

Quick sort & Comparator

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Agenda

Sarthak

— Pivot Partition

Naval Oli

— Quick sort

Balaji S K

— Comparator Problem

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Madhan Kumar M S

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Murali krishna Talluri

Sushant

Divya P

Purusharth A

suyash gupta

Gowtham

Bhaveshkumar

Sanket Giri

Shani Jaiswal

Vimal

Gagan Kumar S

Sridhar Hissaria

Gokul Aditya S

Shradha Srivastava

Abhishek Sharma

GREAT
JOB!

Current DSP



66 %

Q> Given an integer array, consider the first element as pivot and rearrange the elements such that for all i

if $(A[i] < p)$ then it should be present on left side

if $(A[i] > p)$ then it should be present on right side

All elements are distinct.

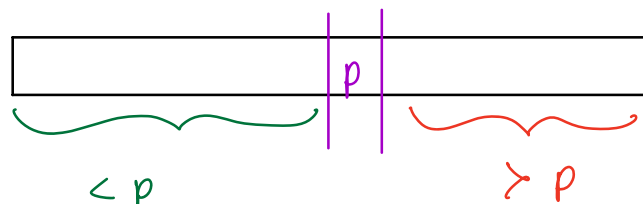
Return the correct position of pivot

$A = 54 \quad 26 \quad 93 \quad 17 \quad 77 \quad 31 \quad 44 \quad 55 \quad 20$

| | | | | | | | | |
|----|----|----|----|----|----|----|----|----|
| 26 | 17 | 31 | 44 | 20 | 54 | 93 | 77 | 55 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

ans 5 \because 5 is the index of 54

first element is p



TC: $O(N \log N)$

Brute force

sort the array.

| | | | | | | | | | |
|-------|----|----|----|----|----|----|----|----|----|
| $A =$ | 54 | 26 | 93 | 17 | 77 | 31 | 44 | 55 | 20 |
| | 17 | 20 | 26 | 31 | 44 | 54 | 55 | 77 | 93 |

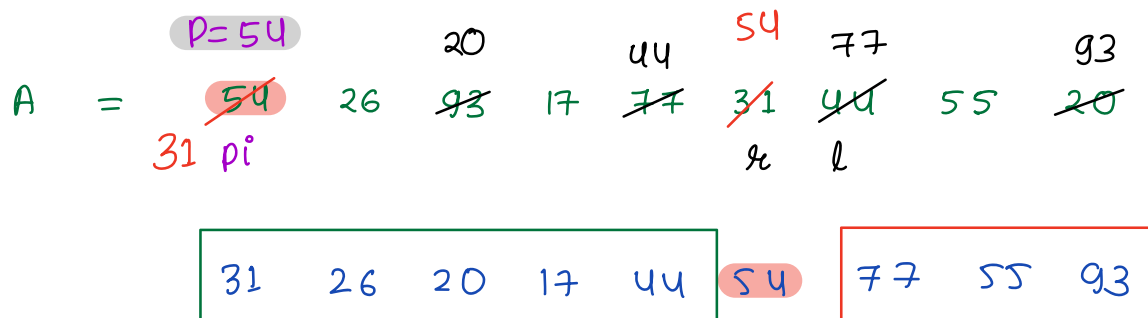
Approach 2

Create an extra array and populate all the smaller elements first {first pass}

then pivot

then all the greater elements {second pass}

TC: $O(N)$



if $A[l] < p$ { $l++$ }

if $A[x] > p$ { $x--$ }

x gives the correct position of pivot.

