HOMEWORK 01, STAT 251

(1) Write down the statistical model for each of the following inference procedures that were introduced in stat 121. For each model, derive the likelihood.
  (a) one-sample t-test based on the sample $y_1, \ldots, y_n$
  (b) independent two-sample t-test based on sample $y_{1i}, \ldots, y_{1n_1}$ from population one and $y_{2i}, \ldots, y_{2n_2}$ from population two.
  (c) one-sample test of proportions based on sample $y_1, \ldots, y_n$ where each realization comes from m trials.

		Progesterone	
		Positive	Negative
Estrogen	Positive	13/28	4/28
	Negative	2/28	9/28

Table 1

(2) Table 1 gives proportions of estrogen- and progesterone-receptor status for 25 tumors. (The proportions are from fictional data, but we'll suppose it was authentic). Let A be the event that a tumor is estrogen-receptor positive. Let B be the event that it is progesterone-receptor positive.
  (a) What is $P(B)$?
  (b) What is $P(A|B)$?
  (c) What is $P(A|B^C)$?
  (d) Use law of total probability to find $P(A)$.
  (e) Use Bayes' rule and your calculations of $P(A|B)$, $P(A|B^c)$, and $P(B)$ to find $P(B|A)$. [In this instance, you can check your answer by calculating $P(B|A)$ directly from the table, and you should do so.]

(3) A young man was diagnosed as having a disease that occurs extremely rarely in young people. Naturally, he was upset. A wise statistician told him it was probably a mistake. She reasoned as follows: No medical test is perfect—there are always incidences of false positives and false negatives. Let D stand for the event that he has the disease and let $+$ stand for the event that an individual responds positively to the test. Assume $P(D) = 1/100,000 = .00001$. (So only one person per one hundred thousand his age has the disease). Also assume $P(+|D) = .992$, and $P(+|D^c) = .011$. (The test is very good compared to many medical tests, giving only 0.8% false negatives and 1.1% false positives.) Find the probability that he has the disease given that he has a positive test response. (After you make this calculation, you will not be surprised to learn that he did not have the disease.)

(4) Using a standard 52 card deck (13 cards of each suit), what is the probability of selecting 5 cards of the same suit (a flush) without replacement? (Hint: The cards could all be spades, or they could all be clubs, or they could all be diamonds, or they could all be hearts, and it would still count as a flush).