        - John has a debt of $120. If he pays back $25 each week, how many weeks will it take him to pay off his debt?

- Difficulty: Application
- Common pitfalls:
    - Errors in arithmetic with fractions or decimals.
    - Misinterpreting the problem to set up the correct operations.
    - Failing to consider the signs of rational numbers.
- Introduced definitions: Rational number
- Concept / Introduction / Definition: 2
- Practice: 4
- Common Pitfall avoidance: 2
- Application/word problem: 8
- Challenging: 2
- Total number of questions for each goal: 18

2. Rewriting decimals as fractions challenge

- Name: Rewriting decimals as fractions challenge
- Description: This skill is a foundational building block for working with rational numbers in different forms. Students will learn how to convert repeating decimals into their equivalent fractional form. This deepens their understanding of rational numbers as numbers that can be expressed as a ratio of two integers.
    - Core Example Questions:
        - Convert 0.3 to a fraction.
        - Express 0.45 as a fraction in simplest form.
        - Show that 0.9 is equivalent to 1.
- Difficulty: Conceptual
- Common pitfalls:
    - Setting up the algebraic steps incorrectly (e.g., multiplying by the wrong power of 10).
    - Errors in solving the resulting linear equation.
    - Not simplifying the final fraction.
- Introduced definitions: Repeating decimal, terminating decimal
- Concept / Introduction / Definition: 3
- Practice: 6
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 3
- Total number of questions for each goal: 14

3. Add and subtract decimals: word problems

- Name: Add and subtract decimals: word problems
- Description: Students will solve real-world problems that require the addition and subtraction of decimals. This skill emphasizes accurate calculation and contextual understanding of decimal operations.
    - Core Example Questions:
        - Sarah bought a book for $12.75 and a pen for $2.50. How much did she spend in total?
        - A swimmer completed a lap in 23.45 seconds. Her previous best was 22.98 seconds. By how much did she improve?
        - The rainfall in April was 3.15 inches and in May was 4.8 inches. What was the total rainfall for both months?
- Difficulty: Application
- Common pitfalls:
    - Incorrectly aligning decimal points during addition or subtraction.
    - Arithmetic errors.
    - Misinterpreting the problem to determine if addition or subtraction is needed.

- Introduced definitions: None
- Concept / Introduction / Definition: 0
- Practice: 3
- Common Pitfall avoidance: 1
- Application/word problem: 5
- Challenging: 0
- Total number of questions for each goal: 9

4. Multiply decimals and whole numbers: word problems

- Name: Multiply decimals and whole numbers: word problems
- Description: This skill focuses on solving real-world problems that involve multiplying decimals by whole numbers. Students need to understand how to apply multiplication in context and accurately calculate the product.
    - Core Example Questions:
        - A car travels 50.5 miles per hour. How far will it travel in 3 hours?
        - Each candy bar costs $0.75. How much will 8 candy bars cost?
        - A recipe calls for 0.6 cups of sugar per batch. If you make 5 batches, how much sugar do you need?
- Difficulty: Application
- Common pitfalls:
    - Incorrect placement of the decimal point in the product.
    - Arithmetic errors.
    - Failing to identify the multiplication operation.
- Introduced definitions: None
- Concept / Introduction / Definition: 0
- Practice: 3
- Common Pitfall avoidance: 1
- Application/word problem: 5
- Challenging: 0
- Total number of questions for each goal: 9

5. Divide decimals by whole numbers: word problems

- Name: Divide decimals by whole numbers: word problems
- Description: This skill involves solving real-world problems where a decimal is divided by a whole number. Students must correctly set up the division and interpret the quotient in the context of the problem.
    - Core Example Questions:
        - A 10.5-meter rope is cut into 5 equal pieces. How long is each piece?
        - If 4 friends equally share a bill of $37.20, how much does each person pay?
        - A 6.8-pound bag of dog food is divided equally among 4 dogs. How much food does each dog get?
- Difficulty: Application
- Common pitfalls:
    - Incorrect placement of the decimal point in the quotient.
    - Arithmetic errors during division.
    - Confusing the dividend and divisor.
- Introduced definitions: None
- Concept / Introduction / Definition: 0
- Practice: 3
- Common Pitfall avoidance: 1
- Application/word problem: 5
- Challenging: 0
- Total number of questions for each goal: 9

6. Estimate sums, differences, and products of decimals

- Name: Estimate sums, differences, and products of decimals
- Description: Students will develop estimation strategies (e.g., rounding to the nearest whole number or tenth) to determine the reasonableness of answers when performing operations with decimals. This helps build number sense and allows students to check their exact calculations.
    - Core Example Questions:
        - Estimate the sum of $1.95+3.08$.
        - Is $15.7-8.2\approx 7$? Explain your reasoning.
        - Estimate the product of 4.8×6.1.
- Difficulty: Conceptual
- Common pitfalls:
    - Rounding incorrectly.
    - Not understanding when an overestimate or underestimate is appropriate.
    - Failing to apply estimation as a check.
- Introduced definitions: Estimate, reasonableness
- Concept / Introduction / Definition: 2
- Practice: 5
- Common Pitfall avoidance: 1
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 9

7. Add, subtract, multiply, and divide decimals: word problems

- Name: Add, subtract, multiply, and divide decimals: word problems
- Description: This skill integrates all four basic operations with decimals into multi-step real-world problems. Students will need to determine the correct sequence of operations and perform accurate calculations.
    - Core Example Questions:
        - Sarah buys 2 pounds of apples at $1.80 per pound and 3 pounds of bananas at $0.75 per pound. How much does she spend in total?
        - A taxi charges a flat fee of $3.50 plus $2.25 per mile. If a ride costs $12.50, how many miles was the ride?
        - A recipe requires 0.5 cups of sugar, but you only have 0.2 cups. How much more sugar do you need? If you want to make 3 batches, how much sugar total will you need?
- Difficulty: Application
- Common pitfalls:
    - Incorrectly identifying the operations required at each step.
    - Errors in multi-step calculations.
    - Not keeping track of units or context.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 2
- Common Pitfall avoidance: 1
- Application/word problem: 8
- Challenging: 2
- Total number of questions for each goal: 14

8. Maps with decimal distances

- Name: Maps with decimal distances

- Description: Students will solve problems involving distances on maps where measurements are given in decimals. This applies their understanding of decimal operations in a practical, visual context.
    - Core Example Questions:
        - On a map, the distance from town A to town B is 4.7 cm. The scale is 1 cm = 2.5 miles. What is the actual distance between the towns?
        - If two cities are 15.6 miles apart and the map scale is 1 inch = 3 miles, what is their distance on the map?
        - A cyclist travels 12.8 miles on day 1 and 15.3 miles on day 2. How much further did they travel on day 2?
- Difficulty: Application
- Common pitfalls:
    - Misinterpreting the map scale.
    - Errors in decimal multiplication or division.
    - Confusing actual distance with map distance.
- Introduced definitions: Scale (on a map)
- Concept / Introduction / Definition: 1
- Practice: 2
- Common Pitfall avoidance: 0
- Application/word problem: 5
- Challenging: 1
- Total number of questions for each goal: 9

9. Evaluate numerical expressions involving decimals

- Name: Evaluate numerical expressions involving decimals
- Description: This skill requires students to apply the order of operations (PEMDAS/GEMDAS) to evaluate numerical expressions that include decimals. This reinforces the importance of procedural accuracy and understanding of mathematical conventions.
    - Core Example Questions:
        - Evaluate: $3.2+0.5\times4$
        - Calculate: $(10-2.5)\div0.5$
        - Find the value of $22+1.5\times3-0.25$.
- Difficulty: Procedural
- Common pitfalls:
    - Incorrectly applying the order of operations.
    - Arithmetic errors with decimals.
    - Overlooking parentheses or exponents.
- Introduced definitions: None (reinforces order of operations)
- Concept / Introduction / Definition: 1
- Practice: 7
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 11

10. Add and subtract fractions: word problems

- Name: Add and subtract fractions: word problems
- Description: Students will solve real-world problems that involve adding and subtracting fractions, including those with unlike denominators. This emphasizes the need for common denominators and accurate fraction arithmetic in context.
    - Core Example Questions:

        - John ran 43 of a mile and then walked 21 of a mile. How far did he travel in total?
        - A recipe calls for 87 cup of sugar. If you only have 41 cup, how much more do you need?
        - Sarah ate 31 of a pizza, and Tom ate 61 of the same pizza. How much pizza did they eat altogether?
- Difficulty: Application
- Common pitfalls:
    - Failing to find a common denominator.
    - Incorrectly adding or subtracting numerators without adjusting the denominator.
    - Arithmetic errors with fractions.
- Introduced definitions: None
- Concept / Introduction / Definition: 0
- Practice: 3
- Common Pitfall avoidance: 1
- Application/word problem: 5
- Challenging: 0
- Total number of questions for each goal: 9

11. Add and subtract mixed numbers: word problems

- Name: Add and subtract mixed numbers: word problems
- Description: This skill extends fraction operations to mixed numbers in real-world contexts. Students will need to convert between mixed numbers and improper fractions as needed, and accurately perform addition and subtraction.
    - Core Example Questions:
        - A ribbon is 321 feet long. If 141 feet are cut off, how much ribbon is left?
        - A baker used 232 cups of flour for one cake and 121 cups for another. How much flour did they use in total?
        - A plant grew 581 inches in one month and 343 inches in the next. How much did it grow in total?
- Difficulty: Application
- Common pitfalls:
    - Errors when borrowing or regrouping with mixed numbers.
    - Not finding a common denominator for the fractional parts.
    - Incorrectly converting between mixed numbers and improper fractions.
- Introduced definitions: Mixed number
- Concept / Introduction / Definition: 0
- Practice: 3
- Common Pitfall avoidance: 1
- Application/word problem: 5
- Challenging: 0
- Total number of questions for each goal: 9

12. Estimate sums and differences of mixed numbers

- Name: Estimate sums and differences of mixed numbers
- Description: Students will use estimation strategies, typically rounding to the nearest whole number, to assess the reasonableness of answers when adding or subtracting mixed numbers. This reinforces number sense and provides a quick way to check calculations.
    - Core Example Questions:
        - Estimate the sum of 431+265.
        - Is $721-341 \approx 4$? Explain your reasoning.
        - Approximately how much flour is needed if a recipe calls for 387 cups and you want to make double the recipe?

- Difficulty: Conceptual
- Common pitfalls:
    - Rounding mixed numbers incorrectly.
    - Not understanding when an overestimate or underestimate is appropriate.
    - Failing to apply estimation as a check.
- Introduced definitions: None
- Concept / Introduction / Definition: 2
- Practice: 5
- Common Pitfall avoidance: 1
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 9

13. Multiply fractions and mixed numbers: word problems

- Name: Multiply fractions and mixed numbers: word problems
- Description: This skill involves solving real-world problems that require multiplying fractions and mixed numbers. Students must understand when to use multiplication in context and accurately calculate the product.
    - Core Example Questions:
        - A recipe calls for 32 cup of sugar. If you want to make 21 of the recipe, how much sugar do you need?
        - A garden is 1021 feet long and 632 feet wide. What is the area of the garden?
        - If a person walks 341 miles per hour, how far will they walk in 221 hours?
- Difficulty: Application
- Common pitfalls:
    - Not converting mixed numbers to improper fractions before multiplying.
    - Arithmetic errors during multiplication of fractions.
    - Failing to simplify the final product.
- Introduced definitions: None
- Concept / Introduction / Definition: 0
- Practice: 3
- Common Pitfall avoidance: 1
- Application/word problem: 5
- Challenging: 0
- Total number of questions for each goal: 9

14. Divide fractions and mixed numbers: word problems

- Name: Divide fractions and mixed numbers: word problems
- Description: Students will solve real-world problems that involve dividing fractions and mixed numbers. This requires understanding the concept of division by fractions (multiplying by the reciprocal) and applying it in context.
    - Core Example Questions:
        - How many 41-cup servings are in a 221-cup container of yogurt?
        - A rope is 541 meters long. If it's cut into pieces that are 43 meter long, how many pieces can be made?
        - You have 121 hours to complete 5 tasks. If each task takes the same amount of time, how long do you have for each task?
- Difficulty: Application
- Common pitfalls:
    - Forgetting to multiply by the reciprocal (dividing instead).
    - Not converting mixed numbers to improper fractions.
    - Arithmetic errors during division.

- Introduced definitions: Reciprocal
- Concept / Introduction / Definition: 0
- Practice: 3
- Common Pitfall avoidance: 1
- Application/word problem: 5
- Challenging: 0
- Total number of questions for each goal: 9

15. Estimate products and quotients of fractions and mixed numbers

- Name: Estimate products and quotients of fractions and mixed numbers
- Description: This skill involves using estimation strategies to determine the reasonableness of answers when multiplying or dividing fractions and mixed numbers. Students will typically round to the nearest whole number to simplify calculations for estimation.
    - Core Example Questions:
        - Estimate the product of 251×443.
        - Approximately how many 31-cup servings are in 687 cups of trail mix?
        - Is 10÷21 closer to 5 or 20? Explain.
- Difficulty: Conceptual
- Common pitfalls:
    - Rounding incorrectly for fractions or mixed numbers.
    - Not understanding how estimation works for multiplication/division.
    - Failing to apply estimation as a check.
- Introduced definitions: None
- Concept / Introduction / Definition: 2
- Practice: 5
- Common Pitfall avoidance: 1
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 9

16. Maps with fractional distances

- Name: Maps with fractional distances
- Description: Students will solve problems involving distances on maps where measurements are given in fractions. This applies their understanding of fractional operations in a practical, visual context, similar to decimal map problems.
    - Core Example Questions:
        - On a map, the distance between two parks is 221 inches. If the map scale is 1 inch = 43 mile, what is the actual distance between the parks?
        - A hiking trail is 841 miles long. On a map, the trail is represented by 321 inches. What is the scale of the map?
        - If the distance between two towns is 1521 miles and the map scale is 1 inch = 241 miles, how far apart are they on the map?
- Difficulty: Application
- Common pitfalls:
    - Misinterpreting the map scale.
    - Errors in fractional multiplication or division.
    - Confusing actual distance with map distance.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 2
- Common Pitfall avoidance: 0
- Application/word problem: 5
- Challenging: 1

- Total number of questions for each goal: 9

17. Evaluate numerical expressions involving fractions

- Name: Evaluate numerical expressions involving fractions
- Description: This skill requires students to apply the order of operations (PEMDAS/GEMDAS) to evaluate numerical expressions that include fractions. This reinforces the importance of procedural accuracy and understanding of mathematical conventions with fractional values.
    - Core Example Questions:
        - Evaluate: $21+43\times31$
        - Calculate: $(65-31)\div21$
        - Find the value of $(52)2+21\times54$.
- Difficulty: Procedural
- Common pitfalls:
    - Incorrectly applying the order of operations.
    - Arithmetic errors with fractions (common denominators, multiplying reciprocals).
    - Overlooking parentheses or exponents.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 7
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 11

18. Multi-step word problems with positive rational numbers

- Name: Multi-step word problems with positive rational numbers
- Description: This skill integrates all four operations with positive rational numbers (integers, fractions, and decimals) into complex, multi-step real-world problems. Students must be able to break down the problem into smaller steps, identify the correct operations, and perform accurate calculations.
    - Core Example Questions:
        - A store sells apples for $1.50 per pound and oranges for $0.75 per pound. If you buy 3 pounds of apples and 2.5 pounds of oranges, how much change do you get from a $10 bill?
        - A recipe calls for 241 cups of flour for one batch of cookies. If you want to make 121 times the recipe, and you already have 21 cup of flour, how much more flour do you need?
        - A car traveled 120 miles in 2.5 hours. If it maintains the same average speed, how far will it travel in 4 hours?
- Difficulty: Application
- Common pitfalls:
    - Difficulty in breaking down complex problems into manageable steps.
    - Choosing the wrong operation at various stages.
    - Arithmetic errors across multiple calculations.
    - Not answering all parts of the question.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 2
- Common Pitfall avoidance: 1
- Application/word problem: 8
- Challenging: 3
- Total number of questions for each goal: 15

19. Write an equation from words

- Name: Write an equation from words
- Description: This skill focuses on translating verbal descriptions of real-world situations into algebraic equations. Students must identify the unknown quantity, assign a variable, and use mathematical operations to represent the relationships described in the problem.
    - Core Example Questions:
        - Write an equation: "Seven less than twice a number is fifteen."
        - A rental car costs $25 per day plus $0.10 per mile. Write an equation to find the number of miles driven if the total cost was $45.
        - The sum of three consecutive integers is 36. Write an equation to represent this.
- Difficulty: Application
- Common pitfalls:
    - Incorrectly translating verbal phrases (e.g., confusing "less than" with subtraction order).
    - Not defining the variable clearly.
    - Omitting parts of the equation or including extraneous information.
- Introduced definitions: Equation, variable
- Concept / Introduction / Definition: 2
- Practice: 5
- Common Pitfall avoidance: 2
- Application/word problem: 5
- Challenging: 1
- Total number of questions for each goal: 15

20. Solve equations using properties

- Name: Solve equations using properties
- Description: This skill requires students to justify each step of solving an equation by explicitly stating the property of equality used (e.g., Addition Property of Equality, Subtraction Property of Equality, Multiplication Property of Equality, Division Property of 1 Equality). This builds a strong conceptual understanding of why algebraic manipulations are valid.
    - Core Example Questions:
        - Solve $x+7=12$. State the property used for each step.
        - Given the equation $4x=20$, what property should you use to solve for x?
        - Explain how the Addition Property of Equality helps solve $x-5=9$.
- Difficulty: Conceptual
- Common pitfalls:
    - Confusing the names of the properties.
    - Not understanding the purpose of each property in isolating the variable.
    - Incorrectly applying a property (e.g., adding a number to one side but not the other).
- Introduced definitions: Properties of equality (Addition, Subtraction, Multiplication, Division)
- Concept / Introduction / Definition: 4
- Practice: 6
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 13

21. Model and solve equations using algebra tiles

- Name: Model and solve equations using algebra tiles

- Description: This skill provides a concrete, visual method for understanding how to solve one- and two-step equations. Students will use algebra tiles to represent both sides of an equation and manipulate them (adding/removing zero pairs, distributing) to isolate the variable.
    - Core Example Questions:
        - Use algebra tiles to model and solve the equation $x+3=5$.
        - Draw algebra tiles to represent $2x-1=5$ and show the steps to solve it.
        - Which set of tile manipulations correctly solves the equation $3x=9$?
- Difficulty: Conceptual
- Common pitfalls:
    - Incorrectly representing negative terms or constants with tiles.
    - Difficulty with visualizing inverse operations (e.g., adding zero pairs to clear constants).
    - Not understanding how to "divide" tiles to solve for x.
- Introduced definitions: None (reinforces previous definitions)
- Concept / Introduction / Definition: 4
- Practice: 6
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 13

22. Write and solve equations that represent diagrams

- Name: Write and solve equations that represent diagrams
- Description: Students will translate visual representations (such as tape diagrams or hanger diagrams) into algebraic equations and then solve them. This bridges the gap between concrete models and abstract algebraic notation.
    - Core Example Questions:
        - Write an equation that represents the given tape diagram, then solve for x. (Diagram showing a total length, with parts labeled x, x, and a constant).
        - A hanger diagram shows a balance with two x blocks and one 5-unit block on one side, and 11-unit blocks on the other. Write and solve the equation.
        - Draw a tape diagram to represent the equation $3x-2=7$.
- Difficulty: Conceptual
- Common pitfalls:
    - Misinterpreting the relationships shown in the diagram.
    - Incorrectly translating the visual model into an equation.
    - Errors in solving the resulting equation.
- Introduced definitions: Tape diagram, hanger diagram
- Concept / Introduction / Definition: 3
- Practice: 5
- Common Pitfall avoidance: 1
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 10

23. Find the mistake: two-step equations

- Name: Find the mistake: two-step equations
- Description: This skill focuses on error analysis in solving two-step linear equations. Students are presented with a solved problem that contains an error and must identify, explain, and correct the mistake. This deepens their understanding of the solution process and common misconceptions.
    - Core Example Questions:

            - A student solved $2x+3=11$ as follows: $2x=8$, $x=4$. What mistake did they make, if any?
            - Identify the error in the following solution: $-3x-5=10 \Rightarrow -3x=5 \Rightarrow x=-35$.
            - Correct the mistake and solve the equation: $5-x=7 \Rightarrow x=7-5 \Rightarrow x=2$.
- Difficulty: Reinforcement
- Common pitfalls:
    - Failing to recognize subtle errors (e.g., sign errors).
    - Not fully understanding the rationale behind each step.
    - Focusing on arithmetic errors rather than conceptual errors.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 0
- Common Pitfall avoidance: 8
- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 11

24. Two-step equations

- Name: Two-step equations
- Description: Students will fluently solve two-step linear equations of the form $px+q=r$ and $p(x+q)=r$, where p, q, and r are rational numbers. This is a core procedural skill for equation solving.
    - Core Example Questions:
        - Solve for x: $3x+5=17$
        - Solve for y: $2y-4=1$
        - Solve for a: $2(a-3)=10$
- Difficulty: Procedural
- Common pitfalls:
    - Incorrectly applying inverse operations (e.g., adding instead of subtracting).
    - Errors in arithmetic (especially with negative numbers).
    - Applying the distributive property incorrectly.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 10
- Common Pitfall avoidance: 3
- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 16

25. Two-step equations with decimals and fractions

- Name: Two-step equations with decimals and fractions
- Description: This skill extends solving two-step equations to include rational coefficients and constants that are decimals or fractions. Students will need to apply their knowledge of rational number operations while solving equations.
    - Core Example Questions:
        - Solve for x: $0.5x+1.2=3.7$
        - Solve for y: $32y-61=65$
        - Solve for z: $2.5(z+1)=7.5$
- Difficulty: Procedural
- Common pitfalls:
    - Errors in arithmetic with decimals and fractions.

