# Two Dimensional Phase Retrieval from Local Measurements

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## ABSTRACT

2D or not 2D, that is the tribe called question

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## 1. INTRODUCTION

In this paper we consider the problem of approximately recovering an unknown two dimensional sample transmission function $q : \mathbb{R}^2 \to \mathbb{C}$ with compact support, $\text{supp}(q) \subset [0,1]^2$, from phaseless Fourier measurements of the form

$$\left|\left(\mathcal{F}\left[aS_{x_0,y_0}q\right]\right)(u,v)\right|^2, \quad (u,v) \in \Omega \subset \mathbb{R}^2, \quad (x_0,y_0) \in \mathcal{L} \subset [0,1]^2 \tag{1}$$

where $\mathcal{F}$ denotes the 2 dimensional Fourier transform, $a : \mathbb{R}^2 \to \mathbb{C}$ is a known illumination function from an illuminating beam, $S_{x_0,y_0}$ is a shift operator defined by $\left(S_{x_0,y_0}q\right)(x,y) := q(x-x_0, y-y_0)$, $\Omega$ is a finite set of sampled frequencies, and $\mathcal{L}$ is a finite set of shifts. When the illuminating beam is sharply focussed one can further assume that $a$ is also (effectively) compactly supported within a smaller region $[0,\delta']^2$ for $\delta' \ll 1$. This is known as the *ptychographic imaging problem* and is of great interest in the physics community (see, e.g., Rodenburg[1]). Herein we will make the further assumption that all the utilized shifts of $q$ also have their supports contained in $[0,1]^2$. That is, that

$$\bigcup_{(x_0,y_0)\in\mathcal{L}} \text{supp}\left(S_{x_0,y_0}q\right) \subseteq [0,1]^2$$

holds. Note that an analogous assumption can always be achieved by dilating $[0,1]^2$.

Discretizing (1) using periodic boundary conditions we obtain a finite dimensional problem aimed at recovering an unknown matrix $Q \in \mathbb{C}^{d\times d}$ from phaseless measurements of the form

$$\left|\frac{1}{d^2}\sum_{j=1}^{d}\sum_{k=1}^{d} A_{j,k}\left(S_\ell Q S_{\ell'}^*\right)_{j,k} \mathbb{e}^{\frac{-2\pi \mathbb{i}}{d}(ju+kv)}\right|^2 \tag{2}$$

where $A \in \mathbb{C}^{d\times d}$ is a known measurement matrix representing our illuminating beam, and $S_\ell \in \mathbb{R}^{d\times d}$ is the discrete circular shift operator defined by $(S_\ell \mathbf{x})_j := x_{j-\ell \text{ mod } d}$ for all $\mathbf{x} \in \mathbb{C}^d$ and $j,\ell \in [d] := \{1,\ldots,d\}$. Herein we will make the simplifying assumption that our original illuminating beam funciton $a$ is not only sharply focused, but also separable. Using this assumption we let weighted measurement matrix be $\frac{1}{d^2}A := \mathbf{ab}^*$ where $\mathbf{a},\mathbf{b} \in \mathbb{C}^d$ both have $a_j = b_j = 0$ for all $j \in [d]\setminus\{1,\ldots,\delta\}$. Here $\delta \in \mathbb{Z}^+$ is much smaller than $d$.

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Using the small support and separability of $\frac{1}{d^2}A := \mathbf{a}\mathbf{b}^*$ we can now rewrite the measurements (2) as

$$\begin{aligned}\left|\sum_{j=1}^{\delta}\sum_{k=1}^{\delta} a_j\overline{b_k}\left(S_\ell Q S_{\ell'}^*\right)_{j,k} \mathbb{e}^{\frac{-2\pi\mathbb{i}}{d}(ju+kv)}\right|^2 &= \left|\sum_{j=1}^{\delta}\sum_{k=1}^{\delta} \overline{\overline{a_j}\mathbb{e}^{\frac{2\pi\mathbb{i}ju}{d}} b_k \mathbb{e}^{\frac{2\pi\mathbb{i}kv}{d}}}\left(S_\ell Q S_{\ell'}^*\right)_{j,k}\right|^2 \\ &= \left|\langle S_\ell Q S_{\ell'}^*, \mathbf{a}_u\mathbf{b}_v^*\rangle_{\mathrm{HS}}\right|^2 \end{aligned} \tag{3}$$

