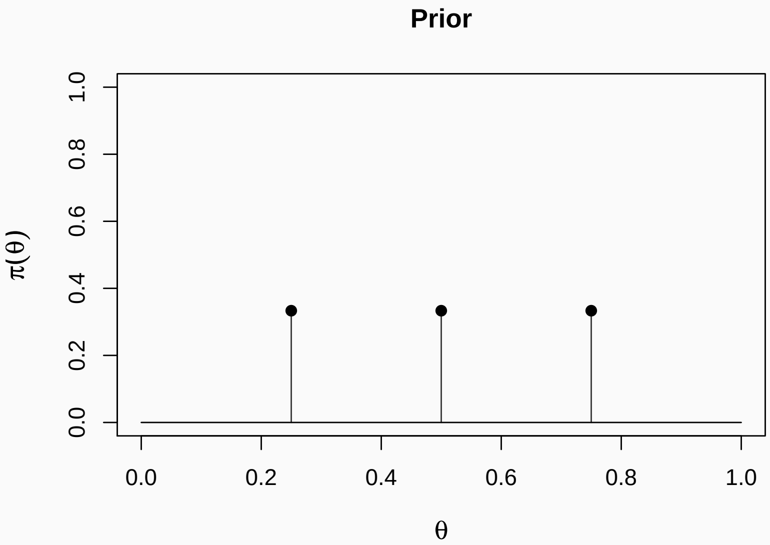
Beta-Binomial Practice

STAT 251, Unit 4C

Recap from Free-Throw Activity

- Parameter of interest: $\theta \equiv$ proportion of BYU students who favor honor code amendment.
- Three guesses on possible values for θ solicited from the class: (.25, .5, .75)
- Prior probability of each value (per class vote): $(1/3, 1/3, 1/3)$
- Then, a Bernoulli observation: it is observed whether or not the first person selected was a "success"; the result is y_1. (Note: If θ were given, $Pr(Y_1 = 1)$ would be θ.)
- Calculated the posterior distribution of θ given the new data
    - Multiplied the prior probability of each value by the corresponding likelihood of the observed data and standardized by dividing by the marginal likelihood
- Included additional data and updated posterior again (and again)

Example with only one observation, $y_1 = 0$



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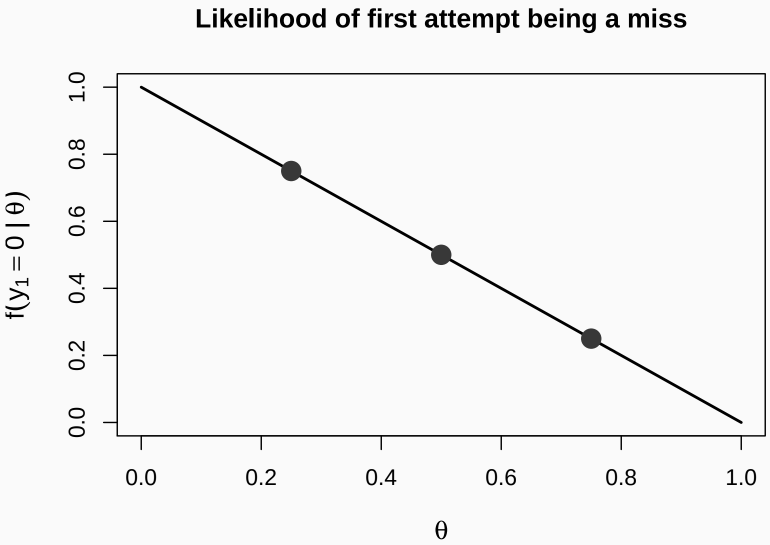

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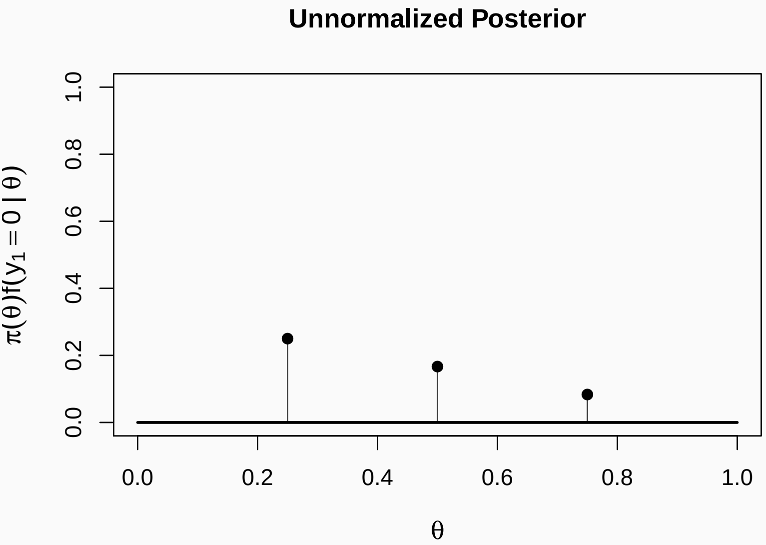

