3) Markov Random Fields.

Energy Function.

a) • x_K is a target variable with two states $\{-1, 1\}$

• all the other variables are fixed.

$$E(x,y) = h\sum_{i\neq k} x_i - \beta\sum_{\{i,j\}\neq k} x_i x_j - \eta\sum_{i\neq k} x_i y_i + h x_K - \beta\sum_{l} x_K x_l - \eta x_K y_K$$

$$= h\sum_{i\neq k} x_i + h x_K - \beta\sum_{\{i,j\}\neq k} x_i x_j - \beta\sum_{l} x_K x_l - \eta\sum_{i\neq k} x_i y_i - \eta x_K y_K$$

• just like in $-\beta\sum_{\{i,j\}} x_i x_j$ x_i and x_j are adjacent and neighbours

Similarly $\boxed{-\beta\sum_{l} x_K x_l}$ → here x_K and x_l are neighbours.

$\boxed{+h x_K - \beta\sum_{l} x_K x_l - \eta x_K y_K}$ → shows dependence of $E(x,y)$ on x_K

∴ $E(x,y)$ can be written as.

$$\boxed{E(x,y) = h x_K - \beta\sum_{l} x_K x_l - \eta x_K y_K}$$

x_K has two states $\{-1, 1\}$

To get change in energy sub $x_K = 1$ & $x_K = -1$ in $E(x,y)$ and get the difference

let $x_K = 1$

$$E(x,y) = h - \beta\sum_{l} x_l - \eta y_K.$$

let $x_K = -1$

$$E(x,y) = -h + \beta\sum_{l} x_l + \eta y_K$$

$$E(x,y) = \left(h - \beta\sum_{l} x_l - \eta y_K\right) - \left(-h + \beta\sum_{l} x_l + \eta y_K\right)$$

$$\boxed{E(x,y) = 2h - 2\beta\sum_{l} x_l - 2\eta y_K}$$

Hence this shows that the difference only depends on quantities ~~that~~ that are local to x_K, which is implied by h, ~~and~~ x_l and y_K.