where $\mathbf{a}_u, \mathbf{b}_v \in \mathbb{C}^d$ are defined by $(a_u)_j := \overline{\mathbb{e}^{\frac{-2\pi\mathbb{i}ju}{d}} a_j}$ and $(b_v)_k := \overline{\mathbb{e}^{\frac{2\pi\mathbb{i}kv}{d}} b_k}$ for all $j,k \in [d]$. Continuing to rewrite (3) we can now see that our discretized measurements will all take the form of

$$\left|\langle S_\ell Q S_{\ell'}^*, \mathbf{a}_u\mathbf{b}_v^*\rangle_{\mathrm{HS}}\right|^2 = \left|\mathrm{Trace}\left(\mathbf{b}_v\mathbf{a}_u^* S_\ell Q S_{\ell'}^*\right)\right|^2 = \left|\mathrm{Trace}\left(S_{\ell'}^*\mathbf{b}_v\left(S_\ell^*\mathbf{a}_u\right)^* Q\right)\right|^2 = \left|\left\langle Q, S_\ell^*\mathbf{a}_u\left(S_{\ell'}^*\mathbf{b}_v\right)^*\right\rangle_{\mathrm{HS}}\right|^2 \tag{4}$$

for a finite set of frequencies $(u,v) \in \Omega \subset \mathbb{R}^2$ and shifts $(\ell,\ell') \in \mathcal{L} \subseteq [d]\times[d]$.

Motivated by ptychographic imaging we propose a new efficient numerical scheme for solving general discrete phase retrieval problems using measurements of type (4) herein. We will next outline our proposed method in §2 below. A preliminary numerical evaluation of the method is then presented in §3.

## 2. AN EFFICIENT METHOD FOR SOLVING THE DISCRETE 2D PHASE RETRIEVAL PROBLEM

In this section we present a lifted formulation[2] of the discrete 2D phase retrieval problem from local measurements of type (4). We then employ this lifted formulation to rapidly solve for $Q \in \mathbb{C}^{d\times d}$ using a modified variant of the BlockPR agorithm.[3,4] More specifically, we will consider the collection of measurements given by

$$y_{(\ell,\ell',u,v)} := \left|\left\langle Q, S_\ell^*\mathbf{a}_u\left(S_{\ell'}^*\mathbf{b}_v\right)^*\right\rangle_{\mathrm{HS}}\right|^2 \tag{5}$$

for all $(\ell,\ell',u,v) \in [d]^2\times\Omega^2$ where $\Omega \subset [d]$ has $|\Omega| = 2\delta-1$. Thus, we collect a total of $D := (2\delta-1)^2\cdot d^2$ measurements where each measurement is due to a vertical and horizontal shift of a rank one illumination pattern $\mathbf{a}_u\mathbf{b}_v^* \in \mathbb{C}^{d\times d}$. As above, the $\langle\cdot,\cdot\rangle_{\mathrm{HS}}$-inner product denotes the Hilbert-Schmidt inner product, and we assume that our measurements are *local* so that $(a_u)_j = (b_v)_j = 0$ for all $j \in [d]\setminus[\delta]$ and all $(u,v) \in \Omega^2$.* Recall that $\delta \ll d$ so that the total number of measurements $D$ is essentially linear in the problem size.

Toward our lifted formulation of the problem we can see that

$$\begin{aligned} y_{(\ell,\ell',u,v)} = \left|\left\langle Q, S_\ell^*\mathbf{a}_u\left(S_{\ell'}^*\mathbf{b}_v\right)^*\right\rangle_{\mathrm{HS}}\right|^2 &= \left|\mathrm{Trace}\left(S_{\ell'}^*\mathbf{b}_v\left(S_\ell^*\mathbf{a}_u\right)^* Q\right)\right|^2 = \left|\left(S_\ell^*\mathbf{a}_u\right)^* Q S_{\ell'}^*\mathbf{b}_v\right|^2 \\ &= \left|\langle S_\ell^*\overline{\mathbf{a}_u}\otimes S_{\ell'}^*\mathbf{b}_v, \mathrm{vec}\left(Q^*\right)\rangle\right|^2 \end{aligned}$$

where $\otimes$ denotes the Kronecker product so that $S_\ell^*\overline{\mathbf{a}_u}\otimes S_{\ell'}^*\mathbf{b}_v \in \mathbb{C}^{d^2}$, and $\mathrm{vec}(\cdot)$ denotes column-wise vectorization of a given matrix so that

$$\mathrm{vec}\left(Q^*\right) = \left(q_{1,1},\ldots,\ q_{1,d},\ q_{2,1},\ldots,\ q_{2,d},\ \ldots,\ q_{d,1},\ldots,\ q_{d,d}\right)^* \in \mathbb{C}^{d^2}.$$