Pseudocode

```
int partition ( A[N] ) {  
    p = A[0]  
    l = 1  
    r = N-1  
  
    while ( l <= r ) {  
        if ( A[l] < p ) { l++ }  
        else if ( A[r] > p ) { r-- }  
        else {  
            swap(A, l, r)  
        }  
    }  
    swap(A, 0, r)  
    return r  
}
```

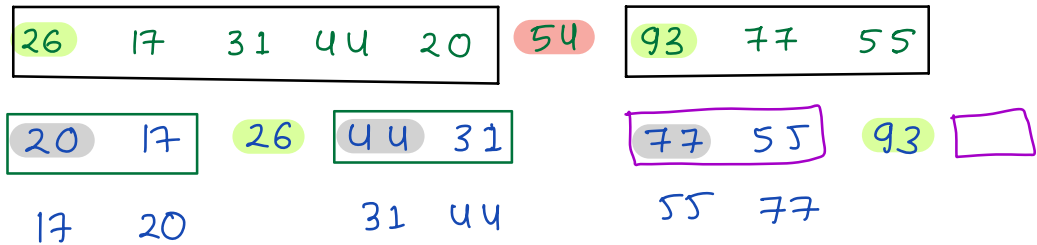
TC: $O(N)$
SC: $O(1)$

HW

Change the above code to handle duplicates?

Quick sort {divide & conquer}

A = 54 26 93 17 77 31 44 55 20



```
void quicksort ( A[N], l, r ) {  
    if ( l < r ) {  
        pi = partition ( A, l, r ) // first element  
                                   as pivot  
        quicksort ( A, l, pi - 1 )  
        quicksort ( A, pi + 1, r )  
    }  
}
```

```
int partition ( A[N], start, end ) {  
    p = A[start]  
    l = start + 1  
    r = end  
  
    while ( l <= r ) {  
        if ( A[l] <= p ) { l++ }  
        else if ( A[r] > p ) { r-- }  
        else {  
            // Handle duplicates  
            swap ( A[l], A[r] )  
            l++  
            r--  
        }  
    }  
    swap ( A[start], A[r] )  
    return r  
}
```

```

    swap(A, l, r)
    swap(A, start, r)
    return r

```

TC: $O(N)$

SC: $O(1)$

Time complexity of Quick sort

1 2 3 4

2 3 4

3 4

4

3 1 2 4

1 2 3 4

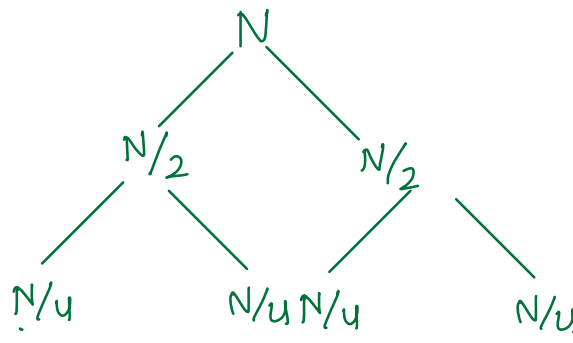
1 2 4

3 splits the array in
~ half

the sub problem
reduces by only one
element.

