

## Binary Search 2

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## AGENDA:

- Search in a rotated sorted array
- Square root
- A<sup>th</sup> magical number .
- Median of two sorted arrays .

Current PSP

65%

Important Announcement

Contest 2 — Reattempt 2 is live

## Recap

1) Identify the search space

$$l = 0$$

$$r = N - 1$$

while ( $l \leq r$ ) {

2) check if mid position can be my ans

$$\text{mid} = l + (r - l) / 2 \quad // \text{avoids overflow.}$$

if ( $A[\text{mid}] == k$ ) return mid

3) Decide whether to go left or right

if ( $A[\text{mid}] > k$ ) {

$r = \text{mid} - 1$       // go left

    else {  $l = \text{mid} + 1$  }      // go right

}

return -1

3 steps to solve any binary search question

1) Identify the search space

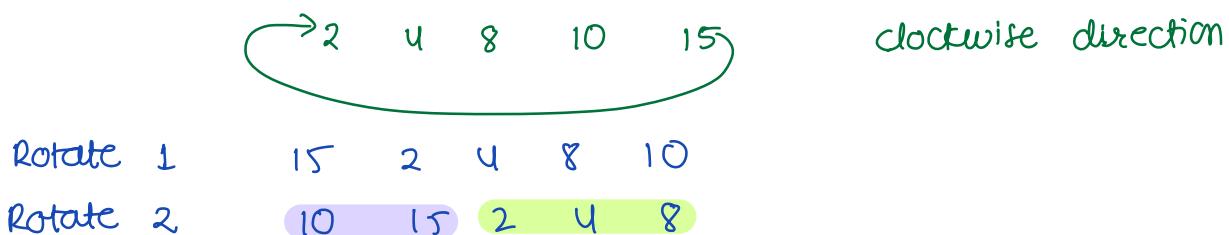
2) check if mid position can be my ans

3) Decide whether to go left or right

Q> Search in a rotated sorted array.

MSFT

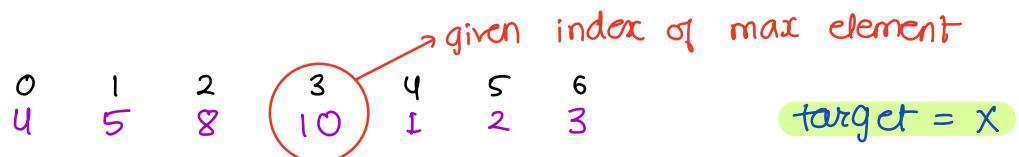
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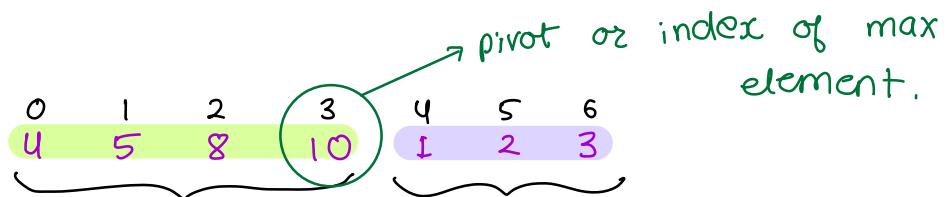
How to figure out if array is already sorted?

Compare first & last value in array

$$A[0] < A[N-1]$$

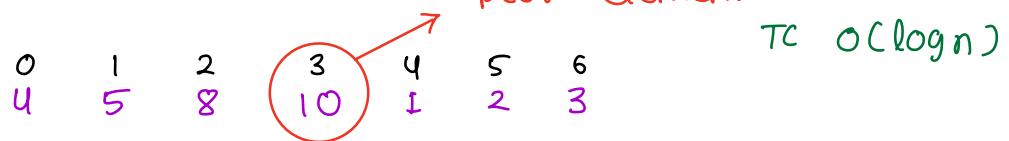


If you are given the index of the largest element  
How will you solve it in  $O(\log N)$ .



two separate sorted arrays.

- If you are not given the pivot. How will you solve it.

