

2. Bayesian Networks.

ia). $A \perp\!\!\!\perp C \mid B, D$.

When conditioned on B and D all the paths are blocked between A and C, hence this is True.

Mathematically.

$$\frac{P(A, B, C, D)}{P(B, D)} = \frac{P(A \mid B, D) P(B \mid A, C) P(C \mid B, D) P(D \mid A, C)}{P(B \mid A, C) P(D \mid A, C)}$$

$$= P(A \mid B, D) P(C \mid B, D)$$

Therefore $A \perp\!\!\!\perp C \mid B, D$ is True

—————x

. $B \perp\!\!\!\perp D \mid A, C$

When conditioned on A and C all the paths are blocked between B and D, hence this is True.

Mathematically.

$$\frac{P(A, B, C, D)}{P(A, C)} = \frac{P(A \mid B, D) P(B \mid A, C) P(C \mid B, D) P(D \mid A, C)}{P(A \mid B, D) P(C \mid B, D)}$$

$$= P(B \mid A, C) P(D \mid A, C).$$

$\therefore B \perp\!\!\!\perp D \mid A, C$ is True

—————x