    - Not clearing fractions (multiplying by the common denominator) when it would simplify the problem.
    - Misplacing decimal points.
- Introduced definitions: None
- Concept / Introduction / Definition: 0
- Practice: 8
- Common Pitfall avoidance: 3
- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 13

26. Two-step equations word problems

- Name: Two-step equations word problems
- Description: Students will translate real-world word problems into two-step linear equations and then solve them. This combines the skill of writing equations from words with the procedural skill of solving two-step equations.
    - Core Example Questions:
        - A gym membership costs $30 to join and $25 per month. If Sarah paid a total of $130, how many months has she been a member?
        - The perimeter of a rectangle is 40 inches. If the length is 12 inches, what is the width?
        - You buy 3 shirts for $15 each and a pair of jeans. If your total bill was $60, how much did the jeans cost?
- Difficulty: Application
- Common pitfalls:
    - Incorrectly setting up the equation from the word problem.
    - Not defining the variable clearly.
    - Errors in solving the two-step equation.
    - Not answering the specific question asked in the word problem.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 2
- Common Pitfall avoidance: 1
- Application/word problem: 6
- Challenging: 2
- Total number of questions for each goal: 12

27. Solve one-step equations

- Name: Solve one-step equations
- Description: This skill focuses on solving one-step linear equations involving rational numbers. Students will apply inverse operations (addition/subtraction, multiplication/division) to isolate the variable.
    - Core Example Questions:
        - Solve for x: $x+8=15$
        - Solve for y: $y-3=-7$
        - Solve for a: $5a=30$
        - Solve for b: $4b=6$
- Difficulty: Introductory
- Common pitfalls:
    - Performing the incorrect inverse operation.
    - Arithmetic errors, especially with negative numbers.
    - Applying the operation to only one side of the equation.
- Introduced definitions: Inverse operations, isolating the variable

- Concept / Introduction / Definition: 2
- Practice: 10
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 15

28. Solve one-step equations with decimals and fractions

- Name: Solve one-step equations with decimals and fractions
- Description: This skill extends one-step equation solving to include coefficients and constants that are decimals or fractions. Students will apply their knowledge of rational number operations in the context of solving equations.
    - Core Example Questions:
        - Solve for x: $x+0.7=2.1$
        - Solve for y: $y-41=83$
        - Solve for a: $2.5a=10$
        - Solve for b: $3b=52$
- Difficulty: Procedural
- Common pitfalls:
    - Arithmetic errors with decimals and fractions.
    - Incorrectly applying inverse operations for fractional or decimal coefficients.
    - Not simplifying fractional answers.
- Introduced definitions: None
- Concept / Introduction / Definition: 0
- Practice: 8
- Common Pitfall avoidance: 3
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 12

29. Solve two-step equations without parentheses

- Name: Solve two-step equations without parentheses
- Description: This skill focuses specifically on solving two-step linear equations of the form $px+q=r$, where no distributive property is initially required. This builds fluency with the standard two-step process.
    - Core Example Questions:
        - Solve for x: $4x-7=13$
        - Solve for y: $-2y+9=1$
        - Solve for z: $3z+5=8$
- Difficulty: Procedural
- Common pitfalls:
    - Adding/subtracting before multiplying/dividing.
    - Sign errors when moving terms across the equality.
    - Arithmetic errors.
- Introduced definitions: None
- Concept / Introduction / Definition: 0
- Practice: 10
- Common Pitfall avoidance: 3
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 14

30. Solve two-step equations with parentheses

- Name: Solve two-step equations with parentheses
- Description: This skill focuses on solving two-step linear equations of the form $p(x+q)=r$, which requires applying the distributive property as the first step before solving the resulting two-step equation.
    - Core Example Questions:
        - Solve for x: $3(x+2)=15$
        - Solve for y: $-5(y-4)=20$
        - Solve for z: $21(z+6)=7$
- Difficulty: Procedural
- Common pitfalls:
    - Failing to distribute the number outside the parentheses to both terms.
    - Distributing incorrectly (especially with negative numbers or fractions).
    - Errors in the subsequent two-step equation solving.
- Introduced definitions: None
- Concept / Introduction / Definition: 0
- Practice: 10
- Common Pitfall avoidance: 3
- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 15

31. Solve two-step equations

- Name: Solve two-step equations
- Description: This skill aims for fluency in solving both forms of two-step linear equations: $px+q=r$ and $p(x+q)=r$. It serves as a comprehensive practice for the core two-step equation solving process.
    - Core Example Questions:
        - Solve: $4x+1=21$
        - Solve: $2(x-5)=14$
        - Solve: $3x-6=-2$
- Difficulty: Procedural
- Common pitfalls:
    - Choosing the wrong first step (e.g., dividing before adding/subtracting).
    - Arithmetic errors with positive and negative numbers.
    - Distributive property errors.
- Introduced definitions: None
- Concept / Introduction / Definition: 0
- Practice: 12
- Common Pitfall avoidance: 4
- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 18

32. Solve two-step equations with fractions

- Name: Solve two-step equations with fractions
- Description: This skill focuses specifically on solving two-step linear equations where coefficients and/or constants are fractions. Students can either work with fractions directly or clear them by multiplying by a common denominator, deepening their flexibility in equation solving.
    - Core Example Questions:
        - Solve for x: $21x+43=45$

        - Solve for y: $32(y-1)=94$
        - Solve for z: $-53z-101=107$
- Difficulty: Procedural
- Common pitfalls:
    - Errors in fraction arithmetic.
    - Incorrectly finding common denominators or reciprocals.
    - Not clearing fractions effectively.
- Introduced definitions: None
- Concept / Introduction / Definition: 0
- Practice: 8
- Common Pitfall avoidance: 3
- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 13

33. Choose two-step equations: word problems

- Name: Choose two-step equations: word problems
- Description: This skill focuses on the critical thinking aspect of translating word problems into equations. Students are given a word problem and a set of possible two-step equations, and they must select the correct one that accurately represents the situation. This emphasizes understanding the problem's structure rather than just calculation.
    - Core Example Questions:
        - A carpenter charges $50 for a service call plus $35 per hour. If the total bill was $155, which equation represents this situation? (A) $50h+35=155$ (B) $35h+50=155$ (C) $85h=155$
        - You have x video games. Your friend has 4 more than twice your number of games, totaling 20 games. Which equation represents this?
        - Which equation correctly models the following: "Three less than half a number is nine."
- Difficulty: Application
- Common pitfalls:
    - Misinterpreting keywords or phrases.
    - Confusing the coefficient with the constant.
    - Not correctly setting up the operations based on the problem description.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 3
- Common Pitfall avoidance: 2
- Application/word problem: 6
- Challenging: 1
- Total number of questions for each goal: 13

34. Solve two-step equations: word problems

- Name: Solve two-step equations: word problems
- Description: This skill combines the ability to write two-step equations from real-world contexts with the procedural skill of solving them. It's a comprehensive application of algebraic problem-solving.
    - Core Example Questions:
        - A taxi charges a $2.50 flat fee plus $1.75 per mile. If your total fare was $11.25, how many miles did you travel?
        - The length of a rectangle is 3 inches more than twice its width. If the perimeter is 42 inches, find the width of the rectangle.

        - Sarah bought 4 notebooks and a binder. The binder cost $7, and her total bill was $23. How much did each notebook cost?
- Difficulty: Application
- Common pitfalls:
    - Failing to set up the correct equation.
    - Arithmetic errors during the solving process.
    - Not understanding what the variable represents in the context of the problem.
    - Forgetting to answer the specific question asked.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 2
- Common Pitfall avoidance: 1
- Application/word problem: 7
- Challenging: 2
- Total number of questions for each goal: 13

35. Solve equations involving like terms

- Name: Solve equations involving like terms
- Description: This skill introduces equations that require an initial step of combining like terms on one or both sides of the equation before proceeding with inverse operations to solve. This builds on students' understanding of equivalent expressions.
    - Core Example Questions:
        - Solve for x: $5x-2x+8=20$
        - Solve for y: $7y+10-3y=26$
        - Solve for a: $6+2a-9=5a-15$
- Difficulty: Procedural
- Common pitfalls:
    - Incorrectly combining like terms (e.g., ignoring signs).
    - Attempting to apply inverse operations before combining like terms.
    - Arithmetic errors.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 8
- Common Pitfall avoidance: 3
- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 14

36. Solve equations: complete the solution

- Name: Solve equations: complete the solution
- Description: Students are presented with partially completed solutions to linear equations and must fill in the missing steps or justifications. This reinforces their understanding of the logical flow and properties involved in solving equations, focusing on process over just the final answer.
    - Core Example Questions:
        - Complete the solution: $3x+7=16 \Rightarrow 3x=\_\_\_ \Rightarrow x=\_\_\_$ (fill in the blanks).
        - Fill in the missing property: $5x-2=13 \Rightarrow 5x=15$ (Property: _____).
        - Identify the next logical step in solving $2(x-4)=10$.
- Difficulty: Reinforcement
- Common pitfalls:

    - Not understanding the correct sequence of steps.
    - Failing to identify the mathematical reasoning (properties) for each step.
    - Arithmetic errors in the missing steps.
- Introduced definitions: None
- Concept / Introduction / Definition: 0
- Practice: 0
- Common Pitfall avoidance: 7
- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 9

Solving Inequalities

1. Testing solutions to inequalities

- Name: Testing solutions to inequalities
- Description: This skill introduces the fundamental concept of inequalities by having students determine whether a given numerical value satisfies a linear inequality. This reinforces the idea that inequalities have a range of solutions, not just a single one.
    - Core Example Questions:
        - Is $x=5$ a solution to $x+3>7$?
        - Does $y=-2$ satisfy the inequality $2y-1\leq -5$?
        - Which of the following values are solutions to $3z<10$: $z=3$, $z=4$, $z=0$?
- Difficulty: Introductory
- Common pitfalls:
    - Incorrectly performing arithmetic operations when substituting values.
    - Misunderstanding the meaning of inequality symbols (e.g., confusing $>$ with $\geq$).
    - Forgetting to consider negative numbers and how they affect inequality comparisons.
- Introduced definitions: Inequality, solution to an inequality
- Concept / Introduction / Definition: 3
- Practice: 6
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 12

2. Relationships between Quantities

- Name: Relationships between Quantities
- Description: This skill focuses on analyzing given numerical relationships to determine if they are proportional or not. Students will use this understanding to begin building equations that represent these relationships. While not directly about inequalities, it builds the foundational understanding of how quantities relate algebraically.
    - Core Example Questions:
        - A recipe calls for 2 cups of flour for every 3 eggs. Is the relationship between flour and eggs proportional? Explain why or why not.
        - A taxi charges a $3 flat fee plus $2 per mile. Is the total cost proportional to the number of miles?
        - Write an equation to represent the relationship between the number of hours worked (h) and earnings (E) if you get paid $15 per hour.
- Difficulty: Conceptual
- Common pitfalls:
    - Confusing proportional relationships with linear relationships that have a non-zero y-intercept.
    - Not understanding that for proportionality, the ratio of quantities must be constant.

    - Errors in identifying the constant of proportionality or initial value.
- Introduced definitions: Proportional relationship, constant of proportionality
- Concept / Introduction / Definition: 3
- Practice: 4
- Common Pitfall avoidance: 1
- Application/word problem: 3
- Challenging: 1
- Total number of questions for each goal: 12

3. Reasoning about Contexts with Tape Diagrams (Part 1)

- Name: Reasoning about Contexts with Tape Diagrams (Part 1)
- Description: This skill involves interpreting tape diagrams that represent word problems and using them as a visual tool to find unknown values. This lays a crucial groundwork for understanding how to set up and solve equations from real-world contexts, especially helpful before formal algebraic equation solving.
    - Core Example Questions:
        - A tape diagram shows a total of 20, divided into two equal parts labeled 'x' and a part labeled '5'. What equation does this represent? What is the value of 'x'?
        - Draw a tape diagram to represent the problem: "Sarah has 3 times as many apples as John. Together they have 16 apples."
        - If a tape diagram shows 'y', 'y', 'y', and '8' making a total of '23', what does 'y' equal?
- Difficulty: Conceptual
- Common pitfalls:
    - Misinterpreting the parts and whole in the tape diagram.
    - Failing to correctly identify the unknown quantity.
    - Struggling to translate the visual model into an arithmetic step.
- Introduced definitions: Tape diagram (reinforces visual model)
- Concept / Introduction / Definition: 3
- Practice: 5
- Common Pitfall avoidance: 1
- Application/word problem: 2
- Challenging: 1
- Total number of questions for each goal: 12

4. Reasoning about Contexts with Tape Diagrams (Part 2)

- Name: Reasoning about Contexts with Tape Diagrams (Part 2)
- Description: Building on Part 1, this skill focuses on connecting tape diagrams to their corresponding algebraic equations. Students will write equations that match given tape diagrams and also match diagrams to pre-written equations, solidifying the link between visual models and abstract algebraic representations.
    - Core Example Questions:
        - Which equation matches the tape diagram that shows 3 equal parts of 'x' and a part of '7' totaling '25'?
        - Draw a tape diagram to represent the equation $2x+4=10$.
        - Write an equation for a tape diagram where a total of 18 is divided into four equal parts labeled 'x' and a remaining part of '2'.
- Difficulty: Conceptual
- Common pitfalls:
    - Incorrectly translating the visual components of the diagram into algebraic terms.
    - Confusing the meaning of terms in an equation with parts of the diagram.
    - Errors in identifying the correct operations.

- Introduced definitions: None
- Concept / Introduction / Definition: 3
- Practice: 5
- Common Pitfall avoidance: 1
- Application/word problem: 2
- Challenging: 1
- Total number of questions for each goal: 12

5. Reasoning about Equations and Tape Diagrams (Part 1)

- Name: Reasoning about Equations and Tape Diagrams (Part 1)
- Description: This skill coordinates tape diagrams, equations of the form x+p=q, and verbal descriptions to interpret solutions within a given context. It emphasizes understanding the meaning of the variable and the solution in a real-world scenario.
    - Core Example Questions:
        - A tape diagram shows a total of 15, with parts 'x' and '7'. Write the equation, solve it, and explain what 'x' means in a scenario where it represents the remaining length of a 15-foot rope after cutting off 7 feet.
        - Given the equation y+10=25, draw a tape diagram, write a word problem that it represents, and explain the solution.
        - If an equation is m−5=12, what would a corresponding tape diagram look like, and what could 'm' represent in a real-world context?
- Difficulty: Conceptual
- Common pitfalls:
    - Struggling to create a coherent word problem that fits the equation/diagram.
    - Misinterpreting the solution in context (e.g., stating x=5 without explaining what 5 represents).
    - Errors in solving the simple one-step equations.
- Introduced definitions: None
- Concept / Introduction / Definition: 2
- Practice: 3
- Common Pitfall avoidance: 1
- Application/word problem: 4
- Challenging: 1
- Total number of questions for each goal: 11

6. Reasoning about Equations and Tape Diagrams (Part 2)

- Name: Reasoning about Equations and Tape Diagrams (Part 2)
- Description: This skill extends the previous one by coordinating tape diagrams, equations of the form px=q, and verbal descriptions to interpret solutions in context. This reinforces understanding of proportional relationships and how they are represented algebraically and visually.
    - Core Example Questions:
        - A tape diagram shows a total of 24, divided into 3 equal parts, each labeled 'y'. Write the equation, solve it, and explain what 'y' means if the total is the cost of 3 identical items.
        - Given the equation 4z=32, draw a tape diagram, write a word problem that it represents, and explain the solution.
        - If a word problem states "A recipe requires 5 times as much flour as sugar, and it uses 15 cups of flour," what equation and tape diagram would represent this? What is the solution?
- Difficulty: Conceptual
- Common pitfalls:

    - Similar to Part 1, struggling to integrate all three representations (diagram, equation, context).
    - Errors in solving simple one-step equations involving multiplication/division.
    - Misinterpreting the meaning of the coefficient in proportional relationships.
- Introduced definitions: None
- Concept / Introduction / Definition: 2
- Practice: 3
- Common Pitfall avoidance: 1
- Application/word problem: 4
- Challenging: 1
- Total number of questions for each goal: 11

7. Distinguishing between Two Types of Situations

- Name: Distinguishing between Two Types of Situations
- Description: This skill focuses on helping students differentiate between situations that lead to equations of the form $x+p=q$ (additive relationships) and those that lead to $px=q$ (multiplicative/proportional relationships). Students will write and categorize equations from given scenarios and tape diagrams.
    - Core Example Questions:
        - Classify the situation as leading to $x+p=q$ or $px=q$: "You paid $5 for a notebook and some pens. The total was $12." Write the equation.
        - Draw a tape diagram for each situation and write the corresponding equation: "Sarah saved $15 more than John, who saved x. Together they saved $45." and "A recipe calls for twice as much sugar as flour. There are 6 cups of sugar."
        - Explain the difference in the structure of problems that lead to equations like $x+8=20$ versus $8x=20$.
- Difficulty: Conceptual
- Common pitfalls:
    - Confusing additive and multiplicative relationships.
    - Incorrectly identifying the operation required to represent a situation.
    - Errors in setting up the equation based on the type of relationship.
- Introduced definitions: Additive relationship, Multiplicative relationship
- Concept / Introduction / Definition: 3
- Practice: 4
- Common Pitfall avoidance: 2
- Application/word problem: 3
- Challenging: 1
- Total number of questions for each goal: 13

8. Reasoning about Solving Equations (Part 1)

- Name: Reasoning about Solving Equations (Part 1)
- Description: This skill uses balanced hanger diagrams to visually represent and solve equations of the form $x+p=q$. Students will understand how maintaining balance by performing the same operation on both sides of the hanger corresponds to applying properties of equality.
    - Core Example Questions:
        - A hanger diagram shows 'x' and 3 unit weights on one side, balancing 8 unit weights on the other. Write the equation and describe the steps to solve it using the diagram.
        - Draw a hanger diagram to represent the equation $y-2=5$. How would you manipulate the diagram to solve for 'y'?

