1.9 Ben Pung

1. $n \lg n$ faster than n

$$\lim_{n\to\infty} \frac{n \lg n}{n} = \lim_{n\to\infty} \lg n = \infty$$

↳ this means $n \lg n$ grows faster. $f \in \omega(g)$

2. $$\lim_{n\to\infty} \frac{n^3}{n^2} = \lim_{n\to\infty} n = \infty$$

$f \in \omega(g)$ which means n^3 faster than n^2

3. $$\lim_{n\to\infty} \frac{2^{2n}}{2^n} = \lim_{n\to\infty} 2^{2n-1} = \infty$$

$f \in \omega(g)$ which means 2^{2n} is faster than 2^n

4. $\ln(n)$ and $\lg(n)$ grow at same rate

$$\lim_{n\to\infty} \frac{\log_e(n)}{\log_2(n)} \overset{\text{derivative}}{\Rightarrow} \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{n \ln(2)}} = \lim_{n\to\infty} \ln(2) = c$$

$f \in \Theta(g)$ meaning $\ln(n)$ and $\lg(n)$ grow same

5. $\lg^2 n$ faster $\lg n$

$$\lim_{n\to\infty} \frac{\lg^2 n}{\lg n} = \lim_{n\to\infty} \lg n = \infty$$

$f \in \omega(g)$

6. $25n$ ~~$\geq$~~ $.5n$

$$\lim_{n\to\infty} \frac{25\cancel{n}}{.5\cancel{n}} = \lim_{n\to\infty} 50 = 50 = c$$

$f \in \Theta(g)$ meaning same $25n$ can $= .5n$

7. $f(1) = 0 \quad < \quad f(n) = k$

$$\lim_{n\to\infty} \frac{0}{k} = 0$$

$f \in o(g)$ meaning 0 is slower than k

8.

$$\lim_{n\to\infty} \frac{\lg n}{\lg(\lg n)} \overset{\text{derivative}}{=} \lim_{n\to\infty} \frac{\frac{1}{n\ln(2)}}{\frac{1}{n\ln(2)} \cdot \left(\frac{1}{\lg(n)\cdot\ln(2)}\right)} = \lim_{n\to\infty} \lg(n)\ln(2)$$

$= \infty \Rightarrow f \in \omega(g) = \lg n$ faster

9. $O(f)$ says $g(n)$ is $O(f(n))$ if $n \geq n_0$
and $g(n) \leq c \cdot f(n)$
$n!$ than a^n, $a > 1$

when $n_0 = 5$, $c = 1$, $a = 2$ assume $a > 1$

when $n! = 120$
$a^n = 32$

therefore $a^n \in O(n!)$

where $n > n_0$

10. let $c=1$, $n_0=2$

so $n! = 1 \cdot 2 = 2$

$n^n = (1)\cdot(2 \cdot 2) = 4$

so $n! \le n^n$ therefore

$\boxed{n! \in O(n^n) \text{ whenever } n > n_0}$

11. 2^{2n} vs n^n

let $n_0 = 2$, $c=1$

$(1)2^{(2)(2)} = 2^4 = 16$

$n^n = 2^2 = 4$

$\boxed{n^n \in O(2^{2n}) \text{ whenever } n > n_0}$

12. if $a > b$

$$\lim_{n\to\infty} \frac{\lg^a n}{\lg^b n} = \lim_{n\to\infty} \lg^{a-b} n = \infty$$

$f(x) \in \omega(g)$ so $\lg^a n$ grows

13. $\lim_{n\to\infty} \frac{A_k \cancel{\lg^k n}}{\cancel{\lg^k n}} = A_k = c = \Theta(n)$

ignore small terms due to polynomial rate

$\boxed{\lg^k n \text{ and } A_k \lg^k n \ldots \text{ grow at same rate}}$

14.
2^{2^n} — super exp
n^n
$(n+1)!$ $(n!)$ — fact
$2^{n \lg n}$
add line 2^n
2^{2n}
$3n^{2.1}$, $n^{2.1}$ — exp
$n^{2.01}$ — poly/quad
$3n$, n, $6n+7$ — line
$n^{1.00001}$
$\lg(n)$, $\ln(n)$, $\lg(n^2)$, $\lg(n^3)$ — log
3 — c