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Advanced Algorithms

6.4

**We hinted at how INDEPENDENT SET and CLIQUE could be shown to be NP-Complete but didn’t construct a formal proof. Do the formal proof for INDEPENDENT SET. Note a formal proof consists of the following:**

**a. A construction f(x)**

**b. A proof that the construction works. You must prove two things, if x is in my starting set then f(x) is in INDEPENDENT SET. If x is not in my starting set then f(x) is not in INDEPENDENT SET.**

1. Start with Vertex Cover (G,k) in VC, G is a graph and k is an integer. (G,k)’s in VC if G has a vertex cover of size k, from (G,k) construct the INDEPENDENT SET problem C, where n is the number of vertices. If C is in VC then there is a set C a subset of the vertices of G, such that C contains one endpoint of every edge and C is of size k. Let V be the vertex set of G. Since C contains on end point of every , V-c, which is of size k cannot have any two vertices with an edge between them. If this were so, one of the vertices must be in C, and this is not possible.
2. Now suppose (G , K) has an independent set of size k. And let S a subset of V be an independent set of G and V-S is a set of size k, and it must be a vertex-cover because if it did contain one endpoint for every edge, there would be an edge between two vertices in S, and this was not possible.

**2. Complete the proof for CLIQUE**

Start with INDEPENDENT SET. Given a member of the INDEPENDENT SET universe (g,k) compute G where G’ is the complement of G. (Every edge NOT in G is present in G’, and every edge in G is absent in G’). Suppose G has an independent set of size k and let S be such a set. There is no edge in G between any pair of vertices in S. Since these edges are missing in G, they are present in G’ and S must be a clique. Now suppose S is a clique of size k in G’. Since the edges of the clique are present in G’ they must be absent in G. Therefore, there is no edge in between any of the vertices of S, and S is an independent set of G’.