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Advanced Algorithms

6.5

**1. Show that the 4-colorability problem is NP-Complete.**

Solved using hint in class:

Given a graph G add an addition vertex v, thus creating G’. Add and edge between v and every vertex in G. Since v cannot have the same color as any vertex of G, the chromatic number of G’ is one larger than the chromatic number of G. G’ s 4-colorable if an only if G is 3-colorable.

**2. The k-colorability is the following: Given a graph G and an integer k, is it possible to color the vertices of G with k colors so that no two adjacent vertices are the same color? Show that the k-colorability problem is NP-complete.**

Using the proof in question A, I know that 4-colorability is a NP-complete if and only if 3-colorability is NP complete. I can then show that k-color is NP-complete if and only-if 3-colorability is NP-complete. That means basically 3-colorability is a subset of k-color ability meaning k is NP-complete as well.

**3. The bounded-3-colorability problem is, given an integer k, is it possible to color the vertices of a graph so that there are no more than k edges that connect vertices of the same color? Show that the bounded-3-colorability problem is NP-complete.**

Yes I think its possible to color the vertices of a graph such that there are no more than k edges that connect vertices of the same color. Same as above, we know that 3-colorability is NP-complete so then we can say that 3colorability is a subset of bounded-3-colorability because 3-colorability assumes no adjacent vertices have the same color but the requirement is less the bounded-colorabilty problem.