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Tutorial 8

Q1 Construct PDA for $L = \{0^n 1^n \mid n \geq 0\}$

If $n = 0, 1, 2, 3 \ldots$

$L = \{\epsilon, 01, 0011, 000111, \ldots\}$

Step 1: Definition of PDA

$PDA \rightarrow (Q, \Sigma, q_0, F, Z_0, \delta)$

Step 2: Logic

For every '0' push 1X onto the stack

For every '1' pop 1X from the stack

Step 3: PDA

For 1st 0 $\delta(q_0, 0, Z_0) = (q_0, XZ_0)$

For remaining 0's $\delta(q_0, 0, X) = (q_0, XX)$

For 1st 1, $\delta(q_0, 1, XX) = (q_1, XZ_0)$

For remaining 1's $\delta(q_1, 1, XZ_0) = \delta(q_1, Z_0)$

$\delta(q_1, \epsilon, Z_0) = (q_f, Z_0)$

Assume 0011

$\delta(q_0, 0011, Z_0)$

$\Rightarrow \delta(q_0, 011, XZ_0)$

$\Rightarrow \delta(q_0, 11, XX)$

$\rightarrow \delta(q_1, 1, XZ_0)$

$\Rightarrow \delta(q_1, \epsilon, Z_0)$

$\Rightarrow (q_f, Z_0)$

Hence string is acceptable

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Q2 Define and design a DPDA accepting balanced string of brackets

$L = \{ '()', '[]', '\{\}', '\{[()]\}', \ldots \}$

Step 1: Definition of PDA

$PDA \rightarrow (Q, \Sigma, q_0, F, \Gamma, Z_0)$

Step 2: Logic

For every '(' push 1X onto the stack
For every '[' push 1Y onto the stack
For every '{' push 1P onto the stack
For every ')' pop 1X from the stack
For every ']' pop 1Y from the stack
For every '}' pop 1P from the stack

Step 3: PDA

For '(' $\delta(q_0, '(', Z_0) = (q_0, XZ_0)$
For '[' $\delta(q_0, '[', XZ_0) = (q_0, YX)$
For '{' $\delta(q_0, '\{', ZYX) = (q_0, PYX)$
For '}' $\delta(q_0, '\}', PYX) = (q_0, YX)$
For ']' $\delta(q_0, ']', YX) = (q_0, XZ_0)$
For ')' $\delta(q_0, ')', X) = (q_0, Z_0)$

$\delta(q_0, \epsilon, Z_0) = (q_2, Z_0)$

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Assume {[()]}

$\delta(q_0, \text{'\{[()]\}'}, Z_0)$
$\Rightarrow \delta(q_0, \text{'[()]\}'}, PZ_0)$
$\Rightarrow \delta(q_0, \text{'()]\}'}, YPZ_0)$
$\Rightarrow \delta(q_0, \text{')]\}'}, XYPZ_0)$
$\Rightarrow \delta(q_0, \text{']\}'}, YPZ_0)$
$\Rightarrow \delta(q_0, \text{'\}'}, PZ_0)$
$\Rightarrow \delta(q_0, \epsilon, Z_0)$
$\Rightarrow \delta(q_3, Z_0)$ Hence string is accepted

Q3 Design a PDA to accept string containing equal no. of 0's and 1's

$L = \{01, 10, 1010, 0011, 1100, \ldots\}$

Step 1: Definition of PDA
PDA $\rightarrow (Q, \Sigma, q_0, F, \Gamma, Z_0)$

Step 2: Logic
If 0 comes first then push X onto the stack
After 0 if again 0 comes push X onto the stack
After 0 if 1 comes, pop X from the stack

If 1 comes first then push Y onto the stack
After 1 if again 1 comes push Y onto the stack
If 0 comes then pop Y from the stack

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Step 3 : PDA

If 0 comes first

$\delta(q_0, 0, Z_0) = (q_0, X, Z_0)$

For remaining 0's $\delta(q_0, 0, X) = (q_0, XX)$

For any 1 $\delta(q_0, 1, X) = (q_0, Z_0)$

If 1 comes first $\delta(q_0, 1, Z_0) = q_0, YZ_0)$

For remaining 1's $\delta(q_0, 1, Y) = (q_0, YY)$

For any 0 $\delta(q_0, 1, Y) = (q_0, Z_0)$

$\delta(q_0, \epsilon, Z_0) = (q_f, Z_0)$

Assume 0101

$\delta(q_0, 0101, Z_0)$

$\Rightarrow \delta(q_0, 101, XZ_0)$

$\Rightarrow \delta(q_0, 01, Z_0)$

$\Rightarrow \delta(q_0, 1, XZ_0)$

$\Rightarrow \delta(q_0, \epsilon, Z_0)$

$\Rightarrow (q_f, Z_0)$

Hence string is accepted

Q4 Design a PDA to accept all the palindrome over $\{a, b, c\}$ of the form $W_c W^r$ and W over $\{a, n\}$

