说明

1. 不准使用计算机。
2. 对一题得4分，错一题倒扣1分。
3. 答案E：若是“以上皆非”或“无法确定”，一律以“***”代替之。

INSTRUCTIONS

1. Calculators are not allowed.
2. 4 marks will be awarded for each correct answer and 1 mark will be deducted for each wrong answer.
3. (E) *** indicates “none of the above”.

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1. 求函数$f(x)=\left|x-3+\dfrac{1}{x-3}\right|-2$的值域，其中$x$为实数。

   Find the range of the function $f(x)=\left|x-3+\dfrac{1}{x-3}\right|-2$ where x is a real number.

   A. $\mathbb{R}$ B. $\mathbb{R}\backslash\{2\}$ C. $\mathbb{R}^+$ D. $\mathbb{R}^+\backslash\{2\}$ E. ***

2. 求级数$\lim\limits_{n\to\infty} n\left(\dfrac{\tan^{-1}\frac{1}{n}}{1^2+n^2}+\dfrac{\tan^{-1}\frac{2}{n}}{2^2+n^2}+\dfrac{\tan^{-1}\frac{3}{n}}{3^2+n^2}+\cdots+\dfrac{\tan^{-1}\frac{n}{n}}{n^2+n^2}\right)$的值。

   Find the value of $\lim\limits_{n\to\infty} n\left(\dfrac{\tan^{-1}\frac{1}{n}}{1^2+n^2}+\dfrac{\tan^{-1}\frac{2}{n}}{2^2+n^2}+\dfrac{\tan^{-1}\frac{3}{n}}{3^2+n^2}+\cdots+\dfrac{\tan^{-1}\frac{n}{n}}{n^2+n^2}\right)$.

   A. $\dfrac{\pi^2}{4}$ B. $\dfrac{\pi^2}{16}$ C. $\dfrac{9\pi^2}{16}$ D. $\dfrac{\pi^2}{32}$ E. ***

3. 若实数x、y满足$\begin{cases}|x|+x-xy=9\\|y|-y+3xy=-7\end{cases}$。求所有$y$的可能值之和。

   Given that real numbers x, y satisfy $\begin{cases}|x|+x-xy=9\\|y|-y+3xy=-7\end{cases}$. Find the sum of possible values of y.

   A. 1 B. -1 C. -6 D. -7 E. ***

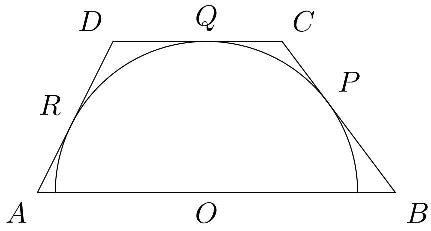
4. 一个矩阵定义为$a_{i,j}=i+j$。求这样一个7行6列的矩阵的所有元素之和。

   A matrix is defined by $a_{i,j}=i+j$. Find the sum of all elements of such matrix with 7 rows and 6 columns.

   A. 42 B. 294 C. 315 D. 588 E. ***

5. 图中，$ABCD$是一个梯形。一个半圆的圆心O在AB上，与BC、CD、DA相切于P、Q、R。若$OA-DQ=CQ=4$，$AD=\sqrt{80}$，求AB。

In the figure, $ABCD$ is a trapezium. A semicircle with center O on AB is tangent to BC, CD, DA at P, Q, R respectively. If $OA-DQ=CQ=4$, $AD=\sqrt{80}$, find AB.



A. $8\sqrt{5}$ B. $10+4\sqrt{5}$ C. 19 D. 21 E. ***

6. 若$\frac{4}{1!13!}+\frac{4}{3!11!}+\frac{4}{5!9!}+\cdots+\frac{4}{13!1!}=\frac{2^k}{14!}$，求$k$的值。

If $\frac{4}{1!13!}+\frac{4}{3!11!}+\frac{4}{5!9!}+\cdots+\frac{4}{13!1!}=\frac{2^k}{14!}$, find the value of k.

A. 11 B. 12 C. 13 D. 14 E. ***

7. 设$\omega=\sin\left(\frac{2\pi}{7}\right)+i\cos\left(\frac{2\pi}{7}\right)$，求$(i-\omega)(i-\omega^2)(i-\omega^3)\ldots(i-\omega^{27})$的值。

Let $\omega=\sin\left(\frac{2\pi}{7}\right)+i\cos\left(\frac{2\pi}{7}\right)$, find the value of $(i-\omega)(i-\omega^2)(i-\omega^3)\ldots(i-\omega^{27})$.