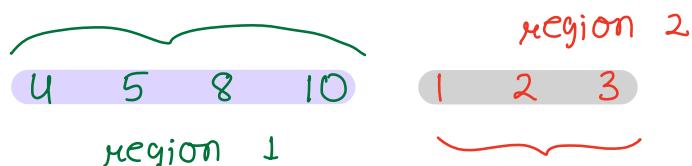


Apply BS on 0 to peak element index

Apply BS on peak element index +1 → last index

overall TC:  $O(\log N)$

\* Do it using one single binary search.



How to identify in which region any value lies?

$x = 8 \rightarrow$  region 1

$x = 3 \rightarrow$  region 2

```
int getRegion (A[], x) {
    if (x >= A[0]) return 1
    return 2
}
```

target =  $x$

Overall Idea :-

1) Identify the search space  $\rightarrow 0 - N-1$

2) check if mid position can be my ans  
 $\text{if } A[\text{mid}] == x \text{ return mid}$

3) decide whether to go left or right

| Region 1 |    |    | Region 2 |   |   |   |   |   |   |    |    |
|----------|----|----|----------|---|---|---|---|---|---|----|----|
| 0        | 1  | 2  | 3        | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 10       | 20 | 30 | 1        | 2 | 3 | 4 | 5 | 6 | 7 | 8  | 9  |

$x=20$

| left | right | mid | mid Region | target Region |           |
|------|-------|-----|------------|---------------|-----------|
| 0    | 11    | 5   | 2          | 1             | go left   |
| 0    | 4     | 2   | 1          | 1             | normal BS |
| 0    | 1     | 0   | 1          | 1             | go left   |
| 1    | 1     | 1   | 1          | 1             | go right  |

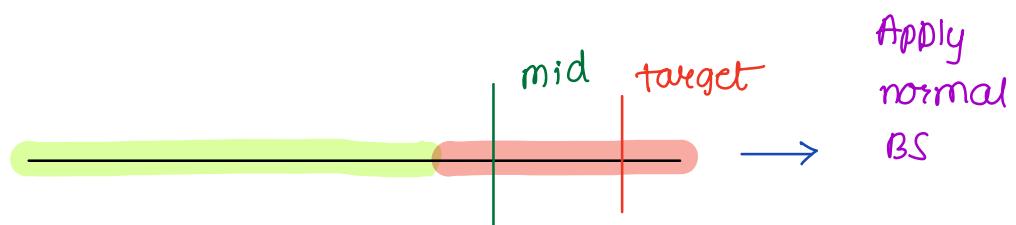
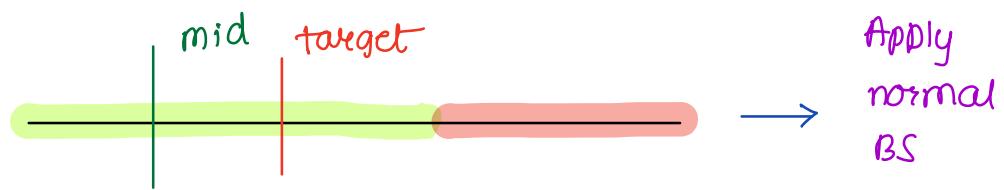
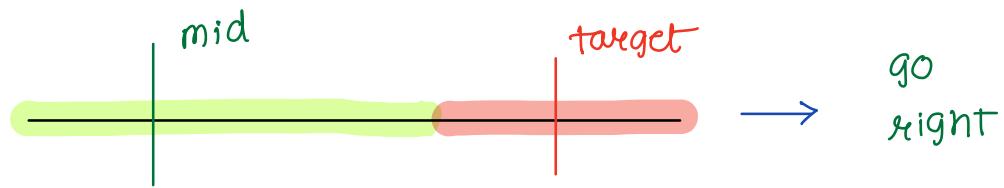
found 20 at index 1