Best case scenario for quicksort

pivot always splits the array in half



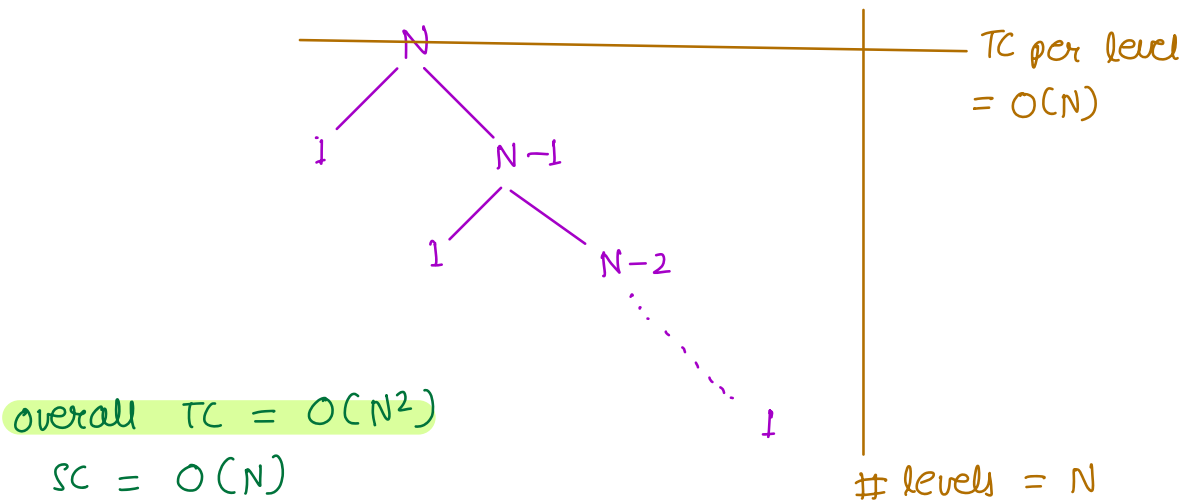
SC: $O(\log N)$

due to recursion
stack.

1

Exactly same as mergesort $N \log N$

Worst case TC for quick sort
pivot is the smallest element.



Randomized Quicksort

54 26 93 17 77 31 44 55 20

Instead of choosing A[0] as pivot

we pick any element at random to be the pivot

54 26 93 17 77 31 44 55 20

Probability of the worst case scenario
pivot is the smallest element.

| # elements | probability to select smallest |
|------------|--------------------------------|
| N | $\frac{1}{N}$ |
| N-1 | $\frac{1}{N-1}$ |
| N-2 | $\frac{1}{N-2}$ |
| \vdots | |
| 1 | 1 |

probability of picking smallest element at random always.

$$\frac{1}{N} * (N-1) * (N-2) \dots 1 = \frac{1}{N!} \approx \text{very small}$$

For the above reason

$$\begin{aligned} \text{TC for randomized quicksort} &= O(N \log N) \\ \text{SC} &= O(\log N) \end{aligned}$$

Size of the largest array that can be sorted? 10^6
value of $\log_2(10^6) \approx 20$

For this reason SC for sorting is considered $O(1)$

Comparator

Break 10:38

A comparator is a function that compares two values and returns a result indicating whether values are
less than
equal to
greater than

For languages — Java, Python, JS, C#, Ruby

$f \rightarrow$ first argument

$s \rightarrow$ second argument

return

-ve \rightarrow f comes before s

+ve \rightarrow f comes after s

0 \rightarrow $f == s$

For languages — C++

$f \rightarrow$ first argument

$s \rightarrow$ second argument

return

true \rightarrow f comes before s

false \rightarrow otherwise.

Sorting based on factors

Given $A[N]$, sort the data in ascending order of count of factors. If the factor count is same sort them on the basis of their magnitude. {asc}

| | | | | | | |
|----------|---|---|---|----|---|---|
| A | = | 9 | 3 | 10 | 6 | 4 |
| #factors | = | 3 | 2 | 4 | 4 | 3 |

Order \longrightarrow

| | | | | |
|---|---|---|---|----|
| 3 | 4 | 9 | 6 | 10 |
| 2 | 3 | 3 | 4 | 4 |

$\xrightarrow{\hspace{10em}}$
Count of factors
in ascending order.

Assuming countFactors function is already present.
TC: $O(N\sqrt{N})$

C++

```
bool compare ( val1 , val2 ) {  
    cnt1 = countFactors ( val1 )  
    cnt2 = countFactors ( val2 )  
  
    if ( cnt1 == cnt2 ) {  
        if ( val1 < val2 ) { return true }  
        else return false  
    }  
  
    return cnt1 < cnt2  
}
```

return

true \longrightarrow f comes before s
false \longrightarrow otherwise.

