No.

Date

(1) $$m(a+bX) = a + b \times m(X)$$

we know: $m(X) = 1/N \sum_{i=1}^{N} x_i$

$$\Rightarrow m(a+bX) = \frac{1}{N}\sum_{i=1}^{N}(a+bx_i) = \frac{1}{N}\left(\sum_{i=1}^{N} a + b\sum_{i=1}^{N} x_i\right)$$

we Know $\sum_{i=1}^{N} a = Na$

$$\therefore m(a+bX) = \frac{1}{N}\left(aN + b\sum_{i=1}^{N} x_i\right) = a + b\left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)$$

$$= \underline{\underline{a + bm(X)}}$$

(2) prove: $\text{cov}(X, a+bY) = b\,\text{cov}(X,Y)$

we know: $m(a+bX) = a + bm(X)$

$$\text{Cov}(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))((a+by_i) - m(a+bY))$$

$$m(a+bY) = a + bm(Y)$$

so, $(a+by_i) - (a+bm(Y)) = b(y_i - m(Y))$

$$\therefore \text{cov}(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))\cdot b(y_i - m(Y))$$

$$\Rightarrow b\cdot\frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(y - m(Y)) = \underline{\underline{b\,\text{cov}(X,Y)}}$$

③ $cov(a+bX, a+bX) = b^2 cov(X,X)$ & $cov(X,X) = s^2$

→ use substitution, where $U = a+bX$, $U_i = a+bx_i$

$$\Rightarrow cov(U,U) = \frac{1}{N}\sum_{i=1}^{N}(U_i - m(U))(U_i - m(U))$$

$m(U) = m(a+bX) = a + bm(X)$ * proved in ①

then, $U_i - m(U) = (a+bx_i) - (a+bm(X))$
$= b(x_i - m(X))$

⇒ pluging in back into $cov(U,U)$

$$\rightarrow = \frac{1}{N}\sum_{i=1}^{N} b^2 (x_i - m(X))^2 = b^2 \cdot \underbrace{\frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(x_i - m(X))}_{\text{detinetion of } cov(X,X)}$$

$\therefore cov(a+bX, a+bX)$
$= \underline{b^2 cov(X,X)}$

and since $s^2 = 1/N \sum_{i=1}^{N}(x_i - m(X))^2$

$cov(a+bX, a+bX) = b^2 s^2$

No.

Date

(4) let $\tilde{x}$ be the median, or median (X)

non-decreasing : if $x \geq x'$, then $g(x) \geq g(x')$

- If g is increasing & one-to-one on the sample, by applying g, it does not change which obsenation sits in the middle of orderd list, only ~~chagn~~ changes the value :

$$\text{median}(g(x)) = g(\text{median}(x))$$

" The transformed median(s) correspond to transforming the original median(s). "

∴ Yes, monoton, non-decreasing transformations carry medians to medians

Quantiles : by the same logic as before, a non-decreasing transformation preserves rank :

$$Q_p(g(x)) = g(Q_p(x)) \text{, non-uniqness}$$

IQR :

$$IQR(X) = Q_{0.75}(X) - Q_{0.25}(X)$$

For non-decreasing g,

$$\begin{aligned} IQR(g(x)) &= Q_{0.75}(g(X)) - Q_{0.25}(g(x)) \\ &= g(Q_{0.75}(x)) - g(Q_{0.25}(x)) \quad \checkmark \\ &\neq g(IQR(X)) \end{aligned}$$

∴ ~~not~~ does NOT apply

Range:

$$\text{range}(X) = \max(X) - \min(X)$$

$$\text{range}(g(X)) = \max(g(X)) - \min(g(X))$$
$$= g(\max(X)) - g(\min(X))$$
$$\neq g(\text{rang}(X))$$, cannot distribute g over

DOES not Apply !!

however for a linear transformation: $g(x) = \cancel{a} + bX$
it becomes $b\,\text{range}(X)$

(5) No, not generally, the mean uses arithmetic averaging where non-linear transformation do not commute w/ averaging

Example, let $X = \{0, 2\}$, $N = 2$, $g(x) = x^2$

non-decreasing from $[0, \infty)$

$m(X) = \frac{0+2}{2} = 1$, $g(m(X)) = 1^2 = 1$

but $m(g(x)) = \frac{0^2 + 2^2}{2} = \frac{0+4}{2} = 2$

$\therefore m(g(X)) \neq g(m(X))$

however for linear $g(x) = a + bX$ then

$$m(g(X)) = m(a + bX) = a + b\,m(X) = g(M(X))$$

From Question 1, proof