$L = \{abcba, abbcbba, bbcbb, bacab \dots\}$

Step 1: Definition of PDA

$PDA \longrightarrow (Q, \Sigma, q_0, F, \Gamma, Z_0)$

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Step 2: Logic
For each 'a' push 1X onto the stack
For each 'b' push 1Y onto the stack
When c comes go to the next state
For each a pop 1X from the stack
For each b pop 1Y from the stack

Step 3: PDA
For first a's $\delta(q_0, a, Z_0) = (q_0, XZ_0)$
For remaining a's $\delta(q_0, a, X) = (q_0, YXZ_0)$

For first b $\delta(q_0, b, Z_0) = (q_0, \cancel{B}, YZ_0)$
For remaining b's $\delta(q_0, b, Y) = (q_0, YYZ_0)$

When c comes $\delta(q_0, c, Z_0) = \delta(q_1, Z_0)$

For a after c $\delta(q_1, a, XZ_0) = \delta(q_1, Z_0)$
For b after c $\delta(q_1, b, YZ_0) = \delta(q_1, Z_0)$

$\delta(q_1, \epsilon, Z_0) = \delta(q_f, Z_0)$
Assume 'abcba'
$\delta(q_0, abcba, Z_0)$
$\Rightarrow \delta(q_0, bcba, XZ_0)$
$\Rightarrow \delta(q_0, cba, YXZ_0)$
$\Rightarrow \delta(q_1, ba, YXZ_0)$
$\Rightarrow \delta(q_1, a, XZ_0)$
$\Rightarrow \delta(q_1, \epsilon, Z_0)$
$\Rightarrow \delta(q_f, Z_0)$
Therefore, string is accepted

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Q5 Design a PDA to accept $(bdb)^n c^n$

$L = \{dbbc, bdbbdbcc, \ldots\}$

Step 1: Definition of PDA

PDA $\longrightarrow (Q, \Sigma, q_0, F, \Gamma, Z_0)$

Step 2: Logic

For every 'bdb' push 1 X into the stack

For every 'c' pop 1 X from the stack

Step 3: PDA

For 1st b $\delta(q_0, b, Z_0) = \delta(q_1, Z_0)$

For d after b $\delta(q_1, d, Z_0) = \delta(q_2, Z_0)$

For b after d $\delta(q_2, b, Z_0) = \delta(q_0, XZ_0)$

For c $\delta(q_0, c, XZ_0) = \delta(q_0, Z_0)$

$\delta(q_0, \epsilon, Z_0) = \delta(q_3, Z_0)$

Assume bdbc

$\delta(q_0, bdbc, Z_0)$

$\Rightarrow \delta(q_1, dbc, Z_0)$

$\Rightarrow \delta(q_2, bc, Z_0)$

$\Rightarrow \delta(q_0, c, XZ_0)$

$\Rightarrow \delta(q_0, \epsilon, Z_0)$

$\Rightarrow \delta(q_3, Z_0)$

$\therefore$ String is accepted

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Q6 Design a PDA to accept $a^n(bdb)^n$
$L = \{ abdb, aabdbbdb \dots \}$

Step 1: Definition of PDA
$PDA \rightarrow (Q, \Sigma, q_0, F, \Gamma, Z_0)$

Step 2: Logic
For every 'a' push 1X onto the stack
For every 'bdb' pop 1X from the stack

Step 3: PDA
For 1st a $\delta(q_0, a, Z_0) = (q_0, XZ_0)$
For remaining a's $\delta(q_0, a, XZ_0) = \delta(q_0, XX)$

