

Tries + Trees



Agenda

1. Trie of Bits
2. Maximum XOR of two elements
3. Maximum XOR of a subarray
4. Flatten a Tree into Linkedlist
5. Swapped Nodes on BST

Hello Everyone

Very Special Good Evening

to all of you 😊😊😊

We will start session

from 9:06 PM

Trie of Bits

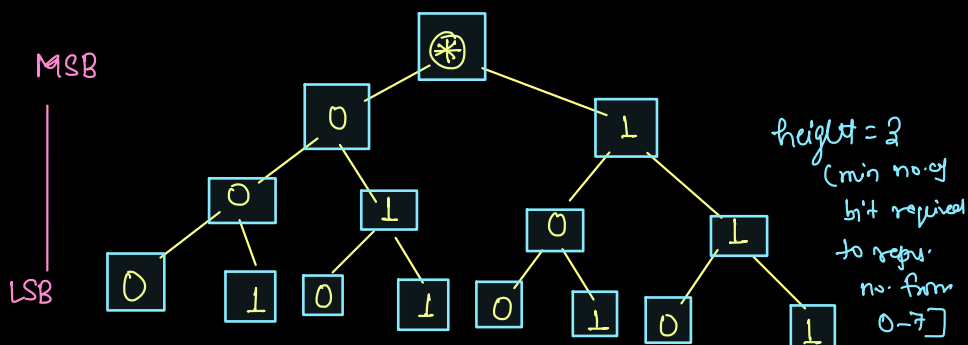
- Hierarchical Data Structure
- prefix Tree

Trie on Bit Representation

Numbers from 0 to 7:

→ min. no. of bits Required to store 0 to 7 = 3 bits

	MSB	LSB
0 →	0	0 0 ✓
1 →	0	0 1 ✓
2 →	0	1 0 ✓
3 →	0	1 1 ✓
4 →	1	0 0 ✓
5 →	1	0 1 ✓
6 →	1	1 0 ✓
7 →	1	1 1 ✓



	Range	height
0-7 →	2^3	3
0-15 →	2^4	4
0-30 →	$0 \text{ to } 2^5 - 1$	height = 5

max = N are available, height = $\log_2 N$

```

Node {
  Node[] child;
  Node() {
    child = new
    Node[2];
  }
}
    
```

Maximum XOR of two elements

Given an integer array A, find the maximum value of $A[i] \oplus A[j]$ for all (i, j) pair

arr: [9, 8, 10, 7]
 0 1 2 3

* Same same puppy shame

Bruteforce Approach: Explore all possible pair & find max result

A	B	$A \oplus B$
0	0	→ 0
0	1	→ 1
1	0	→ 1
1	1	→ 0

$$\begin{array}{r} 9 \rightarrow 1001 \\ 8 \rightarrow 1000 \\ \hline 0001 \end{array} \quad \begin{array}{r} 9 \rightarrow 1001 \\ 10 \rightarrow 1010 \\ \hline 0011 \end{array} \quad \begin{array}{r} 9 \rightarrow 1001 \\ 7 \rightarrow 0111 \\ \hline 1110 \end{array}$$

$$\begin{array}{r} 8 \rightarrow 1000 \\ 10 \rightarrow 1010 \\ \hline 0010 \end{array} \quad \begin{array}{r} 8 \rightarrow 1000 \\ 7 \rightarrow 0111 \\ \hline 1111 \rightarrow \text{max} \end{array} \quad \begin{array}{r} 10 \rightarrow 1010 \\ 7 \rightarrow 0111 \\ \hline 1101 \end{array}$$

ans = 15

Optimised Approach:

$$\begin{array}{r} A \rightarrow 1011101 \\ B \rightarrow 0100010 \end{array}$$

$$\text{max XOR} \equiv \underline{1111111} \\ \text{(Best possibility)}$$

$$\begin{array}{r} A \rightarrow 1011101 \\ B \rightarrow 1100010 \\ \hline 0111111 \\ \hline 2^6 \quad 2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0 \end{array}$$

$$\Rightarrow 2^6 - 1$$

$$\begin{array}{r} A \rightarrow 1011101 \\ B \rightarrow 0011101 \\ \hline 1000000 \\ \hline \uparrow \\ 2^6 \end{array}$$

$$\rightarrow 2^6$$

$2^6 > 2^6 - 1$ \therefore MSB matters a lot

Second number
arr[]: [5, 20, 15, 10, 25, 30]
0 1 2 3 4 5
XOR → 28 30 27

Prepare Trie of these numbers.