$k=60$

|    |    |    |     |    |    |    |
|----|----|----|-----|----|----|----|
| 0  | 1  | 2  | 3   | 4  | 5  | 6  |
| 70 | 80 | 90 | 100 | 40 | 50 | 60 |

| left | right | mid | mid Region | target Region |          |
|------|-------|-----|------------|---------------|----------|
| 0    | 6     | 3   | 1          | 2             | go right |
| 4    | 6     | 5   | 2          | 2             | go right |
| 6    | 6     | 6   | 2          | 2             |          |

Found value at index 6



```

int getRegion ( A[ ] , x ) {
    if ( x >= A[0] )  return 1
    return 2
}
  
```

## Pseudo code

```
int rotatedSearch ( A[N] , target) {  
    left = 0  
    right = n - 1  
    targetRegion = getRegion (target)  
  
    while ( left <= right ) {  
        mid = left + (right - left) / 2  
        if ( A[mid] == target ) return mid  
        3> Decide whether to go left or right  
            midRegion = getRegion (A[mid])  
            if ( midRegion == 1 )  
                if ( targetRegion == 2 ) { // go right  
                    left = mid + 1  
                }  
            else if ( midRegion == 2 )  
                if ( targetRegion == 1 ) { // go left  
                    right = mid - 1  
                }  
            else { // same region  
                // Normal Binary search  
                if ( A[mid] < target ) {  
                    left = mid + 1  
                }  
                else { right = mid - 1 }  
            }  
    }  
    return -1  
}
```

TC:  $O(\log N)$

SC:  $O(1)$

## Square Root of N

Q) Given +ve N, find  $\text{sqrt}(N)$  → **Floor sqrt(N)**

$$\text{sqrt}(25) = 5$$

$$\text{sqrt}(20) = 4$$

$$\text{sqrt}(10) = 3$$

Brute force : **Linear Search.**

$$i = 1 \quad \text{ans} = 0$$

while ( $i * i \leq N$ )

$$\quad \quad \quad \text{ans} = i$$

$$\quad \quad \quad i++$$

}

print(ans)

TC:  ~~$O(N)$~~   
 $O(\sqrt{N})$

$$\sqrt{64} \quad \text{vs} \quad \log_2 64$$

8                            6

Idea : **Binary Search**

1) Search space →  $1 - N$

2) Target → if ( $\text{mid} * \text{mid} == N$ ) return mid.

case 1 :  $\text{mid} * \text{mid} == N \rightarrow \text{return mid}$

case 2 :  $\text{mid} * \text{mid} < N \rightarrow \text{go right}$   
 $\text{ans} = \text{mid}$

case 3 :  $\text{mid} * \text{mid} > N \rightarrow \text{go left.}$

$N = 50$

$$ans = \cancel{-1} \neq 7$$

| left | right | mid |                |          |
|------|-------|-----|----------------|----------|
| 1    | 50    | 25  | $25 * 25 > 50$ | go left  |
| 1    | 24    | 12  | $12 * 12 > 50$ | go left  |
| 1    | 11    | 6   | $6 * 6 < 50$   | go right |
| 7    | 11    | 9   | $9 * 9 > 50$   | go left  |
| 7    | 8     | 7   | $7 * 7 < 50$   | go right |
| 8    | 8     | 8   | $8 * 8 > 50$   | go left  |
| 8    | 7     |     |                |          |

### Pseudocode

```

int sqrt ( N ) {
    left = 1
    right = N
    ans = 0
    while ( left <= right ) {
        mid = left + (right - left) / 2
        if ( mid * mid == N ) return mid
        if ( mid * mid < N ) {
            ans = mid
            left += mid + 1
        } else { right = mid - 1 }
    }
    return ans
}

```

TC:  $O(\log(N))$

SC:  $O(1)$

use long for l, r, mid

Break - 22:26

## A<sup>th</sup> Magical Number

Pre-requisites — gcd

greatest common divisor

$$\text{lcm}(x, y) = \frac{x * y}{\text{gcd}(x, y)}$$

|                |                        |     |         |
|----------------|------------------------|-----|---------|
| Multiples of 1 | $\rightarrow [1, 100]$ | 100 | $100/1$ |
| Multiples of 3 | $\rightarrow [1, 100]$ | 33  | $100/3$ |
| Multiples of 4 | $\rightarrow [1, 100]$ | 25  | $100/4$ |
| Multiples of 6 | $\rightarrow [1, 100]$ | 16  | $100/6$ |