For 1st b $\delta(q_0, b, XZ_0) = (q_1, XZ_0)$
For d after b $\delta(q_1, d, XZ_0) = \delta(q_2, XZ_0)$
For b after d $\delta(q_2, b, XZ_0) = \delta(q_0, Z_0)$
Assume abdb
$\delta(q_0, abdb, Z_0)$
$\Rightarrow \delta(q_0, bdb, XZ_0)$
$\Rightarrow \delta(q_1, db, XZ_0)$
$\Rightarrow \delta(q_2, b, XZ_0)$
$\Rightarrow \delta(q_0, \epsilon, Z_0)$
$\Rightarrow \delta(q_f, Z_0)$
String accepted

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Q7 Design an NPDA to accept an even palindrome of $\Sigma = \{a, b\}$

$L = \{abba, aa, baab, bbbb, abbbba, \ldots\}$

Logic:
For each 'a' push 1X onto the stack
For each 'b' push 1Y onto the stack
After Halfway
For each 'a' pop 1X from the stack
For each 'b' pop 1Y from the stack

Design PDA:
For 'a' $\delta(q_0, a, Z_0) = \delta(q_0, XZ_0)$
For remaining a's $\delta(q_0, a, XZ_0) = \delta(q_0, XX)$
For first b $\delta(q_0, b, Z_0) = \delta(q_0, XZ_0)$
For remaining b's $\delta(q_0, b, YZ_0) = \delta(q_0 YY)$
After halfway $\delta(q_0, \epsilon, Z_0) = \delta(q_1, \epsilon, Z_0)$
For remaining a's $\delta(q_1, a, XZ_0) = \delta(q_1, Z_0)$
For remaining b's $\delta(q_1, b, YZ_0) = \delta(q_1 Z_0)$

$\delta(q_1, \epsilon, Z_0) = \delta(q_3, Z_0)$
Assume abba
$\delta(q_0, abba, Z_0)$
$\Rightarrow \delta(q_0, bba, XZ_0)$
$\Rightarrow \delta(q_0, ba, YXZ_0)$
$\Rightarrow \delta(q_0, a, XZ_0)$
$\Rightarrow \delta(q_0, \epsilon, Z_0)$
$\Rightarrow \delta(q_3, Z_0)$

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Q8 Construct a PDA equivalent to the following grammar

$S \rightarrow aAA$

$A \rightarrow aS / bS / a$

The following grammar has no null production as unit production. It is also in CNF form

CFG	PDA
$G = \{V, T, P, S\}$	$M = (Q, \Sigma, F, S, q_0, I, Z_0)$
$V = \{S, A\}$	$I = \{S, A\}$
$T = \{a, b\}$	$\Sigma = \{a, b\}$
	$\delta(q, \epsilon, S) = (q, S)$
$S \rightarrow aAA$	$\delta(q, a, S) = (q, AA)$
$A \rightarrow aS$	$\delta(q, a, A) = (q, S)$
$A \rightarrow bS$	$\delta(q, b, A) = (q, S)$
$A \rightarrow a$	$\delta(q, a, A) = (q, \epsilon)$

Q10 PDA for the following grammar

$S \rightarrow aSa / bSb / a / b / \epsilon$

Removal of Null Production

Production	New Production
$S \rightarrow aSa$	$S \rightarrow aSa$
	$S \rightarrow aa$
$S \rightarrow bSb$	$S \rightarrow bSb$
	$S \rightarrow bb$
$S \rightarrow a$	
$S \rightarrow b$	

$S \rightarrow aSa / bSb / a / b / aa / bb$

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Converting into GNF form

Production	New Production
$S \to aSa$	$S \to aSS$
	$S \to a$
$S \to bSb$	$S \to bSS$
	$S \to b$
$S \to aa$	$S \to aS$
$S \to bb$	$S \to bS$
$S \to a$	
$S \to b$	

$S \to aSS, bSS, aS, bS, a, b$

CFG	PDA
$G = \{V, T, P, S\}$	$M = \{Q, \Sigma, F, \delta, q_0, I, Z$
$V = \{S\}$	$I = \{S\}$
$T = \{a, b\}$	$\Sigma = \{a, b\}$
	$\delta(q, \epsilon, S) = (q, S)$
$S \to aSS$	$\delta(q, a, S) = (q, SS)$
$S \to bSS$	$\delta(q, b, S) = (q, SS)$
$S \to aS$	$\delta(q, a, S) = (q, S)$
$S \to bS$	$\delta(q, b, S) = (q, S)$
$S \to a$	$\delta(q, a, S) = (q, \epsilon)$
$S \to b$	$\delta(q, b, S) = (q, \epsilon)$