A. 0 B. i C. -27 D. $27i$ E. ***

8. 求2024^2的正因数的个数。

Find the number of positive factors of 2024^2.

A. 7 B. 21 C. 42 D. 63 E. ***

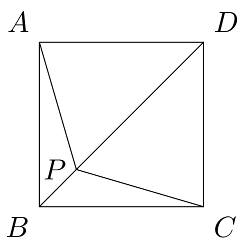
9. 求方程式$x^5-3x^4+3x^3-4x^2-4x+7=0$的实数解的个数。

Find the number of real solutions to $x^5-3x^4+3x^3-4x^2-4x+7=0$.

A. ≤ 2 B. 3 C. 4 D. 5 E. ***

10. 图中，$ABCD$为正方形。P是正方形内一点使得$\angle APD+\angle BPC=180°$。若$\angle BPC>90°$，$PB=7\sqrt{2}$、$PC=25$，求$AB$的长。

In figure 1, $ABCD$ is a square. P is a point in the square such that $\angle APD+\angle BPC=180°$. If $\angle BPC>90°$, and $PB=7\sqrt{2}$, $PC=25$, find the length of AB.



A. 24 B. 26 C. $26\sqrt{2}$ D. 31 E. ***

11. 若x、y、z是三个正数满足

$$(x+y)^2 = 144 + xy$$
$$(y+z)^2 = 289 + yz$$
$$(z+x)^2 = 625 + zx$$

求$xy+yz+zx$的值。

Let x, y, z be three positive reals such that

$$(x+y)^2 = 144 + xy$$
$$(y+z)^2 = 289 + yz$$
$$(z+x)^2 = 625 + zx$$

Find the value of $xy+yz+zx$.

A. 180 B. 360 C. $360\sqrt{3}$ D. 1080 E. ***

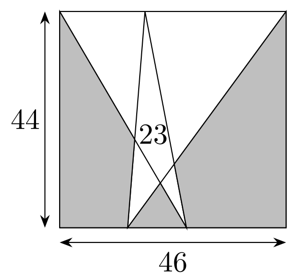
12. 若$\dfrac{3+n}{3n-1}$不是整数，可是$\dfrac{4(3+n)}{3n-1}$是整数，求所有正整数n的可能值之和。

Find the sum of all possible positive integer values of n such that $\dfrac{3+n}{3n-1}$ is not an integer, but $\dfrac{4(3+n)}{3n-1}$ is an integer.

A. 3 B. 10 C. 27 D. 37 E. ***

13. 如图，一个长方形的边长为44及46。中间区域面积为23。求阴影部分的总面积。

In the figure, a rectangle has dimensions 44 and 46. The region at the center has an area of 23. Find the total area of the shaded regions.



A. 989 B. 1035 C. 1495 D. 2001 E. 2047

14. 若a、b、c、d、e为不小于-1的整数。求方程式$a+b+c+d+e=6$的解(a,b,c,d,e)的个数。

Let a, b, c, d, e be integers not less than -1. Determine the number of solutions (a,b,c,d,e) to the equation $a+b+c+d+e=6$.

A. 330 B. 364 C. 1815 D. 1830 E. ***

15. 若$4^a+9^b+144^c = 2^a3^b + 2^{a+2c}3^c + 2^{2c}3^{b+c}$，且$a$、$b$、$c$为正数。则$c=$?

Let $4^a+9^b+144^c = 2^a3^b + 2^{a+2c}3^c + 2^{2c}3^{b+c}$ and a, b, c are positive reals. Then $c=$?

A. $\dfrac{ab}{a+2b}$ B. $\dfrac{ab}{2a+b}$ C. $\dfrac{2ab}{a+b}$ D. $\dfrac{2ab}{2a+b}$ E. ***

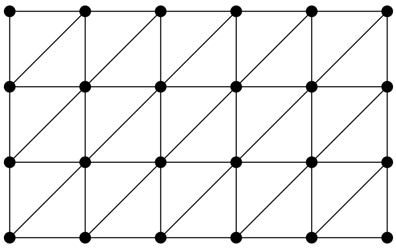
16. 已知曲线$y=\sqrt{2070+x}$与$y=\sqrt{2070+\sqrt{2070+\sqrt{2070+\sqrt{2070+x}}}}$在卡氏坐标上只有一个交点$(h,k)$。求$h+k$的值。

It is known that the graphs $y=\sqrt{2070+x}$ and $y=\sqrt{2070+\sqrt{2070+\sqrt{2070+\sqrt{2070+x}}}}$ have one and only one intersection (h,k) on the Cartesian plane. Find the value of $h+k$.