Multiples of 4 or 6 from  $[1, 100]$  double counted

$$\frac{100}{4} + \frac{100}{6} = 25 + 16 = 41 \quad 12 \quad 24 \quad 36 \dots$$
$$- \frac{100}{12} = -8$$
$$= 33$$

No. of multiples of  $B$  or  $C$  from  $[1 \rightarrow X]$   $= \frac{X}{B} + \frac{X}{C} - \frac{X}{\text{lcm}(B, C)}$

Q) Given  $A, B, C$ , find  $A^{th}$  magical no.

Number is magical if it is divisible by  $B$  or  $C$

Eg :       $B \quad C \quad A$        $an = 12$   
              2    3            8

multiple of  $B = 2 \quad 4 \quad 6 \quad 8 \quad 10 \quad 12 \quad 14 \quad 16$

multiple of  $C = 3 \quad 6 \quad 9 \quad 12 \quad 15 \quad 18 \quad 21 \quad 24$   
1    2    3    4    5    6    7    8    9    10    11    12  
1    2    3    4    5    6    7    8    9    10    11    12  
                8

Eg       $B \quad C \quad A$   
              4    6            5

1    2    3    4    5    6    7    8    9    10    11    12    13    14    15    16  
1            2            3            4            5

\* Brute force

$cnt = 0$   
for  $i \rightarrow 1 \text{ to } \infty$   
    if ( $i \% B \text{ || } i \% C$ )  
         $cnt++$   
    if ( $cnt == A$ ) return  $i$

TC:  $O(\min(B, C) * A)$

Search Space  $\longrightarrow$   $\min(B, C) - \min(B, C) * A$

| Eg : | B | C | A  |
|------|---|---|----|
|      | 4 | 6 | 10 |

Is  $36$  my  $10^{\text{th}}$  magical no. ?  
9 guess

No. of multiples till 36 for 4 or 6

$$= \frac{36}{4} + \frac{36}{6} - \frac{36}{12} = 9 + 6 - 3 \\ = \underline{\underline{12}}$$

36 is my  $12^{\text{th}}$  magical no. dec the guess  
go left.

Is  $24$  my  $10^{\text{th}}$  magical no. ?  
guess  
 $\frac{24}{4} + \frac{24}{6} - \frac{24}{12} = 6 + 4 - 2 = 8$

increase the guess  $\longrightarrow$  go right

Is  $30$  my  $10^{\text{th}}$  magical no. ?  
 $\frac{30}{4} + \frac{30}{6} - \frac{30}{12} = 7 + 5 - 2 \\ = \underline{\underline{10}}$

## Dry Run

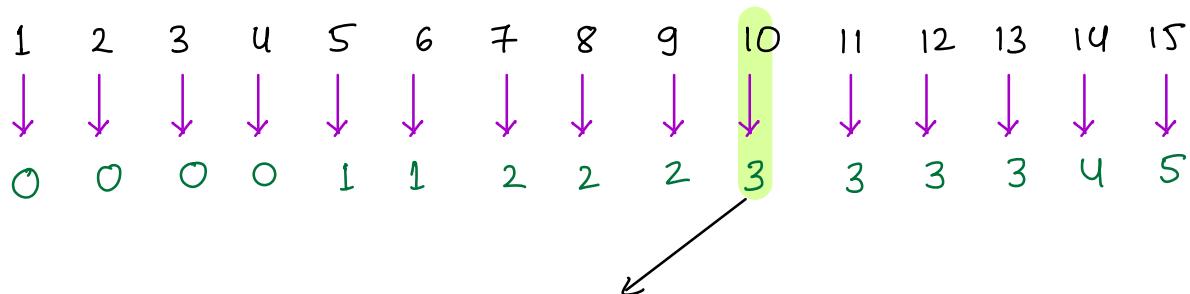
$$\begin{array}{ccc} B & C & A \\ 5 & 7 & 4 \end{array}$$