        - Explain why removing 2 units from both sides of a balanced hanger diagram maintains its balance.
- Difficulty: Conceptual
- Common pitfalls:
    - Not understanding that operations must be applied to *both* sides to maintain balance.
    - Difficulty visualizing subtraction as removing items from a hanger.
    - Incorrectly translating hanger manipulations into algebraic steps.
- Introduced definitions: Balanced hanger diagram
- Concept / Introduction / Definition: 3
- Practice: 5
- Common Pitfall avoidance: 1
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 10

9. Reasoning about Solving Equations (Part 2)

- Name: Reasoning about Solving Equations (Part 2)
- Description: This skill extends the use of balanced hanger diagrams to visualize and solve equations of the form $px=q$. Students will learn how dividing items on a hanger corresponds to the Division Property of Equality.
    - Core Example Questions:
        - A hanger diagram shows 3 'x' blocks on one side, balancing 12 unit weights on the other. Write the equation and describe how to find the value of 'x' using the diagram.
        - Draw a hanger diagram to represent $2y=8$. Explain the steps to solve it.
        - If a hanger has 4 identical bags on one side balancing 20 unit weights, what does each bag weigh? How is this represented by an equation?
- Difficulty: Conceptual
- Common pitfalls:
    - Not understanding how to "divide" or group items on a hanger diagram.
    - Confusing the multiplication operation with addition (e.g., $3x$ as $x+x+x$ vs. $x+3$).
    - Incorrectly translating hanger manipulations into algebraic steps.
- Introduced definitions: None
- Concept / Introduction / Definition: 3
- Practice: 5
- Common Pitfall avoidance: 1
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 10

10. Dealing with Negative Numbers

- Name: Dealing with Negative Numbers
- Description: This skill specifically addresses solving equations of the forms $x+p=q$ and $px=q$ when negative numbers are involved. Students must apply their understanding of operations with integers in the context of solving equations.
    - Core Example Questions:
        - Solve for x: $x+(-5)=12$
        - Solve for y: $-3y=18$
        - Solve for z: $z-(-4)=9$
        - Solve for a: $-2a=7$
- Difficulty: Procedural
- Common pitfalls:

        - Sign errors when adding, subtracting, multiplying, or dividing negative numbers.
        - Forgetting to apply the inverse operation correctly with negative signs.
        - Confusing subtraction with negative values (e.g., $x-5$ vs. $x+(-5)$).
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 8
- Common Pitfall avoidance: 4
- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 15

11. Different Options for Solving One Equation

- Name: Different Options for Solving One Equation
- Description: This skill highlights that there can be multiple valid approaches to solving certain equations, particularly those involving parentheses. Students will compare and choose between expanding using the distributive property or dividing both sides by the coefficient outside the parentheses as initial steps.
    - Core Example Questions:
        - Solve $2(x+3)=10$ by first distributing. Then solve it by first dividing by 2. Which method do you prefer and why?
        - Explain when it might be more efficient to divide first, and when it's better to distribute first, for equations like $a(x+b)=c$.
        - Show two different ways to solve the equation $-4(y-1)=8$.
- Difficulty: Conceptual
- Common pitfalls:
    - Not recognizing that both methods yield the same correct answer.
    - Making arithmetic errors when applying the chosen method.
    - Struggling with fractional or decimal results when dividing first.
- Introduced definitions: None
- Concept / Introduction / Definition: 2
- Practice: 6
- Common Pitfall avoidance: 1
- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 11

12. Using Tape Diagrams and Equations to Solve Problems

- Name: Using Tape Diagrams and Equations to Solve Problems
- Description: This skill integrates the use of tape diagrams and algebraic equations to solve real-world word problems. Students will translate the problem into a diagram, then into an equation, and finally solve for the unknown value, reinforcing the connection between visual models and abstract algebra.
    - Core Example Questions:
        - Sarah has 3 times as many stickers as Tom. Together they have 24 stickers. Draw a tape diagram, write an equation, and solve to find out how many stickers Tom has.
        - A recipe uses 5 fewer cups of sugar than flour. If a total of 17 cups of sugar and flour are used, draw a tape diagram, write an equation, and solve to find the amount of flour.
        - Given a word problem, create a tape diagram and an equation, then solve: "A rectangle's length is twice its width. The perimeter is 36 cm."
- Difficulty: Application

- Common pitfalls:
    - Errors in translating the word problem into an accurate tape diagram.
    - Incorrectly writing the equation from the diagram or problem.
    - Arithmetic errors during the solving process.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 3
- Common Pitfall avoidance: 1
- Application/word problem: 6
- Challenging: 2
- Total number of questions for each goal: 13

13. Solving Problems about Percent Increase or Decrease

- Name: Solving Problems about Percent Increase or Decrease
- Description: Students will solve word problems involving percent increase or decrease by using tape diagrams and equations. This applies their understanding of percentages and rational number operations in a real-world financial or measurement context.
    - Core Example Questions:
        - A shirt costs $20. It's on sale for 25% off. Use a tape diagram and an equation to find the sale price.
        - The population of a town increased by 10%. If the new population is 5500, what was the original population? Use a tape diagram and an equation.
        - A price decreased from $80 to $60. What is the percent decrease? How can a tape diagram illustrate this?
- Difficulty: Application
- Common pitfalls:
    - Confusing the original amount with the new amount after increase/decrease.
    - Calculating the percentage of the wrong value (e.g., 25% off $20 is not 25% of the *final* price).
    - Errors in converting percentages to decimals or fractions.
    - Struggling to set up the correct equation for percent change.
- Introduced definitions: Percent increase, percent decrease
- Concept / Introduction / Definition: 2
- Practice: 2
- Common Pitfall avoidance: 1
- Application/word problem: 6
- Challenging: 2
- Total number of questions for each goal: 13

14. Reintroducing Inequalities

- Name: Reintroducing Inequalities
- Description: This skill reintroduces the concept of inequalities, focusing on writing inequality statements to represent given situations and using substitution to check if values are solutions. This serves as a foundation for solving inequalities systematically.
    - Core Example Questions:
        - Write an inequality for: "You must be at least 16 years old to drive."
        - The maximum capacity of an elevator is 1200 pounds. If w is the weight of people in the elevator, write an inequality.
        - Is $x=10$ a solution to $2x-5>12$? Show your work.
- Difficulty: Introductory
- Common pitfalls:
    - Using the wrong inequality symbol (e.g., using $<$ instead of $\leq$ for "at most").

        - Misinterpreting phrases like "at least," "no more than," "less than," etc.
        - Basic arithmetic errors during substitution.
- Introduced definitions: Inequality, at least, at most, no more than, no less than
- Concept / Introduction / Definition: 3
- Practice: 5
- Common Pitfall avoidance: 2
- Application/word problem: 2
- Challenging: 1
- Total number of questions for each goal: 13

15. Finding Solutions to Inequalities in Context

- Name: Finding Solutions to Inequalities in Context
- Description: This skill builds on "Reintroducing Inequalities" by having students write inequalities to represent real-world situations and then determine a range of valid solutions, emphasizing the meaning of the solution set within the given context.
    - Core Example Questions:
        - A movie ticket costs $8. You have $25 to spend. Write an inequality to represent the number of tickets you can buy and list possible numbers of tickets.
        - To pass a class, you need at least 70% average. You scored 65% and 72% on your first two tests. Write an inequality for your third test score (t) to pass the class, and suggest a possible score for t.
        - The speed limit on a highway is 65 mph. Write an inequality for speed s, and give two possible speeds that are legal and two that are not.
- Difficulty: Application
- Common pitfalls:
    - Incorrectly setting up the inequality.
    - Not understanding that there can be multiple solutions.
    - Providing solutions that are mathematically correct but don't make sense in the real-world context (e.g., negative number of tickets).
- Introduced definitions: Solution set
- Concept / Introduction / Definition: 1
- Practice: 2
- Common Pitfall avoidance: 1
- Application/word problem: 5
- Challenging: 2
- Total number of questions for each goal: 11

16. Efficiently Solving Inequalities

- Name: Efficiently Solving Inequalities
- Description: This skill introduces the method of solving inequalities by first solving the associated equation (replacing the inequality sign with an equals sign) and then testing values to determine the direction of the solution set. This provides a systematic approach to solving inequalities.
    - Core Example Questions:
        - Solve $2x+3>7$ by first solving $2x+3=7$. Then test a value to determine the solution set.
        - Explain why solving the associated equation helps determine the critical point for an inequality.
        - Solve $-3y+1\leq 10$. Show how you use the associated equation and test a point.
- Difficulty: Procedural

- Common pitfalls:
    - Forgetting to test a point or testing a point incorrectly.
    - Not understanding why the inequality sign flips when multiplying or dividing by a negative number.
    - Making arithmetic errors in solving the associated equation.
- Introduced definitions: Associated equation
- Concept / Introduction / Definition: 2
- Practice: 6
- Common Pitfall avoidance: 3
- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 13

17. Inequalities in Context

- Name: Inequalities in Context
- Description: This skill requires students to match an inequality to a real-world scenario, solve the inequality, and then interpret the meaning of the solution set within that context. This integrates problem-solving, algebraic manipulation, and contextual understanding.
    - Core Example Questions:
        - Which inequality represents: "You need to earn at least $100 this week. You already earned $60. How much more do you need to earn (x)? Solve and explain the solution."
        - A car rental costs $40 per day plus $0.15 per mile. If you want to spend no more than $70, write an inequality, solve it, and explain what the solution means.
        - A small business aims to keep its monthly expenses (E) under $1500. If fixed costs are $800 and variable costs are $10 per unit (u), write and solve an inequality, then interpret the solution.
- Difficulty: Application
- Common pitfalls:
    - Incorrectly setting up the inequality from the context.
    - Errors in solving the inequality, particularly with direction of the sign.
    - Failing to fully interpret the solution in the context of the problem.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 2
- Common Pitfall avoidance: 1
- Application/word problem: 6
- Challenging: 2
- Total number of questions for each goal: 12

18. Modeling with Inequalities

- Name: Modeling with Inequalities
- Description: This skill involves writing and solving inequalities for various real-world problems and analyzing their solutions and implications. It often requires multi-step thinking and a deeper understanding of how inequalities constrain possibilities.
    - Core Example Questions:
        - You have $50 to spend at an amusement park. Admission is $15, and each ride costs $3. Write an inequality to represent the number of rides (r) you can go on. Solve it and explain the maximum number of rides.
        - A plumber charges a flat fee of $60 plus $45 per hour. If a customer has a budget of $200, write and solve an inequality to determine the maximum number of hours the plumber can work.

        - A phone plan costs $40 per month plus $0.05 per text message. If you want your bill to be less than $60, write an inequality for the number of text messages (t), solve it, and explain the meaning of your solution.
- Difficulty: Application
- Common pitfalls:
    - Misidentifying the variable or constant terms in the problem.
    - Errors in setting up the inequality equation.
    - Not considering practical constraints on the solution (e.g., number of items must be a whole number).
    - Incorrectly interpreting the solution in the real-world context.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 2
- Common Pitfall avoidance: 1
- Application/word problem: 7
- Challenging: 3
- Total number of questions for each goal: 14

19. Solutions to inequalities

- Name: Solutions to inequalities
- Description: This skill is a fundamental check of understanding. Students will determine whether a given numerical value is a solution to an inequality by substituting the value and evaluating the truth of the resulting statement.
    - Core Example Questions:
        - Is $x=4$ a solution to $3x-2<10$?
        - Does $y=-5$ make the inequality $y+7\geq 2$ true?
        - Which of these numbers is a solution to $12-z>5$: $z=7, z=8, z=6$?
- Difficulty: Introductory
- Common pitfalls:
    - Arithmetic errors during substitution.
    - Misinterpreting the inequality symbol.
    - Failing to check all given values if multiple choices are provided.
- Introduced definitions: None (reinforces existing definitions)
- Concept / Introduction / Definition: 2
- Practice: 6
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 11

20. Graph inequalities on number lines

- Name: Graph inequalities on number lines
- Description: Students will graphically represent the solution set of a linear inequality on a number line. This involves understanding open vs. closed circles and the direction of shading.
    - Core Example Questions:
        - Graph $x>3$ on a number line.
        - Graph $y\leq -1$ on a number line.
        - Show the solution set for $z\geq 0$ on a number line.
- Difficulty: Procedural
- Common pitfalls:

    - Confusing open and closed circles (for < and > vs. ≤ and ≥).
    - Shading in the wrong direction.
    - Incorrectly placing the critical point on the number line.
- Introduced definitions: Number line, open circle, closed circle, shading (for solution set)
- Concept / Introduction / Definition: 3
- Practice: 7
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 13

21. Write inequalities from number lines

- Name: Write inequalities from number lines
- Description: This skill is the inverse of graphing. Students will interpret a given graph on a number line and write the corresponding algebraic inequality. This reinforces their understanding of the relationship between graphical and algebraic representations of inequalities.
    - Core Example Questions:
        - Write an inequality that represents a number line with an open circle at 5 and shaded to the right.
        - Write an inequality for a graph with a closed circle at -2 and shaded to the left.
        - A number line shows all numbers greater than or equal to 0. Write the inequality.
- Difficulty: Conceptual
- Common pitfalls:
    - Incorrectly choosing the inequality symbol based on the circle (open vs. closed).
    - Incorrectly determining the direction of the inequality from the shading.
    - Mixing up the variable with the number.
- Introduced definitions: None
- Concept / Introduction / Definition: 3
- Practice: 6
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 12

22. One-step inequalities

- Name: One-step inequalities
- Description: Students will solve one-step linear inequalities of the form $x+p \leq q$ or $px>q$ (and variations). They will also graph their solution sets on a number line. This combines the procedural solving with graphical representation.
    - Core Example Questions:
        - Solve and graph: $x+5 \geq 12$
        - Solve and graph: $3y<15$
        - Solve and graph: $-2z>4$
- Difficulty: Procedural
- Common pitfalls:
    - Forgetting to flip the inequality sign when multiplying or dividing by a negative number.
    - Arithmetic errors.

    - Errors in graphing (open/closed circle, shading direction).
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 8
- Common Pitfall avoidance: 3
- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 14

23. Two-step inequalities

- Name: Two-step inequalities
- Description: Students will solve two-step linear inequalities of the form $px+q \leq r$ or $p(x+q)>r$ (and variations). They will also graph their solution sets on a number line, requiring them to apply the distributive property if necessary and remember the rule for flipping the inequality sign.
    - Core Example Questions:
        - Solve and graph: $2x-3<7$
        - Solve and graph: $-4y+1 \geq 9$
        - Solve and graph: $3x+2 \leq 5$
- Difficulty: Procedural
- Common pitfalls:
    - Failing to distribute correctly.
    - Forgetting to flip the inequality sign when multiplying or dividing by a negative number at any step.
    - Arithmetic errors with multiple steps.
    - Errors in graphing.
- Introduced definitions: None
- Concept / Introduction / Definition: 0
- Practice: 10
- Common Pitfall avoidance: 4
- Application/word problem: 0
- Challenging: 3
- Total number of questions for each goal: 17

24. Two-step inequality word problems

- Name: Two-step inequality word problems
- Description: Students will translate real-world word problems into two-step linear inequalities, solve them, and interpret the solution in context. This is a comprehensive application of inequality understanding.
    - Core Example Questions:
        - You have $40 to spend at a carnival. Admission is $10, and each game costs $2. Write and solve an inequality to find the maximum number of games you can play.
        - A text messaging plan costs $15 per month plus $0.08 per text. If you want your bill to be no more than $25, write and solve an inequality for the number of texts you can send.
        - The perimeter of a rectangular garden is at most 100 feet. If the length is 30 feet, write and solve an inequality for the width.
- Difficulty: Application
- Common pitfalls:
    - Incorrectly setting up the two-step inequality from the word problem.

    - Errors in solving the inequality (especially with sign flipping).
    - Not interpreting the solution correctly in the real-world context (e.g., fractional games played, negative people).
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 2
- Common Pitfall avoidance: 1
- Application/word problem: 6
- Challenging: 2
- Total number of questions for each goal: 12

25. Solve one-step inequalities

- Name: Solve one-step inequalities
- Description: This skill focuses purely on the algebraic procedure of solving one-step linear inequalities, reinforcing the concept of inverse operations and the special rule for multiplying/ dividing by a negative number.
    - Core Example Questions:
        - Solve for x: $x-7<1$
        - Solve for y: $-5y\geq 20$
        - Solve for z: $3z\leq -4$
- Difficulty: Procedural
- Common pitfalls:
    - Forgetting to flip the inequality sign when multiplying or dividing by a negative.
    - Arithmetic errors.
    - Applying the wrong inverse operation.
- Introduced definitions: None
- Concept / Introduction / Definition: 0
- Practice: 10
- Common Pitfall avoidance: 3
- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 15

26. Graph solutions to one-step inequalities

- Name: Graph solutions to one-step inequalities
- Description: Students will graphically represent the solution sets of one-step inequalities on a number line, demonstrating their understanding of the boundaries and direction of solutions. This skill is a direct application of the solution from the previous step.
    - Core Example Questions:
        - Graph the solution to $x+2>5$.
        - Graph the solution to $4y\leq -8$.
        - Graph the solution to $z-3\geq -1$.
- Difficulty: Procedural
- Common pitfalls:
    - Incorrectly applying open/closed circles.
    - Shading in the wrong direction based on the inequality sign.
    - Errors in determining the critical point if they haven't solved the inequality first.
- Introduced definitions: None
- Concept / Introduction / Definition: 0
- Practice: 8
- Common Pitfall avoidance: 2
- Application/word problem: 0

- Challenging: 1
- Total number of questions for each goal: 11

27. One-step inequalities: word problems

- Name: One-step inequalities: word problems
- Description: Students will translate real-world scenarios into one-step linear inequalities and solve them. This focuses on simpler contextual problems that can be represented with a single operation.
    - Core Example Questions:
        - You need to read at least 50 pages for homework. You've already read 20 pages. Write and solve an inequality to find how many more pages you need to read.
        - A maximum of 8 people can fit in a car. If x represents the number of people, write an inequality.
        - Each box weighs 15 pounds. If the total weight must be less than 100 pounds, write and solve an inequality for the number of boxes (b).
- Difficulty: Application
- Common pitfalls:
    - Choosing the incorrect inequality symbol.
    - Failing to set up the correct one-step inequality.
    - Arithmetic errors.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 2
- Common Pitfall avoidance: 1
- Application/word problem: 5
- Challenging: 1
- Total number of questions for each goal: 10

28. Solve two-step inequalities

- Name: Solve two-step inequalities
- Description: This skill focuses exclusively on the algebraic procedure for solving two-step linear inequalities. It serves as a drill to build fluency and accuracy with the multi-step solving process, including the application of the distributive property and the sign-flipping rule.
    - Core Example Questions:
        - Solve for x: $3x+4 \geq 16$
        - Solve for y: $-2(y-5)<8$
        - Solve for z: $4z-1 \leq 3$
- Difficulty: Procedural
- Common pitfalls:
    - Incorrectly applying the order of inverse operations.
    - Forgetting to flip the inequality sign when multiplying or dividing by a negative number.
    - Arithmetic errors.
    - Distributive property errors.
- Introduced definitions: None
- Concept / Introduction / Definition: 0
- Practice: 12
- Common Pitfall avoidance: 4
- Application/word problem: 0
- Challenging: 3
- Total number of questions for each goal: 19

29. Graph solutions to two-step inequalities

- Name: Graph solutions to two-step inequalities
- Description: Students will graphically represent the solution sets of two-step inequalities on a number line. This requires them to accurately solve the inequality first and then correctly apply the graphing conventions (open/closed circles, shading direction).
    - Core Example Questions:
        - Solve $5x-1<9$ and graph the solution.
        - Solve $-3y+6\geq 12$ and graph the solution.
        - Solve $2x+4\leq 3$ and graph the solution.
- Difficulty: Procedural
- Common pitfalls:
    - Errors in solving the inequality, leading to an incorrect critical point or direction.
    - Confusing open/closed circles.
    - Shading in the wrong direction.
    - Not flipping the inequality sign when necessary, leading to an incorrect graph.
- Introduced definitions: None
- Concept / Introduction / Definition: 0
- Practice: 8
- Common Pitfall avoidance: 3
- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 13

Geometry

Geometric Figures and Relationships

Angles

1. Identify angle relationships (complementary, supplementary, vertical, adjacent)

- Name: Identify angle relationships (complementary, supplementary, vertical, adjacent)
- Description: This skill focuses on the foundational definitions of key angle relationships. Students will learn to recognize and name complementary angles (two angles that add up to 90°), supplementary angles (two angles that add up to 180°), vertical angles (opposite angles formed by the intersection of two lines, which are always congruent), and adjacent angles (angles that share a common vertex and a common side but no common interior points). Mastery of these definitions is crucial for solving problems involving unknown angles.
    - Core Example Questions:
        - Look at the diagram. Are $\angle A$ and $\angle B$ adjacent, complementary, supplementary, or vertical angles?
        - Draw two lines intersecting and label a pair of vertical angles.
        - If $\angle C$ and $\angle D$ are complementary, what does that mean about their measures?
- Difficulty: Introductory
- Common pitfalls:
    - Confusing complementary (90°) and supplementary (180°) angles.
    - Misidentifying vertical angles; sometimes confusing them with adjacent angles.
    - Not understanding that adjacent angles don't necessarily have a specific sum unless they form a straight line or a right angle.

- Introduced definitions: Complementary angles, Supplementary angles, Vertical angles, Adjacent angles, Congruent
- Concept / Introduction / Definition: 5
- Practice: 5
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 13

2. Find measures of complementary and supplementary angles

- Name: Find measures of complementary and supplementary angles
- Description: This skill applies the definitions of complementary and supplementary angles to calculate unknown angle measures. Students will be given one angle measure and need to determine the measure of its complement or supplement. This is a direct application of subtraction from 90◦ or 180◦.
    - Core Example Questions:
        - If ∠A measures 35◦ and ∠A and ∠B are complementary, what is the measure of ∠B?
        - Two angles are supplementary. If one angle measures 110◦, what is the measure of the other angle?
        - What is the complement of a 60◦ angle?
- Difficulty: Core
- Common pitfalls:
    - Subtracting from the wrong total (e.g., subtracting from 90◦ for supplementary angles).
    - Basic arithmetic errors.
    - Not distinguishing between complementary and supplementary in the problem statement.
- Introduced definitions: None (reinforces previous definitions)
- Concept / Introduction / Definition: 1
- Practice: 8
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 12

3. Determine unknown angles using complementary and supplementary relationships (no visual)

- Name: Determine unknown angles using complementary and supplementary relationships (no visual)
- Description: This skill removes the visual aid to ensure students rely solely on the definitions of complementary and supplementary angles. They will be given numerical information and need to apply the correct 90◦ or 180◦ rule to find the unknown angle measure. This helps abstract the concept from a diagram.
    - Core Example Questions:
        - An angle measures 72◦. What is the measure of its supplement?
        - Two angles add up to 90◦. If one angle is 48◦, what is the other angle's measure?