Python

```
A.sort ( key = lambda x : (countFactors(x), x))
```

Java

```
collections.sort ( A , ( f , s )  $\longrightarrow$  {  
    cnt1 = countFactors ( f );  
    cnt2 = countFactors ( s );  
    if ( cnt1 == cnt2 ) return f - s ;  
    return cnt1 - cnt2 ;  
});
```

NOTE : $f - s \longrightarrow$ ascending order on magnitude
 $cnt1 - cnt2 \longrightarrow$ ascending order on #factors

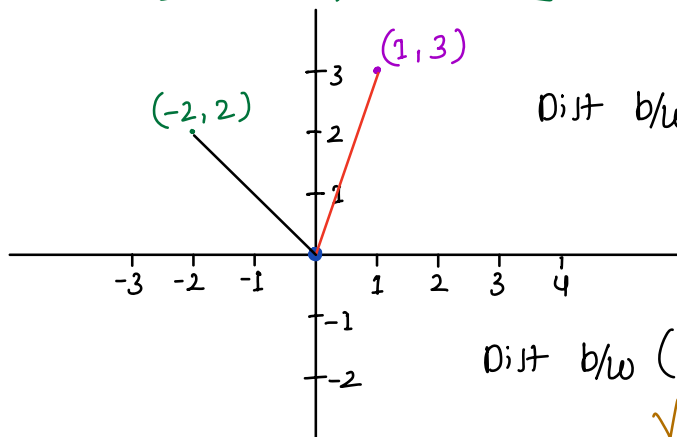
B closest points to origin **

Given array of points where $\text{points}[i] = [x_i, y_i]$ and an integer B.

Return B closest points to the origin $\{0, 0\}$

$$\text{Distance b/w } (x_1, y_1) \text{ \& } (x_2, y_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$\text{points} = [[1, 3], [-2, 2]] \quad B = 1$$



$$\text{Dist b/w } (0, 0) \text{ \& } (1, 3) \\ \sqrt{1^2 + 3^2} = \sqrt{10}$$

$$\text{Dist b/w } (0, 0) \text{ \& } (-2, 2) \\ \sqrt{2^2 + 2^2} = \sqrt{8}$$

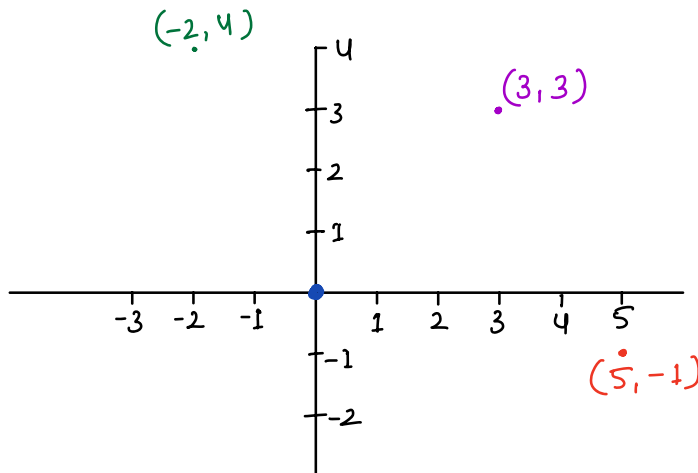
$$\text{sorted points} = [-2, 2], [1, 3]$$

$$\text{ans} = [-2, 2]$$

$$\text{points} = [[3, 3], [5, -1], [-2, 4]]$$

Dist from (0,0) $\sqrt{18}$ $\sqrt{26}$ $\sqrt{20}$

$$B = 2$$



| | | | |
|-------------------|-------------|-------------|-------------|
| sorted points | $[3, 3]$ | $[-2, 4]$ | $[5, -1]$ |
| distance from 0,0 | $\sqrt{18}$ | $\sqrt{20}$ | $\sqrt{26}$ |

$$ans = [[3, 3], [-2, 4]]$$

size of ans == 2

Approach Use comparator

Assume we have two points p_1 & p_2

$$\text{Dist b/w } p_1 \text{ \& origin} = \sqrt{x_1^2 + y_1^2}$$

$$\text{Dist b/w } p_2 \text{ \& origin} = \sqrt{x_2^2 + y_2^2}$$

NOTE : you can ignore sqrt in the distance formula.

Revision Framework

————→ Read before the class { ^{Pre lecture} content to be taught }

————→ Next day after the class ———→ Revise all notes
Revise the assignment.

Bookmark the question which is tough for you
for a revision later

————→ Every Sunday revise all class notes & assignment
for the week.
