Tutorial 5 - Q Learning and Examples

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This problem set consists of two parts

1. We will discuss Q-learning.
2. We will write an algorithm for an agent that uses Q-learning to learn (i) to find the exit in a building, and (ii) to find the shortest path between nodes.

Q Learning

Consider the setting of Markov Decision Processes of previous tutorial, where the state-value function is given by

$$\begin{aligned} \nu_\pi(s) &= E_\pi(G_t|S_t = s) \\ &= E_\pi(R_{t+1} + \gamma \nu_\pi(S_{t+1})|S_t = s), \end{aligned}$$

where the Temporal Differencing (TD) method uses an estimate of the latter expression as a target. In particular, the TD method samples the expected values and it uses the current estimate V instead of true ν_π.

The TD(0) method update is as follows: for action A given by policy π in state S, observe reward R and next state S', and update

$$V(S) \leftarrow V(S) + \alpha \left(R + \gamma V(S') - v(S)\right)$$

However, we are often interested in the action-value function, that is, in estimating $q_\pi(s, a)$ for all (s, a). We can do this using the same TD method as described above.

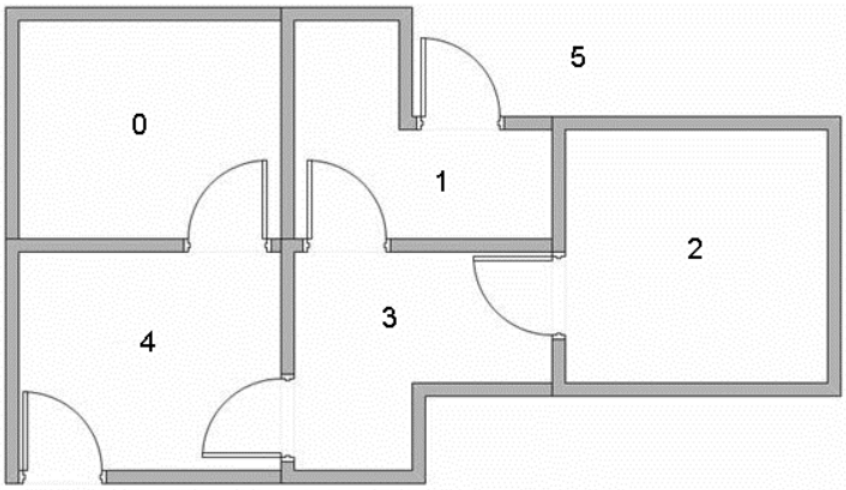
$$Q(S, A) \leftarrow Q(S, A) + \alpha \left(R + \gamma Q(S', A') - Q(S, A)\right).$$

The groundbreaking idea behind Q-learning is to directly approximate the optimal action-value function, independent of the policy. The algorithm for Q-learning is given below (see Sutton and Barto, 2007). Note the difference between the update step for Q-learning and the TD method.

Initialize $Q(s, a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
    Initialize S
    Repeat (for each step of episode):
        Choose A from S using policy derived from Q (e.g., ϵ-greedy)
        Take action A, observe R, S'
        $Q(S, A) \leftarrow Q(S, A) + \alpha\left[R + \gamma \max_a Q(S', a) - Q(S, A)\right]$
        $S \leftarrow S'$;
    until S is terminal

Code: Path-Finding

In this example, we are using Q-learning to find the optimal route from a given room to outside. As an illustration, let's take a look at the figure below.



Suppose we want to know what is the best way to get from Room 2 to Room 5 (outside). We need to provide information to the agent what the rewards or penalties are for attempting to go from one room (state) to another room (state). We do so using a reward matrix R.

$$R_{ij} = \begin{cases} -1, & \text{if you can not go from } i \text{ to } j, \\ 0, & \text{if destination } j \text{ is not the target state,} \\ 100, & \text{if destination } j \text{ is the target state.} \end{cases}$$

For the example above, the reward matrix is specified below.

```
mR <- t(matrix(c(-1, -1, -1, -1, 0, 1,
       -1, -1, -1, 0, -1, 0,
       -1, -1, -1, 0, -1, -1,
       -1, 0, 0, -1, 0, -1,
        0, -1, -1, 0, -1, 0,
       -1, 100, -1, -1, 100, 100), nrow=6, ncol=6, byrow=TRUE))
```

Task 1: write a Q-learning algorithm that learns the Q matrix for a given reward matrix R.

```
####
# q_learning : learn Q matrix using Q-learning algorithm
#
# Arguments :
#   mR : matrix, reward matrix
#   iNepisodes : integer, number of episodes
#   dAlpha : float, learning rate
#   dGamma : float, discount factor
#   iTargetState : integer, target state of the problem
#
# Output :
# mQ_norm : matrix, normalized Q matrix


q_learning <- function(mR, iNepisodes, dAlpha, dGamma, iTargetState) {
  # initialize Q matrix
  mQ <- matrix(rep(0,length(mR)), nrow=nrow(mR))
```

```
  # loop over episodes
  for (i in 1:iNepisodes) {
    # for each episode, choose an initial state at random
    current_state <- sample(1:nrow(mR), 1)

    ## iterate until we get to the iTargetState
    while (1) {

      ## TO DO
      # choose next state from possible actions at current state

      ## TO DO
      # if only one possible action, then choose it; otherwise, choose at random

      ## TO DO
      # update Q value

      # break out of while loop if target state is reached
      if (next_state == iTargetState) break

      # otherwise, set next state as current state and repeat
      current_state <- next_state
    }

  }

  # normalize Q by max value
  mQ_norm <- 100*mQ/max(mQ)

  # return
  results <- list(Q=mQ_norm)
  return(results)
}
```