ele → binary representation

5 → 00101

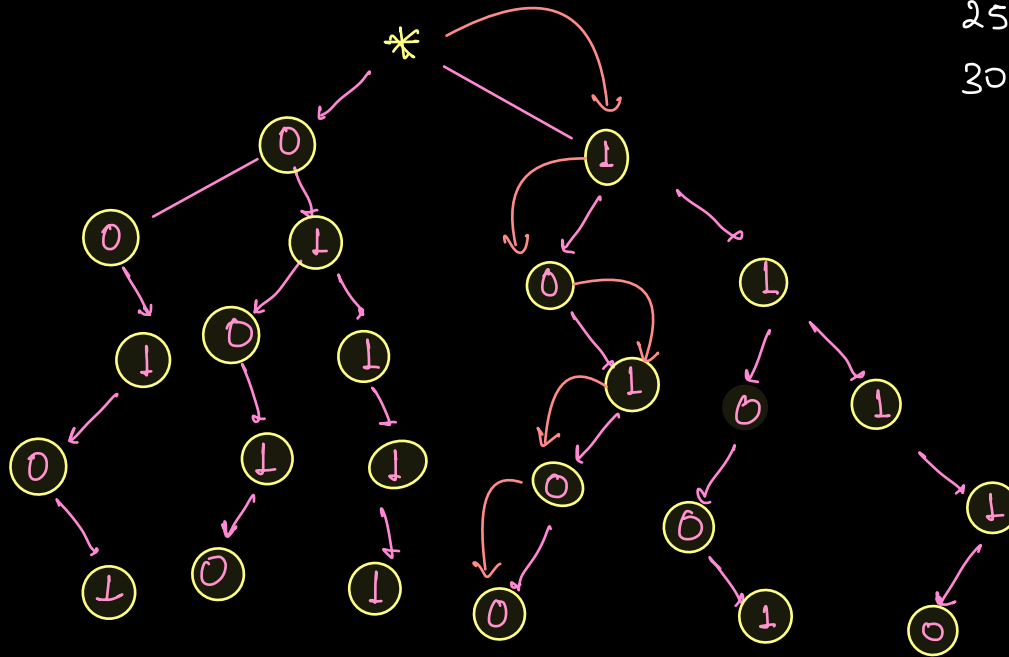
20 → 10100

15 → 01111

10 → 01010

25 → 11001

30 → 11110



5 → 00101
demand → 11010
11100

20 → 10100
demand → 01011
11110

15 → 01111
demand → 10000
11011

... and so on.

TODO: Complete the dry run...

pseudocode:

1. Find max element of given array.
2. Find "min count of bits" Required to make that number

```

int count=0;
while(max != 0) {
    |   max = (max >> 1);
    |   count++;
}
  
```

```

③ Node root = new Node();

for(int ele: arr) {
    |   insert(root, ele, count);  → TODO:
    |
3

```

```

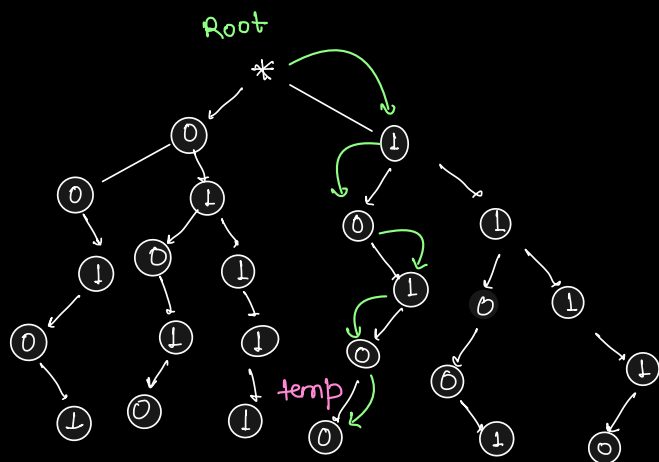
④ int max = 0;
for(int ele: arr) {
    |   max = Math.max(max, bestXORWithEle(root, ele, count))
    |
3
return max;