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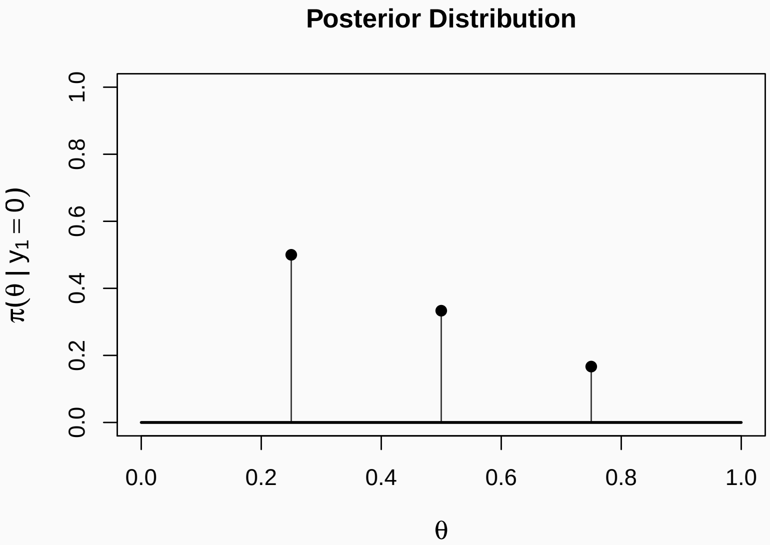
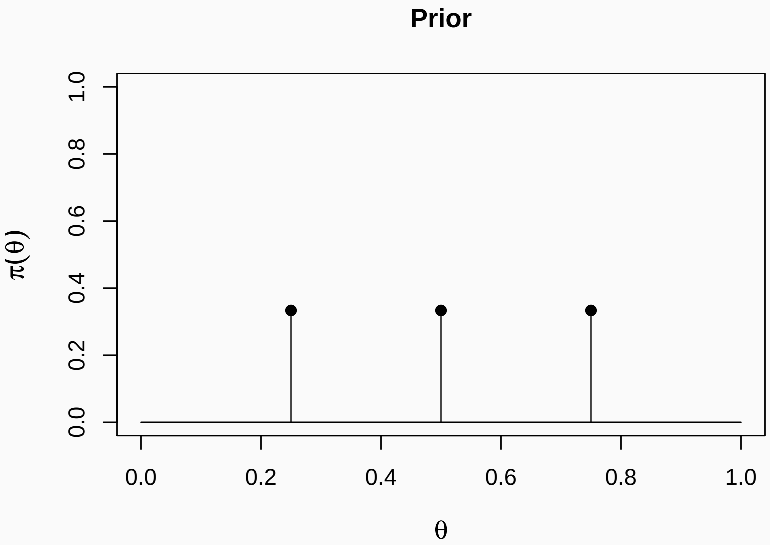


