Ben Pung

Prof Orduz

Advanced Algorithms

31 October 2023

4.5

1.

I show the order the vertices are added to the MST and then the set of fringe vertices, weights, and parents when the vertex is added to the tree. I include the weight and parent of each fringe vertex

I assume we start at vertex A. The parent of that is (-1) because it doesn’t exist.

Vertex added: A (+0) (Total : 0)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |
| B | 6 | A |
| G | 32 | A |
| D | 28 | A |

Vertex added: B (+6) (Total : 6)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |
| E | 16 | B |
| C | 5 | B |
| G | 32 | A |
| D | 28 | A |

Vertex added: C (+5) (Total : 11)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |
| E | 16 | B |
| W | 11 | C |
| F | 1 | C |
| G | 32 | A |
| D | 28 | A |

Vertex added: F (+1) (Total : 12)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |
| E | 16 | B |
| W | 11 | C |
| J | 19 | F |
| D | 28 | A |
| N | 4 | F |
| G | 32 | A |

Vertex added: N (+4) (Total : 16)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |
| E | 16 | B |
| W | 11 | C |
| J | 19 | F |
| M | 15 | N |
| W | 1 | N |
| U | 31 | N |
| G | 32 | A |
| D | 28 | A |

Vertex added: W (note that there were no fringe edges added with W because all nodes around it are in tree) (+1) (Total : 17)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |
| E | 16 | B |
| J | 19 | F |
| M | 15 | N |
| U | 31 | N |
| G | 32 | A |
| D | 28 | A |

Vertex added: M (+15) (Total : 32)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |
| E | 16 | B |
| J | 19 | F |
| L | 27 | M |
| T | 34 | M |
| U | 31 | N |
| G | 32 | A |
| D | 28 | A |

Vertex added: E (+10) (note that some vertices have multiple paths to them now so I include two instances and will remove all when a path is chosen from the minheap) (Total : 42)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |
| D | 20 | E |
| J | 30 | E |
| J | 19 | F |
| L | 27 | M |
| T | 34 | M |
| U | 31 | N |
| G | 32 | A |
| D | 28 | A |

Vertex added: J (+19) (Total : 61)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |
| D | 20 | E |
| L | 27 | M |
| T | 34 | M |
| U | 31 | N |
| G | 32 | A |
| D | 28 | A |
| H | 2 | J |

Vertex added: H (+2) (Total : 63)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |
| D | 20 | E |
| L | 27 | M |
| T | 34 | M |
| U | 31 | N |
| G | 32 | A |
| D | 28 | A |
| D | 36 | H |
| K | 12 | H |
| L | 26 | H |

Vertex added: K (+12) (Total : 75)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |
| D | 20 | E |
| L | 27 | M |
| T | 34 | M |
| U | 31 | N |
| G | 32 | A |
| D | 28 | A |
| D | 36 | H |
| G | 21 | K |
| L | 26 | H |
| P | 7 | K |
| R | 33 | K |

Vertex added: P (+7) (Total : 82)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |
| D | 20 | E |
| L | 27 | M |
| T | 34 | M |
| U | 31 | N |
| G | 32 | A |
| D | 28 | A |
| D | 36 | H |
| G | 21 | K |
| L | 26 | H |
| G | 29 | P |
| R | 33 | K |
| Q | 17 | P |

Vertex added: Q (+17) (Total : 99)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |
| D | 20 | E |
| L | 27 | M |
| T | 34 | M |
| U | 31 | N |
| G | 32 | A |
| D | 28 | A |
| D | 36 | H |
| G | 21 | K |
| L | 26 | H |
| G | 29 | P |
| R | 33 | K |
| R | 13 | Q |
| V | 8 | Q |
| S | 24 | Q |

Vertex added: V (+8) (Total : 107)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |
| D | 20 | E |
| L | 27 | M |
| T | 34 | M |
| U | 31 | N |
| G | 32 | A |
| D | 28 | A |
| D | 36 | H |
| G | 21 | K |
| L | 26 | H |
| G | 29 | P |
| R | 33 | K |
| R | 13 | Q |
| S | 24 | Q |
| T | 18 | V |
| U | 9 | V |

Vertex added: U (+9) (Total : 116)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |
| D | 20 | E |
| L | 27 | M |
| T | 34 | M |
| G | 32 | A |
| D | 28 | A |
| D | 36 | H |
| G | 21 | K |
| L | 26 | H |
| G | 29 | P |
| R | 33 | K |
| R | 13 | Q |
| S | 24 | Q |
| T | 18 | V |
| T | 23 | U |

