

Count & Merge Sort

AGENDA:

- Count Sort
- Merge two sorted arrays.
- Merge Sort
- Inversion Count.
- Stable / Inplace

Current PSP



67%



70%

Rules

- Qs in question tab
- Answers in private

Count sort

Find the smallest no. that can be formed by re-arranging the digits of given no. in an array

Return the smallest no. in the form of an array.

$A[i] \rightarrow [0, 9]$

A = 6 3 4 2 7 2 1
arr = 1 2 2 3 4 6 7

A = 4 2 7 3 9 0
arr = 0 2 3 4 7 9

Brute force sort the array

TC: $O(N \log N)$

A 0 1 2 9 5 1 4 0 6

arr = 0 0 0 1 2 8 9
freq of 0 freq of 1 freq 8 freq 9

Idea \rightarrow use freq array to sort A.
HM as well

$A = 1 \quad 3 \quad 8 \quad 2 \quad 3 \quad 5 \quad 3 \quad 8 \quad 5 \quad 2 \quad 2 \quad 3$
 $\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow$
 freq 0 1 2 3 4 5 6 7 8 9
 ans = 1 2 2 2 3 3 3 3 5 5 8 8

Pseudocode

// calculate freq

freq[10] // \because digits $\rightarrow [0, 9]$

for $i \rightarrow 0$ to $N-1$ {
 freq[A[i]]++
 } TC: $O(N)$

for digit $\rightarrow 0$ to 9 {
 times = freq[digit]
 for $i \rightarrow 1$ to times {
 print(digit)
 }
 } TC: $O(N)$
 Overall TC : $O(N)$

$A = 2 \quad 2 \quad 2 \quad 2$
 freq 0 1 2 3 4 5 6 7 8 9
 0 1 2 3 4 5 6 7 8 9

Q> will count sort work if $A[i]$ is around 10^9
 $[0, 10^9]$

wont work for large $A[i]$ values.

∴ we cannot create $\text{freq}[10^9]$

$4 * 10^9$

4GB

HM will not work for 10^{19} scenario think why?

Can count sort work on negative no.?

$$A = \begin{pmatrix} -2 & 3 & 8 & 3 & -2 & 3 \end{pmatrix}$$

yes count sort can work for negative value

we need to shift the range to non-negative values.

$A = \begin{matrix} -2 & 3 & 8 & 3 & -2 & 3 \\ +2 & +2 & +2 & +2 & +2 & +2 \\ 0 & 5 & 10 & 5 & 0 & 5 \end{matrix}$ $A_i \rightarrow [-9, 9]$

freq

x^2	x^3	
0		1
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		

$$\Rightarrow \begin{array}{cccccc} 0 & 0 & 5 & 5 & 5 & 10 \\ -2 & -2 & -2 & -2 & -2 & -2 \\ \hline -2 & -2 & 3 & 3 & 3 & 8 \end{array}$$
$$A = \begin{matrix} & -2 & -3 & 3 & 5 & 9 & -9 & -2 & -3 \end{matrix}$$

↓
make the smallest value
pointing to index 0

A = -2 -3 3 5 9 -9 -2 -3
 +9 +9 +5 +9 +9 +9 +9 +9
 7 6 12 14 18 0 7 6

Apply count sort here and convert back the
output by subtracting 9

freq [max - min + 1]

Approach 2 → Use hashmap and no need for any
conversion of input & output.

A = -2 3 8 3 -2 3

HM = -2 : 2
 3 : 3
 8 : 1 } Create freq HM

```
for val → min val to max val {
    times = hm.getOrDefault(val, 0)
    for i → 1 to times {
        print(val)
    }
}
```

NOTE: Only we count sort if $\text{max} - \text{min} + 1 \leq 10^6$

Always avoid count sort and use in-built sort.

Merge two sorted arrays

Given an integer array where all odd elements are sorted and all even elements are sorted.

