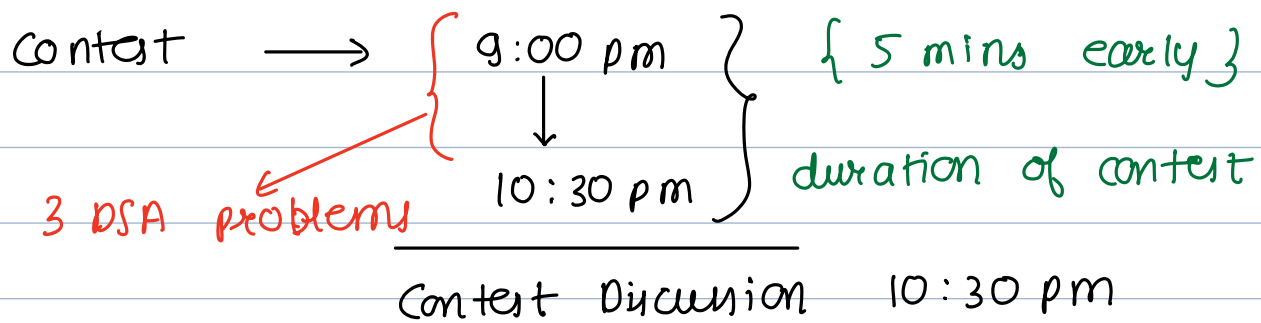


Recursion 2

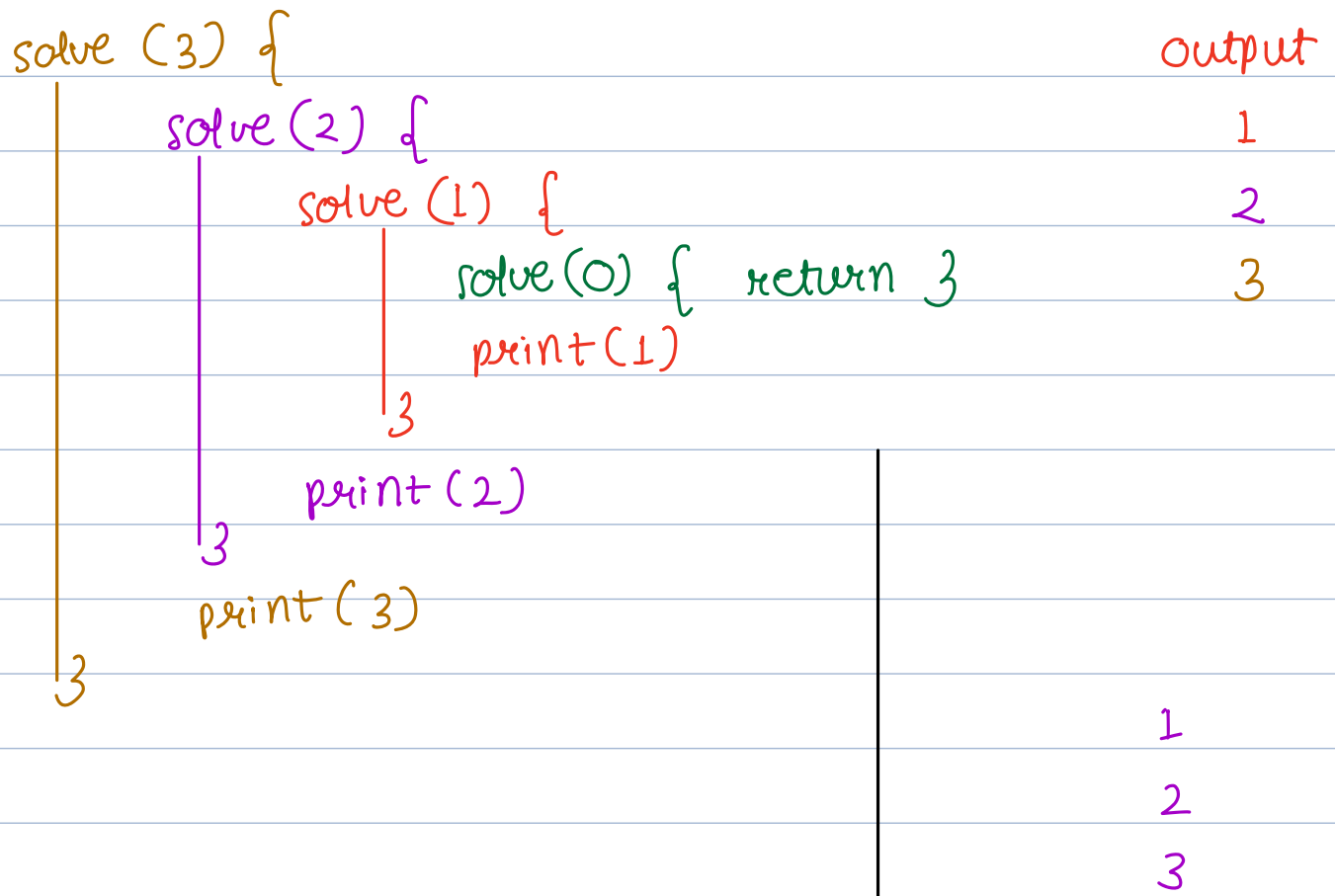
Content

- Quizzes on Recursion
- Tower of Hanoi
- Balanced Parenthesis



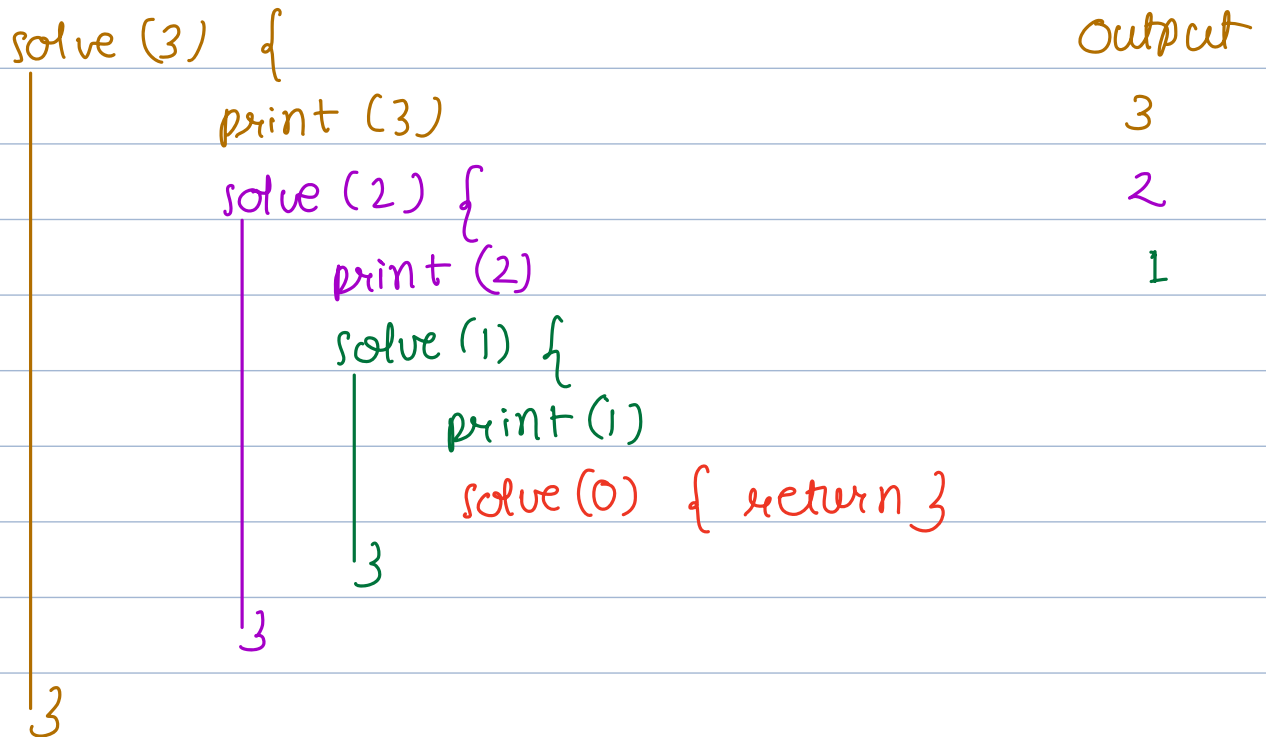
What is the output of the following code for N = 3?

```
void solve(int N){  
    if(N == 0)  
        return;  
    solve(N-1);  
    print(N);  
}
```



What is the output of the following code for N = 3?

```
void solve(int N){  
    if(N == 0)  
        return;  
    print(N);  
    solve(N-1);  
}
```