        - Is an angle of 130◦ complementary to an angle of 50◦? Why or why not?
- Difficulty: Procedural
- Common pitfalls:
    - Misremembering which sum (90◦ or 180◦) corresponds to which term.
    - Simple calculation errors.
    - Not being able to visualize the relationship without a diagram.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 7
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 11

4. Determine unknown angles using vertical angle relationships

- Name: Determine unknown angles using vertical angle relationships
- Description: This skill focuses on identifying and using the property of vertical angles (that they are congruent or equal in measure) to find unknown angle measures in diagrams involving intersecting lines.
    - Core Example Questions:
        - In the diagram, if $\angle 1$ measures 70◦, what is the measure of $\angle 3$ (which is vertical to $\angle 1$)?
        - Two lines intersect. If one angle formed is 125◦, what is the measure of the angle directly opposite it?
        - Explain why vertical angles are always congruent.
- Difficulty: Core
- Common pitfalls:
    - Confusing vertical angles with adjacent angles.
    - Assuming vertical angles add up to 90◦ or 180◦ instead of being equal.
    - Not recognizing vertical angles in more complex diagrams.
- Introduced definitions: None (reinforces previous definitions)
- Concept / Introduction / Definition: 1
- Practice: 7
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 11

5. Write and solve simple equations for unknown angles

- Name: Write and solve simple equations for unknown angles
- Description: This skill integrates algebra with angle relationships. Students will write and solve one-step and simple multi-step linear equations to find unknown angle measures based on complementary, supplementary, vertical, and adjacent angle relationships, often presented in diagrams with expressions involving variables.
    - Core Example Questions:
        - Two angles are complementary. One angle is x◦ and the other is $2x$◦. Write an equation and solve for x.
        - In a diagram, two angles form a straight line. One is $(y+20)$◦ and the other is 70◦. Write an equation and find y.
        - Vertical angles measure $(3z-5)$◦ and 85◦. Set up an equation and solve for z.

- Difficulty: Procedural
- Common pitfalls:
    - Incorrectly setting up the equation (e.g., adding to 90◦ instead of 180◦).
    - Algebraic errors in solving the one- or two-step equation.
    - Sign errors, especially with negative numbers.
- Introduced definitions: None (applies equation-solving skills to geometry)
- Concept / Introduction / Definition: 1
- Practice: 8
- Common Pitfall avoidance: 3
- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 14

6. Solve multi-step problems with angle relationships

- Name: Solve multi-step problems with angle relationships
- Description: This skill challenges students to solve more complex problems involving multiple angle relationships within a single figure. Students must identify the relationships, set up appropriate equations (sometimes requiring more than one step to find intermediate angles), and solve for unknown angles. This requires synthesizing several concepts.
    - Core Example Questions:
        - In a diagram, $\angle 1$ and $\angle 2$ are supplementary. $\angle 2$ and $\angle 3$ are vertical angles. If $\angle 1 = 60^\circ$, find the measure of $\angle 3$.
        - Two intersecting lines create four angles. If one angle is x°, the adjacent angle is $(x+20)^\circ$. Find the measures of all four angles.
        - An angle and its complement are in a ratio of 2:3. Find the measure of each angle.
- Difficulty: Synthesis
- Common pitfalls:
    - Not breaking the problem down into smaller, solvable steps.
    - Incorrectly identifying the sequence of angle relationships to use.
    - Algebraic errors in multi-step equations.
    - Overlooking specific definitions or properties in a complex diagram.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 3
- Common Pitfall avoidance: 2
- Application/word problem: 1
- Challenging: 4
- Total number of questions for each goal: 11

7. Write and solve simple equations for unknown angles in figures

- Name: Write and solve simple equations for unknown angles in figures
- Description: This skill specifically focuses on setting up and solving one-step or two-step algebraic equations to find unknown angle measures directly from provided figures. These figures will visually depict supplementary, complementary, vertical, and adjacent angles, allowing students to practice translating visual information into algebraic statements. This is similar to Skill 5 but emphasizes working directly from visual figures.
    - Core Example Questions:
        - (Diagram showing a right angle divided into x° and 30°) Write an equation and solve for x.

        - (Diagram showing two intersecting lines, with one angle labeled $4x^\circ$ and its vertical angle labeled 100°) Write an equation and find x.
        - (Diagram showing a straight line with two adjacent angles: y° and $(2y-15)^\circ$) Set up an equation and solve for y.
- Difficulty: Procedural
- Common pitfalls:
    - Incorrectly identifying the angle relationship from the figure.
    - Mistakes in writing the equation (e.g., adding angles that aren't related in that way).
    - Algebraic errors in solving the equation.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 8
- Common Pitfall avoidance: 3
- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 14

8. Solve multi-step problems involving angle relationships

- Name: Solve multi-step problems involving angle relationships
- Description: This skill provides extensive practice in applying the full range of angle relationships (complementary, supplementary, vertical, and adjacent) within multi-step problems to find unknown angle measures. These problems often involve a sequence of deductions, requiring students to identify one relationship, solve for an angle, and then use that angle to find another. It reinforces the procedural application of multiple concepts in combination. This is a drill-focused version of Skill 6, emphasizing quantity and variety of problems.
    - Core Example Questions:
        - Given a diagram with three intersecting lines, find the measures of all labeled unknown angles if one angle is 50°.
        - Angles A and B are supplementary. Angle B and Angle C are complementary. If Angle A=140°, find Angle C.
        - Two angles form a linear pair. One angle is $4x^\circ$ and the other is $(x+50)^\circ$. Find the measure of each angle.
- Difficulty: Procedural
- Common pitfalls:
    - Not systematically working through the problem.
    - Making calculation errors at any step that then propagate through subsequent steps.
    - Misidentifying angle pairs or their properties.
- Introduced definitions: None
- Concept / Introduction / Definition: 0
- Practice: 10
- Common Pitfall avoidance: 4
- Application/word problem: 0
- Challenging: 3
- Total number of questions for each goal: 17

Triangles

1. Classify triangles by sides and angles

- Name: Classify triangles by sides and angles
- Description: This foundational skill requires students to recall and apply the classifications of triangles based on their side lengths and angle measures. For side lengths, categories include equilateral (all sides equal), isosceles (at least two sides equal), and scalene (no sides equal). For angle measures, categories include acute (all angles less than 90◦), right (one angle exactly 90◦), and obtuse (one angle greater than 90◦). Students should be able to classify a given triangle by both criteria.
    - Core Example Questions:
        - A triangle has side lengths of 5 cm, 5 cm, and 7 cm. Classify it by its sides.
        - A triangle has angles measuring 30◦, 60◦, and 90◦. Classify it by its angles.
        - Can a triangle be both obtuse and equilateral? Explain why or why not.
- Difficulty: Introductory
- Common pitfalls:
    - Confusing isosceles and equilateral definitions.
    - Misidentifying angle types (acute, obtuse, right).
    - Forgetting that the sum of angles in a triangle is 180◦.
- Introduced definitions: Equilateral triangle, Isosceles triangle, Scalene triangle, Acute triangle, Right triangle, Obtuse triangle
- Concept / Introduction / Definition: 4
- Practice: 6
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 13

2. Triangle side length rules (Triangle Inequality Theorem)

- Name: Triangle side length rules (Triangle Inequality Theorem)
- Description: This skill introduces the Triangle Inequality Theorem, which states that the sum of the lengths of any two sides of a triangle must be greater than the length of the third side. Students will apply this theorem to determine if three given side lengths can actually form a triangle.
    - Core Example Questions:
        - Can a triangle have side lengths of 3 cm, 4 cm, and 8 cm? Explain your reasoning.
        - Which of the following sets of side lengths can form a triangle: (A) 2, 5, 7 (B) 6, 8, 10 (C) 1, 2, 4?
        - If two sides of a triangle are 5 inches and 10 inches long, what are the possible whole number lengths for the third side?
- Difficulty: Conceptual
- Common pitfalls:
    - Only checking one or two pairs of sides, not all three.
    - Confusing "greater than" with "greater than or equal to."
    - Arithmetic errors when adding side lengths.
- Introduced definitions: Triangle Inequality Theorem
- Concept / Introduction / Definition: 3
- Practice: 7
- Common Pitfall avoidance: 2
- Application/word problem: 1
- Challenging: 2
- Total number of questions for each goal: 15

3. Ordering triangle sides and angles

- Name: Ordering triangle sides and angles
- Description: This skill focuses on understanding and applying the direct relationship between the lengths of sides and the measures of angles in a triangle: the longest side is always opposite the largest angle, and the shortest side is always opposite the smallest angle.[1] Students will use this relationship to order sides given angle measures or vice versa.
    - Core Example Questions:
        - A triangle has angles measuring 40◦, 60◦, and 80◦. List the sides in order from shortest to longest.
        - A triangle has sides measuring 10 cm, 7 cm, and 12 cm. List the angles in order from smallest to largest.
        - If angle A is the largest angle in a triangle, what can you say about the side opposite angle A?
- Difficulty: Conceptual
- Common pitfalls:
    - Incorrectly identifying opposite sides and angles.
    - Confusing the order (e.g., largest angle opposite shortest side).
    - Arithmetic errors if required to find a missing angle before ordering.
- Introduced definitions: Opposite side/angle
- Concept / Introduction / Definition: 2
- Practice: 6
- Common Pitfall avoidance: 1
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 10

4. Constructing triangles with given conditions (sides)

- Name: Constructing triangles with given conditions (sides)
- Description: This hands-on skill involves using a ruler and compass (or protractor) to construct triangles when given specific side lengths. Students will physically attempt the construction and then determine if a unique triangle, multiple triangles, or no triangle can be formed, connecting back to the Triangle Inequality Theorem.
    - Core Example Questions:
        - Use a ruler and compass to construct a triangle with side lengths 4 cm, 5 cm, and 6 cm. Is this a unique triangle?
        - Attempt to construct a triangle with side lengths 2 inches, 3 inches, and 6 inches. What happens, and why?
        - Given side lengths 3 cm, 4 cm, and 5 cm, draw the triangle. What type of triangle is it (by angles)?
- Difficulty: Procedural
- Common pitfalls:
    - Inaccurate measurement or drawing.
    - Not understanding how to use a compass to mark arc lengths.
    - Failing to connect the construction outcome to the Triangle Inequality Theorem.
- Introduced definitions: Compass, Unique triangle
- Concept / Introduction / Definition: 2
- Practice: 5
- Common Pitfall avoidance: 1
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 9

5. Constructing triangles with given conditions (angles)

- Name: Constructing triangles with given conditions (angles)
- Description: This skill involves constructing triangles using a ruler and protractor when given specific angle measures. Students will discover that when only angles are given (e.g., AAA condition), it's possible to form multiple similar triangles of different sizes, or no triangle at all if the angles don't sum to 180◦.
    - Core Example Questions:
        - Use a protractor and ruler to draw a triangle with angles 45◦, 45◦, and 90◦. Can you draw another triangle with these same angles but a different size?
        - Attempt to construct a triangle with angles 100◦, 50◦, and 40◦. What conclusion can you draw?
        - Draw a triangle with angles 60◦,60◦,60◦. What type of triangle is it?
- Difficulty: Procedural
- Common pitfalls:
    - Inaccurate measurement with the protractor.
    - Drawing sides that don't connect to form a closed figure.
    - Not recognizing that angles must sum to 180◦.
    - Failing to understand the concept of similar triangles when multiple triangles can be formed.
- Introduced definitions: Similar triangles
- Concept / Introduction / Definition: 2
- Practice: 5
- Common Pitfall avoidance: 1
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 9

6. Constructing triangles with given conditions (sides and angles)

- Name: Constructing triangles with given conditions (sides and angles)
- Description: This advanced construction skill combines side lengths and angle measures (e.g., ASA, SAS, SSS, AAS conditions). Students will use a ruler and protractor to draw triangles based on these combinations and identify if a unique triangle, multiple triangles, or no triangle can be formed. This helps reinforce the congruence postulates indirectly.
    - Core Example Questions:
        - Construct a triangle with sides 5 cm and 7 cm, and the included angle between them measuring 60◦. Is this a unique triangle?
        - Draw a triangle with angles 30◦ and 70◦, and the side between them measuring 8 cm.
        - Can you construct a unique triangle if you are given three angles? Explain why or why not using an example.
- Difficulty: Procedural
- Common pitfalls:
    - Inaccurate measurements.
    - Misinterpreting "included" angle or side.
    - Struggling to follow the construction steps precisely.
    - Failing to correctly determine if a unique, multiple, or no triangle can be formed.
- Introduced definitions: Included angle, Included side
- Concept / Introduction / Definition: 2
- Practice: 6
- Common Pitfall avoidance: 1

- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 11

Three-Dimensional Figures

1. Identify bases of three-dimensional figures

- Name: Identify bases of three-dimensional figures
- Description: This skill focuses on recognizing and correctly identifying the base(s) of various three-dimensional figures, particularly prisms and pyramids. Students will learn that the base of a prism is one of two identical and parallel faces, while the base of a pyramid is the single face that the lateral faces meet at. Understanding the base is crucial for calculating volume and surface area.
    - Core Example Questions:
        - For a rectangular prism, how many bases does it have, and what shape are they?
        - What is the shape of the base of a triangular pyramid?
        - If a prism has a hexagonal base, how many lateral faces does it have?
- Difficulty: Introductory
- Common pitfalls:
    - Confusing a lateral face with a base in prisms.
    - Incorrectly assuming a pyramid has two bases.
    - Not understanding that the "base" of a prism doesn't always have to be the bottom face.
- Introduced definitions: Base (of 3D figure), Prism, Pyramid, Lateral face
- Concept / Introduction / Definition: 4
- Practice: 6
- Common Pitfall avoidance: 1
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 12

2. Cross sections of 3D objects (basic)

- Name: Cross sections of 3D objects (basic)
- Description: This skill involves visualizing and describing the two-dimensional shapes that result from slicing right rectangular prisms and right rectangular pyramids with a plane. Students will explore horizontal, vertical, and diagonal slices and learn how the orientation of the slice affects the shape of the cross-section.
    - Core Example Questions:
        - If you slice a right rectangular prism horizontally, what shape is the cross-section?
        - Describe the shape of the cross-section when you slice a right rectangular pyramid vertically through its apex.
        - What shape is formed when you slice a rectangular prism diagonally?
- Difficulty: Conceptual
- Common pitfalls:
    - Difficulty visualizing the 2D shape within a 3D object.
    - Confusing the effects of different slice orientations.
    - Incorrectly identifying polygons (e.g., calling a rectangle a square when it's not guaranteed).

- Introduced definitions: Cross-section, Horizontal slice, Vertical slice, Diagonal slice, Plane
- Concept / Introduction / Definition: 3
- Practice: 6
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 13

Area, Volume, and Surface Area

Circles

1. Identify parts of a circle (radius and diameter)

- Name: Identify parts of a circle (radius and diameter)
- Description: This foundational skill introduces students to the basic components of a circle. They will identify and define the radius, diameter, and center of a circle, and understand their crucial relationship: the diameter is twice the length of the radius (d=2r).
    - Core Example Questions:
        - Draw a circle and label its center, a radius, and a diameter.
        - If the radius of a circle is 7 cm, what is its diameter?
        - Explain the difference between a radius and a diameter.
- Difficulty: Introductory
- Common pitfalls:
    - Confusing radius and diameter.
    - Misidentifying the center.
    - Not understanding that all radii (and diameters) of a given circle are equal in length.
- Introduced definitions: Circle, Center, Radius, Diameter
- Concept / Introduction / Definition: 4
- Practice: 6
- Common Pitfall avoidance: 1
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 12

2. Explore the relationship between circumference and diameter (informal derivation of pi)

- Name: Explore the relationship between circumference and diameter (informal derivation of pi)
- Description: This skill guides students to investigate and understand the proportional relationship between a circle's circumference (the distance around the circle) and its diameter. Through hands-on activities or visual examples, they will discover that the ratio C/d is a constant, leading to an informal derivation of pi (π) as approximately 3.14 or 22/7.
    - Core Example Questions:
        - Measure the circumference and diameter of three different circular objects. Calculate the ratio of circumference to diameter for each. What do you notice?
        - If the circumference of a circle is about 3 times its diameter, what does that number represent?
        - Explain why π is an important constant for circles.
- Difficulty: Conceptual
- Common pitfalls:
    - Inaccurate measurements if hands-on.
    - Not understanding that the relationship is a ratio.

    - Struggling with the concept of an irrational number like π.
- Introduced definitions: Circumference, Pi (π)
- Concept / Introduction / Definition: 3
- Practice: 4
- Common Pitfall avoidance: 1
- Application/word problem: 2
- Challenging: 1
- Total number of questions for each goal: 11

3. Calculate circumference of a circle

- Name: Calculate circumference of a circle
- Description: Students will apply the formulas for the circumference of a circle ($C=\pi d$ or $C=2\pi r$) to solve problems. This skill requires understanding when to use the diameter or radius and substituting values correctly, often using $\pi \approx 3.14$ or leaving answers in terms of π.
    - Core Example Questions:
        - Find the circumference of a circle with a diameter of 10 cm. (Use $\pi \approx 3.14$)
        - What is the circumference of a circle with a radius of 7 inches? (Leave your answer in terms of π)
        - A circular track has a circumference of 150π meters. What is its radius?
- Difficulty: Procedural
- Common pitfalls:
    - Using the wrong formula (e.g., $C=\pi r2$ which is area).
    - Confusing diameter and radius in the formula.
    - Calculation errors, especially when using approximate values for π.
    - Not following instructions for rounding or leaving in terms of π.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 8
- Common Pitfall avoidance: 2
- Application/word problem: 1
- Challenging: 2
- Total number of questions for each goal: 14

4. Calculate circumference of parts of circles (semicircles, quarter circles)

- Name: Calculate circumference of parts of circles (semicircles, quarter circles)
- Description: This skill extends the concept of circumference to parts of circles, specifically semicircles (half circles) and quarter circles. Students must remember to include the straight edge(s) that form the diameter or radii when calculating the perimeter of these shapes.
    - Core Example Questions:
        - Find the perimeter of a semicircle with a diameter of 14 cm. (Use $\pi \approx 22/7$)
        - A quarter circle has a radius of 10 meters. Calculate its perimeter. (Use $\pi \approx 3.14$)
        - Explain why the perimeter of a semicircle isn't just half the circumference of a full circle.
- Difficulty: Procedural
- Common pitfalls:
    - Forgetting to add the length of the straight edge(s) (diameter or radii).
    - Incorrectly calculating the fraction of the circumference.
    - Arithmetic errors with fractions or decimals.
- Introduced definitions: Semicircle, Quarter circle
- Concept / Introduction / Definition: 1

- Practice: 6
- Common Pitfall avoidance: 2
- Application/word problem: 1
- Challenging: 1
- Total number of questions for each goal: 11

5. Solve problems involving circumference and rotations

- Name: Solve problems involving circumference and rotations
- Description: This skill applies circumference to real-world scenarios, particularly involving rotational motion. Students will use the radius or diameter of a circular object (like a wheel or gear) to determine the distance it travels in a given number of rotations. This connects circumference to linear distance.
    - Core Example Questions:
        - A bicycle wheel has a diameter of 26 inches. How far does the bicycle travel if the wheel makes 10 full rotations? (Use $\pi \approx 3.14$)
        - A car tire has a radius of 1 foot. If the car travels 314 feet, how many times does the tire rotate? (Use $\pi \approx 3.14$)
        - If a gear with a circumference of 20 cm makes 5 rotations per second, what is the speed of a point on its edge in cm/second?
- Difficulty: Application
- Common pitfalls:
    - Incorrectly calculating the circumference.
    - Forgetting that one rotation equals one circumference.
    - Division/multiplication errors with the number of rotations.
    - Not understanding the relationship between rotational and linear distance.
- Introduced definitions: Rotation
- Concept / Introduction / Definition: 1
- Practice: 3
- Common Pitfall avoidance: 1
- Application/word problem: 5
- Challenging: 2
- Total number of questions for each goal: 12

6. Estimate the area of a circle

- Name: Estimate the area of a circle
- Description: This skill focuses on developing an intuitive understanding of the area of a circle by using informal methods, such as approximating it with a square or counting squares on a grid. This helps students grasp the concept of area for a curved shape before formal formula introduction.
    - Core Example Questions:
        - Draw a circle with a radius of 3 units on grid paper. Estimate its area by counting squares.
        - Explain how you could use a square to roughly estimate the area of a circular pizza.
        - If a circle fits inside a square with side length 10 cm, what is the maximum possible area of the circle? What would be a reasonable estimate for the circle's actual area?
- Difficulty: Conceptual
- Common pitfalls:
    - Inaccurate counting of partial squares on a grid.
    - Not understanding how to relate the dimensions of an approximating square to the circle's radius/diameter.