| left | right | mid |   |
|------|-------|-----|---|
| 5    | 20    | 12  | $\frac{12}{5} + \frac{12}{7} - \frac{12}{35} = 3$ |
| 13   | 20    | 16  | $\frac{16}{5} + \frac{16}{7} - \frac{16}{35} = 5$ |
| 13   | 15    | 14  | $\frac{14}{5} + \frac{14}{7} - \frac{14}{35} = 4$ |

$$\begin{array}{ccc} B & C & A \\ 5 & 7 & 3 \end{array}$$

Is 12 my 3<sup>rd</sup> magical no?

$$\frac{12}{5} + \frac{12}{7} - \frac{12}{35} = 2+1 = \underline{\underline{3}}$$



First occurrence of 3

Even if you get value == A go left to find first occurrence of A.

## Pseudo code

```
int magical ( A, B, C ) {
```

```
    left =
```

```
    right =
```

```
    while ( left <= right ) {
```

```
        mid = left + (right - left) / 2
```

H.W

complete the  
Pseudocode

## Median of Array

middle value in sorted form of array

Median  $\rightarrow$  1, 2, 3, 4, 5  $an = 5$

Median  $\rightarrow$  5, 4, 1, 3, 2

Sort the array 1 2 3 4 5  $an = 5$

Median  $\rightarrow$  1, 2, 3, 4, 5, 6

$$\frac{3+4}{2} \quad an = 3.5$$

Given two sorted arrays A & B. Find median of combined A+B array

Eg.  $A[] = 1, 4, 5$

$B[] = 2, 3 \quad an = 3$

Eg.  $A[] = 1, 2, 3$

$A+B = 1 2 3 4$

$B[] = 4$

$an = 2.5$

Eg.  $A[] = 1, 3, 4, 7, 10, 12$

$B[] = 2, 3, 6, 15$

$A+B = \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 3 & 4 & 6 & 7 & 10 & 12 & 15 \end{matrix}$

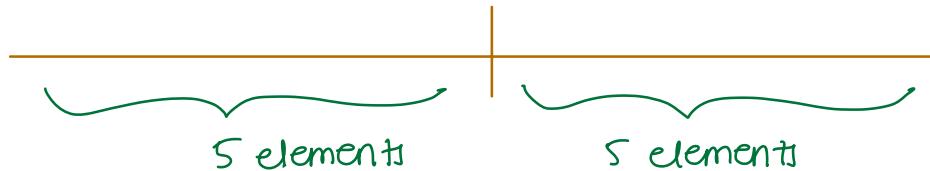
$$an = \frac{4+6}{2} = 5$$

Brute force merge A & B then find median

Case I Assume 4 elements from A

$$A[] = 1, 3, 4, 7, 10, 12 \quad } 10 \text{ elements}$$

$$B[] = 2, 3, 6, 15$$



$$\begin{matrix} 1 & 3 & 4 & 7 \\ 2 & & & \end{matrix} \xrightarrow{\quad} \begin{matrix} 10 & 12 \\ 3 & 6 & 15 \end{matrix}$$

$2 < 10$  but  $7 \not< 3$  go left.

Case II Assume 2 elements from A

$$\begin{matrix} 1 & 3 \\ 2 & 3 \end{matrix} \xrightarrow{\quad} \begin{matrix} 4 & 7 & 10 & 12 \\ 15 \end{matrix}$$

$6 \not< 4$        $3 < 15$  go right.