What is the output of the following code for N = -3?

```
void solve(int N){  
    if(N == 0)  
        return;  
    print(N);  
    solve(N-1);  
}
```

solve(-3) {

print(-3)

solve(-4) {

print(-4)

solve(-5) {

print(-5)

stack overflow

output

-3

-4

-5

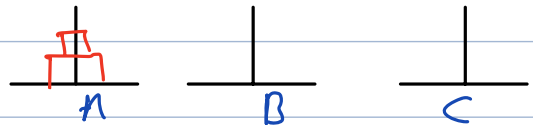
⋮

solve(-5)

solve(-4)

solve(-3)

Tower of Hanoi ***



There are n disks placed on a tower A
{different sizes}

Move all disks from tower A to C using B

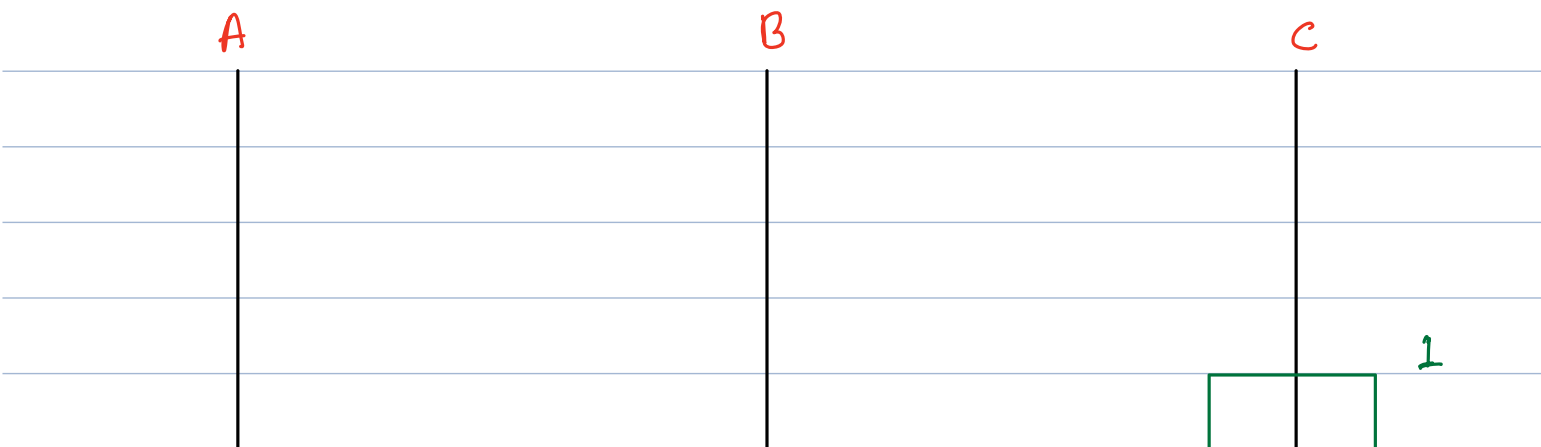
Constraints

- Only 1 disk can be moved at a time
- Larger disk cannot be placed on a small disk at any step.

Print the movement of disks from A to C in minimum steps.