Sort the entire array { better than $n \log n$ }

A = 2 5 4 8 11 13 10 15 21

A

B

2 4 8 10
~~/~~ ~~/~~ ~~/~~ ~~/~~ i

5 11 13 15 21
~~/~~ j

C = 2 4 5 8 10 11 13 15 21

Pseudocode

// step 1 segregate even into A[]
odd into B[]

```
int[] merge ( A[N] , B[M] ) {  
    C [N+M]  
    i = 0 // A  
    j = 0 // B  
    k = 0 // C  
  
    while ( i < N && j < M ) {  
        if ( A[i] <= B[j] ) {  
            C[k] = A[i]  
            i++  
        }  
        else {  
            C[k] = B[j]  
            j++  
        }  
        k++  
    }  
    while ( i < N ) C[k++] = A[i++]  
    while ( j < M ) C[k++] = B[j++]  
    return C  
}
```

```

        i++
        k++
    } else {
        C[k] = B[j]
        j++
        k++
    }
}

// while A or B contains elements put them
// in C
while (i < N) {
    C[k] = A[i]
    i++
    k++
}

while (j < M) {
    C[k] = B[j]
    j++
    k++
}

return C
}

```

TC: $O(N+M)$
 SC: $O(N+M)$

void merge (A[N], l, m, r)
 merge A[] from l to m and m+1 to r
 sorted sorted.

A = 100

1	2	9	10	3	4	6	7	12
<i>l</i>			<i>m</i>					<i>r</i>

 2

Break 10:28

H.W.