Continuing, we can further see that

$$\begin{aligned} y_{(\ell,\ell',u,v)} &= \langle S_\ell^*\overline{\mathbf{a}_u}\otimes S_{\ell'}^*\mathbf{b}_v, \mathrm{vec}\left(Q^*\right)\rangle\,\overline{\langle S_\ell^*\overline{\mathbf{a}_u}\otimes S_{\ell'}^*\mathbf{b}_v, \mathrm{vec}\left(Q^*\right)\rangle} \\ &= \left(\mathrm{vec}\left(Q^*\right)\right)^* S_\ell^*\overline{\mathbf{a}_u}\otimes S_{\ell'}^*\mathbf{b}_v\left(S_\ell^*\overline{\mathbf{a}_u}\otimes S_{\ell'}^*\mathbf{b}_v\right)^*\mathrm{vec}\left(Q^*\right) \\ &= \mathrm{Trace}\left(\mathrm{vec}\left(Q^*\right)\left(\mathrm{vec}\left(Q^*\right)\right)^* S_\ell^*\overline{\mathbf{a}_u}\otimes S_{\ell'}^*\mathbf{b}_v\left(S_\ell^*\overline{\mathbf{a}_u}\otimes S_{\ell'}^*\mathbf{b}_v\right)^*\right) \\ &= \left\langle \mathrm{vec}\left(Q^*\right)\left(\mathrm{vec}\left(Q^*\right)\right)^*,\ S_\ell^*\overline{\mathbf{a}_u}\otimes S_{\ell'}^*\mathbf{b}_v\left(S_\ell^*\overline{\mathbf{a}_u}\otimes S_{\ell'}^*\mathbf{b}_v\right)^*\right\rangle_{\mathrm{HS}}. \end{aligned}$$

*For any $n \in \mathbb{Z}^+$ we will define $[n] := \{1,2,3,\ldots,n\} \subset \mathbb{Z}^+$.

Let $\mathcal{M} : \mathbb{C}^{d^2 \times d^2} \mapsto \mathbb{R}^D$ be the linear measurement operator defined by

$$(\mathcal{M}(Z))_{(\ell,\ell',u,v)} := \left\langle Z,\ S_\ell^* \overline{\mathbf{a}_u} \otimes S_{\ell'}^* \mathbf{b}_v \left(S_\ell^* \overline{\mathbf{a}_u} \otimes S_{\ell'}^* \mathbf{b}_v\right)^* \right\rangle_{\mathrm{HS}}. \tag{6}$$

We can now see that our measurements $\mathbf{y}$ defined in (5) result from $\mathcal{M}\left(\mathrm{vec}\,(Q^*)\,(\mathrm{vec}\,(Q^*))^*\right)$. This linearized relationship between $\mathbf{y}$ and $\mathrm{vec}\,(Q^*)\,(\mathrm{vec}\,(Q^*))^*$ forms the basis of our lifted problem formulation. Our new objective is to solve for $\mathrm{vec}\,(Q^*)\,(\mathrm{vec}\,(Q^*))^*$ using our measurements $\mathbf{y}$ by inverting the linear measurement operator $\mathcal{M}$. Once we have solved for $\mathrm{vec}\,(Q^*)\,(\mathrm{vec}\,(Q^*))^*$ we can then find its principal eigenvector in order to compute $\mathrm{vec}\,(Q^*)$ (and therefore $Q$) up to a global phase multiple.

