Backlog Coverage Session

Agenda

1. Z- Algorithm → KMP
2. DSU – Disjoint set Union
3. Krushkal Algo
   Expectation: MST
   Prims Algo

Hello Everyone
Very Special Good Evening
to all of you 😊😊😊
We will start session
from 9:06 PM

Z- Algorithm:

Requirment:

* no. of occurrence of pattern in text.
* indexes of pattern which is available in text.

for eg: pattern → a b a

text → a b a b a d a b a b a e
0 1 2 3 4 5 6 7 8 9 10 11

no. of occurrence of pattern → 4
Starting indices → [0, 2, 6, 8]

Brute force:

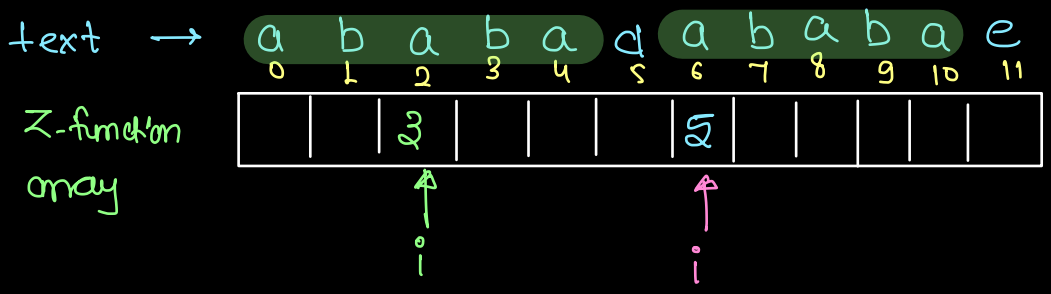
Using sliding window we can do that.

→ for more reference of brute for visit → pattern matching session.

When we solved KMP, we take help of LPS → length of longest prefix suffix

(NOTE: excluding complete string)

similarly for Z-Algo, we have to use Z-function



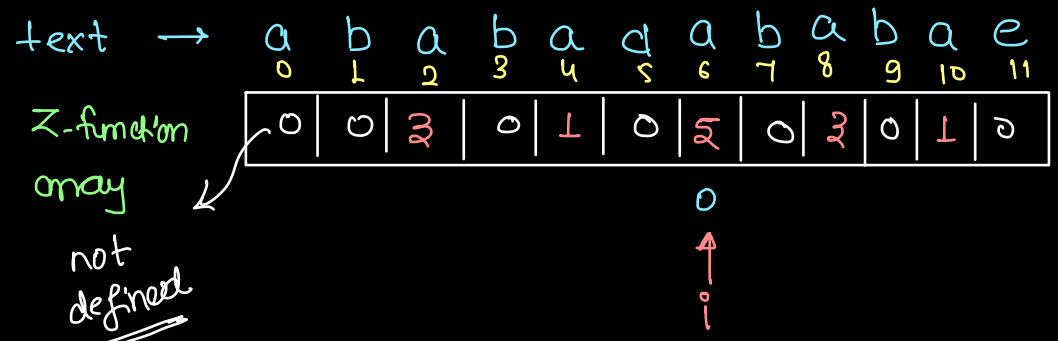
length of longest substring which is starting from index 'i' and it is also proper prefix.

proper prefix

str = "abac"

a
ab
aba
abac

proper prefix



pseudocode:

```
int[] zfunction(string s){
    int[] z = new int[n]; // value of z[i], initially 0.
    for(int i=1; i<n; i++){
        while( s[z[i]] == s[i+z[i]] ){
            z[i]++;
        }
    }
    return z;
}
```

worst case: str - a a a a a a

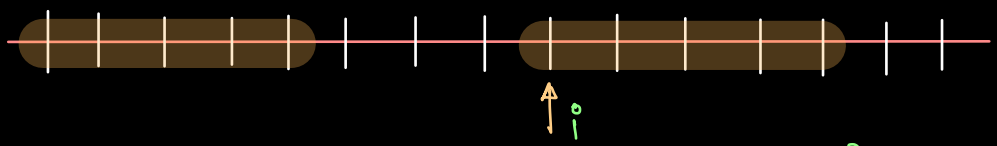
for (n-1) (n-2) (n-3) ... 1

total str = 1+2+3+...+(n-1)

