2.4) Lower bound for sorting average

$$\sum_{j=1}^{i-1} \frac{1}{i} j + \frac{1}{i}(i-1)$$

$$= \frac{1}{i}\frac{(i(i-1))}{2} + \frac{2}{2} - \frac{1}{i} = \frac{i+1}{2} - \frac{1}{i}$$

For all keys:

$$\sum_{i=2}^{n}\left(\frac{i+1}{2} - \frac{1}{i}\right) = \frac{n(n+1)}{4} - \frac{1}{2} + \frac{2n}{4} - \frac{1}{2} - \sum_{i=2}^{n}\frac{1}{i}$$

$$= \frac{n^2}{4} + \frac{3n}{4} - 1 - \sum_{i=2}^{n} \in \boxed{\Theta(n^2)}$$

The average order is still $O(n^2)$ for the lower bound. Even with a 75% chance of keys sorted there will still be times when the list is unsorted.