A. 46 B. 47 C. 92 D. 94 E. ***

17. 右图为一个3×5的方格。若小明在点A沿着直线走到点B，每次只能向右走、向上走或向右上走，有多少种走法?

The figure shows a 3×5 grid. If Xiao Ming wants to travel from A to B, he is only allowed to move up, move right, or move diagonally upright, how many ways are there?



A. 236 B. 246 C. 250 D. 266 E. ***

18. 若直线$4x-3y=5$与y轴相交于P。C_1与C_2为两个与此直线相切于P，半径为3的不同的圆。此外，圆C于y轴相切于P且半径为3。若C与C_1交于P及Q、C与C_2交于P及R，求QR的长。

If the line $4x-3y=5$ intersects the y-axis at P. C_1 and C_2 are two distinct circles tangent to this line at P and have radii 3. Moreover, C is a circle tangent to the y-axis at P and has radius 3. If C and C_1 intersect at P and Q, C and C_2 intersect at P and R, find the length of QR.

A. 3 B. 6 C. $3\sqrt{2}$ D. $3\sqrt{3}$ E. ***

19. 求$(\sqrt{2}-1)\displaystyle\prod_{n=2}^{2024}\left(\frac{n^2+n-1}{\sqrt{n^2+n}+1}+\frac{1}{\sqrt{n+1}+\sqrt{n}}\right)$的值。

Find the value of $(\sqrt{2}-1)\displaystyle\prod_{n=2}^{2024}\left(\frac{n^2+n-1}{\sqrt{n^2+n}+1}+\frac{1}{\sqrt{n+1}+\sqrt{n}}\right)$.

A. 2023! B. 2024! C. $44\times 2023!$ D. $45\times 2024!$ E. ***

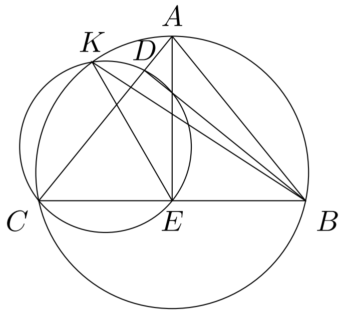
20. 若630名高三学生规定要参加至少一个课外活动。每个学生至多能同时参加三个课外活动。若每一个课外活动有110个人参加，每两个课外活动有24人参加，每三个课外活动有3个人参加。已知共有n个不同的课外活动，求n的值。

Assume that 630 senior three students must participate in at least one extracurricular activity. Each student can participate in at most three extracurricular activities simultaneously. Each extracurricular activity has 110 participants, every two extracurricular activities have 24 participants in common, and every three extracurricular activities have 3 participants in common, then given that there are n different extracurricular activities, find the value of n.

A. 5 B. 6 C. 8 D. 9 E. ***

21. 图中，$\triangle ABC$是一个等腰三角形，$AB = AC$，$\angle BAC = 78°$。D、E在AC、BC上使得$BD \perp AC$，$AE \perp BC$。若经过A、B、C的圆与经过C、D、E的圆相交于K，求$\angle BKE$。

In the figure, $\triangle ABC$ is an isosceles triangle, $AB = AC$, $\angle BAC = 78°$. D, E are on AC, BC such that $BD \perp AC$, $AE \perp BC$. If the circle passing through A, B, C and the circle passing through C, D, E intersect again at K, find the value of $\angle BKE$.



A. 26° B. 27° C. 30° D. 39° E. ***

22. 若$\dfrac{a^2b^2}{a^2+b^2} = \dfrac{4}{5}$，$\dfrac{b^2c^2}{b^2+c^2} = \dfrac{36}{13}$及$\dfrac{c^2a^2}{c^2+a^2} = \dfrac{10}{9}$，求$a^2+b^2+c^2$的值。

If $\dfrac{a^2b^2}{a^2+b^2} = \dfrac{4}{5}$, $\dfrac{b^2c^2}{b^2+c^2} = \dfrac{36}{13}$ and $\dfrac{c^2a^2}{c^2+a^2} = \dfrac{10}{9}$, find the value of $a^2+b^2+c^2$.

A. 9 B. 10 C. 14 D. 19 E. ***

23. 若实数数列a_1、a_2、a_3、...、a_{24}中每一项a_k满足$a_k^2 = 1$。则$a_1+a_2+a_3+\cdots+a_{24}$之值有多少种可能？

If the sequence of real numbers $a_1, a_2, a_3, \ldots, a_{24}$ satisfies, for any a_k, we have $a_k^2 = 1$. How many possible values are there for $a_1+a_2+a_3+\cdots+a_{24}$?