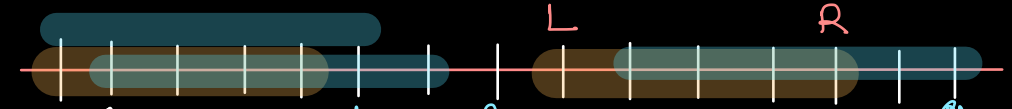
$= \frac{n(n-1)}{2}$

TC: $O(n^2)$

How to optimise - Z-function: (Assumption, vertical lins are char)



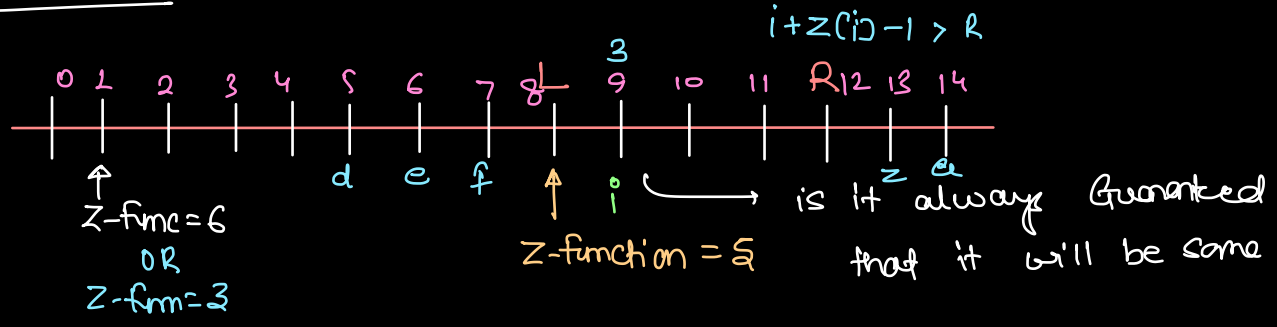
Z-function = 5 → first 5 chars are equal to substrng of length 5 which is starting from index i



→ is it always Guaranteed that it will be same

NOTE: for this particular point we are not sure about Z-func & chars since it is beyond the range of R.
But we are sure within the Range of Z-function will be (4) (R-i+1)

pseudo code: (optimise)




```
int[] zfunction(string s){
    int[] z = new int[n]; // value of z[i], initially 0.
    int l=0, r=0;
    for(int i=1; i<n; i++){
        if( i<=R)
            z[i]= Min( z[i-l] , R-i+1)
        while( s[z[i]] == s[i+z[i]]){
            z[i]++;
        }
        if( i+z[i]-1 >R){
            L= i;
            R= i+z[i]-1;
        }
    }
    return z;
}
```

overall str by for-loop ⇒ n

overall " " while-loop ⇒ n

total str = 2n

⇒ T.C : O(n)

S.C : O(n)

	a	b	a	b	a	d	a	b	a	b	a	e
	0	1	2	3	4	5	6	7	8	9	10	11
Z-fnction	0	0	3	0	1	0	5	0	3	0	1	0

L = 0 2 6

R = 0 4 10

i = 1 2 3 4 5 6 7 8 9 10 11 12

```
int[] zfunction(String S){
    int[] z = new int[n];
    int l=0, r=0;
    for(int i=1; i<n; i++){
        if( i<=R)
            z[i] = Min(z[i-l], R-i+1)
        while( s[z[i]] == s[i+z[i]] ){
            z[i]++;
        }
        if( i+z[i]-1 > R){
            l = i;
            R = i+z[i]-1;
        }
    }
    return z;
}
```