    - Confusing area with circumference.
- Introduced definitions: Area (of a circle)
- Concept / Introduction / Definition: 3
- Practice: 5
- Common Pitfall avoidance: 1
- Application/word problem: 1
- Challenging: 1
- Total number of questions for each goal: 11

7. Informal derivation of the area of a circle formula

- Name: Informal derivation of the area of a circle formula
- Description: This skill aims to help students understand the relationship between the area and circumference of a circle and informally derive the formula for the area of a circle ($A=\pi r2$). This often involves visualizing a circle cut into wedges and rearranged into a shape resembling a rectangle, where the length relates to half the circumference and the width relates to the radius.
    - Core Example Questions:
        - Imagine cutting a circle into many small wedges and rearranging them. What shape does it start to resemble? How does this shape relate to the circumference and radius of the original circle?
        - Explain why the formula for the area of a circle uses the radius squared, while circumference uses the radius (or diameter) to the first power.
        - If the rearranged wedges form a "rectangle," what are the approximate dimensions of that rectangle in terms of the circle's radius and circumference?
- Difficulty: Conceptual
- Common pitfalls:
    - Difficulty visualizing the rearrangement of wedges.
    - Not connecting the dimensions of the "rectangle" back to the circle's components.
    - Struggling to see why $\pi r2$ makes sense from the derivation.
- Introduced definitions: None
- Concept / Introduction / Definition: 4
- Practice: 2
- Common Pitfall avoidance: 0
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 7

8. Calculate area of a circle

- Name: Calculate area of a circle
- Description: Students will apply the formula for the area of a circle ($A=\pi r2$) to solve problems. This requires correctly identifying the radius (or converting diameter to radius), squaring the radius, and multiplying by π (either as an approximation or leaving in terms of π).
    - Core Example Questions:
        - Find the area of a circle with a radius of 5 cm. (Use $\pi \approx 3.14$)
        - What is the area of a circular garden with a diameter of 14 feet? (Use $\pi \approx 22/7$)
        - If the area of a circular tabletop is 36π square inches, what is its radius?
- Difficulty: Procedural
- Common pitfalls:
    - Using the diameter instead of the radius in the formula.
    - Forgetting to square the radius.

    - Using the circumference formula instead of the area formula.
    - Calculation errors, especially with exponents or π approximations.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 8
- Common Pitfall avoidance: 2
- Application/word problem: 1
- Challenging: 2
- Total number of questions for each goal: 14

9. Calculate area of parts of circles (semicircles, quarter circles)

- Name: Calculate area of parts of circles (semicircles, quarter circles)
- Description: This skill extends the calculation of area to parts of circles, specifically semicircles and quarter circles. Students will calculate the area of these fractional parts of a circle, understanding that the area is simply a fraction of the full circle's area. Unlike circumference, there are no straight edges to add for area.
    - Core Example Questions:
        - Find the area of a semicircle with a radius of 6 cm. (Use $\pi \approx 3.14$)
        - A quarter circular piece of paper has a diameter of 20 inches. What is its area? (Use $\pi \approx 3.14$)
        - Explain how to find the area of a 270° sector of a circle with a radius of 8 units.
- Difficulty: Procedural
- Common pitfalls:
    - Incorrectly calculating the fraction of the area (e.g., dividing by 4 for a semicircle).
    - Forgetting to square the radius before multiplying by the fraction.
    - Confusing area calculations with perimeter calculations for parts of circles (e.g., adding diameter for area).
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 6
- Common Pitfall avoidance: 2
- Application/word problem: 1
- Challenging: 1
- Total number of questions for each goal: 11

10. Solve real-world problems involving area and circumference of circles

- Name: Solve real-world problems involving area and circumference of circles
- Description: This critical thinking skill requires students to determine whether circumference or area is more appropriate for solving various real-world problems involving circles. They must analyze the context of the problem to decide if they need the distance around (circumference) or the space inside (area).
    - Core Example Questions:
        - You want to put a fence around a circular garden. Do you need to calculate the garden's area or circumference? Explain.
        - You're buying a circular rug for your bedroom. Do you need to know its area or circumference to make sure it fits?
        - A pizza company is advertising the size of its pizzas by their diameter. If you want to know how much pizza you're getting, which measurement (area or circumference) is more relevant?
- Difficulty: Application
- Common pitfalls:

    - Misinterpreting the problem's context.
    - Confusing "distance around" with "space inside."
    - Not justifying their choice of area or circumference.
- Introduced definitions: None
- Concept / Introduction / Definition: 2
- Practice: 3
- Common Pitfall avoidance: 2
- Application/word problem: 5
- Challenging: 1
- Total number of questions for each goal: 13

11. Calculate area of compound figures involving circles

- Name: Calculate area of compound figures involving circles
- Description: This challenging skill involves calculating the area of complex shapes that include fractions of circles or multiple circular components. Students must decompose these compound figures into simpler shapes (rectangles, triangles, semicircles, etc.), calculate the area of each component, and then add or subtract to find the total area.
    - Core Example Questions:
        - Find the area of a figure shaped like a square with a semicircle on one side. The square has side length 10 cm.
        - A running track is shaped like a rectangle with a semicircle at each end. If the rectangle is 100 meters long and 60 meters wide, find the total area of the track.
        - Calculate the area of the shaded region in a square with a circle inscribed within it (the circle touches all four sides of the square).
- Difficulty: Synthesis
- Common pitfalls:
    - Incorrectly decomposing the figure.
    - Errors in calculating the area of individual components.
    - Forgetting to add or subtract correctly.
    - Misidentifying radii/diameters from the compound figure's dimensions.
- Introduced definitions: Compound figure
- Concept / Introduction / Definition: 1
- Practice: 3
- Common Pitfall avoidance: 2
- Application/word problem: 4
- Challenging: 3
- Total number of questions for each goal: 13

Area, Volume, and Surface Area of Prisms

1. Find the area of triangles and quadrilaterals (review)

- Name: Find the area of triangles and quadrilaterals (review)
- Description: This skill serves as a review of finding the area of common two-dimensional shapes, which is essential foundational knowledge for working with prisms. Students will apply formulas for the area of triangles (A=21bh), rectangles (A=lw), parallelograms (A=bh), and trapezoids (A=21h(b1+b2)).
    - Core Example Questions:
        - Calculate the area of a triangle with a base of 8 inches and a height of 5 inches.
        - A parallelogram has a base of 12 cm and a height of 7 cm. What is its area?

            - Find the area of a trapezoid with bases of 6 meters and 10 meters, and a height of 4 meters.
    - Difficulty: Foundational Review
    - Common pitfalls:
        - Using the wrong formula for a given shape.
        - Confusing base and height in triangles and parallelograms.
        - Arithmetic errors.
        - Forgetting the 21 for triangles and trapezoids.
    - Introduced definitions: None (review of previous concepts)
    - Concept / Introduction / Definition: 0
    - Practice: 8
    - Common Pitfall avoidance: 2
    - Application/word problem: 0
    - Challenging: 1
    - Total number of questions for each goal: 11

2. Find the area of composite two-dimensional figures

- Name: Find the area of composite two-dimensional figures
- Description: This skill builds on finding areas of basic shapes by having students calculate the area of complex two-dimensional figures. They will do this by decomposing them into simpler shapes like triangles, quadrilaterals, and sometimes parts of circles. This involves identifying the component shapes, finding their individual areas, and then adding or subtracting as necessary.
    - Core Example Questions:
        - Find the area of an L-shaped figure that can be divided into two rectangles.
        - A polygon is composed of a rectangle and a triangle. Calculate its total area.
        - Find the area of the shaded region in a large rectangle with a smaller rectangle cut out of its center.
- Difficulty: Procedural
- Common pitfalls:
    - Incorrectly decomposing the figure.
    - Missing a component shape or double-counting an area.
    - Errors in calculating the area of individual components.
    - Not correctly identifying dimensions of the decomposed shapes.
- Introduced definitions: Composite figure
- Concept / Introduction / Definition: 1
- Practice: 6
- Common Pitfall avoidance: 2
- Application/word problem: 2
- Challenging: 2
- Total number of questions for each goal: 13

3. Calculate the surface area of cubes and right prisms using nets

- Name: Calculate the surface area of cubes and right prisms using nets
- Description: This skill introduces surface area by having students visualize or create nets of cubes and right prisms. They will then sum the areas of all the two-dimensional faces shown in the net to find the total surface area. This concrete approach helps build understanding before moving to formulas.
    - Core Example Questions:
        - Draw a net for a rectangular prism with dimensions 3x4x5. Calculate its surface area.
        - A cube has side lengths of 6 cm. Draw its net and find its surface area.
        - Given the net of a triangular prism, calculate its total surface area.

- Difficulty: Conceptual
- Common pitfalls:
    - Forgetting to include all faces in the net.
    - Incorrectly calculating the area of individual faces.
    - Errors in summing all the face areas.
    - Difficulty visualizing the 3D shape from its 2D net.
- Introduced definitions: Surface area, Net (of a 3D figure)
- Concept / Introduction / Definition: 2
- Practice: 6
- Common Pitfall avoidance: 2
- Application/word problem: 1
- Challenging: 2
- Total number of questions for each goal: 13

4. Find the volume of right prisms

- Name: Find the volume of right prisms
- Description: Students will calculate the volume of right prisms, including cubes and rectangular prisms, using the formula V=Bh, where B is the area of the base and h is the height of the prism. This emphasizes that the volume is the base area extended through the height.
    - Core Example Questions:
        - Find the volume of a rectangular prism with length 10 cm, width 4 cm, and height 5 cm.
        - A cube has a side length of 7 inches. What is its volume?
        - The base of a right prism is a rectangle with an area of 24 cm2. If the height of the prism is 8 cm, what is its volume?
- Difficulty: Procedural
- Common pitfalls:
    - Confusing volume with surface area.
    - Using perimeter of the base instead of area of the base.
    - Incorrectly identifying the height of the prism.
    - Arithmetic errors.
- Introduced definitions: Volume, Area of the base (B)
- Concept / Introduction / Definition: 1
- Practice: 8
- Common Pitfall avoidance: 2
- Application/word problem: 1
- Challenging: 1
- Total number of questions for each goal: 13

5. Find the volume of prisms with non-rectangular bases

- Name: Find the volume of prisms with non-rectangular bases
- Description: This skill extends volume calculations to prisms with bases that are other polygons (e.g., triangular prisms, trapezoidal prisms). Students will still use the formula V=Bh, but they will need to first calculate the area of the non-rectangular base by decomposing it into simpler shapes or applying the appropriate area formula.
    - Core Example Questions:
        - Find the volume of a triangular prism with a base triangle having a base of 6 ft and height of 4 ft, and a prism height of 10 ft.
        - A prism has a trapezoidal base with parallel sides of 5 cm and 9 cm, a height of 4 cm, and the prism's height is 12 cm. Calculate its volume.
        - How would you find the volume of a prism whose base is an irregular polygon?

- Difficulty: Procedural
- Common pitfalls:
    - Errors in calculating the area of the non-rectangular base.
    - Confusing the height of the prism with the height of the base shape.
    - Not using the correct formula for the area of the base.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 6
- Common Pitfall avoidance: 2
- Application/word problem: 1
- Challenging: 2
- Total number of questions for each goal: 12

6. Find the surface area of cubes and right prisms

- Name: Find the surface area of cubes and right prisms
- Description: Students will calculate the surface area of cubes and right prisms using either the net method (as introduced in Skill 3) or by applying a formulaic approach (e.g., SA=2B+Ph for prisms, where P is perimeter of base). This skill emphasizes efficient calculation and reinforces the understanding of all faces contributing to the total area.
    - Core Example Questions:
        - Find the surface area of a rectangular prism with dimensions 5 in by 8 in by 3 in.
        - Calculate the surface area of a cube with side length 9 meters.
        - A triangular prism has a base that is a right triangle with legs of 3 cm and 4 cm, and a hypotenuse of 5 cm. The height of the prism is 10 cm. Find its surface area.
- Difficulty: Procedural
- Common pitfalls:
    - Forgetting to calculate the area of all six faces (for rectangular prisms) or all relevant faces (for other prisms).
    - Calculation errors, especially when dealing with different face dimensions.
    - Confusing volume with surface area.
    - Errors in calculating the area of the base shapes for triangular or other polygonal prisms.
- Introduced definitions: None
- Concept / Introduction / Definition: 0
- Practice: 10
- Common Pitfall avoidance: 3
- Application/word problem: 1
- Challenging: 3
- Total number of questions for each goal: 17

7. Solve real-world problems involving area and volume of 2D and 3D objects

- Name: Solve real-world problems involving area and volume of 2D and 3D objects
- Description: This comprehensive application skill requires students to solve real-world and mathematical problems that involve calculating areas and volumes of both two- and three-dimensional objects. These problems often require multiple steps, including identifying the appropriate shape, using the correct formula, and interpreting the results in context.
    - Core Example Questions:
        - A swimming pool is 12 meters long, 6 meters wide, and 2 meters deep. How much water is needed to fill it? (Volume)

            - You want to paint a rectangular wall that is 8 feet high and 15 feet long. If one can of paint covers 120 square feet, how many cans do you need? (Area)
            - A cylindrical well has a diameter of 3 feet and is 20 feet deep. How much water can it hold? (Volume, requires knowing cylinder volume)
- Difficulty: Application
- Common pitfalls:
    - Choosing the wrong measurement (area vs. volume).
    - Not being able to extract relevant dimensions from the word problem.
    - Calculation errors.
    - Not interpreting the answer in context (e.g., number of cans needed).
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 3
- Common Pitfall avoidance: 2
- Application/word problem: 6
- Challenging: 3
- Total number of questions for each goal: 15

8. Solve real-world problems involving surface area of 3D objects

- Name: Solve real-world problems involving surface area of 3D objects
- Description: This skill focuses specifically on solving real-world problems that require calculating the surface area of three-dimensional objects, including cubes and right prisms. Problems will involve scenarios like determining material costs for construction, wrapping presents, or painting surfaces, emphasizing the practical application of surface area.
    - Core Example Questions:
        - A gift box is a rectangular prism with dimensions 10 inches by 6 inches by 4 inches. How much wrapping paper is needed to cover it completely?
        - A carpenter is building a wooden chest in the shape of a cube with side length 2 feet. If wood costs $1.50 per square foot, how much will the wood cost for the chest?
        - A room is 12 ft long, 10 ft wide, and 8 ft high. If you need to paint the walls and ceiling (but not the floor), what is the total area you need to paint?
- Difficulty: Application
- Common pitfalls:
    - Incorrectly calculating the surface area of the given object.
    - Not understanding what parts of the surface area are relevant to the problem (e.g., painting walls but not floor).
    - Calculation errors, especially involving unit conversions or costs.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 2
- Common Pitfall avoidance: 2
- Application/word problem: 6
- Challenging: 3
- Total number of questions for each goal: 14

9. Determine when to use area, volume, or surface area in problem solving

- Name: Determine when to use area, volume, or surface area in problem solving
- Description: This crucial analytical skill requires students to differentiate between scenarios where area, volume, or surface area is the appropriate measure to solve a real-world problem. They must understand the distinct meaning of each measurement and how it relates to different types of practical questions.

    - Core Example Questions:
        - For a swimming pool, would you use area, volume, or surface area to find out how much water it can hold? Explain.
        - If you want to know how much carpet is needed for a room, which measurement should you use?
        - A company is designing a new shipping box. They need to know how much material to buy for the box and how many small items can fit inside. Which measurements (area, volume, surface area) are relevant for each of these questions?
- Difficulty: Conceptual/Application
- Common pitfalls:
    - Confusing the terms area, volume, and surface area.
    - Not clearly articulating why a specific measurement is appropriate.
    - Misinterpreting the "verb" in a word problem (e.g., "fill" implies volume, "cover" implies surface area, "fit on" implies area).
- Introduced definitions: None
- Concept / Introduction / Definition: 3
- Practice: 3
- Common Pitfall avoidance: 3
- Application/word problem: 5
- Challenging: 2
- Total number of questions for each goal: 16

Here's the breakdown of the "Statistics and Probability" section, with detailed descriptions for each skill:

Statistics and Probability

Sampling and Inference

Understanding Samples

1. Populations and samples

- Name: Populations and samples
- Description: This fundamental skill introduces students to the core concepts of population and sample in statistics. Students will learn to differentiate between the entire group of interest (the population) and a subset of that group from which data is collected (the sample). This distinction is critical for understanding statistical inference.
    - Core Example Questions:
        - In a study of all students in a school, if you survey 50 students, what is the population and what is the sample?
        - A company wants to know the average height of trees in a forest. They measure 100 trees. Identify the population and the sample.
        - Explain why a census (surveying everyone) might be considered a sample of itself.
- Difficulty: Introductory
- Common pitfalls:
    - Confusing the definitions of population and sample.
    - Incorrectly identifying the complete group of interest.
    - Thinking the sample must be a large percentage of the population.
- Introduced definitions: Population, Sample
- Concept / Introduction / Definition: 4
- Practice: 6
- Common Pitfall avoidance: 1

- Application/word problem: 2
- Challenging: 1
- Total number of questions for each goal: 14

2. Understand populations, samples, and why sampling might be used

- Name: Understand populations, samples, and why sampling might be used
- Description: Building on the previous skill, students will explain the concepts of populations and samples more deeply and, crucially, identify situations where sampling is appropriate for gathering information. This includes understanding the practical reasons for sampling, such as cost, time, and feasibility.
    - Core Example Questions:
        - Why would a food company test a sample of potato chips for saltiness instead of every single chip?
        - A scientist wants to study the health of all fish in a very large lake. Why would they choose to take a sample of fish instead of catching every fish?
        - Describe a situation where you might want to survey an entire population instead of just a sample.
- Difficulty: Conceptual
- Common pitfalls:
    - Struggling to articulate the practical advantages of sampling.
    - Not recognizing scenarios where a census is impractical or impossible.
    - Confusing sampling with surveying every member of a group.
- Introduced definitions: Census
- Concept / Introduction / Definition: 3
- Practice: 4
- Common Pitfall avoidance: 1
- Application/word problem: 4
- Challenging: 1
- Total number of questions for each goal: 13

3. Identify representative, random, and biased samples

- Name: Identify representative, random, and biased samples
- Description: This skill introduces critical characteristics of samples. Students will distinguish between representative samples (accurately reflecting the characteristics of the population), random samples (every member of the population has an equal chance of being selected), and biased samples (favoring certain outcomes or groups). They will understand the implications of these characteristics for drawing valid inferences.
    - Core Example Questions:
        - A principal wants to know what students think about the cafeteria food. She surveys every 10th student entering the school. Is this a random sample? Is it likely representative?
        - A TV station asks viewers to call in and vote for their favorite show. Is this sample likely biased? Why or why not?
        - Give an example of how you could get a random sample of people living in your town.
- Difficulty: Conceptual
- Common pitfalls:
    - Confusing "random" with "haphazard" or "convenient."
    - Not recognizing common sources of bias (e.g., voluntary response, convenience sampling).
    - Failing to connect the type of sample to the validity of conclusions.
- Introduced definitions: Representative sample, Random sample, Biased sample, Convenience sample, Voluntary response sample

- Concept / Introduction / Definition: 4
- Practice: 6
- Common Pitfall avoidance: 3
- Application/word problem: 3
- Challenging: 2
- Total number of questions for each goal: 18

4. Evaluate whether a sample is representative of a population using dot plots

- Name: Evaluate whether a sample is representative of a population using dot plots
- Description: This skill focuses on using dot plots as a visual tool to evaluate whether a sample is representative of a given population. Students will compare the distribution, center, and spread of data in a sample's dot plot to what is known or expected about the population's characteristics, identifying if the sample seems to accurately reflect the larger group.
    - Core Example Questions:
        - A dot plot shows the ages of a population of customers. Another dot plot shows the ages of a sample. Do the dot plots suggest the sample is representative? Explain.
        - Given a dot plot of a sample, describe what kind of population it might represent, or if it appears biased.
        - Two dot plots represent samples from the same population. Which sample appears to be more representative and why?
- Difficulty: Analytical
- Common pitfalls:
    - Only looking at one aspect (e.g., just the mean) instead of the overall distribution.
    - Misinterpreting the shape, center, or spread of data in dot plots.
    - Drawing conclusions based on small discrepancies that might be due to natural sampling variability.
- Introduced definitions: Dot plot (as a tool)
- Concept / Introduction / Definition: 2
- Practice: 5
- Common Pitfall avoidance: 2
- Application/word problem: 3
- Challenging: 2
- Total number of questions for each goal: 14

5. Recognize how random sampling leads to representative samples and valid inferences

- Name: Recognize how random sampling leads to representative samples and valid inferences
- Description: This skill emphasizes the importance of random sampling. Students will explain *why* random sampling is crucial for producing representative samples and, consequently, for supporting valid statistical inferences about a population. They will understand that randomness helps minimize bias and ensures that conclusions drawn from the sample can be generalized to the larger group.
    - Core Example Questions:
        - Why is selecting names from a hat considered a better way to get a representative sample than just asking your friends?
        - If a sample is not random, how might that affect any conclusions you try to make about the population?
        - A researcher wants to know the average height of all trees in a forest. Why is it important that their sample of trees is randomly selected?
- Difficulty: Conceptual

- Common pitfalls:
    - Still confusing random with convenient.
    - Not fully grasping the link between randomness, representativeness, and the ability to generalize findings.
    - Overstating what a single random sample can guarantee (it reduces bias, but doesn't eliminate all variability).
- Introduced definitions: Valid inference, Generalize
- Concept / Introduction / Definition: 3
- Practice: 4
- Common Pitfall avoidance: 2
- Application/word problem: 3
- Challenging: 2
- Total number of questions for each goal: 14

6. Valid claims

- Name: Valid claims
- Description: This higher-order thinking skill requires students to evaluate statistical claims made in various contexts. They will determine if these claims are valid by critically assessing whether the underlying sample was representative and random. This involves applying all previous concepts about sample quality to real-world statements.
    - Core Example Questions:
        - A news report states, "75% of Americans prefer Brand X cereal, based on an online poll." Is this a valid claim? Why or why not?
        - A company claims that its new lightbulb lasts 20% longer. They tested 100 lightbulbs from their factory. Is this a valid claim? What information would you need to be more confident?
        - Someone claims that 80% of students in your school like pizza. They surveyed 10 students from the school's pizza club. Evaluate the validity of this claim.
- Difficulty: Analytical/Critical Thinking
- Common pitfalls:
    - Accepting claims at face value without questioning the sampling method.
    - Focusing on the percentage or number presented rather than the methodology.
    - Failing to identify specific sources of bias or lack of randomness.
- Introduced definitions: None
- Concept / Introduction / Definition: 2
- Practice: 3
- Common Pitfall avoidance: 4
- Application/word problem: 6
- Challenging: 3
- Total number of questions for each goal: 18

Making Inferences

1. Making inferences from random samples

- Name: Making inferences from random samples
- Description: This skill moves from evaluating samples to actively drawing conclusions and making predictions about a population based on data from a random sample. Students will learn that while a sample won't perfectly mirror a population, a properly collected random sample allows for reasonable estimates and generalizations.
    - Core Example Questions:
        - A random sample of 20 students at a school shows that 12 prefer reading. If there are 300 students in the school, estimate how many prefer reading.