output

A = 100

1	2	3	4	6	7	9	10	12
<i>l</i>			<i>m</i>					<i>r</i>

 2

Merge sort { divide and conquer }

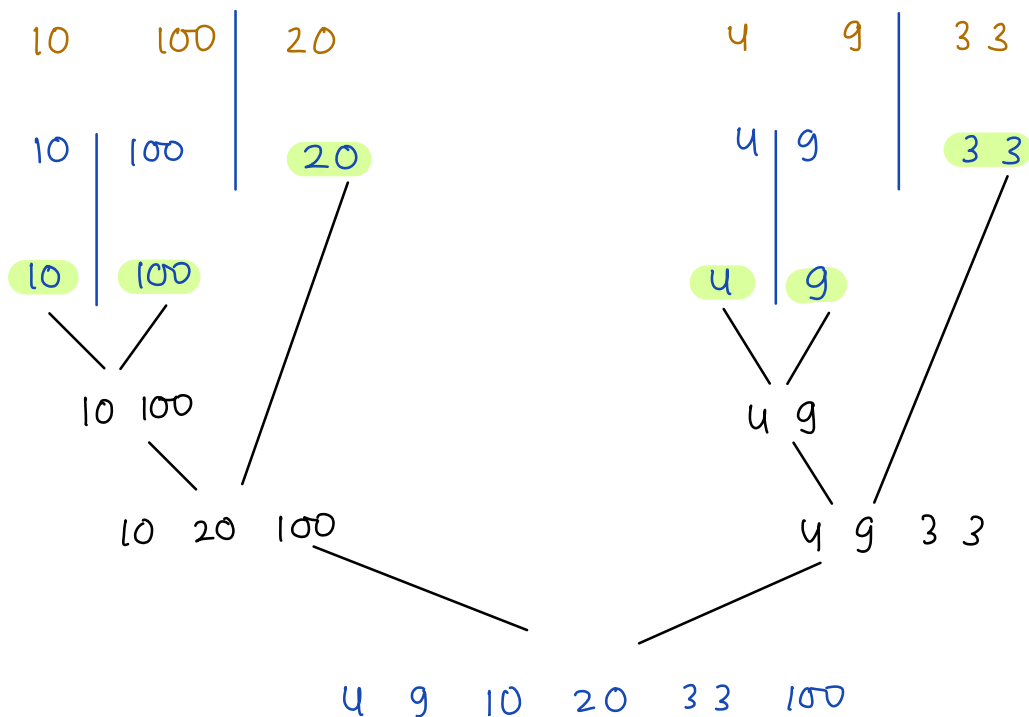
Sujoy { Professor }

{ sort exam papers }

10 100 20 4 9 3 3

Balaji

Aditya



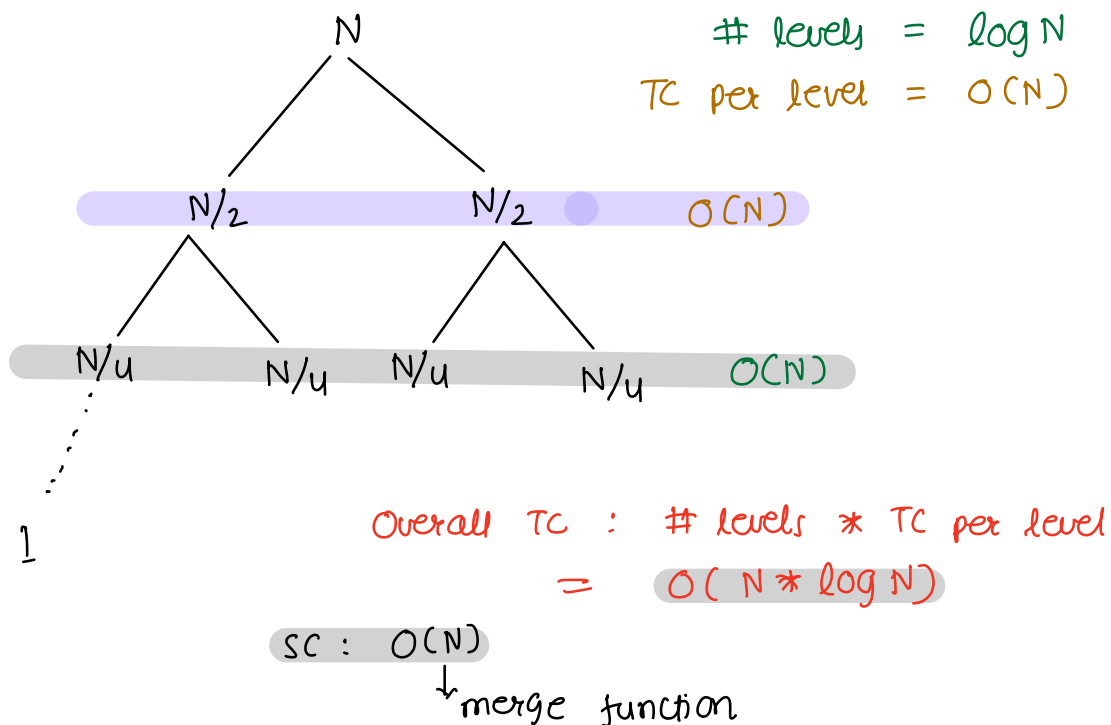
Pseudocode

```
// sort A[] from index l to index r
void mergeSort ( A[] , l , r ) {
    // Base condition
    if ( l == r ) return

    m = (l + r) / 2

    mergeSort ( A[] , l , m )
    mergeSort ( A[] , m+1 , r )

    merge ( A[] , l , m , r ) // merges two sorted
                              // array l to m
                              // m+1 to r
}
```

