Graphs : DSU, Kruskal and Bipartite

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Agenda

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1. Is Knight Reachable
2. Nearest Hospital
3. Bipartite Graph
4. Graph Coloring
5. Chromatic Number
6. Max number of edges
7. Friend's Graph

Hello Everyone
Very Special Good Evening
to all of you 😊😊😊
We will start session
from 9:06 PM

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Is Knight Reachable

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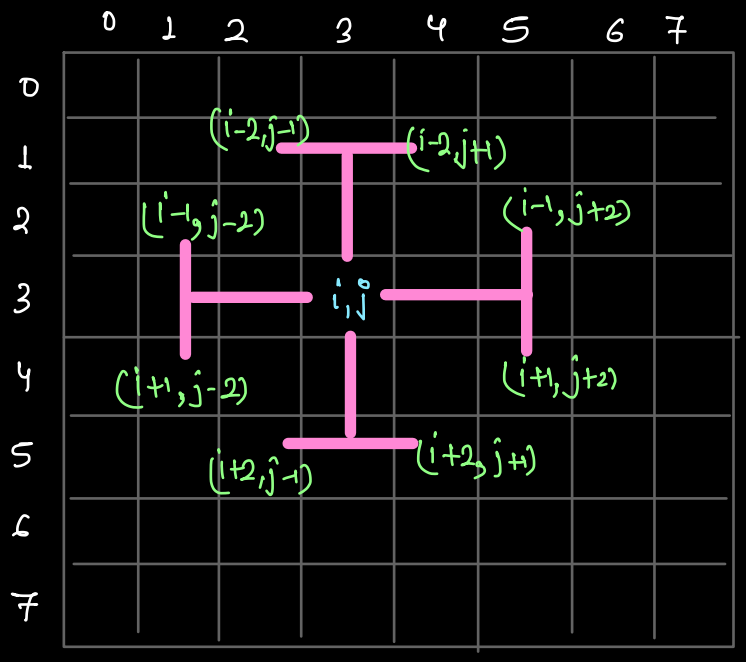
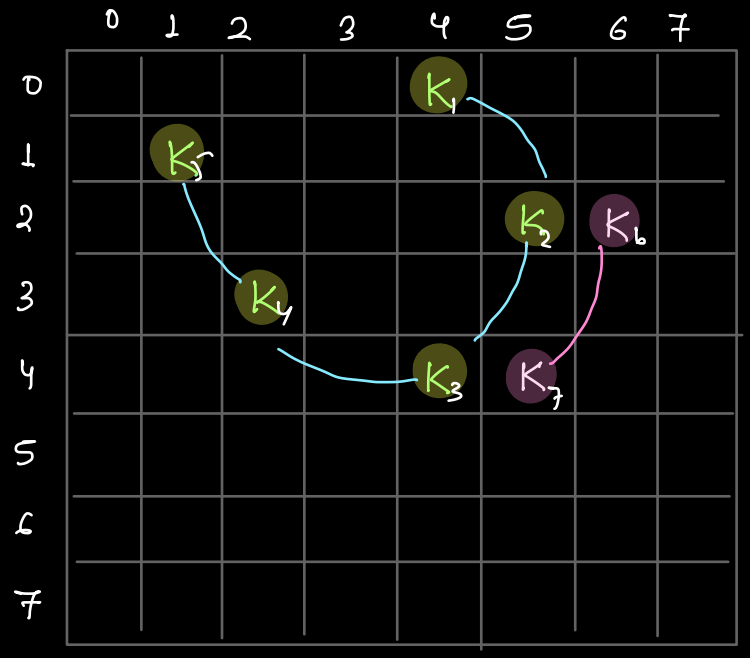
Given a N * N chessboard with K Knights placed on it.
If a Knight is reachable from the other Knight, they can swap their positions
Find the number of ways the Knights can rearrange themselves.