Example with $y = 9$ successes in n=11 trials



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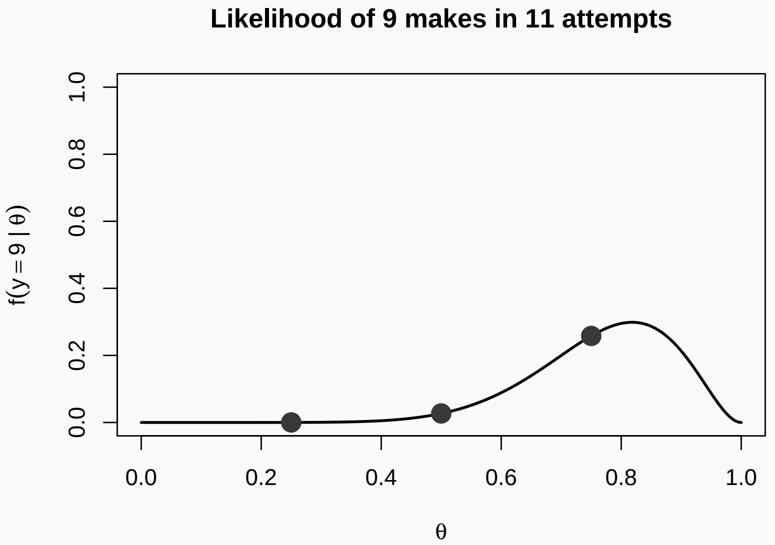
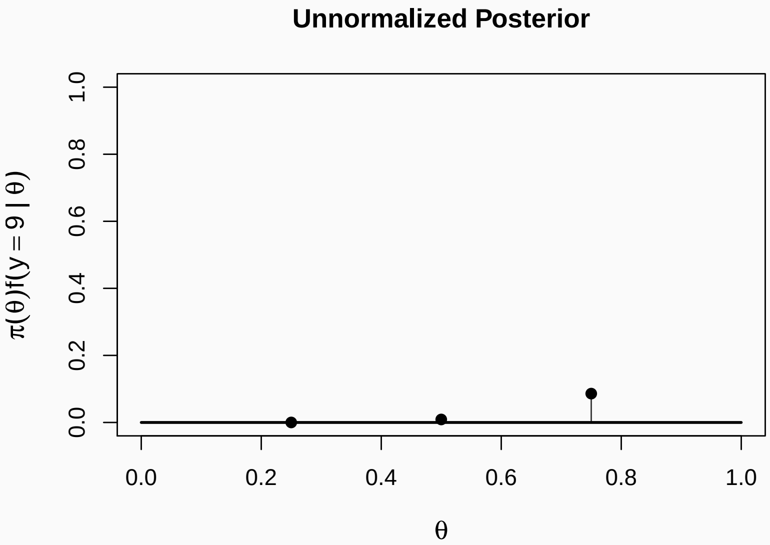


Example with $9 = 2$ successes in n=11 trials



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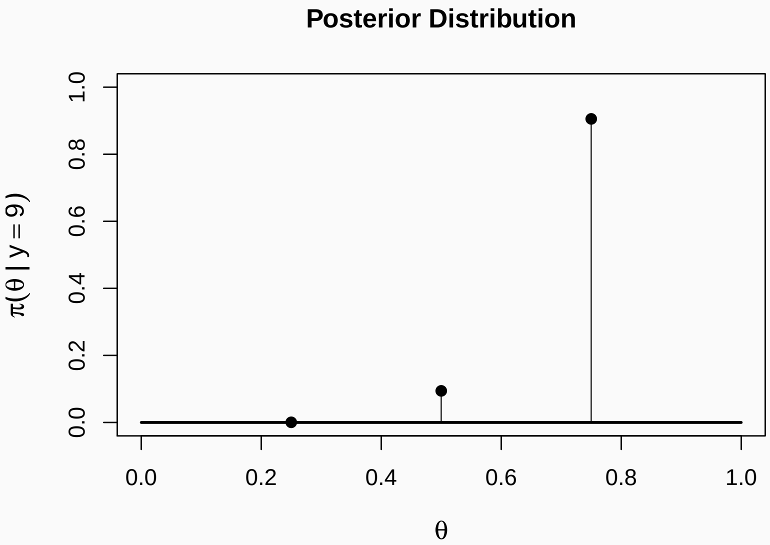


A continuous prior distribution for θ

Why do we want the beta distribution as a prior? (Correct support for θ, and conjugate prior—simplifies determining the posterior distribution)

To choose a beta distribution, we need to choose the values of a and b that will make the prior distribution reflect our beliefs about θ. Recall three possible strategies:

- If we believe all values of θ in the interval $[0, 1]$ are equally likely, choose a=b=1. This is the *uniform* distribution.
- If we have prior information equivalent to c observed successes and d observed failures, choose a=c and b=d.
- Choose the mean and variance for what we believe θ to be, then solve for a and b. Recall that for the Beta(a,b) distribution, the mean is $\frac{a}{a+b}$ and the variance is $\frac{ab}{(a+b)^2(a+b+1)}$, and thus $a = \frac{\mu^2 - \mu^3 - \mu\sigma^2}{\sigma^2}$ and $b = \frac{a - a\mu}{\mu}$.

A continuous prior distribution for θ (cont.)

The mean and variance for the discrete prior we used in our example 0.5 and 0.04167, respectively (see Unit 2B for a reminder on calculating the mean and variance of a pmf).