        - A biologist randomly catches 50 fish in a pond and finds that 10 of them have a certain tag. If there are 800 fish in the pond, how many would you expect to have the tag?
        - Based on a random sample, a company estimates that 2% of its products are defective. If they produce 10,000 products, how many defects would they predict?
- Difficulty: Procedural/Application
- Common pitfalls:
    - Making predictions outside the reasonable range of the data.
    - Arithmetic errors when scaling from sample to population.
    - Stating inferences with 100% certainty rather than as estimates or predictions.
- Introduced definitions: Inference, Prediction
- Concept / Introduction / Definition: 2
- Practice: 5
- Common Pitfall avoidance: 1
- Application/word problem: 5
- Challenging: 2
- Total number of questions for each goal: 15

2. Make inferences from multiple samples

- Name: Make inferences from multiple samples
- Description: This skill enhances the understanding of sampling variability. Students will draw inferences about a population by analyzing and comparing multiple random samples of the same size, observing the variability in estimates. This helps them understand that different samples from the same population will yield slightly different results, but the overall trend provides a good estimate.
    - Core Example Questions:
        - Three different random samples of 20 students from a school showed that 12, 13, and 11 preferred reading, respectively. How would you use this information to make a better estimate for the whole school?
        - A factory takes 5 random samples of 100 items and finds 2, 3, 1, 2, and 4 defective items. What is your best estimate for the percentage of defective items in the entire production?
        - Why is it helpful to look at multiple samples rather than just one when making inferences about a population?
- Difficulty: Analytical
- Common pitfalls:
    - Getting confused by the slight differences between samples.
    - Not understanding how to combine information from multiple samples (e.g., using the average of sample means).
    - Ignoring sampling variability and assuming any difference means a bad sample.
- Introduced definitions: Sampling variability
- Concept / Introduction / Definition: 2
- Practice: 4
- Common Pitfall avoidance: 2
- Application/word problem: 3
- Challenging: 2
- Total number of questions for each goal: 13

3. Estimating Population Proportions

- Name: Estimating Population Proportions
- Description: This skill focuses on using proportions from random samples to infer information about a population, specifically focusing on proportions of a certain characteristic. Students

will use sample percentages or fractions to estimate the corresponding percentage or fraction in the larger population.
    - Core Example Questions:
        - In a random sample of 50 people, 30 reported owning a smartphone. Estimate the percentage of the population that owns a smartphone.
        - A poll surveys 100 likely voters and finds 55 plan to vote for Candidate A. If there are 20,000 voters in the city, how many would you predict will vote for Candidate A?
        - A random sample of 20 bags of candy contains 5 red candies. Based on this, what proportion of all candies are red?
- Difficulty: Procedural/Application
- Common pitfalls:
    - Errors in calculating percentages or proportions.
    - Incorrectly scaling the sample proportion to the population size.
    - Confusing counts with proportions.
- Introduced definitions: Proportion (in statistics)
- Concept / Introduction / Definition: 1
- Practice: 6
- Common Pitfall avoidance: 1
- Application/word problem: 5
- Challenging: 2
- Total number of questions for each goal: 15

4. More about Sampling Variability (Optional)

- Name: More about Sampling Variability (Optional)
- Description: This optional, deeper skill explores sampling variability by having students compare distributions of sample means using dot plots. By examining multiple samples, they can visually observe how much sample means vary around the true population mean, leading to a more nuanced understanding of uncertainty in statistical inference.
    - Core Example Questions:
        - Given dot plots of means from 10 different samples, describe the spread of these sample means.
        - If the true population mean is known, how do the sample means on the dot plot cluster around it?
        - Why might some sample means be far from the true population mean, and what does that tell you about a single sample?
- Difficulty: Conceptual/Advanced (Optional)
- Common pitfalls:
    - Misinterpreting the distribution of sample means.
    - Not understanding that variability is expected but that larger samples tend to reduce it.
    - Confusing the variability of the sample itself with the variability of the sample *means*.
- Introduced definitions: Distribution of sample means, Sampling distribution
- Concept / Introduction / Definition: 3
- Practice: 4
- Common Pitfall avoidance: 1
- Application/word problem: 1
- Challenging: 3
- Total number of questions for each goal: 12

5. Generate a random sample and use it to make population inferences

- Name: Generate a random sample and use it to make population inferences

- Description: This skill provides a practical experience in the sampling process. Students will perform a simple random sampling procedure (e.g., using a random number generator, drawing names from a hat) from a given small population or hypothetical scenario. They will then use the collected data to make inferences about the larger population, effectively going through a mini statistical study.
    - Core Example Questions:
        - You have a bag with 20 colored marbles (5 red, 8 blue, 7 green). Describe how you would randomly select a sample of 5 marbles. If your sample has 1 red, 3 blue, 1 green, predict the number of red marbles in the bag.
        - Given a list of 30 student names, explain how to use a random number generator to select a random sample of 10 students. If 7 of your sample students like science, predict how many total students in the list like science.
        - Conduct a random sample of 15 "students" from a class list to estimate the proportion of students who prefer dogs over cats.
- Difficulty: Procedural/Application
- Common pitfalls:
    - Not following proper random sampling techniques.
    - Errors in collecting or recording data from their own sample.
    - Incorrectly scaling their sample findings to the population.
- Introduced definitions: Simple random sampling, Random number generator
- Concept / Introduction / Definition: 2
- Practice: 3
- Common Pitfall avoidance: 1
- Application/word problem: 5
- Challenging: 2
- Total number of questions for each goal: 13

Analyzing Data

Measures of Center and Spread

1. Calculate mean, median, mode, and range

- Name: Calculate mean, median, mode, and range
- Description: This foundational data analysis skill requires students to calculate measures of center (mean, median, mode) and a basic measure of variability (range) for given numerical data sets. This involves understanding the definition and calculation method for each measure.
    - Core Example Questions:
        - Find the mean, median, mode, and range of the data set: {5, 2, 8, 2, 7, 6}.
        - A student's test scores are 85, 92, 78, 92, 83. Calculate the mean, median, and mode.
        - Which measure of center would be best for the data set {1, 2, 3, 100}? Why?
- Difficulty: Procedural
- Common pitfalls:
    - Arithmetic errors, especially for the mean.
    - Not ordering data first for median.
    - Misidentifying the mode (e.g., if there are multiple modes or no mode).
    - Calculating range incorrectly (e.g., not largest - smallest).
- Introduced definitions: Mean, Median, Mode, Range, Measure of center, Measure of spread/ variability
- Concept / Introduction / Definition: 2
- Practice: 8
- Common Pitfall avoidance: 2

- Application/word problem: 1
- Challenging: 1
- Total number of questions for each goal: 14

2. Interpret charts and graphs to find mean, median, mode, and range

- Name: Interpret charts and graphs to find mean, median, mode, and range
- Description: Students will extract information from various charts and graphs (e.g., dot plots, bar graphs, stem-and-leaf plots) to determine measures of center and spread. This skill emphasizes data visualization and the ability to read and analyze data presented graphically.
    - Core Example Questions:
        - Given a dot plot of student test scores, find the median score.
        - Look at the bar graph showing favorite colors. What is the mode of favorite colors?
        - From a stem-and-leaf plot showing daily temperatures, calculate the range of temperatures.
- Difficulty: Analytical
- Common pitfalls:
    - Misreading values from the graph axes or scales.
    - Incorrectly counting frequencies from the graph.
    - Applying calculation methods for mean/median/mode/range to graphical data without converting to a list of numbers first.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 7
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 11

3. Mean, median, mode, and range: find the missing number

- Name: Mean, median, mode, and range: find the missing number
- Description: This problem-solving skill requires students to work backward. They will solve problems to find a missing number in a data set given its mean, median, mode, or range. This involves setting up equations (for mean) or using logical deduction (for median/mode/range).
    - Core Example Questions:
        - A data set has five numbers: {3, 5, 8, 10, x}. If the mean is 7, what is x?
        - The data set {10, 15, x, 20, 25} has a median of 18. What is the value of x?
        - The range of the data set {1, 5, x, 12, 15} is 14. What are the possible values for x?
- Difficulty: Problem Solving
- Common pitfalls:
    - Algebraic errors when solving for the missing number (especially for the mean).
    - Not ordering data correctly when finding the median.
    - Forgetting that the missing number might be at an extreme end for range problems.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 6
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 3
- Total number of questions for each goal: 12

4. Changes in mean, median, mode, and range

- Name: Changes in mean, median, mode, and range
- Description: This analytical skill examines how changes in data points affect the mean, median, mode, and range. Students will predict or calculate how these measures of center and spread shift when a new data point is added, an existing one is changed, or an outlier is introduced.
    - Core Example Questions:
        - If you add an extremely high score to a data set, how will it likely affect the mean? The median?
        - A data set is {1, 2, 3, 4, 5}. If the 5 is changed to 10, how does the range change? How does the mean change?
        - If an outlier is removed from a data set, how would you expect the mean absolute deviation to change?
- Difficulty: Conceptual/Analytical
- Common pitfalls:
    - Not understanding which measures are resistant to outliers (median) and which are highly affected (mean, range).
    - Incorrectly recalculating the measures after changes.
    - Making generalizations without considering the magnitude of the change or the original data set.
- Introduced definitions: None
- Concept / Introduction / Definition: 2
- Practice: 5
- Common Pitfall avoidance: 2
- Application/word problem: 2
- Challenging: 2
- Total number of questions for each goal: 13

5. Calculate quartiles and interquartile range

- Name: Calculate quartiles and interquartile range
- Description: This skill introduces more advanced measures of spread. Students will calculate quartiles (Q1, Q2, Q3) and the interquartile range (IQR) for a set of numerical data. This involves ordering the data and finding the medians of the lower and upper halves, providing a measure of variability less affected by outliers than the range.
    - Core Example Questions:
        - For the data set {1, 3, 4, 6, 7, 9, 10, 11, 13}, find Q1, Q2 (median), Q3, and the IQR.
        - What does the interquartile range tell you about a data set?
        - Calculate the quartiles and IQR for {10, 12, 15, 18, 20, 22}.
- Difficulty: Procedural
- Common pitfalls:
    - Not ordering the data first.
    - Incorrectly identifying the median of the entire set or the halves.
    - Including the median in both halves when calculating Q1 and Q3 (if the median is a data point itself).
    - Arithmetic errors in calculating IQR (Q3−Q1).
- Introduced definitions: Quartiles (Q1,Q2,Q3), Interquartile Range (IQR)
- Concept / Introduction / Definition: 2
- Practice: 7
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 13

6. Identify an outlier

- Name: Identify an outlier
- Description: This skill focuses on recognizing outliers in a data set and understanding their potential impact on measures of center and spread. Students will learn informal rules (e.g., values much greater/less than the rest) or simple numerical criteria to identify extreme values and discuss how they skew statistics.
    - Core Example Questions:
        - In the data set {10, 12, 11, 15, 8, 50, 13}, which number appears to be an outlier?
        - How might an outlier affect the mean of a data set compared to its median?
        - Why is it important to identify outliers before analyzing a data set?
- Difficulty: Conceptual/Analytical
- Common pitfalls:
    - Misidentifying values that are just large/small but not truly outliers.
    - Not understanding *why* outliers affect measures like mean more than median.
    - Failing to consider the context of the data when identifying outliers.
- Introduced definitions: Outlier
- Concept / Introduction / Definition: 2
- Practice: 5
- Common Pitfall avoidance: 2
- Application/word problem: 1
- Challenging: 2
- Total number of questions for each goal: 12

7. Calculate mean absolute deviation

- Name: Calculate mean absolute deviation
- Description: This skill introduces a more robust measure of variability. Students will calculate the mean absolute deviation (MAD) for a set of numerical data. This involves finding the mean, then the absolute difference of each data point from the mean, and finally the average of those absolute differences. MAD provides a sense of the typical distance of data points from the mean.
    - Core Example Questions:
        - Calculate the MAD for the data set: {2, 4, 6, 8}.
        - Find the mean and MAD of the following test scores: 70, 80, 90, 100.
        - Explain in your own words what the mean absolute deviation tells you about a data set.
- Difficulty: Procedural
- Common pitfalls:
    - Arithmetic errors at any step (calculating mean, subtracting, taking absolute value, averaging).
    - Forgetting to take the absolute value.
    - Confusing MAD with standard deviation (if introduced) or range.
- Introduced definitions: Mean Absolute Deviation (MAD)
- Concept / Introduction / Definition: 2
- Practice: 7
- Common Pitfall avoidance: 2
- Application/word problem: 0
- Challenging: 2
- Total number of questions for each goal: 13

Comparing Populations

1. Comparing distributions

- Name: Comparing distributions
- Description: This skill involves the informal comparison of two numerical data distributions, often presented visually (e.g., as dot plots or histograms). Students will assess similarities and differences in variability (spread) and measure the difference in centers (mean or median), beginning to draw conclusions about whether the populations represented by the distributions are different.
    - Core Example Questions:
        - Given dot plots for the heights of boys and girls in a class, describe how their distributions compare in terms of center and spread.
        - Two groups of students took the same test. Their scores are shown in two histograms. Which group performed better on average? Which group had more consistent scores?
        - Compare the spread of two data sets presented side-by-side.
- Difficulty: Analytical
- Common pitfalls:
    - Only focusing on one aspect (center or spread) rather than both.
    - Making precise statements when only informal comparison is possible.
    - Misinterpreting visual cues on the graphs.
- Introduced definitions: Distribution (of data), Visual overlap
- Concept / Introduction / Definition: 2
- Practice: 6
- Common Pitfall avoidance: 2
- Application/word problem: 2
- Challenging: 2
- Total number of questions for each goal: 14

2. Justify differences between populations based on their means expressed as a multiple of the mean absolute deviation

- Name: Justify differences between populations based on their means expressed as a multiple of the mean absolute deviation
- Description: This advanced comparative skill requires students to assess the difference between the centers (means) of two distributions by expressing it as a multiple of a measure of variability (specifically, the Mean Absolute Deviation or MAD). This provides a more formal way to describe how much the means differ relative to the spread of the data, helping to determine if the difference is "significant."
    - Core Example Questions:
        - Group A has a mean height of 60 inches and a MAD of 2 inches. Group B has a mean height of 65 inches and a MAD of 2.5 inches. How many MADs apart are their means? What does this tell you?
        - Two brands of batteries have average lifetimes. Brand X has a mean of 50 hours and MAD of 5 hours. Brand Y has a mean of 58 hours and MAD of 4 hours. How many MADs greater is the mean of Brand Y than Brand X?
        - Explain why expressing the difference in means as a multiple of MAD is a useful way to compare two populations.
- Difficulty: Synthesis/Analytical
- Common pitfalls:
    - Incorrectly calculating the difference in means or the MAD.
    - Errors in dividing the difference by the MAD.
    - Not being able to interpret what the resulting multiple means in terms of "significant" difference.
    - Confusing which MAD to use if they are different for the two groups.
- Introduced definitions: None

- Concept / Introduction / Definition: 2
- Practice: 4
- Common Pitfall avoidance: 2
- Application/word problem: 3
- Challenging: 3
- Total number of questions for each goal: 14

3. Compare populations using measures of center and spread

- Name: Compare populations using measures of center and spread
- Description: Students will draw informal comparative inferences about two populations by utilizing their measures of center (mean, median) and measures of variability (range, IQR, MAD), usually derived from random samples. This integrates the calculation of these measures with the ability to compare two data sets comprehensively.
    - Core Example Questions:
        - Two classes took the same test. Class A's scores have a mean of 80 and an IQR of 10. Class B's scores have a mean of 75 and an IQR of 5. Compare the performance of the two classes.
        - Given the median and range of two different groups of data (e.g., reaction times of two age groups), make an inference about which group is faster and which is more consistent.
        - Based on their means and MADs, which of two groups of plants seems to grow taller and which has more consistent growth?
- Difficulty: Application/Analytical
- Common pitfalls:
    - Only comparing one measure (e.g., just the mean) and ignoring spread.
    - Making definitive statements instead of informal inferences.
    - Misinterpreting what each measure (mean, median, range, IQR, MAD) signifies.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 4
- Common Pitfall avoidance: 2
- Application/word problem: 5
- Challenging: 2
- Total number of questions for each goal: 14

4. Estimating Population Measures of Center

- Name: Estimating Population Measures of Center
- Description: This skill focuses on the core idea of inference: using the mean and MAD of a sample to infer characteristics about the center of a population. Students will understand that the sample mean provides the best point estimate for the population mean, and the MAD gives a sense of how much individual data points in the population might vary around that estimated center.
    - Core Example Questions:
        - A random sample of heights of 30 oak trees has a mean of 15 meters and a MAD of 1.5 meters. Estimate the average height of all oak trees in the forest.
        - If the mean test score of a random sample of students is 78, what is your best estimate for the mean test score of all students?
        - Explain how the MAD of a sample helps you understand the likely spread of the entire population's data around its mean.
- Difficulty: Conceptual/Application
- Common pitfalls:
    - Confusing sample statistics with population parameters (e.g., assuming sample mean *is* population mean rather than an estimate).