```

BestXORWithEle(Node root, int ele, int bits) {

ele = 15 bits = 5

ele → 0 1 1 1 1
4 3 2 1 0
XOR → 1 1 0 1 1



bit index	bit	required	av1?
4	0	1	T → ✓
3	1	0	T → ✓
2	1	0	False ×
1	1	0	T → ✓
0	1	0	T → ✓

→ * Set the bit in xor
* move to that node

BestXORWithEle(Node root, int ele, int bits) {

```

    int xor = 0; , Node temp = root;
    for(int bi = bits-1; bi >= 0; bi--) {
        // find bit at bit index in ele
        required bit (xb) int bit = (ele & (1 << bi) != 0) ? 1 : 0;
        if(temp.child[rb] != null) {
            xor = (xor | (1 << bi)); → set the bit
            temp = temp.child[rb];
        }
        else {
            temp = temp.child[1-rb];
        }
    }
    return xor;
}

```

T.C: $O(n * x)$

S.C: $O(n * x)$

$x \rightarrow$ no. of bits required in max element

Maximum XOR of a subarray

Given an integer array A, find subarray with maximum XOR value

arr: [1, 4, 3]
 0 1 2

ele \rightarrow bits

1 \rightarrow 001

4 \rightarrow 100

3 \rightarrow 011

all subarrays:

[1] \rightarrow 1

[1, 4] \rightarrow 5

[1, 4, 3] \rightarrow 6

[4] \rightarrow 4

[4, 3] \rightarrow 7 max = 7, ans

[3] \rightarrow 3

Brute force: consider all possible subarrays & solve it for start to end

T.C: $O(n^3)$

consider all possible subarray: \rightarrow

a	b	c	d	e	f	g	h
0	1	2	3	4	5	6	7

prefix XOR[5] \rightarrow a ^ b ^ c ^ d ^ e ^ f

prefix XOR[2] \rightarrow a ^ b ^ c

XOR from 3 to 5 \rightarrow prefix XOR[5] ^ prefix XOR[2]

\rightarrow ~~a ^ b ^ c ^ d ^ e ^ f~~ ^ ~~a ^ b ^ c~~

\rightarrow d ^ e ^ f

iterate on all possible value of i, j & calculate max xor result

XOR from (i) to (j) \rightarrow pxor[j] ^ pxor[i-1]
 $\underbrace{\quad}_{i-1 \geq 0}$

T.C: $O(n^2)$

S.C: $O(n)$

Optimised Approach:

arr: [a, b, c, d, e, f]

prefix XOR \rightarrow pxor[]: [e1, e2, e3, e4, e5, e6]

XOR b/w i, j \rightarrow pxor[j] \wedge pxor[i-1]

max \rightarrow all possible pair of (i, j) \rightarrow max XOR pair in prefix of XOR array.

pseudocode:

```
int n = arr.length;
```

```
int[] pxor = new int[n];
```

```
pxor[0] = arr[0];
```

```
for(int i=1; i<n; i++) {
```

```
    pxor[i] = pxor[i-1]  $\wedge$  arr[i];
```

```
}
```

Now use max XOR pair function \rightarrow previous problem
and pass array as pxor.

T.C: $O(n \times x)$

S.C: $O(n \times x)$

10:44 - 10:50 pm

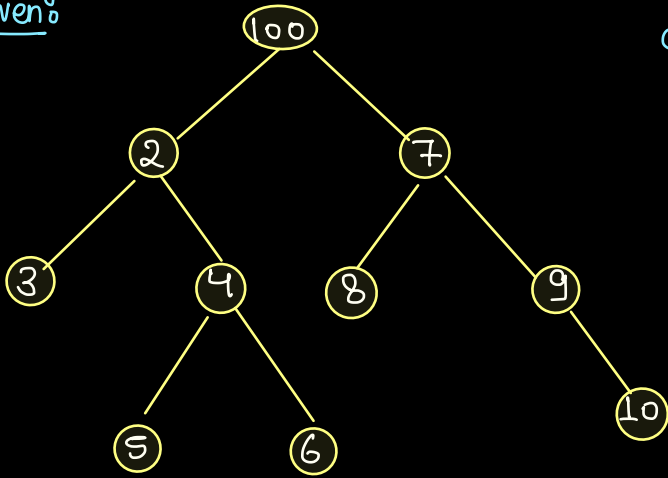
Break

$x \rightarrow$ max bit count in pxor array.