The beta(a=2.5, b=2.5) has about the same mean and variance. (See next slide)

```r
# a function to find (a, b) in the Beta distribution
# that produces a desired mean and variance.
get.beta.ab <- function(mu, sigma2){
  a <- (mu^2-mu^3-mu*sigma2)/sigma2
  b <- (a-a*mu)/mu
  if (is.nan(a) | is.na(a) | a<=0 |
      b<=0 | is.na(b) | is.nan(b)) {
    print("New choice for mu and sigma^2 required")
  } else  return(list(a=a, b=b))
}
out <- get.beta.ab(0.5, 0.04167)
a <- out$a
b <- out$b
c(a,b)
```

```
## [1] 2.49976 2.49976
```

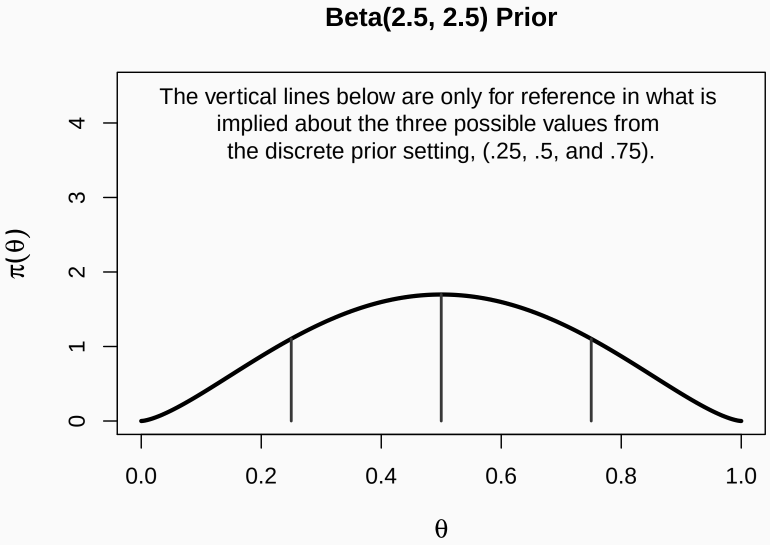
```
# Some mean/variance pairs are impermissible
get.beta.ab(0.5, 2)

## [1] "New choice for mu and sigma^2 required"
```

Note that the variance of the beta(a,b) distribution must be in the interval $[0, 1/4)$, and the mean must be in the interval $[0, 1]$.

Tip: To determine what you think $Var(\theta)$ might be, one starting point is to recognize that the *standard deviation* is (very) roughly the distance between the true θ and your believed value of θ (i.e., $E(\theta)$), or in other words **the standard deviation of θ is about how far off you expect your best guess to be**.

w/ one obs., $y_1 = 0$, and Beta(2.5, 2.5) prior



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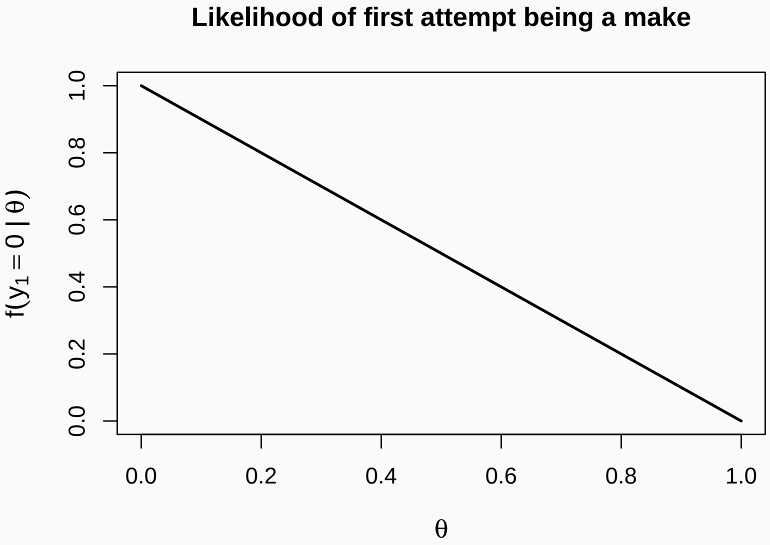