Pattern matching using Z-Algo:

pattern → a b a

text → a b a b a d a b a b a e

0 1 2 3 4 5 6 7 8 9 10 11

pat + # + text ⇒ a b a # a b a b a d a b a b a e

z-function → 0 0 1 0 3 0 3 0 1 0 3 0 3 0 1 0

Substring with length 3 is same as Proper prefix

pseudo code:

```
int countOfPatternInText(String pat, String text){
    String str = pat + "#" + text;
    int[] z = zfunction(str);          → n
    for(int ele : z){                  → n
        if(ele == pat.length){
            count++;
        }
    }
    return count;
}
```

Tc: O(n+m)

Sc: O(n+m)

n → pat length

m → text length

10:21 - 10:35

Break

DSU - Disjoint Set Union

→ union → Merging their leaders
→ find → leader

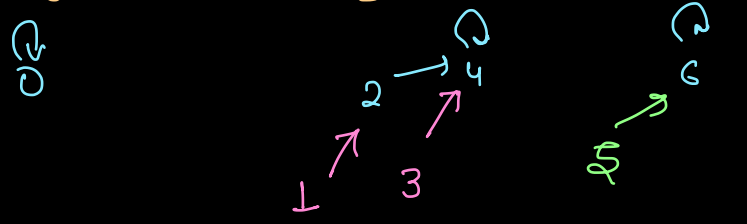
for managment of leaders:

DSU follows transitivity.

→ a similar to b
b " " c
⇒ a " " c

parent:

0	2	4	4	4	6	6
0	1	2	3	4	5	6



Edges

u	v
1	2
3	4
1	4
5	6

(2-4)

u = 1 → find leader = 2
v = 4 → find leader = 4

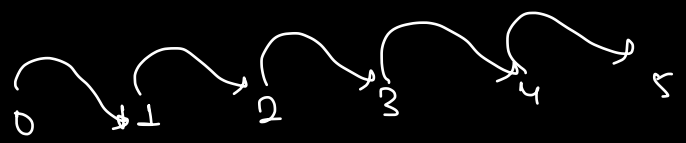
u = 5 → leader(5) = 5
v = 6 → leader(6) = 6

```
int[] parent;  ⟶ parent array globally available
```

```
int find(int x) {
    if( par[x] == x) {
        return x;
    }
    int temp = find(par[x]);
    return temp;
}
```

```
void union(int x, int y) {
    int px = find(x);
    int py = find(y);
    if( px != py){
        // merge them
        par[px] = py;
    }
}
```

worst situation:



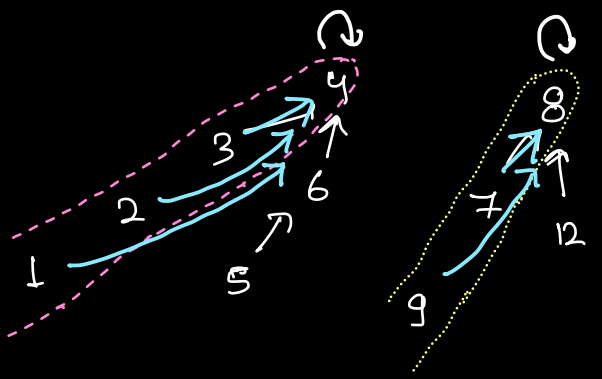
travel for parent of 0 to 4 → 1000 times.

→ Repeat n-1 iterations every time.

* Concept for optimation of DSU.

