HOMEWORK 4, STAT 251

Do the following using R. You must also turn in a copy of your R code.

(1) Plot the $Binomial(n = 47, \theta = 0.8)$ pmf. Make sure the plot is properly labeled.

(2) What is the expected value of a Binomial(n=47, $\theta = 0.8$) random variable?

(3) What is the standard deviation of a $Binomial(n = 47, \theta = 0.8)$ random variable?

The next four problems are interrelated.

(4) Following are data from a clinical trial involving chemotherapy for testicular cancer. It is considered a success if a patient survived five years past the clinical trial. 3 subjects survived for five years past the trial, while 8 did not. What is the value of the (binomial) likelihood that 3 successes occur in 11 trials if θ (the probability of survival) is 0.5?

(5) Refer to question 4. What is the value of the likelihood if θ is 0.31?

(6) Refer to question 4. What is the value of the likelihood if θ is 0.27?

(7) Refer to question 4. Justify why you cannot find a value for θ that makes the likelihood larger than when $\theta = 3/11$.

(8) Plot a beta (3, 5) pdf and a beta (5, 11) pdf on the same graph; the beta(3,5) should be in black and the beta (5,11) in gray. Also, be sure to properly format this graphic (x- and y-axes, overall title, and a legend).

(9) What is the height of the curve of a beta (1, 8) distribution at x = .24? That is, what is the pdf evaluated at x=.24?

(10) What is the probability a beta (1, 8) random variable is less than 0.13?

(11) What is the probability a beta (3, 9) random variable is greater than .4?

(12) What is the probability a beta (18,4.4) random variable is between 0.6 and 0.7?

(13) At what value of x is the probability that a beta (4, 7) random varable is less than x equal to .71? That is, for what x is $Pr(beta(4, 7) \text{ random variable } < x) = .71$?

(14) At what value of x is the probability that a beta (12.2, 25.7) random variable is less than x equal to .2? That is, for what x is $Pr(beta(12.2, 25.7) \text{ random variable } < x) = .2$?

(15) What is the expected value of a beta (3.1,4.8) random variable?

(16) What is the variance of a beta (3, 5) random variable?

(17) What is the mode of a beta(2.8, 2.1) random variable?

(18) Show that the mode of a beta(a,b) random variable is $\frac{a-1}{a+b-2}$ when $a > 1$ and $a + b > 2$.

(19) Refer again to question 4. In general terms, what is the posterior distribution for θ given the results of the clinical trial described in question 4 and assuming a beta(a,b) prior distribution for θ.

(20) Refer again to question 4. For this problem and assuming a beta(a,b) prior, what is $f(y) = \int_0^1 f(y|\theta)\pi(\theta)d\theta$?

(21) Refer again to question 4. Create one figure with two plots (i.e., use `par(mfrow=c(1,2))`). Each plot will contain two curves, a red one representing the prior distribution of θ and a blue representing the posterior distribution of θ. For the left plot, use a beta(1,1) prior distribution. Comment on what this prior distribution implies regarding prior beliefs of the analyst and the impact it has on the posterior. For the right plot, use a beta(25,1) prior distribution. Comment on what this prior distribution implies regarding prior beliefs of the analyst and the impact it has on the posterior.