    - Not understanding what MAD specifically estimates for the population.
    - Making overly precise inferences.
- Introduced definitions: Population mean, Population MAD
- Concept / Introduction / Definition: 2
- Practice: 4
- Common Pitfall avoidance: 1
- Application/word problem: 3
- Challenging: 2
- Total number of questions for each goal: 12

5. Compare populations using samples and reasoning about variability

- Name: Compare populations using samples and reasoning about variability
- Description: This skill culminates the "Comparing Populations" section. Students will apply reasoning about measures of center and spread (and sampling variability) to determine if two populations are meaningfully different based on their samples. This involves synthesizing multiple concepts to make informed judgments about the statistical significance of observed differences between samples.
    - Core Example Questions:
        - Two samples of plant growth show Sample A with a mean of 10 cm and a MAD of 1 cm, and Sample B with a mean of 12 cm and a MAD of 1.5 cm. Do these samples suggest the populations of plants grow meaningfully different? Explain your reasoning.
        - A company claims its new product is better than the old one. They tested 20 old products (mean life 100 hrs, MAD 5 hrs) and 20 new products (mean life 105 hrs, MAD 10 hrs). Does the data strongly support their claim? Why or why not?
        - Describe a situation where two samples might have different means, but you wouldn't conclude that their populations are truly different.
- Difficulty: Synthesis/Critical Thinking
- Common pitfalls:
    - Drawing strong conclusions from small differences that could be due to natural variability.
    - Not adequately considering both center and spread in their reasoning.
    - Failing to connect sample data to inferences about populations.
    - Lack of a clear justification for their conclusion.
- Introduced definitions: Statistical significance (informal introduction)
- Concept / Introduction / Definition: 2
- Practice: 3
- Common Pitfall avoidance: 3
- Application/word problem: 5
- Challenging: 4
- Total number of questions for each goal: 17

Here's the detailed breakdown of the "Probability" section, with descriptions for each skill:

Probability

Understanding Probability

1. Comparing probabilities

- Name: Comparing probabilities
- Description: This foundational skill introduces the numerical representation of probability. Students will learn to compare the likelihood of different chance events based on their

probabilities, which are always values between 0 and 1. They will understand that a higher probability indicates a more likely event.
    - Core Example Questions:
        - Which event is more likely: an event with a probability of 0.75 or an event with a probability of 0.6? Explain.
        - If the probability of rain is 0.2 and the probability of sunshine is 0.9, which is more likely?
        - Order the following probabilities from least likely to most likely: 1/2, 0.25, 0.9, 0.
- Difficulty: Introductory
- Common pitfalls:
    - Confusing larger numbers with less likelihood if thinking of odds (e.g., 1 in 10 vs. 1 in 2).
    - Difficulty with comparing fractions, decimals, and percentages if not in a consistent format.
    - Not understanding that 0 means impossible and 1 means certain.
- Introduced definitions: Probability (numerical value), Likelihood
- Concept / Introduction / Definition: 3
- Practice: 6
- Common Pitfall avoidance: 1
- Application/word problem: 1
- Challenging: 1
- Total number of questions for each goal: 12

2. Describe the likelihood of events and order events from least likely to most likely

- Name: Describe the likelihood of events and order events from least likely to most likely
- Description: This skill focuses on the qualitative description of the likelihood of events using terms like impossible, unlikely, equally likely, likely, and certain. Students will also practice ordering events based on their perceived or calculated probability, translating numerical probabilities into descriptive language and vice versa.
    - Core Example Questions:
        - Describe the likelihood of rolling a 7 on a standard six-sided die.
        - If there's a 50% chance of an event happening, how would you describe its likelihood?
        - Order the following events from least likely to most likely: flipping a coin and getting tails, rolling an even number on a die, drawing a red card from a standard deck, picking a specific day of the week.
- Difficulty: Conceptual
- Common pitfalls:
    - Misusing the qualitative terms (e.g., calling an unlikely event "impossible").
    - Struggling to assign probabilities to common events without a model.
    - Difficulty with events that are equally likely.
- Introduced definitions: Impossible, Unlikely, Equally likely, Likely, Certain
- Concept / Introduction / Definition: 3
- Practice: 5
- Common Pitfall avoidance: 1
- Application/word problem: 3
- Challenging: 1
- Total number of questions for each goal: 13

3. Simple probability (uniform model)

- Name: Simple probability (uniform model)
- Description: This skill introduces the concept of a uniform probability model, where all outcomes of a chance experiment are equally likely. Students will develop such a model by assigning equal probability to all outcomes and then use it to find probabilities of simple events.
    - Core Example Questions:
        - What is the probability of rolling a 3 on a standard six-sided die?
        - If you pick a letter randomly from the word "MATH," what is the probability of picking the letter "A"?
        - Describe a uniform probability model for drawing a single card from a standard deck of 52 cards.
- Difficulty: Procedural
- Common pitfalls:
    - Not correctly identifying all possible outcomes.
    - Not understanding that "equally likely" means each outcome has the same chance.
    - Errors in writing probabilities as fractions or decimals.
- Introduced definitions: Uniform probability model, Outcome, Event
- Concept / Introduction / Definition: 2
- Practice: 7
- Common Pitfall avoidance: 1
- Application/word problem: 2
- Challenging: 1
- Total number of questions for each goal: 13

4. List the sample space of a chance experiment and calculate probability when all outcomes are equally likely

- Name: List the sample space of a chance experiment and calculate probability when all outcomes are equally likely
- Description: Students will learn to explicitly identify and list all possible outcomes in a sample space for a given chance experiment. Once the sample space is defined, they will calculate theoretical probabilities for equally likely events by determining the ratio of favorable outcomes to total outcomes.
    - Core Example Questions:
        - List the sample space for flipping two coins. What is the probability of getting two heads?
        - A bag contains 3 red, 4 blue, and 5 green marbles. List the sample space for picking one marble. What is the probability of picking a blue marble?
        - If you roll a standard six-sided die, list the sample space. What is the probability of rolling an even number?
- Difficulty: Procedural
- Common pitfalls:
    - Missing some outcomes when listing the sample space.
    - Not understanding how to handle repeated outcomes (e.g., two heads, one head and one tail, etc.).
    - Errors in calculating the fraction (favorable/total).
- Introduced definitions: Sample space, Theoretical probability, Favorable outcome
- Concept / Introduction / Definition: 2
- Practice: 7
- Common Pitfall avoidance: 2
- Application/word problem: 2
- Challenging: 2
- Total number of questions for each goal: 15

5. Simulate real-world situations with simple experiments to reflect event probabilities

- Name: Simulate real-world situations with simple experiments to reflect event probabilities
- Description: This practical skill involves designing and conducting simple simulations to model real-world chance events and then estimating their probabilities based on the results of the simulation. This helps bridge the gap between theoretical probability and actual outcomes.
    - Core Example Questions:
        - Describe how you could use a coin flip to simulate whether a customer buys a drink (assuming a 50% chance).
        - How would you use a die to simulate the probability of 1 in 6 cars being red? Conduct 20 trials and estimate the probability.
        - Design a simulation using a spinner or random number generator to model a basketball player who makes 70% of her free throws.
- Difficulty: Application/Procedural
- Common pitfalls:
    - Designing a simulation that doesn't accurately reflect the given probability.
    - Not conducting enough trials for a reasonable estimate.
    - Misinterpreting the results of their own simulation.
- Introduced definitions: Simulation, Random number generator (as tool)
- Concept / Introduction / Definition: 2
- Practice: 3
- Common Pitfall avoidance: 1
- Application/word problem: 5
- Challenging: 2
- Total number of questions for each goal: 13

6. Sample spaces of simple events

- Name: Sample spaces of simple events
- Description: This is a focused revisit of listing sample spaces. Students will primarily identify all possible outcomes for simple chance events, ensuring they can systematically enumerate every single possibility before calculating probabilities. This is a foundational step for all probability calculations.
    - Core Example Questions:
        - List the sample space for choosing a day of the week.
        - What is the sample space for the colors of a spinner that has equal sections of red, blue, green, and yellow?
        - List all possible outcomes when rolling a standard six-sided die.
- Difficulty: Foundational
- Common pitfalls:
    - Forgetting to list an outcome.
    - Listing outcomes that are not possible.
    - Not understanding that "simple" refers to single events, not compound events.
- Introduced definitions: None (reinforces sample space concept)
- Concept / Introduction / Definition: 1
- Practice: 8
- Common Pitfall avoidance: 1
- Application/word problem: 0
- Challenging: 1
- Total number of questions for each goal: 11

7. Probability of simple events

- Name: Probability of simple events
- Description: Building on defining sample spaces, students will calculate the probability of simple events by applying the theoretical probability formula: P(event)=total number of possible outcomesnumber of favorable outcomes. This skill is a direct application of basic probability principles.
    - Core Example Questions:
        - What is the probability of picking a vowel from the letters A, B, C, D, E?
        - A bag has 5 red balls, 3 blue balls, and 2 green balls. What is the probability of picking a red ball?
        - What is the probability of rolling a number less than 3 on a standard six-sided die?
- Difficulty: Procedural
- Common pitfalls:
    - Incorrectly counting favorable outcomes or total outcomes.
    - Reducing fractions incorrectly.
    - Confusing the number of outcomes with the probability value itself.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 8
- Common Pitfall avoidance: 2
- Application/word problem: 1
- Challenging: 1
- Total number of questions for each goal: 13

8. Make predictions using theoretical probability

- Name: Make predictions using theoretical probability
- Description: This skill connects theoretical probability to practical application. Students will use theoretical probability to make predictions about the frequency of events over a given number of trials. This involves multiplying the theoretical probability by the number of trials.
    - Core Example Questions:
        - If you flip a coin 50 times, how many times would you theoretically expect to get heads?
        - If you roll a standard six-sided die 30 times, how many times would you predict you'd roll a 4?
        - A spinner has 4 equal sections: red, blue, green, yellow. If you spin it 200 times, how many times would you expect it to land on blue?
- Difficulty: Application
- Common pitfalls:
    - Not understanding that predictions are *estimates*, not guarantees.
    - Arithmetic errors in multiplication.
    - Confusing theoretical probability with actual experimental results.
- Introduced definitions: Frequency (of an event)
- Concept / Introduction / Definition: 1
- Practice: 3
- Common Pitfall avoidance: 1
- Application/word problem: 5
- Challenging: 2
- Total number of questions for each goal: 12

9. Probability models (non-uniform from observed frequencies)

- Name: Probability models (non-uniform from observed frequencies)
- Description: This skill introduces non-uniform probability models. Students will develop a probability model by observing and analyzing frequencies from data generated by a chance

process. This involves calculating experimental probabilities based on collected data, where outcomes may not be equally likely.
    - Core Example Questions:
        - A student recorded the color of 100 cars passing by and found 30 red, 40 blue, 20 white, and 10 black. Create a probability model for the color of the next car. Is this a uniform model?
        - A survey asked 50 people their favorite ice cream flavor, and 25 said vanilla, 15 chocolate, and 10 strawberry. Create a probability model for favorite ice cream flavor.
        - Why might a probability model based on observed frequencies not be uniform?
- Difficulty: Conceptual/Procedural
- Common pitfalls:
    - Not expressing frequencies as probabilities (summing to 1 or 100%).
    - Failing to identify when a model is non-uniform.
    - Errors in calculating experimental probabilities.
- Introduced definitions: Non-uniform probability model, Observed frequency
- Concept / Introduction / Definition: 2
- Practice: 4
- Common Pitfall avoidance: 1
- Application/word problem: 4
- Challenging: 2
- Total number of questions for each goal: 13

10. Probability of simple events and opposite events

- Name: Probability of simple events and opposite events
- Description: This skill focuses on calculating probabilities of simple events and their complements (opposite events). Students will learn that the probability of an event happening plus the probability of it *not* happening always equals 1 (P(E)+P(not E)=1).
    - Core Example Questions:
        - If the probability of rolling a 6 on a die is 1/6, what is the probability of NOT rolling a 6?
        - The probability of rain tomorrow is 0.3. What is the probability that it will not rain tomorrow?
        - In a bag of marbles, the probability of picking a blue marble is 2/5. What is the probability of not picking a blue marble?
- Difficulty: Procedural
- Common pitfalls:
    - Forgetting that the sum of an event and its complement is 1.
    - Arithmetic errors in subtracting from 1.
    - Not correctly identifying the "opposite event."
- Introduced definitions: Complement (of an event), Opposite event
- Concept / Introduction / Definition: 1
- Practice: 7
- Common Pitfall avoidance: 1
- Application/word problem: 2
- Challenging: 1
- Total number of questions for each goal: 12

11. Probability of mutually exclusive events and overlapping events

- Name: Probability of mutually exclusive events and overlapping events
- Description: This skill introduces distinguishing between and calculating probabilities of mutually exclusive events (events that cannot happen at the same time) and overlapping

events (events that share common outcomes). Students will learn to use the addition rule for probability, adjusting for overlaps.
    - Core Example Questions:
        - Are rolling an even number and rolling an odd number on a die mutually exclusive? Why or why not? What is the probability of rolling an even OR an odd number?
        - Are drawing a red card and drawing a king from a deck of cards mutually exclusive? Why or why not? What is the probability of drawing a red card OR a king?
        - If P(A)=0.4, P(B)=0.5, and P(A and B)=0.2, what is P(A or B)?
- Difficulty: Conceptual/Procedural
- Common pitfalls:
    - Confusing mutually exclusive with independent events.
    - Not correctly identifying overlapping outcomes.
    - Forgetting to subtract the overlap when calculating the probability of overlapping events (P(A or B)=P(A)+P(B)−P(A and B)).
- Introduced definitions: Mutually exclusive events, Overlapping events
- Concept / Introduction / Definition: 2
- Practice: 5
- Common Pitfall avoidance: 2
- Application/word problem: 3
- Challenging: 2
- Total number of questions for each goal: 14

Experimental and Theoretical Probability

1. Experimental probability

- Name: Experimental probability
- Description: This skill focuses on calculating the experimental probability of a chance event based on collected data and observed frequencies from actual experiments. Students will understand that experimental probability is determined by conducting trials and recording results, often expressed as total number of trialsnumber of times event occurs.
    - Core Example Questions:
        - A coin is flipped 20 times, and it lands on heads 12 times. What is the experimental probability of landing on heads?
        - A spinner is spun 50 times. It lands on red 15 times, blue 20 times, and green 15 times. What is the experimental probability of landing on blue?
        - You observed 100 cars pass by, and 25 were SUVs. What is the experimental probability that the next car will be an SUV?
- Difficulty: Procedural
- Common pitfalls:
    - Confusing experimental probability with theoretical probability.
    - Arithmetic errors in division.
    - Not correctly identifying the number of times the event occurred or the total number of trials.
- Introduced definitions: Experimental probability, Trial
- Concept / Introduction / Definition: 1
- Practice: 8
- Common Pitfall avoidance: 1
- Application/word problem: 2
- Challenging: 1
- Total number of questions for each goal: 13

2. Making predictions with probability

- Name: Making predictions with probability
- Description: This skill broadly covers using either experimental or theoretical probability to make predictions about the approximate relative frequency of future events. It emphasizes that probability helps us estimate how often something will happen over many trials, not just the next single trial.
    - Core Example Questions:
        - If the theoretical probability of rolling a 1 is 1/6, how many times would you predict you'd roll a 1 in 60 rolls?
        - Based on an experiment where a student successfully made 8 out of 10 free throws, how many would they predict to make in their next 50 attempts?
        - A weather forecast states there's a 40% chance of rain. If this forecast is accurate, and it rains on 4 out of 10 days with a 40% chance, how many days would you predict rain for a month (30 days) with such forecasts?
- Difficulty: Application
- Common pitfalls:
    - Not distinguishing between theoretical and experimental context for predictions.
    - Assuming predictions are guaranteed outcomes.
    - Arithmetic errors when scaling.
- Introduced definitions: Relative frequency
- Concept / Introduction / Definition: 1
- Practice: 3
- Common Pitfall avoidance: 1
- Application/word problem: 5
- Challenging: 2
- Total number of questions for each goal: 12

3. Determine the likelihood of an event using results from previous experiments

- Name: Determine the likelihood of an event using results from previous experiments
- Description: This skill focuses on using data from past experiments to estimate the likelihood of future events. Students will calculate experimental probabilities from provided data and then use these to make informal judgments about how likely an event is to occur in the future.
    - Core Example Questions:
        - A baseball player got 15 hits in his last 50 at-bats. Based on this, what is the likelihood he will get a hit in his next at-bat? (Express as a probability)
        - In a survey, 70 out of 100 people said they prefer coffee over tea. What is the estimated likelihood that a randomly chosen person will prefer coffee?
        - A factory found 3 defective items in a batch of 100. What is the estimated likelihood of finding a defective item in the next batch of 100?
- Difficulty: Procedural/Application
- Common pitfalls:
    - Confusing likelihood (qualitative) with probability (quantitative).
    - Not correctly calculating the experimental probability from the given data.
    - Overstating the certainty of the prediction.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 5
- Common Pitfall avoidance: 1
- Application/word problem: 3
- Challenging: 1
- Total number of questions for each goal: 11

4. Generalize that the cumulative relative frequency approaches the probability of an event as experiments are repeated

- Name: Generalize that the cumulative relative frequency approaches the probability of an event as experiments are repeated
- Description: This key conceptual skill introduces the Law of Large Numbers. Students will understand and apply the concept that as the number of trials in a chance experiment increases, the cumulative relative frequency (experimental probability) of an event tends to converge to its theoretical probability. This explains why conducting many trials provides a better estimate.
    - Core Example Questions:
        - If you flip a coin 10 times and get 8 heads, the relative frequency is 0.8. If you flip it 1000 times and get 505 heads, the relative frequency is 0.505. What does this tell you about the relationship between experimental and theoretical probability?
        - Explain why a simulation with 1000 trials is generally more reliable than a simulation with 10 trials.
        - What is the "Law of Large Numbers" in your own words, and how does it relate to probability?
- Difficulty: Conceptual
- Common pitfalls:
    - Expecting convergence to be immediate or perfect.
    - Not understanding "relative frequency" or "cumulative."
    - Confusing the law of large numbers with the "gambler's fallacy" (believing past outcomes influence future independent ones).
- Introduced definitions: Law of Large Numbers, Cumulative relative frequency, Convergence
- Concept / Introduction / Definition: 3
- Practice: 3
- Common Pitfall avoidance: 2
- Application/word problem: 2
- Challenging: 3
- Total number of questions for each goal: 13

5. Use results from repeated experiments to estimate the probability of an event

- Name: Use results from repeated experiments to estimate the probability of an event
- Description: This skill builds on experimental probability by having students calculate and use estimated probabilities based on the results of repeated chance experiments. It emphasizes that collecting data from multiple trials provides a more robust estimate of probability than a single trial.
    - Core Example Questions:
        - A dart is thrown at a target 20 times. It hits the bullseye 4 times. If it's thrown 100 more times and hits the bullseye 21 more times, what is the best estimate of the probability of hitting the bullseye?
        - A spinner was spun 100 times. Results: Red: 30, Blue: 45, Green: 25. What is the estimated probability of landing on Blue?
        - You tested 5 batches of 20 lightbulbs each. The number of defective bulbs per batch was 1, 0, 2, 0, 1. What is the estimated probability that a randomly chosen lightbulb is defective?
- Difficulty: Procedural/Application
- Common pitfalls:
    - Not combining data from all repeated experiments correctly.
    - Arithmetic errors in calculating the combined experimental probability.
    - Overstating the certainty of the estimate.

- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 5
- Common Pitfall avoidance: 1
- Application/word problem: 4
- Challenging: 2
- Total number of questions for each goal: 13

6. Make predictions using experimental probability

- Name: Make predictions using experimental probability
- Description: Similar to skill 2 but specifically focused on experimental probability, students will use experimental probability to predict outcomes in a series of future events. This solidifies the connection between observed frequencies and future expectations.
    - Core Example Questions:
        - If a basketball player has made 75 out of 100 free throws in practice, how many would you predict he'll make in his next 20 attempts during a game?
        - A survey found that 60% of people prefer Brand A soda. If 500 people are at a sporting event, how many would you predict would prefer Brand A?
        - In a quality control check, 5 out of 200 items were found to be faulty. If a company produces 1000 items, how many would you predict are faulty?
- Difficulty: Application
- Common pitfalls:
    - Calculation errors when multiplying the experimental probability by the number of future trials.
    - Treating the experimental probability as a rigid guarantee rather than an estimate.
    - Not being able to set up the proportion correctly.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 3
- Common Pitfall avoidance: 1
- Application/word problem: 5
- Challenging: 2
- Total number of questions for each goal: 12

7. Use collected data to find probabilities and make predictions

- Name: Use collected data to find probabilities and make predictions
- Description: This comprehensive skill combines data analysis with probability. Students will analyze collected data to determine probabilities (both simple and potentially more complex ones) and then make predictions about future occurrences based on those probabilities. This involves both calculating experimental probabilities and using them to forecast.
    - Core Example Questions:
        - A school surveyed 200 students about their favorite sport: 80 chose soccer, 60 chose basketball, 40 chose baseball, and 20 chose track. What is the probability that a randomly selected student prefers basketball? If there are 800 students in the school, how many would you predict prefer soccer?
        - You roll a weighted die 100 times. Results: 1 (10 times), 2 (15 times), 3 (20 times), 4 (25 times), 5 (18 times), 6 (12 times). What is the probability of rolling a number greater than 4? If you roll it 50 more times, how many times would you expect to roll a 2?
        - A local weather station collected data on sunny days in June over the last 10 years. Out of 300 total days, 210 were sunny. Based on this data, what is the probability of a sunny day in June? How many sunny days would you predict for next June?