1. Path Compression
2. Union by Rank

Situation:

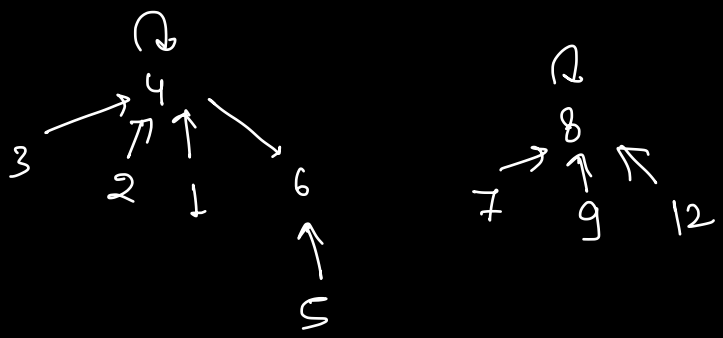


parent(1) → parent of set.

parent(9)

Path Comparison.

find function

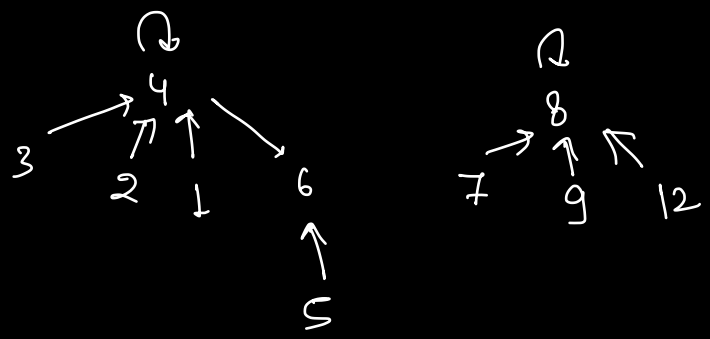


Union by Rank:

leader

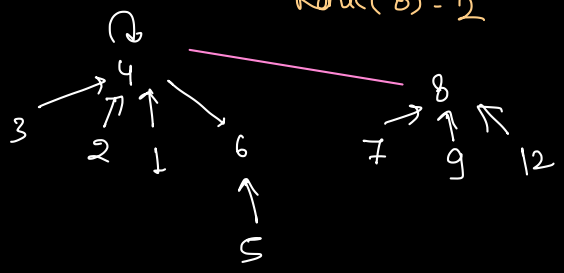
4 → 6 → 4

8 → 12 → 8



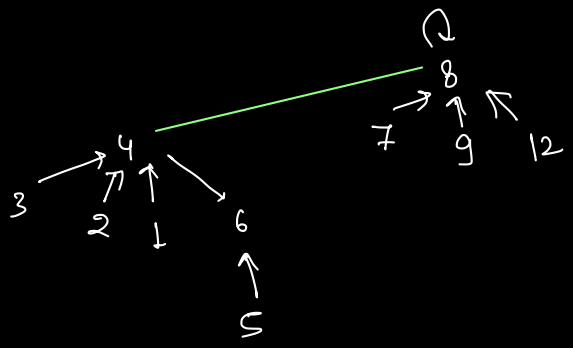
I

Rank is height Rank(4) = 3
Rank(8) = 2



this is better possibility

II



NOTE: union by Rank
make leader which have higher Rank available.
↳ prepare one Rank array as well

parent:

0	2	4	4	4	6	6
0	1	2	3	4	5	6

Rank:

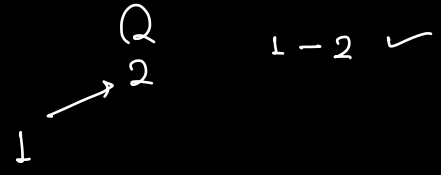
1	1	1	1	1	1	1
0	1	2	3	4	5	6

initial Rank = 1

path Compression:

```
int[] parent;

int find(int x) {
    if( par[x] == x) {
        return x;
    }
    int temp = find(par[x]);
    par[x] = temp; // path compression
    return temp;
}
```



parent ultimate find in O(1)
union is also in O(1)