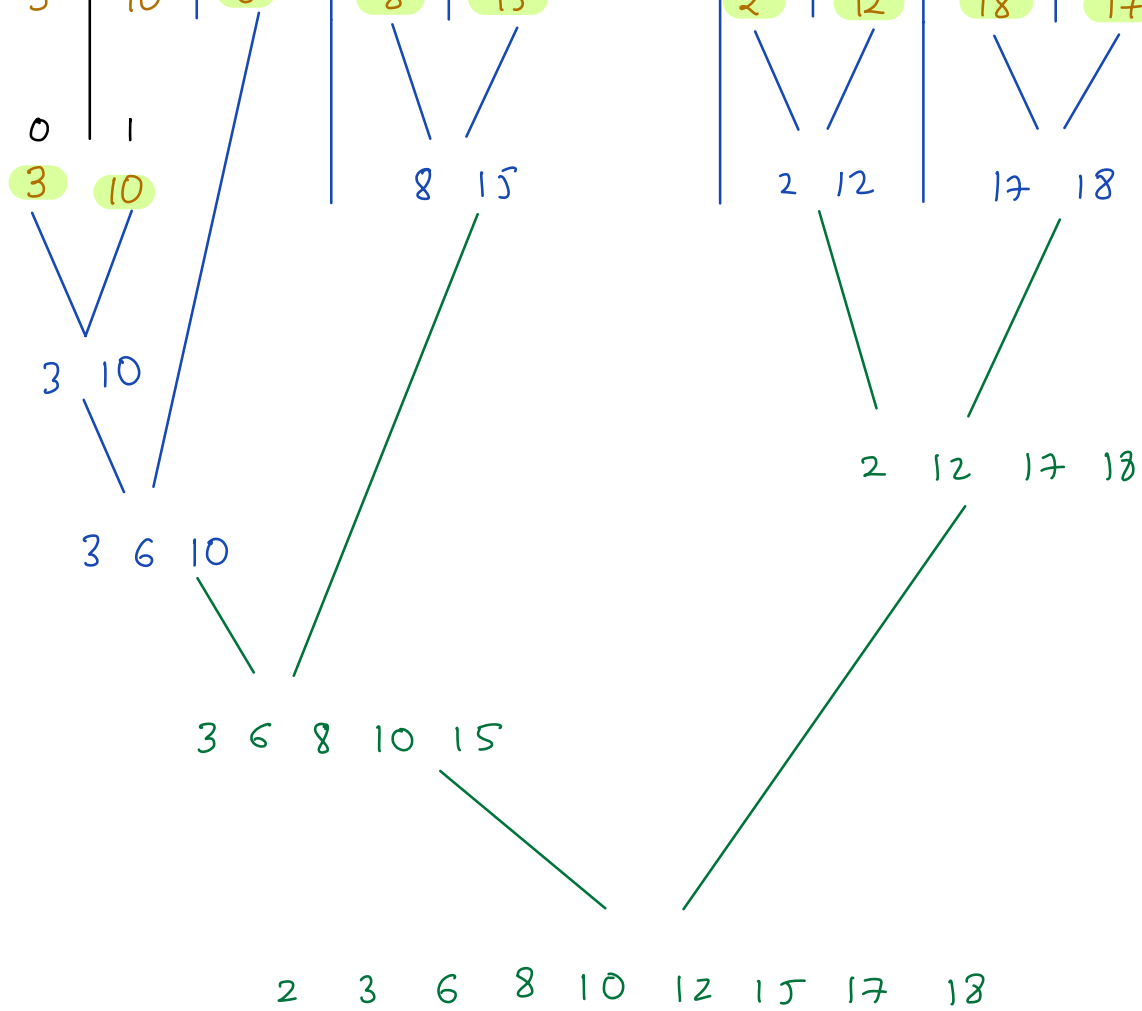


	0	1	2	3	4	5	6	7	8
A =	3	10	6	8	15	2	12	18	17

0	1	2	3	4	5	6	7	8
3	10	6	8	15	2	12	18	17

0	1	2	3	4	5	6	7	8
3	10	6	8	15	2	12	18	17

0	1	2	3	4	5	6	7	8
3	10	6	8	15	2	12	18	17



Calculate no. of pairs such that $A[i] > B[j]$

Given two array, $A[n]$ & $B[m]$

Calculate the no. of pairs i, j such that $A[i] > B[j]$

$A = 7 \ 3 \ 5$
 $B = 2 \ 0 \ 6$

$(7 \ 2) \ (7 \ 0) \ (7 \ 6) \ (3 \ 2) \ (3 \ 0) \ (5 \ 2) \ (5 \ 0)$

7 pairs

Bruteforce

count = 0

```
for i → 0 to N-1 {  
  for j → 0 to M-1 {  
    if (A[i] > B[j]) count++  
  }  
}
```

$A = 7 \ 3 \ 5$
 $B = 2 \ 0 \ 6$

sort A & B

$A = 3 \ 5 \ 7$
 $B = 0 \ 2 \ 6$
 i
 j

ans = +3
+3
+1

Pseudocode

count = 0

i = 0

j = 0

sort array A & B

```
while (i < N && j < M) {  
    if (A[i] > B[j]) {  
        count += N - i  
        j++  
    }  
    else {  
        i++  
    }  
}  
print(count)
```