- Difficulty: Synthesis/Application
- Common pitfalls:
    - Errors in calculating the initial probabilities from the collected data.
    - Arithmetic errors in making predictions.
    - Not choosing the appropriate probability (experimental vs. theoretical) for the given context.
    - Misinterpreting the question and calculating something other than what is asked.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 3
- Common Pitfall avoidance: 2
- Application/word problem: 6
- Challenging: 3
- Total number of questions for each goal: 15

8. Probability models

- Name: Probability models
- Description: This skill involves the comprehensive process of constructing and using probability models to determine the probabilities of events. Students will also compare these theoretical probabilities to observed frequencies from experiments, and identify potential discrepancies or reasons for differences (e.g., small sample size, biased events).
    - Core Example Questions:
        - Create a theoretical probability model for rolling a fair six-sided die. If you roll the die 60 times and get twenty 1s, twenty 2s, and twenty 3s, how does this compare to your theoretical model? What might explain the difference?
        - A spinner is divided into 4 equal sections: Red, Blue, Green, Yellow. What is the theoretical probability model? If you spin it 10 times and get Red 5 times, Blue 2 times, Green 2 times, Yellow 1 time, explain how your experimental results compare to the theoretical model.
        - Design a probability model for drawing a specific card from a well-shuffled deck. Conduct an experiment (e.g., drawing and replacing a card 20 times) and compare your observed frequencies to your model.
- Difficulty: Synthesis/Analytical
- Common pitfalls:
    - Not correctly constructing the theoretical model (e.g., assuming outcomes are equally likely when they are not).
    - Failing to accurately record observed frequencies.
    - Struggling to articulate explanations for discrepancies between theoretical and experimental results (e.g., randomness, insufficient trials, bias).
- Introduced definitions: Probability model (general)
- Concept / Introduction / Definition: 2
- Practice: 3
- Common Pitfall avoidance: 2
- Application/word problem: 4
- Challenging: 3
- Total number of questions for each goal: 14

Compound Events

1. Probabilities of compound events

- Name: Probabilities of compound events
- Description: This skill focuses on calculating the probabilities of compound events by systematically listing and organizing all possible outcomes. Students will learn to use various

tools such as organized lists, tables (arrays), and tree diagrams to represent the sample space and then determine the probability of specific multi-step events.
    - Core Example Questions:
        - You flip a coin twice. Use a tree diagram to list all possible outcomes. What is the probability of getting exactly one head?
        - A student chooses one outfit from 3 shirts (red, blue, green) and 2 pairs of pants (khaki, denim). Use a table to list all possible outfit combinations. What is the probability of choosing a red shirt and denim pants?
        - You roll a 6-sided die and flip a coin. List all possible outcomes. What is the probability of rolling an even number and flipping tails?
- Difficulty: Procedural
- Common pitfalls:
    - Missing some outcomes when creating lists, tables, or tree diagrams.
    - Incorrectly counting favorable outcomes from the sample space.
    - Errors in calculating the fraction (favorable/total).
    - Not understanding how to represent the sample space for sequential events.
- Introduced definitions: Compound event
- Concept / Introduction / Definition: 2
- Practice: 7
- Common Pitfall avoidance: 2
- Application/word problem: 2
- Challenging: 1
- Total number of questions for each goal: 14

2. Sample spaces for compound events

- Name: Sample spaces for compound events
- Description: This skill specifically focuses on the initial step for analyzing compound events: identifying and systematically listing all possible outcomes for compound chance events. This is a critical prerequisite for calculating their probabilities. Students will be encouraged to think about structured ways to list outcomes.
    - Core Example Questions:
        - List the sample space for rolling two standard six-sided dice.
        - A bag contains a red, a blue, and a green marble. You draw one marble, replace it, and draw another. List the sample space of possible pairs of draws.
        - What is the sample space for a family having three children, considering the gender of each child?
- Difficulty: Conceptual/Procedural
- Common pitfalls:
    - Missing some outcomes or listing duplicate outcomes.
    - Not distinguishing between outcomes like (Heads, Tails) and (Tails, Heads) when order matters, or treating them as the same when they should be distinct.
    - Struggling to organize the listing of outcomes for more complex events.
- Introduced definitions: None (reinforces sample space, extends to compound)
- Concept / Introduction / Definition: 2
- Practice: 8
- Common Pitfall avoidance: 2
- Application/word problem: 1
- Challenging: 2
- Total number of questions for each goal: 15

3. The counting principle

- Name: The counting principle

- Description: This skill introduces the fundamental counting principle, a method for determining the total number of outcomes for compound events without necessarily listing them all. Students will learn that if there are 'm' ways for one event to occur and 'n' ways for a second event to occur, then there are m×n ways for both events to occur. This principle extends to more than two events.
    - Core Example Questions:
        - A restaurant offers 5 appetizers, 10 main courses, and 4 desserts. How many different three-course meals can be ordered?
        - How many different outcomes are possible when flipping a coin 4 times?
        - You have 3 pairs of shoes, 4 shirts, and 2 pairs of pants. How many different outfits can you make?
- Difficulty: Procedural
- Common pitfalls:
    - Adding instead of multiplying the number of options.
    - Not correctly identifying the number of options for each stage of the compound event.
    - Confusing the counting principle with probability calculation.
- Introduced definitions: Fundamental counting principle
- Concept / Introduction / Definition: 2
- Practice: 7
- Common Pitfall avoidance: 1
- Application/word problem: 3
- Challenging: 2
- Total number of questions for each goal: 15

4. Understand probability of compound events as a fraction of sample space

- Name: Understand probability of compound events as a fraction of sample space
- Description: This skill reinforces the core definition of theoretical probability applied to compound events. Students will solidify their understanding that the probability of a compound event is the ratio of the number of favorable outcomes to the total number of equally likely outcomes in the entire sample space. This skill focuses on the conceptual connection.
    - Core Example Questions:
        - If there are 36 possible outcomes when rolling two dice, and 5 of them result in a sum of 8, what is the probability of rolling a sum of 8?
        - Explain why the probability of an event is always a fraction between 0 and 1 (inclusive).
        - If the sample space for a compound event has 12 outcomes, and 3 of them are favorable, what is the probability?
- Difficulty: Conceptual
- Common pitfalls:
    - Forgetting that outcomes must be equally likely for this definition to apply directly.
    - Struggling to articulate the concept of probability as a ratio.
    - Mistaking the count of outcomes for the probability itself.
- Introduced definitions: None (reinforces existing definitions in compound context)
- Concept / Introduction / Definition: 3
- Practice: 4
- Common Pitfall avoidance: 1
- Application/word problem: 1
- Challenging: 1
- Total number of questions for each goal: 10

5. Represent sample spaces for compound events (lists, tables, trees)

- Name: Represent sample spaces for compound events (lists, tables, trees)

- Description: This skill focuses specifically on the visual and organized representation of sample spaces for compound events. Students will practice creating organized lists, tables (arrays), and tree diagrams as methods to systematically display all possible outcomes, a crucial step before probability calculation.
    - Core Example Questions:
        - Draw a tree diagram to show all possible outcomes when a family has two children (consider gender).
        - Create a table to show all possible sums when rolling two standard six-sided dice.
        - List all possible outcomes when choosing a flavor (chocolate, vanilla, strawberry) and a topping (sprinkles, fudge).
- Difficulty: Procedural/Visualization
- Common pitfalls:
    - Incomplete or disorganized lists/tables/diagrams.
    - Not understanding which representation is most suitable for a given scenario.
    - Errors in tracing branches of a tree diagram.
- Introduced definitions: Tree diagram, Table (for sample space), Organized list
- Concept / Introduction / Definition: 2
- Practice: 7
- Common Pitfall avoidance: 2
- Application/word problem: 1
- Challenging: 2
- Total number of questions for each goal: 14

6. Identify outcomes for compound events described in everyday language

- Name: Identify outcomes for compound events described in everyday language
- Description: This skill requires students to translate a verbal description of a compound event into its specific favorable outcomes within a given sample space. It assesses their ability to interpret the language of probability problems and correctly select the relevant outcomes.
    - Core Example Questions:
        - If you flip two coins, what outcomes are included in the event "at least one head"? (Sample space: {HH, HT, TH, TT})
        - When rolling two dice, what are the outcomes that result in a sum greater than 10?
        - From a sample space of breakfast orders (e.g., {Eggs & Bacon, Cereal & Milk, Toast & Jam, Pancakes & Syrup}), which outcomes represent "a meal with syrup"?
- Difficulty: Conceptual/Interpretive
- Common pitfalls:
    - Misinterpreting the meaning of phrases like "at least," "at most," "exactly," "not," etc.
    - Not being thorough in identifying *all* favorable outcomes.
    - Confusing individual outcomes with the event itself.
- Introduced definitions: None
- Concept / Introduction / Definition: 2
- Practice: 6
- Common Pitfall avoidance: 3
- Application/word problem: 3
- Challenging: 2
- Total number of questions for each goal: 16

7. Use simulation to estimate the probability of multi-step events

- Name: Use simulation to estimate the probability of multi-step events

- Description: This skill extends the concept of simulation to more complex scenarios. Students will design and execute simulations to estimate probabilities of multi-step events (compound events). This involves choosing appropriate tools (e.g., dice, spinners, random number generators) and conducting a sufficient number of trials.
    - Core Example Questions:
        - Describe a simulation you could use to estimate the probability of getting two heads in a row when flipping a coin twice. Conduct 20 trials and report your estimate.
        - A company makes 30% of its products defective. Design a simulation to estimate the probability of getting exactly one defective product in a sample of two.
        - How would you use a random number generator to simulate rolling a die and flipping a coin, and estimate the probability of rolling a 3 and flipping heads?
- Difficulty: Application/Procedural
- Common pitfalls:
    - Designing an inappropriate simulation for the given multi-step event.
    - Not conducting enough trials for a reasonable estimate.
    - Errors in recording or interpreting the simulation results.
    - Confusing the estimated probability with the theoretical probability.
- Introduced definitions: Multi-step event
- Concept / Introduction / Definition: 2
- Practice: 3
- Common Pitfall avoidance: 1
- Application/word problem: 5
- Challenging: 3
- Total number of questions for each goal: 14

8. Find sample space using tables, trees, or organized lists, and determine total possible outcomes for compound events

- Name: Find sample space using tables, trees, or organized lists, and determine total possible outcomes for compound events
- Description: This skill combines two essential aspects: systematically listing all possible outcomes (sample space) for compound events using tables, tree diagrams, or organized lists, and then determining the total number of possible outcomes. It emphasizes the comprehensive creation and counting of the sample space.
    - Core Example Questions:
        - Use a tree diagram to find all possible outcomes when choosing a shirt (red, blue) and pants (black, white). How many total outcomes are there?
        - Create a table to show the sample space for selecting a card from a deck (hearts or spades) and then rolling a die (1-6). How many total outcomes are possible?
        - List the sample space for ordering a pizza with 1 type of crust (thin, thick) and 2 toppings chosen from pepperoni, mushrooms, and olives (you can choose the same topping twice). How many outcomes are there?
- Difficulty: Procedural
- Common pitfalls:
    - Errors in constructing the full sample space.
    - Miscounting the total number of outcomes from the generated sample space.
    - Not using the specified method (table, tree, list) or using it incorrectly.
- Introduced definitions: None (reinforces previous definitions in combination)
- Concept / Introduction / Definition: 1
- Practice: 7
- Common Pitfall avoidance: 2
- Application/word problem: 2

- Challenging: 2
- Total number of questions for each goal: 14

9. Use sample space to calculate the probability of an event in multi-step experiments

- Name: Use sample space to calculate the probability of an event in multi-step experiments
- Description: Given a defined sample space for a multi-step (compound) experiment, students will calculate the probability of specific events by identifying the favorable outcomes within that sample space and applying the ratio formula. This skill assumes the sample space is already known or can be easily constructed.
    - Core Example Questions:
        - When rolling two standard dice, what is the probability of rolling a sum of 7? (Assume the sample space of 36 outcomes is known)
        - From the sample space for flipping three coins ({HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}), what is the probability of getting exactly two heads?
        - A spinner has 4 equally likely sections (A, B, C, D). You spin it twice. What is the probability of spinning A, then B?
- Difficulty: Procedural
- Common pitfalls:
    - Incorrectly identifying favorable outcomes from the sample space.
    - Arithmetic errors when calculating the probability fraction.
    - Not understanding that each outcome in the sample space must be equally likely.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 8
- Common Pitfall avoidance: 2
- Application/word problem: 2
- Challenging: 2
- Total number of questions for each goal: 15

10. Design and perform a multi-step simulation to estimate probability of a compound event

- Name: Design and perform a multi-step simulation to estimate probability of a compound event
- Description: This advanced simulation skill requires students to plan and carry out multi-step simulations to approximate the probability of compound events. This involves choosing appropriate tools, defining a single trial, performing multiple trials, and then calculating the experimental probability to estimate the theoretical probability.
    - Core Example Questions:
        - A student wants to know the probability of getting two heads in three coin flips. Design a simulation using a coin and perform 20 trials. What is your estimated probability?
        - A certain type of seed has an 80% chance of sprouting. Design a simulation to estimate the probability that exactly 2 out of 3 seeds will sprout.
        - Describe a simulation using a 6-sided die to estimate the probability of rolling an even number and then rolling a 5.
- Difficulty: Synthesis/Application
- Common pitfalls:
    - Inappropriate simulation design (e.g., not representing the probabilities correctly).
    - Not conducting enough trials for a reliable estimate.
    - Errors in recording results or calculating the experimental probability.
    - Misinterpreting the results and making incorrect conclusions.

- Introduced definitions: None
- Concept / Introduction / Definition: 2
- Practice: 2
- Common Pitfall avoidance: 2
- Application/word problem: 5
- Challenging: 3
- Total number of questions for each goal: 14

11. Sample spaces for compound events

- Name: Sample spaces for compound events
- Description: This skill is a direct re-emphasis on identifying all possible outcomes for compound chance events, similar to skill 2 but perhaps with a broader range of complexity or a focus on systematic listing. It is a foundational skill for understanding compound probability.
    - Core Example Questions:
        - List the sample space for choosing a letter from "APPLE" and then choosing a number from {1, 2, 3}.
        - What is the sample space for ordering a pizza with 1 size (small, medium, large) and 1 topping (cheese, pepperoni)?
        - You draw two cards from a deck of 4 cards (A, B, C, D) without replacement. List the sample space.
- Difficulty: Procedural
- Common pitfalls:
    - Missing outcomes, especially in "without replacement" scenarios.
    - Not being systematic in listing to avoid duplicates or omissions.
    - Difficulty with larger sample spaces.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 8
- Common Pitfall avoidance: 1
- Application/word problem: 1
- Challenging: 2
- Total number of questions for each goal: 13

12. Compound events: find the number of outcomes

- Name: Compound events: find the number of outcomes
- Description: This skill focuses on applying the fundamental counting principle to efficiently determine the total number of possible outcomes for compound events without necessarily listing the entire sample space. It reinforces the concept of multiplication for multiple stages of an event.
    - Core Example Questions:
        - How many different license plates are possible if they consist of 3 letters followed by 3 numbers (repetition allowed)?
        - If you have 5 choices for ice cream flavor, 3 choices for cone type, and 4 choices for topping, how many different ice cream combinations can you create?
        - A security code is 4 digits long. How many different codes are possible if digits can be repeated?
- Difficulty: Procedural
- Common pitfalls:
    - Adding instead of multiplying.
    - Not correctly accounting for "repetition allowed" vs. "no repetition."
    - Misidentifying the number of options for each stage.
- Introduced definitions: None (reinforces counting principle)

- Concept / Introduction / Definition: 1
- Practice: 7
- Common Pitfall avoidance: 1
- Application/word problem: 3
- Challenging: 2
- Total number of questions for each goal: 14

13. Compound events: find the number of sums

- Name: Compound events: find the number of sums
- Description: This specialized skill within compound events focuses on determining the number of possible sums when combining outcomes from two events, most commonly two dice rolls. Students will practice systematically listing or recognizing the range of sums possible.
    - Core Example Questions:
        - When rolling two standard six-sided dice, what is the smallest possible sum? What is the largest possible sum?
        - List all the unique sums you can get when flipping two coins, where heads is 1 and tails is 0.
        - If you draw two numbered balls from a bag (e.g., 1-5), and add their numbers, what are the possible sums?
- Difficulty: Procedural
- Common pitfalls:
    - Missing some possible sums.
    - Not understanding that some sums can be formed in multiple ways (though the question asks for *number of sums*, not frequency).
    - Arithmetic errors.
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 6
- Common Pitfall avoidance: 1
- Application/word problem: 1
- Challenging: 2
- Total number of questions for each goal: 11

14. Find the number of outcomes: word problems

- Name: Find the number of outcomes: word problems
- Description: This skill applies the fundamental counting principle (or systematic listing) to solve word problems that require finding the total number of outcomes for compound events. Students will interpret real-world scenarios and translate them into a structure for determining the total number of possibilities.
    - Core Example Questions:
        - A pizza shop offers 3 types of crust, 8 types of toppings, and 2 cheese options. How many different single-topping pizzas can be made?
        - You are choosing a password that must have 2 letters followed by 3 numbers. How many different passwords are possible if letters and numbers can be repeated?
        - How many different routes are there from City A to City C if there are 4 routes from A to B, and 3 routes from B to C?
- Difficulty: Application
- Common pitfalls:
    - Misinterpreting the problem, especially concerning repetition or specific constraints.
    - Setting up the multiplication incorrectly.
    - Failing to identify all the stages of the compound event.

- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 4
- Common Pitfall avoidance: 2
- Application/word problem: 6
- Challenging: 3
- Total number of questions for each goal: 16

15. Probability of compound events

- Name: Probability of compound events
- Description: This broad skill encompasses the calculation of probabilities for various compound events. It integrates the concepts of sample space, favorable outcomes, and the use of tools like tables or tree diagrams to determine the probability of multi-step scenarios. This often involves both identifying the total outcomes and the specific outcomes of interest.
    - Core Example Questions:
        - What is the probability of rolling a double (e.g., two 4s) when rolling two standard six-sided dice?
        - You have a bag with 3 red and 2 blue marbles. You pick one, don't replace it, then pick another. What is the probability of picking two red marbles?
        - A coin is flipped and a die is rolled. What is the probability of getting heads AND an even number?
- Difficulty: Procedural/Analytical
- Common pitfalls:
    - Incorrectly determining the total number of outcomes.
    - Not correctly identifying all favorable outcomes.
    - Errors in setting up or solving the probability fraction.
    - Confusing independent and dependent events (though this is explicitly covered in later skills, it's often a source of error here).
- Introduced definitions: None
- Concept / Introduction / Definition: 1
- Practice: 8
- Common Pitfall avoidance: 3
- Application/word problem: 3
- Challenging: 3
- Total number of questions for each goal: 18

16. Which simulation represents the situation?

- Name: Which simulation represents the situation?
- Description: This skill focuses on critical evaluation of simulations. Students will identify appropriate simulations to model given real-world chance situations, determining if a proposed simulation (e.g., using a coin, die, spinner, or random numbers) accurately reflects the probabilities and conditions of the event being modeled.
    - Core Example Questions:
        - A basketball player makes 60% of his shots. Which simulation method would be best to model his next shot: a coin flip, a 6-sided die, or a spinner with 10 sections? Explain.
        - You want to simulate the probability of a family having a boy and a girl. Which of these correctly models the situation: flipping two coins or rolling a die and assigning odd/even to boy/girl?
        - A company has a 1 in 5 chance of winning a contract. Which simulation is most appropriate: rolling a 5-sided die, or drawing a card from a deck of 10 cards numbered 1-10, where 1-2 wins?
- Difficulty: Conceptual/Application

- Common pitfalls:
    - Choosing a simulation that doesn't match the event's probability.
    - Not understanding how to map random outcomes to the event's outcomes.
    - Overlooking subtle details in the problem description (e.g., replacement vs. no replacement).
- Introduced definitions: None
- Concept / Introduction / Definition: 2
- Practice: 4
- Common Pitfall avoidance: 3
- Application/word problem: 4
- Challenging: 2
- Total number of questions for each goal: 15

17. Identify independent and dependent events

- Name: Identify independent and dependent events
- Description: This crucial skill introduces the distinction between independent events (where the outcome of one event does not affect the probability of the other) and dependent events (where the outcome of the first event changes the probability of the second). Understanding this difference is fundamental for calculating compound probabilities correctly.
    - Core Example Questions:
        - Are rolling a 6 on a die and flipping a coin and getting heads independent or dependent events? Explain.
        - You draw a card from a deck and don't replace it, then draw another card. Are these events independent or dependent? Why?
        - Your favorite team wins their first game, then plays a second game. Is the outcome of the second game dependent on the first?
- Difficulty: Conceptual
- Common pitfalls:
    - Confusing correlation with dependence.
    - Not understanding that "without replacement" implies dependence.
    - Making assumptions about events that are actually independent (e.g., previous coin flips affecting next).
- Introduced definitions: Independent events, Dependent events
- Concept / Introduction / Definition: 3
- Practice: 6
- Common Pitfall avoidance: 2
- Application/word problem: 4
- Challenging: 2
- Total number of questions for each goal: 17

18. Probability of independent and dependent events

- Name: Probability of independent and dependent events
- Description: This advanced skill focuses on calculating probabilities of independent and dependent compound events. Students will apply the multiplication rule for probability, understanding how to adjust the probability of the second event for dependent scenarios.
    - Core Example Questions:
        - What is the probability of flipping a coin twice and getting two heads in a row?
        - A bag contains 5 red and 5 blue marbles. You pick a marble, don't replace it, then pick another. What is the probability of picking two blue marbles?
        - What is the probability of rolling a 4 on a die and then rolling an even number on a second roll?
- Difficulty: Procedural/Analytical

- Common pitfalls:
    - Not distinguishing between independent and dependent problems.
    - Incorrectly adjusting probabilities for dependent events (e.g., forgetting to change the total number of outcomes or favorable outcomes after the first draw).
    - Arithmetic errors in multiplying probabilities.
    - Struggling with multi-stage events.
- Introduced definitions: Conditional probability (informal introduction)
- Concept / Introduction / Definition: 2
- Practice: 6
- Common Pitfall avoidance: 3
- Application/word problem: 4
- Challenging: 3
- Total number of questions for each goal: 18