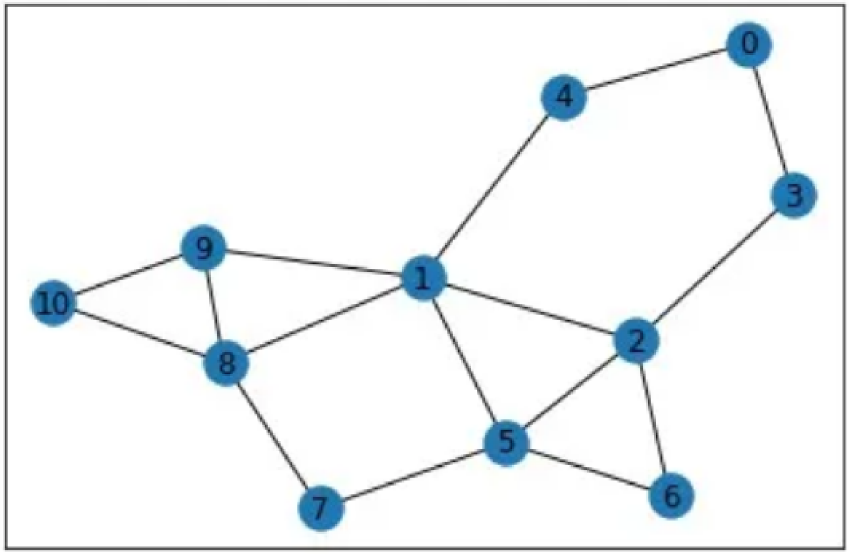
Task 2: find the optimal paths for target state 6. Interpret the Q-matrix.

```
## TO DO
```

Code: Shortest Path

In this example, we will use Q-learning to solve the shortest path problem. We implement an epsilon-greedy policy for the agent. Note that other policies are possible here. You can try for yourself to implement other policies.

We will run an instance of the shortest-path problem. In this case we try to find the shortest path from node 0 to node 10, see figure below. Arriving at node 10 gives us a reward equal to 100. To exclude walking on non-existing edges, we set their Q-value to -100. Possible edges are set to 0. Below we setup the reward matrix and initial Q-matrix to define the edges.




```r
# build 11x11 reward matrix of zeros
mR <- matrix(0, 11, 11)
mR[c(9,10),11] <- 100 # arriving at node 11 yields reward 100

# initialize Q matrix
# impossible edges to -100, possible edges to 0
mQ <- matrix(-100,11,11)
mQ[c(4,5),1] <- 0; mQ[c(3,5,6,9,10),2] <- 0; mQ[c(2,4,6,7),3] <- 0;
mQ[c(1,3),4] <- 0; mQ[c(1,2),5] <- 0; mQ[c(2,3,7,8),6] <- 0;
mQ[c(3,6),7] <- 0; mQ[c(6,9),8] <- 0; mQ[c(2,8,10,11),9] <- 0;
mQ[c(2,9,11),10] <- 0; mQ[c(9,10),11] <- 0;
```

Task 3: Finish the code that performs an epsilon-greedy step for a given Q-matrix and a current node. The function should choose the action according to the highest Q-value of possible arms, where we allow for exploration.

```r
####
# eps_greedy_next : perform epsilon-greedy for Q-learning
#
# Arguments :
#   mQ : matrix, Q-matrix
#   current : integer, current node
#   eps : float, exploration rate
#
# Output :
# next_state : integer, next node

eps_greedy_next <- function(mQ, current, eps){
  ## TO DO


  # return the chosen arm
  return(next_state)
}
```

Task 4: Finish the code that performs Q-learning using epsilon-greedy policy. The function should repeat the following: start at random node, make a walk to next node, and update the Q-matrix.

```r
####
# q_learning_eps : learn Q matrix using Q-learning algorithm according epsilon-greedy policy
#
# Arguments :
#   mR : matrix, reward matrix
#   mQ : matrix, initial Q-matrix
#   iNepisodes : integer, number of episodes
#   dAlpha : float, learning rate
#   dGamma : float, discount factor
#   eps : float, exploration rate for epsilon-greedy policy
#
# Output :
# mQ_norm : matrix, normalized Q matrix

q_learning_eps <- function(mR, mQ, iNepisodes, dAlpha, dGamma, eps) {
  # loop over episodes
  for (i in 1:iNepisodes) {
    ## TO DO
    # for each episode, choose an initial state at random

    ## TO DO
    # choose next state from possible actions at current state

    ## TO DO
    # update Q value
  }

  # return
  return(mQ)
}
```

The final Q-values are:

```r
# run q_learning_eps
mQ <- q_learning_eps(mR, mQ, 50000, 0.8, 0.8,0.3)
round(mQ)
```

Task 5: Finish the code to find the shortest path, using Q-learning with epsilon-greedy policy. The function should choose the node with highest Q-value until it reaches the end node.

```r
####
# shortest_path : find shortest path using Q-learning algorithm
#
# Arguments :
#   mQ : matrix, learned Q-matrix
#   start : integer, start node
#   end : integer, target node
#
# Output :
# path : vector, path of nodes

shortest_path <- function(mQ, start, end){
  # initialize path at start
  path <- c(start)
```

```r
  # get next node from start by maximizing Q-value
  next_node <- which.max(mQ[start,])

  # update path
  path <- c(path, next_node)

  # until end is reached, update path according to Q-matrix
  ## TO DO

  # return path
  return(path)
}

# find shortest path from 1 to 11
shortest_path(mQ, 1, 11)
```