1.10 Ben Pury

1. $B(n) = B(n-1) + n \qquad B(1) = 1$

$B(2) = 3$
$B(3) = 6$
$B(4) = 10$

$\sum_i$

$\boxed{\text{pattern} = \frac{n(n+1)}{2}}$

$B(n) = \frac{(n)(n+1)}{2}$

2 $k(n) = k(n-1) + n^2 \quad , \quad k(0) = 0$

$k(0) = 0$
$k(1) = (k(0)) + 1 = 1$
$k(2) = k(1) + 4 = 5$

$k(7) = 7^2 + 6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2 \qquad \sum_i i^2$

$= \boxed{\frac{n(n+1)(2n+1)}{6}}$

3. $T(n) = 2T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + 1 \quad , \quad T(1) = 0 \quad , \quad T(0) = 0$

$4T\left(\frac{n}{4}\right) + 2$

$8T\left(\frac{n}{8}\right) + 3$

$2^k T\left(\frac{n}{2^k}\right) + k$

when $k = \lg n$: $2^{\lg n} \cancel{T\left(\frac{n}{n}\right)}^{0} + \boxed{\lg(n)}$

$T(n) = \lg n$

4. $R(n) = R(\lfloor \frac{n}{2} \rfloor) + n$, $R(1) = 0$, $R(0) = 0$

$R(n) = R(\lfloor \frac{n}{4} \rfloor) + 2n$

$R(n) = R(\lfloor \frac{n}{8} \rfloor) + 3n$

$R(n) = R(\frac{n}{2^k}) + kn$

When $k = \lg n$, $R(\frac{n}{n})^{\to 0} + (\lg n)(n)$

$= \boxed{n \lg(n)}$

5. $T(n) = 64T(\lfloor \frac{n}{2} \rfloor) + n$

$n^{\log_2 64} = n^6 > n$ so $\boxed{\Theta(n^6)}$

6. $T(n) = 81T(\frac{n}{3}) + n^2$

$n^{\log_3 81} = n^4 > n^2$ so $\boxed{O(n^4)}$

7 $T(n) = 7T(\frac{n}{3}) + n^3$

$n^{\log_3 7} < n^3$ so $\boxed{\Theta(n^3)}$

8. $T(\frac{n}{2}) + 12n$

$n^{\log_2 1} = n^0 = 1 < 12n$ so $\boxed{\Theta(n)}$

9. $16T(\frac{n}{4}) + n^2$

$n^{\log_4 16} = n^2 = n^2$ so $\boxed{\Theta(n^2 \lg(n))}$