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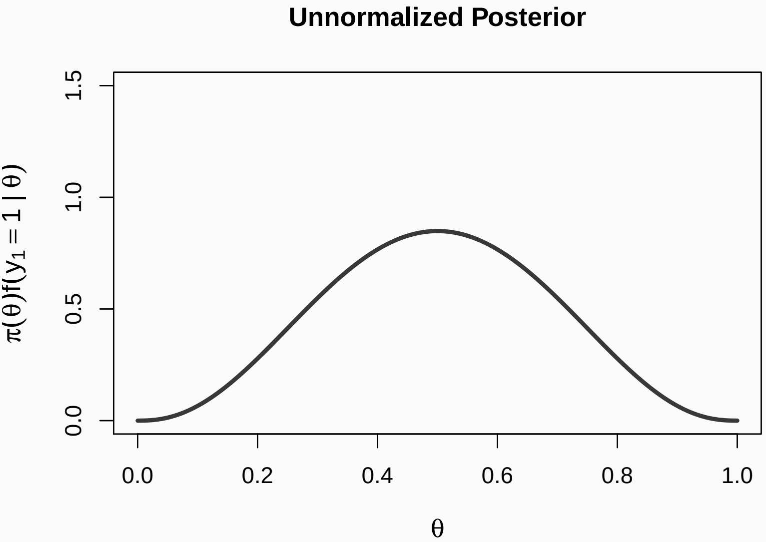

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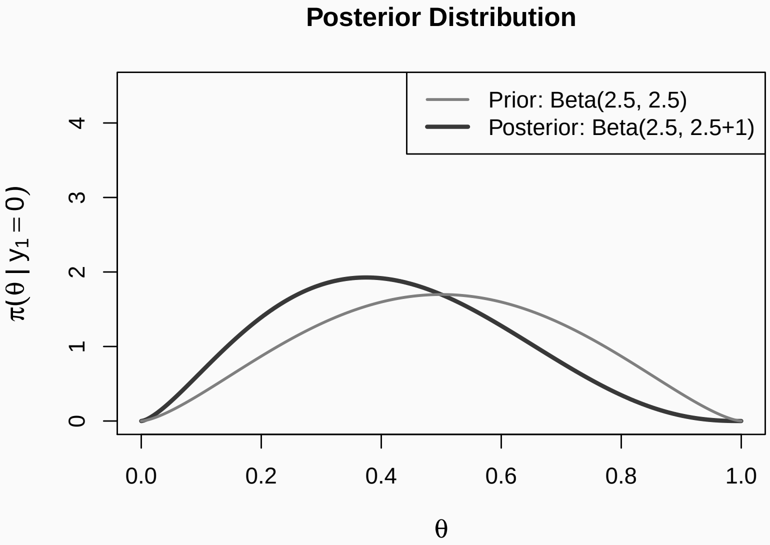
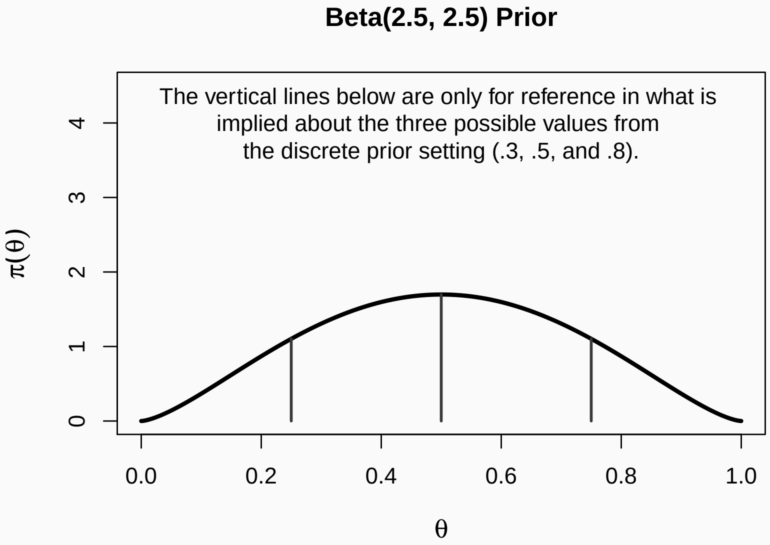