### 2.1 Inverting the Linear Measurement Operator $\mathcal{M}$

Recall that $D = (2\delta - 1)^2 \cdot d^2 \ll d^4$ from above so that our number of measurements (6) is severely underdetermined for arbitrary $Z \in \mathbb{C}^{d^2 \times d^2}$. Define $E_{j,k} \in \mathbb{C}^{d \times d}$ by

$$\left(E_{j,k}\right)_{h,l} = \begin{cases} 1, & (j,k) = (h,l) \\ 0, & \text{otherwise} \end{cases}.$$

Toward circumventing the generally underdetermined nature of our measurements we observe that the local supports of both $\mathbf{a}_u$ and $\mathbf{b}_v$ ensure that $\mathcal{M}\left(\mathrm{vec}\,(E_{j,k})\,(\mathrm{vec}\,(E_{j',k'}))^*\right) = \mathbf{0}$ whenever either $|j - j'| \geq \delta$ or $|k - k'| \geq \delta$ holds. As we result we can see that $\mathcal{M}\left(\mathcal{P}\left(Z\right)\right) = \mathcal{M}\left(Z\right)$ holds for all $Z \in \mathbb{C}^{d^2 \times d^2}$ where $\mathcal{P} : \mathbb{C}^{d^2 \times d^2} \mapsto \mathbb{C}^{d^2 \times d^2}$ is the orthogonal projector onto the span of $\mathcal{B} := \left\{\mathrm{vec}\,(E_{j,k})\,(\mathrm{vec}\,(E_{j',k'}))^* \;\middle|\; |j - j'| < \delta, |k - k'| < \delta\right\}$.[†] Furthermore, the dimension of $\mathcal{P}\left(\mathbb{C}^{d^2 \times d^2}\right) = \mathrm{span}(\mathcal{B})$ is $D$ by construction. As a result, it is conceivable that the composition of $\mathcal{M}$ and $\mathcal{P}$ restricted to $\mathcal{P}\left(\mathbb{C}^{d^2 \times d^2}\right)$, $\mathcal{M}\big|_{\mathcal{P}} : \mathrm{span}(\mathcal{B}) \mapsto \mathbb{R}^D$, is invertible on $\mathrm{span}(\mathcal{B})$. Indeed, numerical experiments indicate this turns out to be the case for many different choices of local pairs $\left\{(\mathbf{a}_u, \mathbf{b}_v) \;\middle|\; (u,v) \in \Omega^2\right\} \subset \mathbb{C}^d \times \mathbb{C}^d$ as long as $|\Omega| \geq 2\delta - 1$.

### 2.2 Computing the Phases of the Entries of $\mathrm{vec}\,(Q^*)$ after Inverting $\mathcal{M}\big|_{\mathcal{P}}$

Assuming that $\mathcal{M}\big|_{\mathcal{P}}$ is invertible so that we can recover $\mathcal{P}\left(\mathrm{vec}\,(Q^*)\,(\mathrm{vec}\,(Q^*))^*\right)$ from our measurements $\mathbf{y}$, we are still left with the problem of how to recover $\mathrm{vec}\,(Q^*)$ from $\mathcal{P}\left(\mathrm{vec}\,(Q^*)\,(\mathrm{vec}\,(Q^*))^*\right)$. Our first step in solving for $\mathrm{vec}\,(Q^*)$ will be to compute all the phases of the entries of $\mathrm{vec}\,(Q^*)$ from $\mathcal{P}\left(\mathrm{vec}\,(Q^*)\,(\mathrm{vec}\,(Q^*))^*\right)$. Thankfully, this can be solved as an angular synchronization problem[5] using the variant utilized by BlockPR.[4,6] Let $\mathbb{1} \in \mathbb{C}^{d^2 \times d^2}$ be the vector of all ones, and $\mathrm{sgn} : \mathbb{C} \mapsto \mathbb{C}$ be

$$\mathrm{sgn}(z) = \begin{cases} \dfrac{z}{|z|}, & z \neq 0 \\ 1, & \text{otherwise} \end{cases}.$$

We now define $\tilde{Q} \in \mathbb{C}^{d^2 \times d^2}$ by

$$\tilde{Q}_{j,k} := \begin{cases} \mathrm{sgn}\left(\left[\mathcal{P}\left(\mathrm{vec}\,(Q^*)\,(\mathrm{vec}\,(Q^*))^*\right)\right]_{j,k}\right), & \left[\mathcal{P}\left(\mathbb{1}\mathbb{1}^*\right)\right]_{j,k} \neq 0 \\ 0, & \text{otherwise} \end{cases}. \tag{7}$$

As we shall see, the principal eigenvector of $\tilde{Q}$ will provide us with all of the phases of the entries of $\mathrm{vec}\,(Q^*)$.