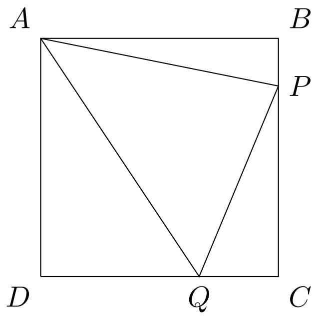
A. 24 B. 25 C. 48 D. $\dbinom{24}{2}$ E. ***

24. 求极限$\displaystyle\lim_{x\to\infty} \frac{\sqrt{2(9x)^6+\sqrt{3(9x)^{12}+\sqrt{(9x)^{24}+1}}}}{\sqrt{2(9x)^6}+\sqrt{3(9x)^6}+\sqrt{(9x)^6+1}}$的值。

Find the limit $\displaystyle\lim_{x\to\infty} \frac{\sqrt{2(9x)^6+\sqrt{3(9x)^{12}+\sqrt{(9x)^{24}+1}}}}{\sqrt{2(9x)^6}+\sqrt{3(9x)^6}+\sqrt{(9x)^6+1}}$.

A. $\dfrac{2+\sqrt{2}+\sqrt{6}}{2}$ B. $\dfrac{2+\sqrt{2}-\sqrt{6}}{2}$ C. $\dfrac{-2+\sqrt{2}+\sqrt{6}}{2}$ D. $\dfrac{-2-\sqrt{2}+\sqrt{6}}{2}$ E. ***

25. 如图所示为一个正方形$ABCD$。P、Q在BC、CD上使得$BP=9$，$DQ=30$且$\angle PAQ=45°$。求$\triangle PAQ$的面积。
The figure shows a square $ABCD$. P, Q are points on BC, CD such that $BP=9, DQ=30$ and $\angle PAQ=45°$. Find the area of $\triangle PAQ$.



A. $\dfrac{1677}{2}$ B. 882 C. $\dfrac{1763}{2}$ D. $\dfrac{1755}{2}$ E. ***

26. 若$N=1^2\times3^2\times5^2\times7^2\times\cdots\times2025^2$是由1到2025的奇数的平方的乘积。若$N$可以被$539^k$整除，求$k$的最大值。
Given that $N=1^2\times3^2\times5^2\times7^2\times\cdots\times2025^2$ is the product of the squares of all odd numbers from 1 to 2025. If N can be divided by 539^k, find the largest possible value of k.

A. 100 B. 101 C. 202 D. 340 E. ***

27. 有多少位五位正整数满足：偶位数数字之和是奇数，奇位数数字之和是偶数?
How many five-digit positive integers such that: the sum of even digits is odd, and the sum of odd digits is even?

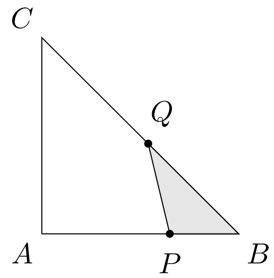
A. 22500 B. 25000 C. 27500 D. 50000 E. ***

28. 若$f(x)=f(b-ax)$，及$\displaystyle\int_0^b xf(x)\mathrm{d}x=k\int_0^{b/a}xf(x)\mathrm{d}x$，求$\displaystyle\int_0^{b/a}xf(x)\mathrm{d}x$的值。
If $f(x)=f(b-ax)$ and $\displaystyle\int_0^b xf(x)\mathrm{d}x=k\int_0^{b/a}xf(x)\mathrm{d}x$, find the value of $\displaystyle\int_0^{b/a}xf(x)\mathrm{d}x$.

A. $\displaystyle\int_0^b\frac{bf(x)}{a^2+k}\mathrm{d}x$ B. $\displaystyle\int_0^b\frac{bf(x)}{a^2-k}\mathrm{d}x$ C. $\displaystyle\int_0^a\frac{bf(x)}{a^2+k}\mathrm{d}x$ D. $\displaystyle\int_0^a\frac{bf(x)}{a^2-k}\mathrm{d}x$ E. ***

29. 如图，ABC是一个等腰直角三角形的步道，$AB = BC = 5$。若P、Q两人在三角形的边上，且P往某个方向绕着步道行走5个单位后刚好抵达Q。假设P走过的这个路程为ℓ，求ℓ与PQ所围成的区域的最大面积。

In the figure, ABC is a right isosceles triangular pathway, $AB = BC = 5$. If P, Q are two people on the pathway, and that P will meet Q after walking 5 units along the pathway in some direction. Let the trail walked by P be ℓ, find the maximum area enclosed by ℓ and PQ.