Example

$N = 1$

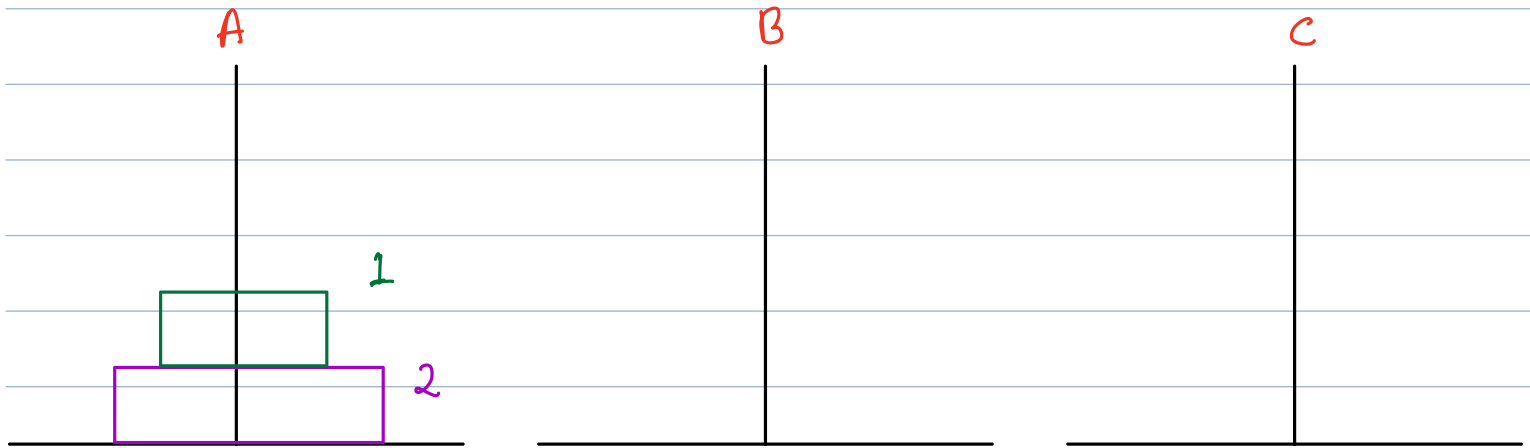


Disk 1 $A \rightarrow C$

$N = 2$

$TOH(2, A, B, C)$

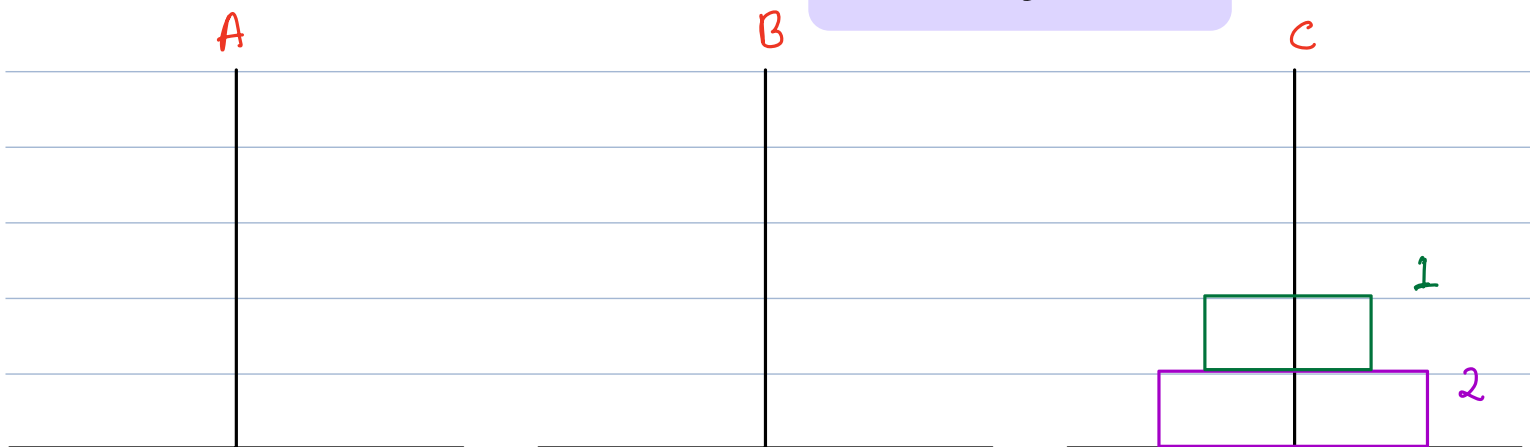
Initial State



steps \longrightarrow

D 1 $A \longrightarrow B$
D 2 $A \longrightarrow C$
D 1 $B \longrightarrow C$

Final state



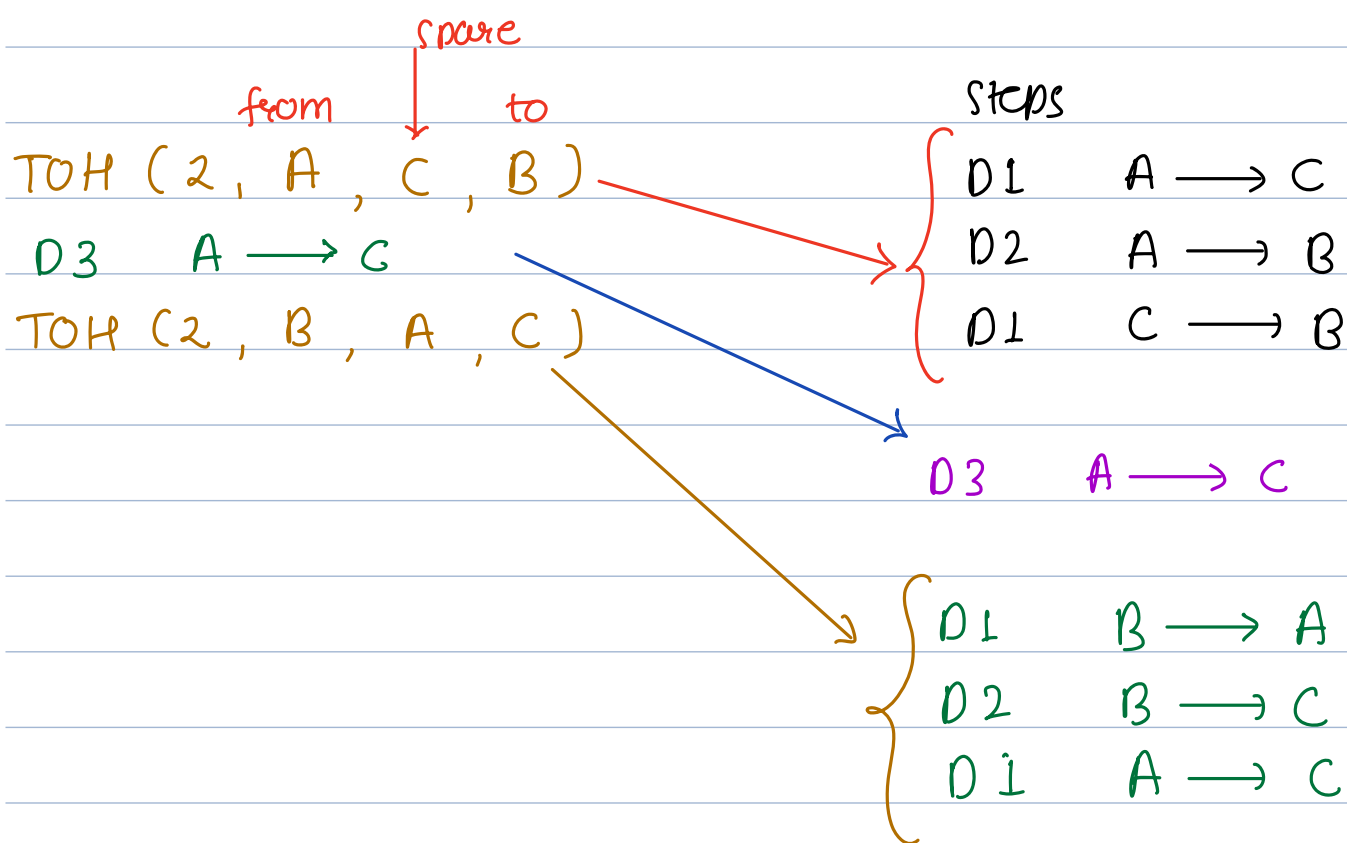
Assumption \longrightarrow $TOH(N, A, B, C)$

\longrightarrow Move N disks from tower A to tower C using a spare tower B

$N = 3$



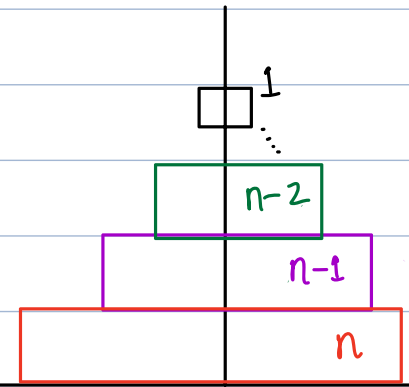
We moved 2 disks from tower A to tower B using C as spare tower.



A

B

C



fr sp to
TOH (N , A , B , C)

step 1

// Move N-1 disks from A to B
using spare C

step 2

TOH (N-1 , A , C , B)

print (D-N , A \rightarrow C)

// Move N-1 disks from B to C
using spare A

TOH (N-1 , B , A , C)

Base case

if $N == 0$ return

Pseudocode

```

void TOH ( N, A, B, C ) {
    if (N==0) return
    TOH (N-1, A, C, B)
    print ( D-N, A → C )
    TOH (N-1, B, A, C )
}

```

from to
 spare
 ↓

N	steps	
1	1	$2 - 1$
2	3	$4 - 1$
3	7	$8 - 1$
4	7 1 7 = 15	$16 - 1$
5		$32 - 1$
⋮		
n	$2^n - 1$	