Case III Assume 3 elements from A

$$\begin{matrix} 1 & 3 & 4 \\ 2 & 3 & \end{matrix} \xrightarrow{\quad} \begin{matrix} 7 & 10 & 12 \\ 6 & 15 \end{matrix}$$

$3 < 7$      $4 < 6$

NOTE — We are binary searching on no. of element to pick from A

$A = 7 \quad 12 \quad 14 \quad 15$   
 $B = 1 \quad 2 \quad 3 \quad 4 \quad 9 \quad 11$ 
10 elements

$\underbrace{\hspace{100px}}$  5 elements  
 $\underbrace{\hspace{100px}}$  5 elements

|               |                 |              |   |    |    |    |
|---------------|-----------------|--------------|---|----|----|----|
| $O$           | $\text{len}(A)$ |              |   |    |    |    |
| $\text{left}$ | $\text{right}$  | $\text{mid}$ |   |    |    |    |
| 0             | 4               | 2            | 7 | 12 | 14 | 15 |
|               |                 |              | 1 | 2  | 3  | 4  |
|               |                 |              |   |    | 9  | 11 |

$$3 < 14 \quad 12 \not< 4$$

Take len values from A  
 go left

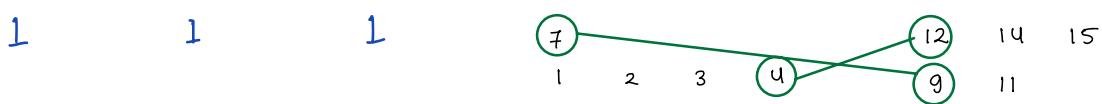
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$0 \quad 1 \quad 0$   
-∞  
 1 2 3 4 9 7 12 14 15

$$9 \not< 7$$

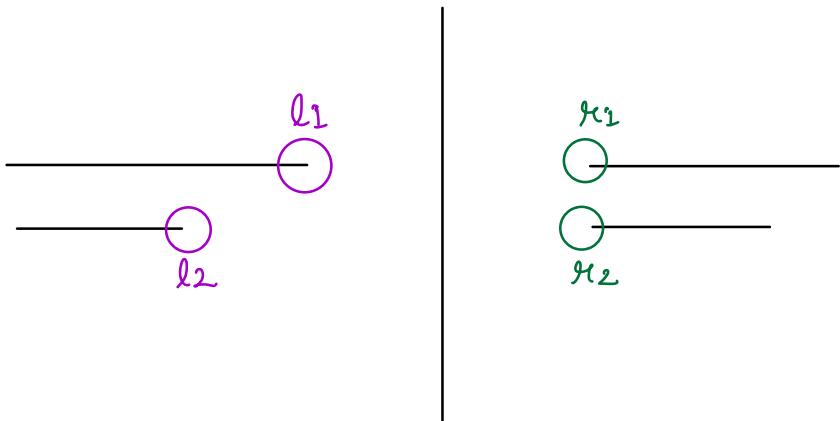
Take more values from A  
 go right

---



we found out the right  
 partitioning or split  
 $7 < 9 \quad 8 \quad 4 < 12$

$$\text{median} = \frac{\max(7, 4) + \min(12, 9)}{2} = 8$$



$l_1 \leq r_2$      $\&$      $l_2 \leq r_1$

## Pseudo code

```
double findMedian ( A[], B[] ) {  
    lengthA = A.length  
    lengthB = B.length  
  
    totalLength = lengthA + lengthB  
    halfLength = totalLength / 2  
    left = 0  
    right = lengthA // min (lengthA, halfLength)  
  
    while ( left <= right ) {  
        mid = left + (right - left) / 2  
        // mid = no. of elements to be taken from A  
        takeB = halfLength - mid  
  
        l1 = A[mid - 1]  
        l2 = B[takeB - 1]  
        r1 = A[mid]  
        r2 = B[takeB]  
  
        if ( l1 <= r2 && l2 <= r1 ) {  
            // Partition is correct  
            l = max (l1, l2)  
            r = min (r1, r2)  
            if ( totalLength / 2 == 0 ) {  
                return (l+r)/2  
            }  
        }  
    }  
}
```

TC:  $O(\log(\text{lenA}))$   
SC:  $O(1)$

*handle edge cases using safeguard*

```
    } else {  
        } return r  
    }  
    } else if ( l1 > r2 ) {  
        } right = mid - 1  
    } else {  
        } left = mid + 1  
    }  
}
```

## Doubt Session

Odd

$$\min(7, 9) = \underline{\underline{7}}$$

9 elements

$$\frac{9}{2} = 4$$

