1.4 Ben Pung

1. 7 heads: $\left(\frac{1}{2}\right)^7 = \boxed{\frac{1}{128} \text{ for all 7 heads}}$

2. ${}_7C_5 = \frac{7!}{(5! \cdot (7-5)!)} = 21$

use combination for this problem

$$P(X=5) = (21) \cdot \left(\frac{1}{2}\right)^5 \cdot (1-.5)^{(7-5)}$$

$$= \frac{21}{2^7} = \boxed{\frac{21}{128}} = \boxed{.1641}$$

3. Expected value: $\Sigma\, x\, p(x)$

$$= \left(\frac{1}{3}\right) + \frac{2}{6} + \frac{3}{6} + \frac{4}{9} + \frac{5}{9} + \frac{6}{9}$$

$$= \frac{3}{9} + \frac{3}{9} + \frac{1}{2} + \frac{4}{9} + \frac{5}{9} + \frac{6}{9}$$

$$= \frac{21}{9} + \frac{1}{2} = \frac{7}{3} + \frac{1}{2} = \boxed{\frac{17}{6}}$$

4. a

$$P(x=0) = \binom{8}{0}\left(\frac{1}{2}\right)^0 (1-.5)^8 = 1\left(\tfrac{1}{2}\right)^0\left(\tfrac{1}{2}\right)^8 = \boxed{\frac{1}{256}}$$

$$x=1 = \binom{8}{1}\left(\frac{1}{2}\right)^1 (1-.5)^7 = 8\left(\tfrac{1}{2}\right)^1\left(\tfrac{1}{2}\right)^7 = \boxed{\frac{1}{32}}$$

$$x=2 = \binom{8}{2}\left(\frac{1}{2}\right)^2 (1-.5)^6 = 28\left(\tfrac{1}{2}\right)^2\left(\tfrac{1}{2}\right)^6 = \boxed{\frac{7}{64}}$$

$$x=3 = \binom{8}{3}\left(\frac{1}{2}\right)^3 (1-.5)^5 = 56\left(\tfrac{1}{2}\right)^3\left(\tfrac{1}{2}\right)^5 = \boxed{\frac{7}{32}}$$

$$x=4 = \binom{8}{4}\left(\frac{1}{2}\right)^4 (1-.5)^4 = 70\left(\tfrac{1}{2}\right)^4\left(\tfrac{1}{2}\right)^4 = \boxed{\frac{35}{128}}$$

$$x=5 = \binom{8}{5}\left(\frac{1}{2}\right)^5 (1-.5)^3 = 56\left(\tfrac{1}{2}\right)^5\left(\tfrac{1}{2}\right)^3 = \boxed{\frac{7}{32}}$$

$1: \frac{3}{4}$

b. $0: \frac{1}{4}$

Use binomial probability distribution from Bernoulli

$$\boxed{\binom{8}{n} \cdot \left(\frac{3}{4}\right)^{n} \cdot \left(\frac{1}{4}\right)^{8-n} \quad \text{where } n = \#\ 1\text{'s}}$$