Induction (2)   Ben Pang

1.3

a. $\sum_{i=1}^{n} (6i^2+2i) = 2n^3+4n^2+2n$

Prove 0:

$$\sum_{i=0}^{0} 6(0)^2 + 2(0) = 0 = 2(0)^3 + 4(0)^2 + 2(0)$$ ✓

Prove n+1:

$$\sum_{i=1}^{n+1} 6i^2+2i = \sum_{i=1}^{n} 6i^2+2i + 6(n+1)^2 + 2(n+1)$$

$$= 2n^3+4n^2+2n + 6n^2+12n+6 + 2n+2$$

$$= \boxed{2n^3+10n^2+16n+8} \longrightarrow \text{(F factored below)}$$

$$= 2(n^3+3n^2+3n+1) + 4(n^2+2n+1) + 2(n+1)$$

$$= \boxed{2(n+1)^3 + 4(n+1)^2 + 2(n+1)}$$

b. Prove 0:

$$\sum_{i=0}^{n} \left(\frac{5}{4}\right)^i = \frac{5^1 - 4^1}{1} = 1$$ ✓

Prove n+1:

$$\sum_{i=0}^{n+1} \left(\frac{5}{4}\right)^i = \sum_{i=0}^{n} \left(\frac{5}{4}\right)^i + \left(\frac{5}{4}\right)^{n+1} = \left(\frac{5^{n+1}-4^{n+1}}{4^n}\right) + \left(\frac{5}{4}\right)^{n+1} =$$

$$= \frac{4(5^{n+1}-4^{n+1})}{4^{n+1}} + \frac{5^{n+1}}{4^{n+1}} =$$

$$\frac{(5)(5)(5^n) - 4(4)(4^n)}{4(4^n)} = \boxed{\frac{5^{((n+1)+1)} - 4^{((n+1)+1)}}{4^{(n+1)}}}$$

This is the same as $\frac{5^{n+1} + 4^{n+1}}{4^n}$ when $n = n+1$