Working toward that goal we may note that

$$\tilde{Q} = \mathrm{diag}\left(\mathrm{sgn}\left(\mathrm{vec}\,(Q^*)\right)\right)\mathcal{P}\left(\mathbb{1}\mathbb{1}^*\right)\mathrm{diag}\left(\overline{\mathrm{sgn}\left(\mathrm{vec}\,(Q^*)\right)}\right) \tag{8}$$

[†]Note that $\mathcal{P}$ can also be described as a restriction operator onto the indices associated with the elements of $\mathcal{B}$. Our periodic boundary conditions also imply that, e.g., $|j - j'| < \delta \Leftrightarrow \exists h \in \mathbb{Z}$ with $|h| < \delta$ s.t. $j' + h \equiv j \bmod d$.

where sgn is applied component-wise to vectors, and where $\operatorname{diag}(\mathbf{x}) \in \mathbb{C}^{d^2 \times d^2}$ is diagonal with $(\operatorname{diag}(\mathbf{x}))_{j,j} := x_j$ for all $\mathbf{x} \in \mathbb{C}^{d^2}$ and $j \in [d^2]$. After noting that both $\operatorname{diag}(\operatorname{sgn}(\operatorname{vec}(Q^*)))$ and $\operatorname{diag}\left(\overline{\operatorname{sgn}(\operatorname{vec}(Q^*))}\right)$ are unitary diagonal matrixes we can further see that the spectral structure of $\tilde{Q}$ is primarily determined by $\mathcal{P}(\mathbb{1}\mathbb{1}^*)$. The following theorem completely characterizes the eigenvalues and eigenvectors of $\mathcal{P}(\mathbb{1}\mathbb{1}^*)$.

THEOREM 1. *Let $F \in \mathbb{C}^{d\times d}$ be the unitary discrete Fourier transform matrix with $F_{j,k} := \frac{1}{\sqrt{d}} \mathbb{e}^{2\pi \mathbb{i} \frac{(j-1)(k-1)}{d}}$ $\forall j,k \in [d]$, and let $D \in \mathbb{C}^{d\times d}$ be the diagonal matrix with $D_{j,j} = 1 + 2\sum_{k=1}^{\delta-1} \cos\left(\frac{2\pi(j-1)k}{d}\right)$ $\forall j \in [d]$. Then,*

$$\mathcal{P}(\mathbb{1}\mathbb{1}^*) = (F\otimes F)(D\otimes D)(F\otimes F)^*.$$

*In particular, the principal eigenvector of $\mathcal{P}(\mathbb{1}\mathbb{1}^*)$ is $\mathbb{1}$ and its associated eigenvector is $(2\delta-1)^2$.*

*Proof.* From the definition of $\mathcal{P}$ we have that

$$\mathcal{P}(\mathbb{1}\mathbb{1}^*) = \sum_{j=1}^{d} \sum_{|j-j'|<\delta} \sum_{k=1}^{d} \sum_{|k-k'|<\delta} \operatorname{vec}(E_{j,k})(\operatorname{vec}(E_{j',k'}))^*$$

where the second and fourth sums are over the $j',k' \in [d]$ that are within $\delta$ of $j,k \in [d]$ modulo $d$. Let $\mathbf{e}_j \in \mathbb{C}^d$ be the standard basis vector with

$$(e_j)_k = \begin{cases} 1, & k=j \\ 0, & \text{otherwise.} \end{cases}$$

for all $j,k \in [d]$. Using standard properties of the Kronecker product (see, e.g., Horn and Johnson[7]) one can see that

$$\operatorname{vec}(E_{j,k})(\operatorname{vec}(E_{j',k'}))^* = \mathbf{e}_k \otimes \mathbf{e}_j (\mathbf{e}_{k'} \otimes \mathbf{e}_{j'})^* = \mathbf{e}_k \mathbf{e}_{k'}^* \otimes \mathbf{e}_j \mathbf{e}_{j'}^* = E_{k,k'} \otimes E_{j,j'}.$$

As a consequence we now have that

$$\mathcal{P}(\mathbb{1}\mathbb{1}^*) = \sum_{j=1}^{d} \sum_{|j-j'|<\delta} \sum_{k=1}^{d} \sum_{|k-k'|<\delta} E_{k,k'} \otimes E_{j,j'} = \left(\sum_{k=1}^{d} \sum_{|k-k'|<\delta} E_{k,k'}\right) \otimes \left(\sum_{j=1}^{d} \sum_{|j-j'|<\delta} E_{j,j'}\right). \quad (9)$$