w/y=9, n=11, and Beta(2.5, 2.5) prior



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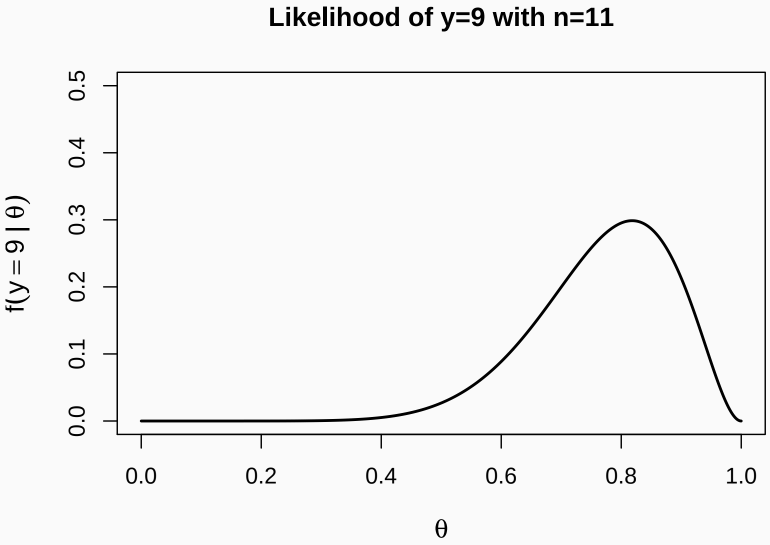

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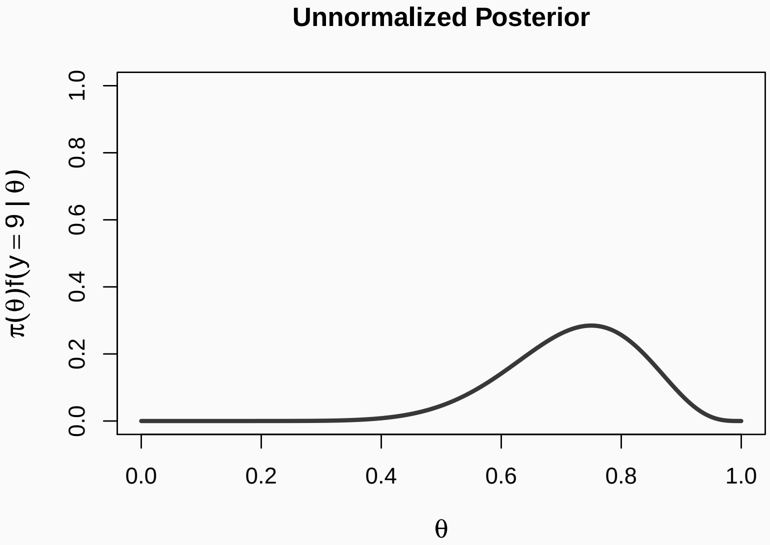

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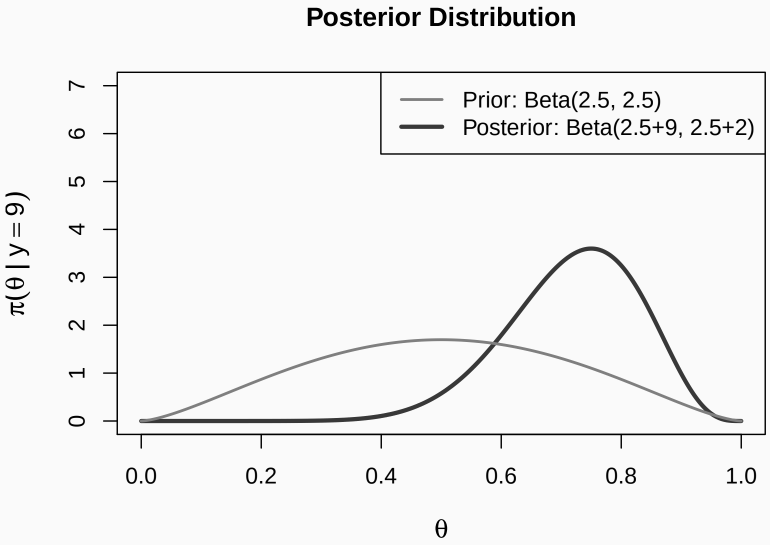

Home Field Advantage

The home-field advantage refers to the tendency for teams to perform better at their home court/field/stadium/pitch/track etc. than anywhere else.

For example, BYU football has a potential advantage when playing at Lavell Edwards Stadium because:

- The crowd is overwhelmingly cheering for BYU.
- The BYU players are more familiar with nuances of the field, such as the firmness of the soil.
- The BYU players have adapted to playing at an atypically high altitude (most opponents are from campuses with a lower sea level).

Home Field Advantage (cont.)

Let θ be defined as the probablity of the home team winning in a randomly selected NFL game.

- Come up with a prior on θ, and justify your choice.
- When this is done, raise your hand and I will give you the relevant data. After seeing this data, determine the posterior distribution and a 90% credible interval.
- Plot your prior and posterior on the same graph.
- Determine the posterior probability that θ is more than a half.