A. $\frac{125}{16}$ B. $\frac{25}{8}$ C. 4 D. $\frac{25\sqrt{2}}{16}$ E. ***

30. 求$\frac{x^2}{4} + \frac{y^2}{9} = 1$与$y = 4\sin(\pi x)$在卡氏坐标上的交点数量。

Find the number of intersection of the graphs $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and $y = 4\sin(\pi x)$ on the Cartesian plane.

A. 6 B. 8 C. 9 D. 10 E. ***

31. 若a、b、c是正数且$a^3 + b^2 + c = 13$，求ab^2c^3的最大值。

Given a, b, c are positive numbers such that $a^3 + b^2 + c = 13$, find the largest value of ab^2c^3.

A. 27 B. 81 C. 2187 D. 6561 E. ***

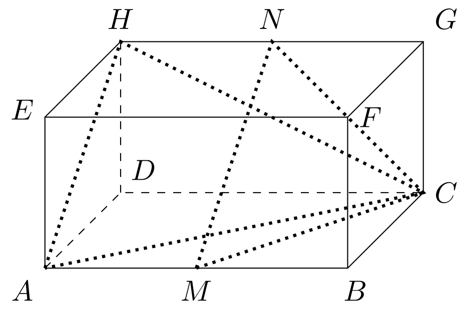
32. 7个人各掷一枚硬币。这枚硬币掷出正面的概率为p。已知刚好3个人掷出正面的概率为$\frac{560}{2187}$，则恰好2人掷出正面的概率为多少？

7 people each toss a coin. The probability of tossing a head of the coin is p. It is given that the probability of exactly 3 people tossing heads is $\frac{560}{2187}$, then the probability of exactly 2 people tossing heads is?

A. $\binom{7}{2}p^5(1-p)^2$ B. $p^2(1-p)^5$ C. $\frac{224}{729}$ D. $\frac{280}{2187}$ E. ***

33. 图中，$ABCD - EFGH$是一个长方体。其中$AB : BC : AE = 2 : 1 : 1$。若M、N为AB、GH的中点，且平面CMN与CHA的夹角为θ，求$\cos\theta$。

In the figure, $ABCD - EFGH$ is a regular cuboid. Given that $AB : BC : AE = 2 : 1 : 1$. If M, N are the midpoints of AB, GH respectively, the angle between planes CMN and CHA is θ, find the value of $\cos\theta$.



A. $\dfrac{\sqrt{3}}{3}$ B. $\dfrac{5\sqrt{3}}{9}$ C. $\dfrac{5\sqrt{3}}{3}$ D. $\dfrac{7\sqrt{3}}{3}$ E. ***

34. 若$\dfrac{2x}{y} = \dfrac{y^2}{z} = \dfrac{\sqrt{z}}{T}$，其中$x$、$y$、$z$、$T$为正实数，则$x$与$T$的关系为?
If $\dfrac{2x}{y} = \dfrac{y^2}{z} = \dfrac{\sqrt{z}}{T}$, and that x, y, z, T are positive reals, then the relation between x and T is?

A. $T = \dfrac{1}{2x}$ B. $T = 2x$ C. $T = \dfrac{1}{\sqrt{2x}}$ D. $T = \sqrt{2x}$ E. ***

35. 已知三次多项式$P(x)$有三个相异的实数根α_1、α_2、α_3且$\alpha_1 > \alpha_2 > \alpha_3$且$x^3$的系数为1。若$P'(\alpha_1) = -12$、$P'(\alpha_2) = 4$、$P'(\alpha_3) = 27$，求$(\alpha_1 - \alpha_2)(\alpha_2 - \alpha_3)(\alpha_3 - \alpha_1)$的值。
It is known that the polynomial of degree three $P(x)$ has three distinct real roots α_1, α_2, α_3 and $\alpha_1 > \alpha_2 > \alpha_3$ and the coefficient of x^3 equals 1. If $P'(\alpha_1) = -12$, $P'(\alpha_2) = 4$, $P'(\alpha_3) = 27$, find the value of $(\alpha_1 - \alpha_2)(\alpha_2 - \alpha_3)(\alpha_3 - \alpha_1)$.

A. 6 B. -6 C. 36 D. -36 E. ***