Vertex added: R (+13) (Total : 129)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |
| D | 20 | E |
| L | 27 | M |
| T | 34 | M |
| G | 32 | A |
| D | 28 | A |
| D | 36 | H |
| G | 21 | K |
| L | 26 | H |
| G | 29 | P |
| S | 24 | Q |
| T | 18 | V |
| T | 23 | U |
| L | 2 | R |

Vertex added: L (+2) (Total : 131)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |
| D | 20 | E |
| T | 34 | M |
| G | 32 | A |
| D | 28 | A |
| D | 36 | H |
| G | 21 | K |
| G | 29 | P |
| S | 24 | Q |
| T | 18 | V |
| T | 23 | U |
| S | 3 | T |

Vertex added: S (+3) (Total : 134)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |
| D | 20 | E |
| T | 34 | M |
| G | 32 | A |
| D | 28 | A |
| D | 36 | H |
| G | 21 | K |
| G | 29 | P |
| T | 25 | S |
| T | 18 | V |
| T | 23 | U |

Vertex added: T (+18) (Total : 152)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |
| D | 20 | E |
| G | 32 | A |
| D | 28 | A |
| D | 36 | H |
| G | 21 | K |
| G | 29 | P |

Vertex added: D (+20) (Total : 172)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |
| G | 32 | A |
| G | 21 | K |
| G | 29 | P |
| G | 14 | D |

Vertex added: G (+14) (Total : 186)

|  |  |  |
| --- | --- | --- |
| Fringe Vertex | Weight of edge between parent | Parent |

Done!

MST is:

A->B

B->C

C->F

F->N

N->W

N->M

C->E

F->J

J->H

H->K

K->P

P->Q

Q->V

V->U

Q->R

R->L

L->S

V->T

E->D

D->G

This covers every node once! Total is 186.

2. Kruskals algorithm

minheap of edges sorted by lowest cost first:

(NODE, NODE) “WEIGHT OF EDGE”

(C,F) 1

(W,N) 1

(H,J) 2

(R, L ) 2

(L,S) 3

(F,N) 4

(B,C) 5

(A,B) 6

(K,P) 7

(Q,V) 8

(V,U) 9

(E,C) 10

(C, W) 11

(K,H) 12

(Q,R) 13

(G,D) 14

(M,N) 15

(B,E) 16

(P,Q) 17

(T,V) 18

(J,F) 19

(D,E) 20

(G,K) 21

(F,W) 22

(T,U) 23

(Q,S) 24

(S,T) 25

(H,L) 26

(L,M) 27

(A,D) 28

(G,P ) 29

(E,J) 30

(N,U) 31

(A,G) 32

(K, R) 33

(M, T) 24

(J,M) 35

(D,H) 36

So the algorithm just pops the shortest edge off the stack if the two edges are not in the same tree. Treat each node as its own tree to start. You can merge two or more large trees later if that happens.

Added:

(C,F) 1

Added:

(W,N) 1

Added:

(H,J) 2

Added:

(R, L ) 2

Added:

(L,S) 3

Added:

(F,N) 4

Added:

(B,C) 5

Added:

(A,B) 6

Added:

(K,P) 7

Added:

(Q,V) 8

Added:

(V,U) 9

Added:

(E,C) 10

SKIPPED:

(C, W) 11

Added:

(K,H) 12

Added:

(Q,R) 13

Added:

(G,D) 14

Added:

(M,N) 15

SKIPPED:

(B,E) 16

Added:

(P,Q) 17

Added:

(T,V) 18

Added:

(J,F) 19

Added:

(D,E) 20

WE SKIPPED BELOW BECAUSE WE ARE DONE

(G,K) 21

(F,W) 22

(T,U) 23

(Q,S) 24

(S,T) 25

(H,L) 26

(L,M) 27

(A,D) 28

(G,P ) 29

(E,J) 30

(N,U) 31

(A,G) 32

(K, R) 33

(M, T) 24

(J,M) 35

(D,H) 36

The MST is a total value of 186 that excludes skipped values.

I drew out the MST in MSPAINT (excuse the horrible drawing skills). I showed that in BLUE (DIJKSTRA-PRIM) and in RED (KRUSKALS) have the same path.

A diagram of a hexagon with red lines and white letters

Description automatically generated