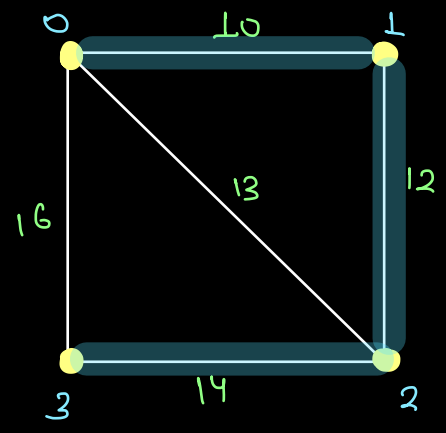
union by Rank

```
void union(int x, int y) {
    int px = find(x);
    int py = find(y);
    if( px != py) {
        if(rank[px] > rank[py]){
            par[py] = px;
        }
        else if( rank[px] < rank[py]){
            par[px] = py;
        }
        else {
            par[px] = py;
            rank[py]++;
        }
    }
}
```

Krushkal's Algorithm:

MST → min. spanning Tree
make sure that (n-1) edges are there.

Tree
n-nodes
Edge ? → (n-1)

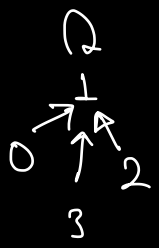


Connect all vertex with min possible cost
⇒ MST

0 – 1 @ 10
1 – 2 @ 12
0 – 2 @ 13
0 – 3 @ 16
3 – 2 @ 14

} n-1 edges are relevant

Sort edgs on the basis of their weight in ↑ing order



3 edgs

0 – 1 @ 10 ✓
1 – 2 @ 12 ✓
~~0 – 2 @ 13~~ ✓ → same parent they are already connected.
3 – 2 @ 14 ✓
~~0 – 3 @ 16~~ ✓ → same parents they are already connected.

10 + 12 + 14 ⇒ 36

```java
import java.util.*;

public class Main {

  public static class Pair {
    int u;
    int v;
    int wt;

    Pair(int u, int v, int wt) {
      this.u = u;
      this.v = v;
      this.wt = wt;
    }
  }

  // edges[i][0] -> u, edges[i][1] -> v, edges[i][2] -> wt
  public static int minCostOfSpanningTree(int[][] edges, int V) {
    // 1. sort the edges on the basis of weight
    Pair[] arr = new Pair[edges.length];
    for(int i = 0; i < edges.length; i++) {
      arr[i] = new Pair(edges[i][0], edges[i][1], edges[i][2]);
    }

    // sort on the basis of weight
    Arrays.sort(arr, new Comparator<Pair>() {
      public int compare(Pair a, Pair b) {
        return a.wt - b.wt;
      }
    });

    // initialised leader and rank array
    leaders = new int[V];
    rank = new int[V];
    for(int i = 0; i < V; i++) {
      leaders[i] = i;
      rank[i] = 1;
    }

    int cost = 0;
    for(int i = 0; i < arr.length; i++) {
      Pair edge = arr[i];

      boolean flag = union(edge.u, edge.v);
      if(flag == true) {
        cost += edge.wt;
      }
    }
    return cost;
  }

  public static int[] leaders;
  public static int[] rank;

  public static boolean union(int x, int y) {
    int lx = find(x);
    int ly = find(y);

    if(lx == ly)
      return false;

```

```java
    // merger is possible here
    if(rank[lx] > rank[ly]) {
      leaders[ly] = lx;
    } else if(rank[lx] < rank[ly]) {
      leaders[lx] = ly;
    } else {
      leaders[lx] = ly;
      rank[ly]++;
    }
    return true;
  }

  public static int find(int x) {
    if(leaders[x] == x)
      return x;

    int tmp = find(leaders[x]);
    leaders[x] = tmp;
    return tmp;
  }

  public static void main(String[] args) {
    int[][] edges = {
      {0, 1, 10},
      {1, 2, 12},
      {0, 2, 13},
      {0, 3, 16},
      {3, 2, 14},
    };

    System.out.println(minCostOfSpanningTree(edges, 4));
  }
}
```