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Advanced Algorithms

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4.1 Graphs

\*Instead of drawing out the graphs I put all node with the directed edges and then their adjacent nodes

1. Create a graph from the binary numbers 000, 001, 010, 011, 100, 101, 110, and 111. Place an edge between two numbers if they have the same number of 1’s. What sort of graph is this?

Start -edge> {adjacent nodes}

000 -> { }

001 -> { 100, 010}

010 - > {100, 001}

100 -> { 010, 001}

011 -> { 101, 110}

101 -> {011, 110}

110 -> {011, 101}

111 -> {}

This is a disconnected cyclical undirected graph.

2. The set {a,b,c} has the following subsets: {}. {a}. {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c}. If set x is a subset of set y, place a directed edge from x to y. Graphs of this sort are called latices, because each collection of subsets has a least upper bound and a greatest lower bound.

{} -> {}

{a} -> {a,b} , {a,c} , {a,b,c}

{b} -> {b,c} , { a,b}, {a,b,c}

{c} -> {a,c} , {b,c}, {a,b,c}

{a,b,} -> {a,b,c}

{b,c} -> {a,b,c}

{a,c} -> {a,b,c}

3. In the graph of problem 2, what is the least upper bound of the set { {a}, {a,b} }?

The least upper bound of the given set { {a} , {a,b} } **is {a,b}.**

4. The characteristic function of a subset is a function on the base set to the set {0.1}. If an element is in the subset, the characteristic function has value 1. If not, it has value 0. Give the characteristic functions for the sets of problem 2. Write the functions as 3-bit numbers, where the low-order digit corresponds to a, the middle digit corresponds to b, and the high-order digit corresponds to c.  
{} =000

{a} = 100

{b} = 010

{c} = 001

{a,b} = 110

{b,c} = 011

{a,c} = 101

{a,b,c} = 111

5. The characteristic functions of the subsets of any set with 64 or fewer elements can be stored in a 64-bit number. We can use the characteristic functions as representations of the subsets. If this is done, how can one find the union and intersection of two subsets?

In order to find the union and intersection of two 64 bit numbers we can use bitwise operations. To find the union we can use the (OR) and in order to find the intersection we can use (AND).