Bernoulli random variables:

- **Experiment:** Any experiment with two outcomes, success and failures:

$$X = \begin{cases} 0 & \text{if outcome is a failures} \\ 1 & \text{if outcome is a success} \end{cases}$$

- **Parameters:** $p =$ probability of success
- $\mathfrak{X} = \mathrm{Range}(X) = \{0, 1\}$
- **Probability mass function:**

$$P(X = x) = p^x (1-p)^{1-x}, \quad x = 0, 1.$$

- **Expected value, variance**

$$\mathrm{E}(X) = p, \quad \mathrm{Var}(X) = p(1-p).$$

Binomial random variables:

- **Experiment:** Repeat n independent Bernoulli trials, each with success probability p

$$X = \text{ \# of success in } n \text{ trials.}$$

- **Parameters:** $n =$ # of trials $p =$ probability of success in a single Bernoulli trial
- $\mathfrak{X} = \mathrm{Range}(X) = \{0, 1, 2, \ldots, n-1, n\}$
- **Probability mass function:**

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \ldots, n.$$

- **Expected value, variance**

$$\mathrm{E}(X) = np, \quad \mathrm{Var}(X) = np(1-p).$$

Geometric random variables:

- **Experiment:** Repeat Bernoulli trials (independently) until first success.

$$X = \text{ \# of failures before first sucess.}$$

- **Parameters:** $p =$ probability of success in a single Bernoulli trial
- $\mathfrak{X} = \mathrm{Range}(X) = \{0, 1, 2, 3, \dots\}$
- **Probability mass function:**

$$P(X = x) = p(1-p)^x, \quad x = 0, 1, 2, \dots.$$

- **Expected value, variance**

$$\mathrm{E}(X) = \frac{1-p}{p}, \quad \mathrm{Var}(X) = \frac{1-p}{p^2}.$$

Negative binomial random variables:

- **Experiment:** Repeat Bernoulli trials (independently) until the rth success.

$$X = \text{ \# of failures before the } r\text{th sucess.}$$

- **Parameters:** $r =$ number of successes after which we terminate the sequence of Bernoulli trials, $p =$ probability of success in a single Bernoulli trial
- $\mathfrak{X} = \mathrm{Range}(X) = \{0, 1, 2, 3, \dots\}$
- **Probability mass function:**

$$P(X = x) = \binom{x+r-1}{x} p^r (1-p)^x, \quad x = 0, 1, 2, \dots.$$

- **Expected value, variance**

$$\mathrm{E}(X) = \frac{r(1-p)}{p}, \quad \mathrm{Var}(X) = \frac{r(1-p)}{p^2}.$$

Poisson random variable:

- **Experiment:** The random variable arises from experiments which can be approximated by Binomial experiments with large n, small p and $np \to \lambda > 0$. It is generally used to model the number of times a certain event occurs in a given time frame or a given area.
- **Parameters:** $\lambda > 0$, referred to as the "rate" parameter.
- $\mathfrak{X} = \mathrm{Range}(X) = \{0, 1, 2, 3, \dots\}$
- **Probability mass function:**

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, \dots.$$

- **Expected value, variance**

$$\mathrm{E}(X) = \lambda, \quad \mathrm{Var}(X) = \lambda.$$

Hypergeometric random variable:

- **Experiment:** Draw n objects from N objects of two types (I and II). The objects are drawn without replacement.
- $X =$ # of objects of Type I.
- **Parameters:**

  $N =$ total number of objects

  $n =$ number of objects drawn

  $k =$ total number of objects of Type I
- $\mathfrak{X} = \mathrm{Range}(X) = \{0, 1, 2, \ldots, k\}$
- **Probability mass function:**

$$P(X = x) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}, \quad x = 0, 1, 2, \ldots, k.$$

- **Expected value, variance**

$$\mathrm{E}(X) = \frac{nk}{N}, \quad \mathrm{Var}(X) = n\left(\frac{k}{N}\right)\left(1 - \frac{k}{N}\right)\left(\frac{N-n}{N-1}\right).$$