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Advanced Algorithms

6.3

**1. Given the 3-CNF expression (a’+b’+c’)(a+b+c) (a’+b’+c) (a+b’+c), how many stars are there, and how many points in each star?**

For any given star there is 2n points with n being the number of clauses. In this case there are 4 clauses meaning that there are 8 points.

For the number of stars we need 1 for each variable. I see three variables, (a,b,c). In this case then I would then need 3 total stars.

**2. List the ordered triples in each star. (Draw them if you wish, but a simple list will do.) This is the component that corresponds to the concept “A variable must be either true or false, but not both.”**

Star 1 (a is the outside literal):

(a1, b1 , c1), (a2, b2, c2), (a3 , b3, c3), (a4,b4, c4)

(a1’, b1, c2), (a2’, b2, c3) , (a3’, b3, c4), (a4’, b4, c1)

Star 2 (b is the outside literal):

(b1, a1 , c1), (b2, a2, c2), (b3 , a3, c3), (b4, a4, c4)

(b1’, a1, c2), (b2’, a2, c3) , (b3’, a3, c4), (b4’, a4, c1)

Star 3 (c is the outside literal)::

(c1, a1 , b1), (c2, a2, b2), (c3 , a3, b3), (c4, a4, b4)

(c1’, a1, b2), (c2’, a2, b3) , (c3’, a3, b4), (c4’, a4, b1)

**3. List the ordered triples in the satisfaction component. This is the component that corresponds to the concept of “one true literal in every clause.”**

So there are 4 clauses so I need to have 4 “propellers”. In each propeller there is a blade for each variable in the clause. There will be a shared s and t for each blade in the clause. I number the clauses 1 to 4.

Here is the CNF from problem: (a’+b’+c’)(a+b+c) (a’+b’+c) (a+b’+c). Ordered Triples are as follows for the propeller:

Clause 1:

(a1’, s1, t1), ( b1’, s1, t1), (c1’, s1, t1)

Clause 2:

(a2, s2, t2), ( b2, s2, t2), (c2, s2, t2)

Clause 3:

(a3’, s3, t3), ( b3’, s3, t3), (c3, s3, t3)

Clause 4:

(a4, s4, t4), ( b4’, s4, t4), (c4, s4, t4)

**4. How many triples are there in the garbage collection component? List some of them, enough to verify you know how to do it.**

So I think that there is a blade in the garbage collection component for each unique variable we created in the propellers and stars above. I think that we have (a-c) (1-4) in each stack for a total of ((3 variables \*4 numbers for each clause) \* 2 options (true/false)) = 24 triples in the collection component for each stack). Then we need n\*(m-1) stacks which is 4\*(3-1) = 8 stacks. Then we have 8 \* 24 = 192 total triples in the entire garbage collector including each stack.

3 examples are:

(a1 , t1, s1)

(a1’, t1, s1)

(b1, s1, t1) and so on for EACH STACK.

5. List the elements of the sets X, Y, and Z.

a1, a2, a3, a4, a1’,a2’,a3’,a4’ -> X

b1, b2, b3, b4, b1’,b2’,b3’,b4’ -> Y

c1, c2, c3, c4, c1’,c2’,c3’,c4’ -> Z

**6. If the expression is satisfiable (a=false,c=true,b=true works) explain how this can give a complete matching in the 3DM problem created from the expression.**

If that expression works at least in partial match, using the garbage collector would allow us to make it a complete match by removing the unused points. If there is no complete match then there aren’t enough garbage collectors to expand the partial match to a complete match. Each garbage collector stack can each one free point on the star so we would only make as many as needed n\*(m-1). So in total, we only need a partial match from the satisfied expression.