Let $T_\delta \in \mathbb{C}^{d\times d}$ be the matrix with entries

$$(T_\delta)_{j,k} = \begin{cases} 1, & |j-k| \mod d < \delta \\ 0, & \text{otherwise.} \end{cases}.$$

Using the definition of $T_\delta$ together with (9) we get that $\mathcal{P}(\mathbb{1}\mathbb{1}^*) = T_\delta \otimes T_\delta$. Thankfully the eigenvectors and eigenvalues of $T_\delta$ are known (see Lemma 1 of Iwen, Preskitt, Saab, and Viswanathan[4]). In particular, $T_\delta = FDF^*$ which then yields the desired result by Theorem 4.2.12 of Horn and Johnson.[7] ∎

Theorem 1 in combination with (8) makes it clear that $\operatorname{sgn}(\operatorname{vec}(Q^*))$ will be the principal eigenvector of $\tilde{Q}$. As a result, we can rapidly compute the phases of all the entries of $\operatorname{vec}(Q^*)$ by using, e.g., a shifted inverse power method[8] in order compute the eigenvector of $\tilde{Q}$ corresponding to the eigenvalue $(2\delta-1)^2$.

## 2.3 Computing the Magnitudes of the Entries of $\operatorname{vec}(Q^*)$ after Inverting $\mathcal{M}\big|_{\mathcal{P}}$

Having found the phases of each entry of $\operatorname{vec}(Q^*)$ using $\mathcal{P}\left(\operatorname{vec}(Q^*)(\operatorname{vec}(Q^*))^*\right)$ it only remains to find each entry's magnitude as well. This is comparably easy to achieve. Note that the set $\mathcal{B}$ above always contains $\operatorname{vec}(E_{j,k})(\operatorname{vec}(E_{j,k}))^*$ for all $j,k \in [d]$. As a result, $\mathcal{P}\left(\operatorname{vec}(Q^*)(\operatorname{vec}(Q^*))^*\right)$ is guaranteed to always provide the diagonal entries of $\operatorname{vec}(Q^*)(\operatorname{vec}(Q^*))^*$ for all $\delta \geq 1$. And, the diagonal entries of $\operatorname{vec}(Q^*)(\operatorname{vec}(Q^*))^*$ are exactly the squared magnitudes of each entry in $\operatorname{vec}(Q^*)$. Combined with the phase information recovered above in §2.2 we are finally able to reconstruct every entry of $\operatorname{vec}(Q^*)$ up to a global phase. See Algorithm 1 for complete pseudocode.

**Algorithm 1** Two Dimensional Phase Retrieval from Local Measurements

**Input:** Measurements $\mathbf{y} \in \mathbb{R}^D$ as per (5)

**Output:** $X \in \mathbb{C}^{d \times d}$ with $X \approx \mathbb{e}^{-\mathbb{i}\theta} Q$ for some $\theta \in [0, 2\pi]$

1: Compute the Hermitian matrix $P = \left( \left( \mathcal{M}\big|_{\mathcal{P}} \right)^{-1} \mathbf{y} \right)/2 + \left( \left( \mathcal{M}\big|_{\mathcal{P}} \right)^{-1} \mathbf{y} \right)^{*}/2 \in \mathcal{P}\left( \mathbb{C}^{d^2 \times d^2} \right)$ as an estimate of $\mathcal{P}\left( \mathrm{vec}\left( Q^* \right) \left( \mathrm{vec}\left( Q^* \right) \right)^* \right)$.

2: Form the matrix of phases, $\tilde{P} \in \mathcal{P}\left( \mathbb{C}^{d^2 \times d^2} \right)$, by normalizing the non-zero entries of $P$ as per (7). We expect that $\tilde{P} \approx \tilde{Q}$.

3: Compute the principal eigenvector of $\tilde{P}$ and use it to compute $U_{j,k} \approx \mathrm{sgn}\left( Q_{j,k} \right)\ \forall j, k \in [d]$ as per §2.2.

4: Use the diagonal entries of $P$ to compute $M_{j,k} \approx \left| Q_{j,k} \right|^2$ for all $j, k \in [d]$ as per §2.3.

5: Set $X_{j,k} = \sqrt{M_{j,k}} \cdot U_{j,k}$ for all $j, k \in [d]$ to form $X$

## 3. NUMERICAL EVALUATION

We will now demonstrate the efficiency and robustness of Algorithm 1.


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