TC : $O(N \log N + M \log M + N + M)$

SC : $O(1)$

Inversion Count

Given $A[n]$, calculate no. of pairs (i, j) such that

$$i < j \quad \&\& \quad A[i] > A[j]$$

$A =$

	0	1	2	3	4
	10	3	8	15	6

i	j	A_i	A_j	$A_i > A_j$
0	1	10	3	✓
0	2	10	8	✓
0	3	10	15	X
0	4	10	6	✓
1	2	3	8	X
1	3	3	15	X
1	4	3	6	X
2	3	8	15	X
2	4	8	6	✓
3	4	15	6	✓

ans = 5

Bruteforce

count = 0

TC: $O(N^2)$

```
for i → 0 to N-1 {  
  for j → i+1 to N-1 {  
    if (  $A[i] > A[j]$  ) count ++  
  }  
}
```

A = ⁰5 ¹2 ²6 ³1

$i < j$ && $A_i > A_j$

5 2 2 1 6 1
5 1

ans = 4

A = ⁰5 ¹3 ²1 ³4 ⁴2

5 3 3 1 4 2
5 1 3 2
5 4
5 2

ans = 7

Hint : Exactly same as merge sort.

During the merge function we logic from
previous question to count inversion.

H.W.

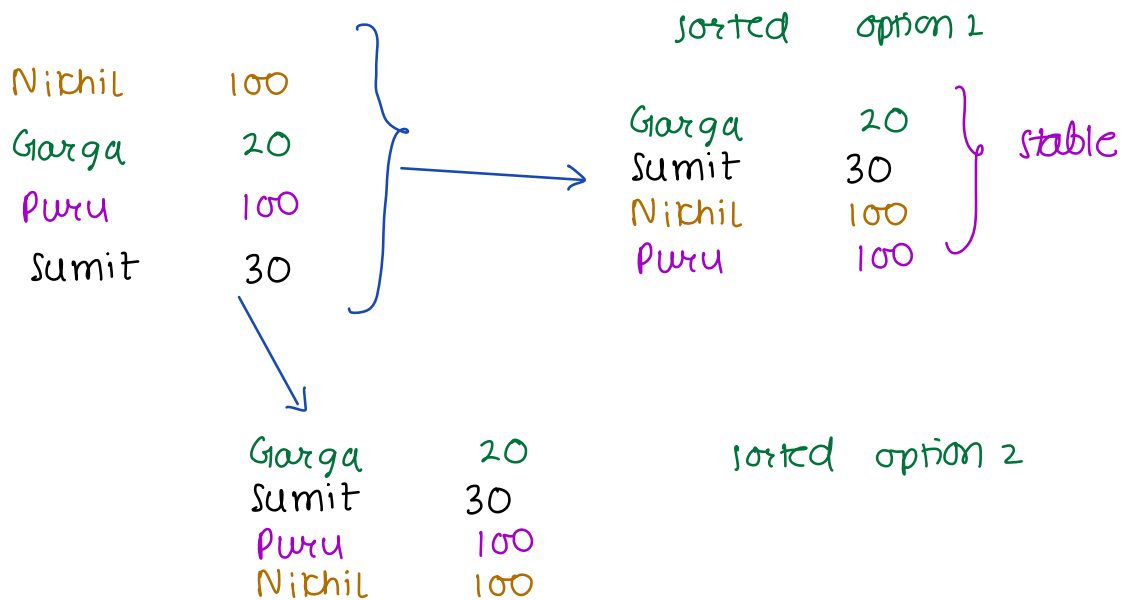
stable sort

Inplace sort

sorting without using extra space SC: $O(1)$

Is merge sort inplace sorting algo?

NO



If the relative order b/w input and output is maintained for same marks

Eg. Nikhil & puru

HW → figure out if merge sort is stable or not.

Doubt session

$$A = 2 \quad 2 \quad 2 \quad 2$$

freq 0 1 2 3 4 5 6 7 8 9

$\emptyset \swarrow \nearrow \nearrow \nearrow \nearrow$

$$TC : O(10 + N)$$