K=7

Group 1

$K_1\ K_2\ K_3\ K_4\ K_5$

we can place group 1 in $5!$ ways

Group 2

$K_6\ K_7$

we can place Group2 in $2!$ way.

$K_1\ K_2\ K_3\ K_4\ K_5$ $\quad$ $K_6\ K_7$

$K_2\ K_1\ K_4\ K_3\ K_5$ $\quad$ $K_7\ K_6$

$K_1\ K_4\ K_2\ K_5\ K_3$

⋮

Total possibility = $5! * 2! = 120 * 2 = 240$ Ans

$\underbrace{K_1\ K_2\ K_3}_{C_1}$ $\quad$ $\underbrace{K_4\ K_5\ K_6\ K_7}_{C_2}$ $\quad$ $\underbrace{K_8\ K_9\ K_{10}}_{C_3}$ $\quad$ $\underbrace{K_{11}\ K_{12}}_{C_4}$

$C_1!$ $\quad$ $C_2!$ $\quad$ $C_3!$ $\quad$ $C_4!$

Total ways = $C_1! * C_2! * C_3! * C_4!$

total ways = $Count(C_1)! * Count(C_2)! * Count(C_3)! * \cdots$

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Nearest Hospital

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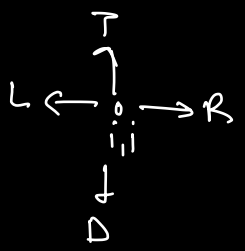
Given a N * M Matrix, the cells of the matrix is either marked as R (Residence) or H (Hospital) For every residence, find the distance to the nearest Hospital.
From a particular cell, you can move to any adjacent cell (diagonal moves not allowed).

	0	1	2	3
0	R_1	R_2	R_3	H_1
1	R_4	R_5	H_2	H_3
2	R_6	H_4	H_5	R_7

	0	1	2	3
0	3	2	1	0
1	2	1	0	0
2	1	0	0	1



optimise Approach:

Multisource BFS:

Steps of BFS → (Add source to queue)

* Remove
* print
* mark
* work
* Add unvisited nbr

Graph Coloring

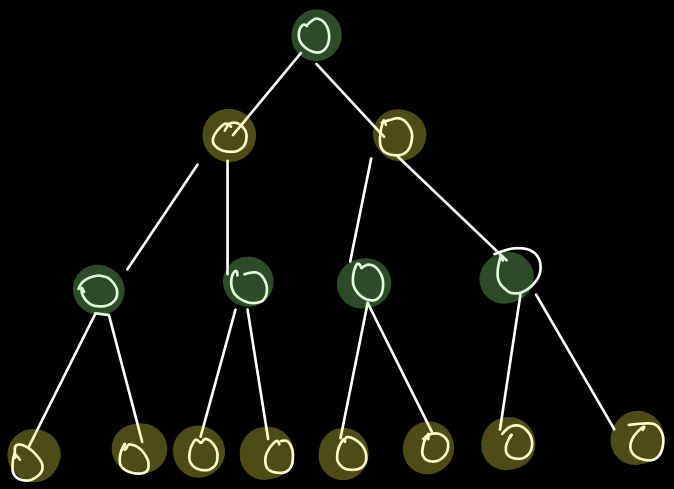
Francis Guthrie (1852)

Chromatic Number + Bipartite Graph

min no. of colors required to paint all nodes in a graph such that no two adjacent nods have same color.

Required color number → Chromatic Number

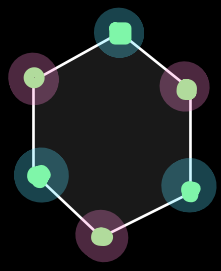
① Tree



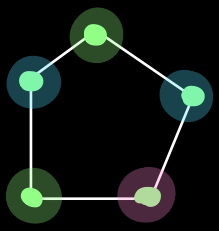
Colour Req. = 2

Chromatic Number = 2

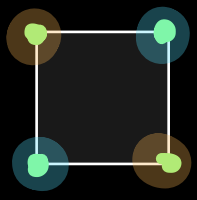
② Cycle in Graph:



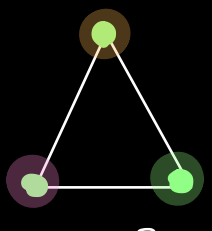
Chromatic no = 2



CN = 3



CN = 2



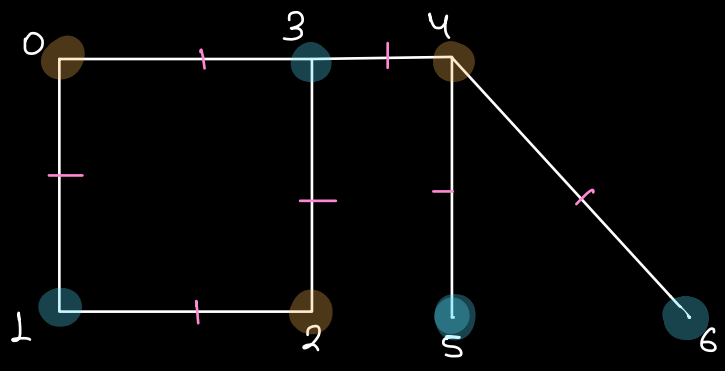
CN = 3

In general, CN of cycle graph = $2 + (N \% 2)$

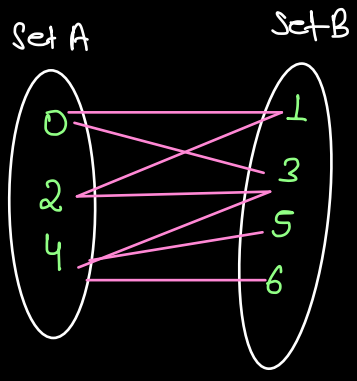
Bipartite Graph:

→ Any graph with Chromatic no = 2 is Bipartite.

→ A graph is called bipartite if we can divide all the nodes into two sets, such that all edges are across the set.

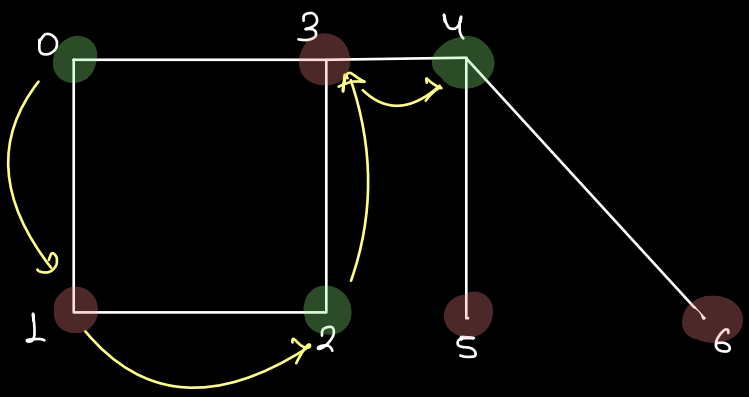


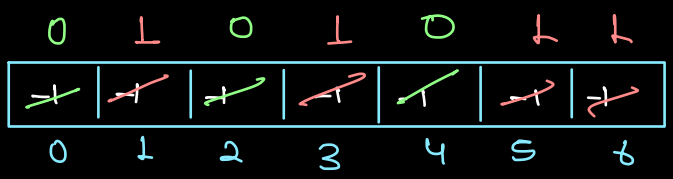
CN = 2



Bipartite Graph.