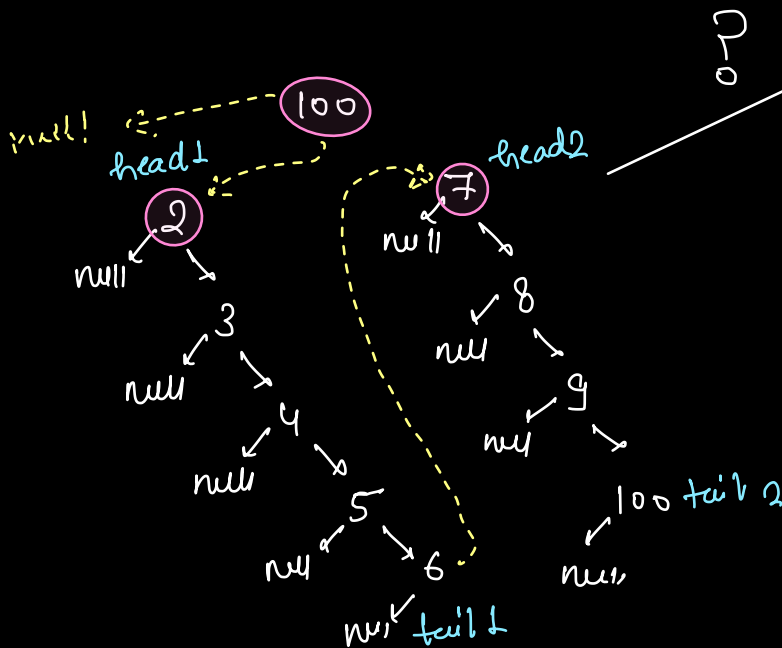
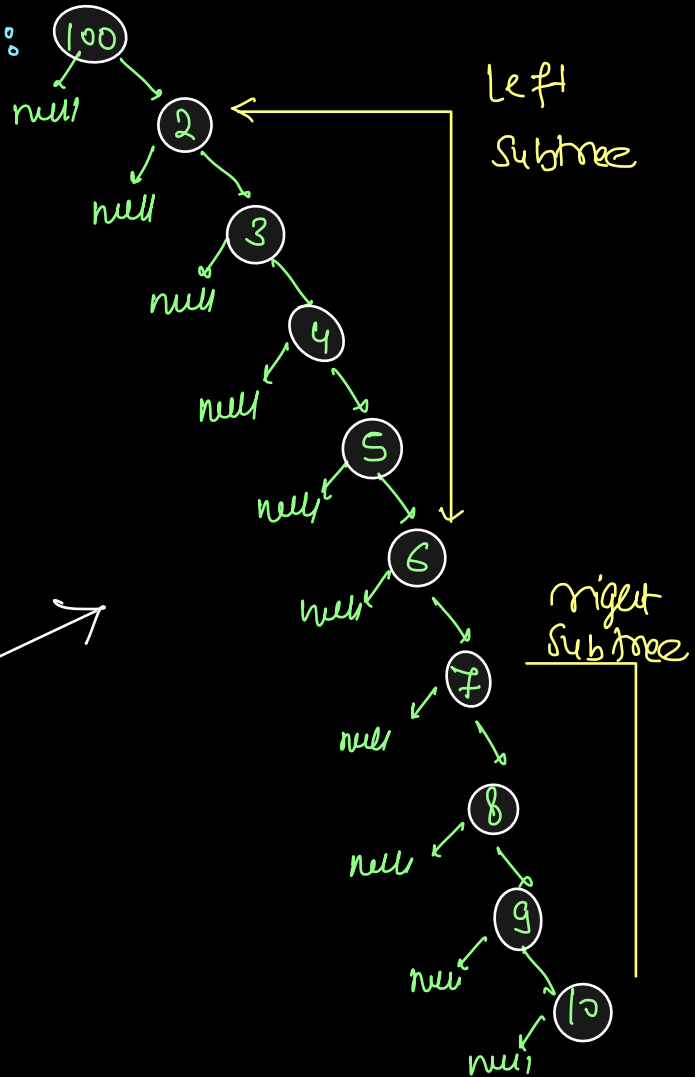
Flatten a Tree into Linkedlist

Flatten the given binary tree to linked list in a preorder manner such that right child will become next and left child for all nodes should be null

Given:



Output:



tail1.right = head1;
root.right = head1;
root.left = null;

return type from traversal \rightarrow pair \Rightarrow

```
class pair {
    Node head;
    Node tail;
}
```

pair flatten (Node root) {

if (root == null) {
 | head tail
 | return new pair (null, null);
 |
 }
 3

pair lp = flatten (root.left);

pair rp = flatten (root.right);

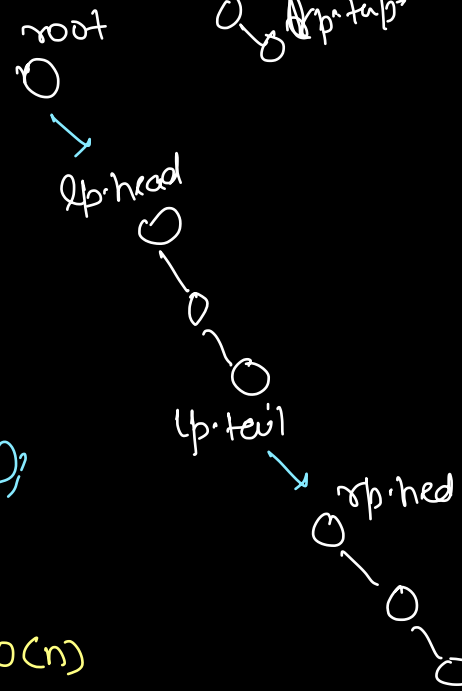
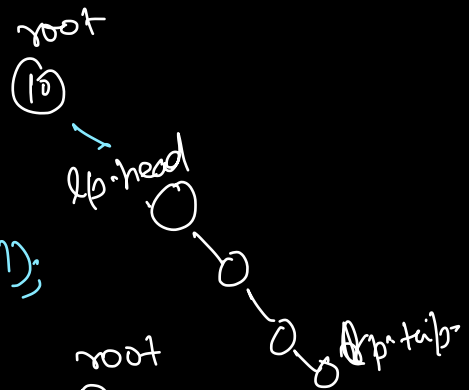
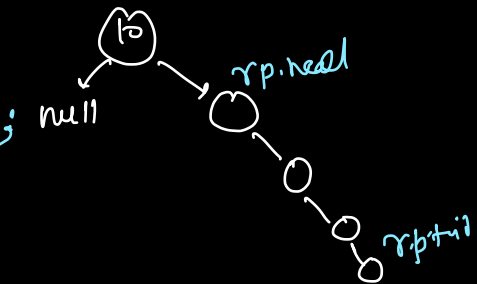
if (lp.head == null & rp.head == null) {
 | return new pair (root, root);
 |
 }
 3

else if (lp.head == null) {
 | return new pair (root, rp.tail);
 | null
 }
 3

else if (rp.head == null) {
 | root.left = null;
 | root.right = lp.head;
 | return new pair (root, lp.tail);
 }
 3

else {

root.right = null;
 lp.tail.right = rp.head;
 root.left = null;
 root.right = lp.head;
 return new pair (root, rp.tail);
 }
 3



T.C: $O(n)$
 S.C: $O(n+1)$;

↳ Recursive space

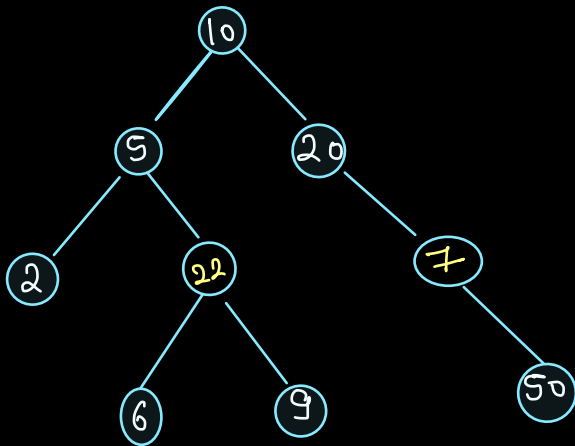
→ in place change in Tree

TODO: Dry Run

2

Swapped Nodes on BST

Given a BST where exactly 2 nodes are swapped find the two nodes (distinct nodes)



inorder is sorted.

Inorder $\rightarrow [2, 5, 6, 22, 9, 10, 20, 7, 50]$
first second

→ inorder traversal,

Inorder using morris traversal

→ prev pointer

→ curr pointer

Already covered in Tree section:

→ Revise it from there:

X

X