Check if given graph is Bipartite Graph or not?





pseudocode:

```
col[N]  ∀i  col[i] = -1

col[src] = 0

boolean dfs(graph, src) {
    for(int nbr: graph[src]) {
        if( col[nbr] == col[src]) { return false; }
        else if( col[nbr] == -1) {
            col[nbr] = 1 - col[src]; // opposite of src
            boolean res = dfs(graph, nbr);
            if(res == false) { return false; }
        }
    }
    return true;
}
```

```
for(int i=0; i<N; i++) {
    if(col[i] == -1) {
        if( dfs(graph, i) == false)
            return false;
    }
}
```

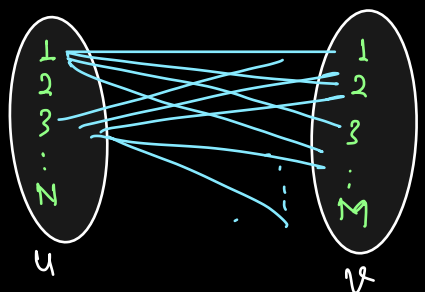
In main function.

T.C: O(V+E)

S.C: O(V+E)

Max number of edges - 1

Given a graph which is Bipartite. The Graph is Divided into 2 disjoint and independent sets u and v with N and M nodes respectively. What is the maximum number of edges we can have ?

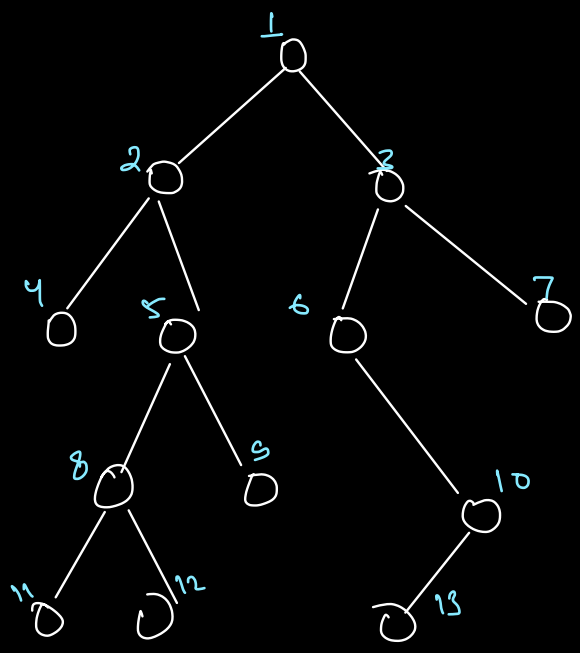


$= N * M$

no. of edge $= n * m$

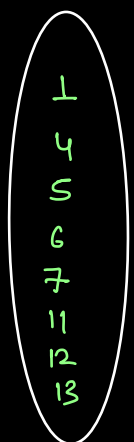
Max number of edges - 2

Given a tree with N nodes find the maximum number of edges that can be added to the tree so that it is still bi-partite graph.

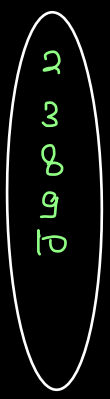


if no. of node = N

then max no. of edge = N - 1 (in tree)



no. of node = 8



no. of node = 5

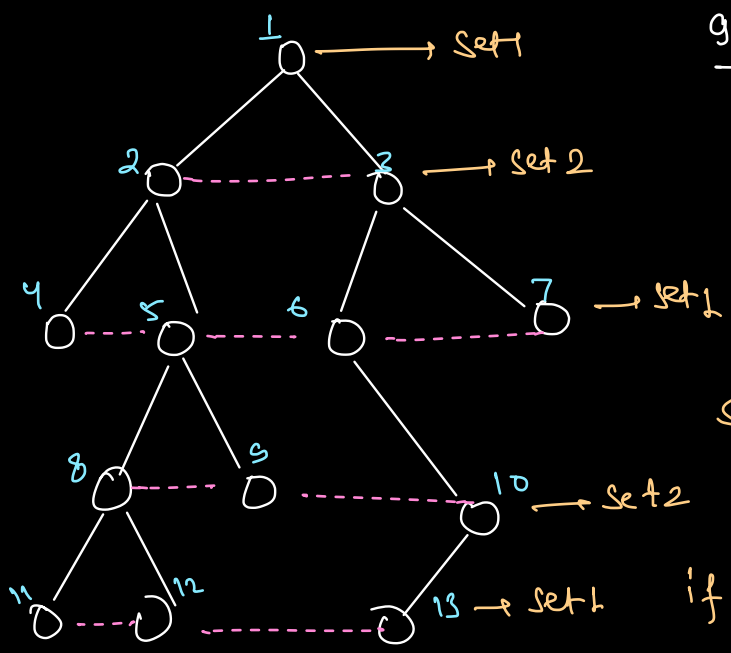
max no. of edge (in bipartite) $= n * m$

$= 8 * 5 = 40$

current edge (in tree) $= n - 1 \Rightarrow 13 - 1 = 12$

Any (extra possible) $= (n * m) - (N - 1)$

$= 40 - 12 = 28$ Ans



On tree :

no. of nodes = N = 13

no. of edge = N-1

= 13-1 = 12

Segregate set of nodes using level order

if level is odd → set 1

if " " even → set 2

count of set 1 nodes = x = 8

" " set 2 " = y = 5

$$ans = x * y - (N-1)$$

$$= 8 * 5 - (12)$$

$$= 40 - 12 = 28$$ By

T.C : O(N) } level order

S.C : O(N)

↳ Queue for level order

Saturday → Instagram

↳ Sunday.

10:12 – 10:22pm

Break

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Friend's Graph

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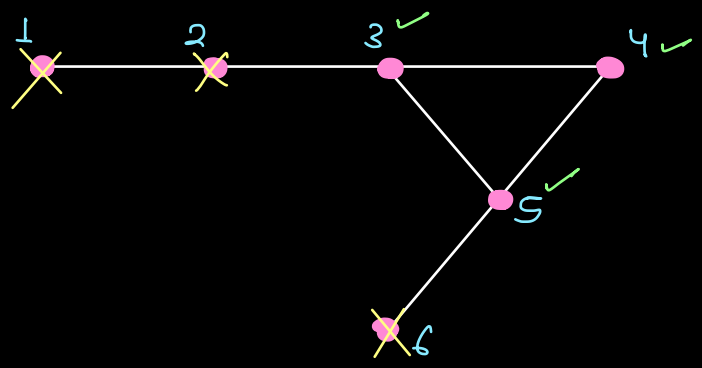
Given a friendship graph of N persons. You can only attend the party if you have minimum K number of friends attending the party.
Find total number of people that can attend the party.



K = 2

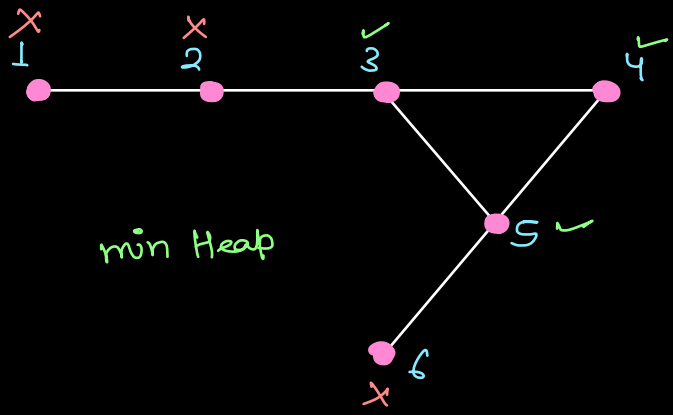
Count (ans) = 3

1 → 2
2 → 1, 3
3 → 2, 4, 5
4 → 3, 5
5 → 3, 4, 6
6 → 5

node Vs. find count

1 → 1 ✓ 1
2 → 2 ✗ 1
3 → 3 ✗ 2
4 → 2
5 → 3 ✗ 2
6 → 1

Count = 3 Ans



Main idea ⇒ using min priority Queue.

left overs → Z-Algo
→ DSU
→ Kruskal
→