N = 2

```

TOH (2, A, B, C) {
    TOH (1, A, C, B) {
        TOH (0, A, B, C) { return }
        print (1, A → B)
        TOH (0, C, A, B) { return }
    }
    print (2, A → C)
}

```

from to
 spare
 ↓

```

void TOH ( N, A, B, C ) {
    if (N==0) return
    TOH (N-1, A, C, B)
    print ( D-N, A → C )
    TOH (N-1, B, A, C )
}

```

from to
 spare
 ↓

```

TOH(1, B, A, C) {
    TOH(0, B, C, A) { return }
    print(1 B → C)
    TOH(0, A, B, C) { return }
}

```

output

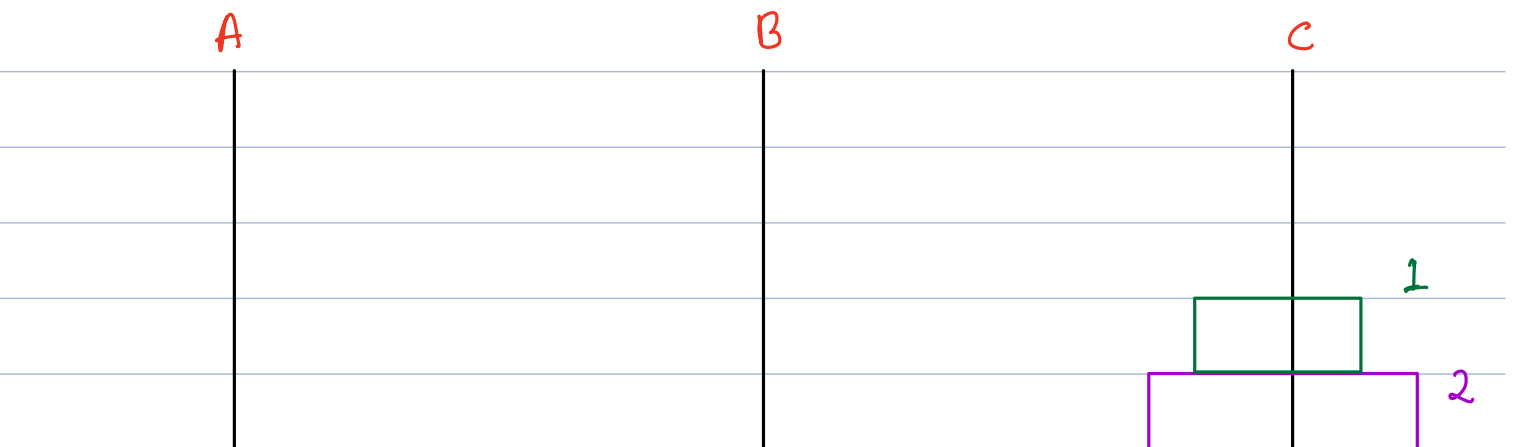
```

1 A → B
2, A → C
1 B → C

```

TOH(2, A, B, C)

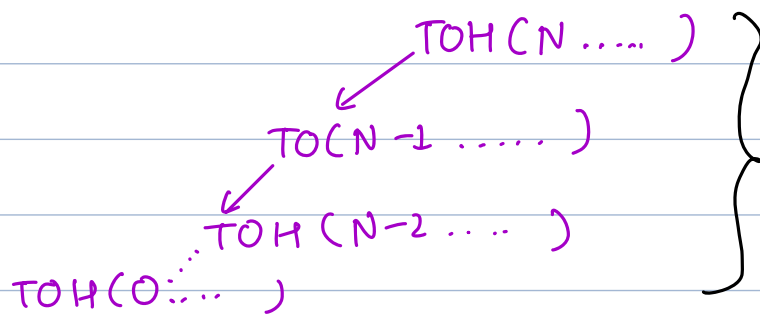
Initial State



Break : 22 : 34

TC : $O(2^N)$

SC : $O(N)$ → recursive stack space.



N+1 function calls
at max.

Valid Parenthesis

Print all valid parenthesis of length $2*N$ for a given N
 at any instant $\left. \begin{array}{l} \text{open} \geq \text{close} \\ \text{open} == \text{close} \end{array} \right\} \text{cond for valid parenthesis}$
 $== N$

Example

Output

$$N = 1$$

()

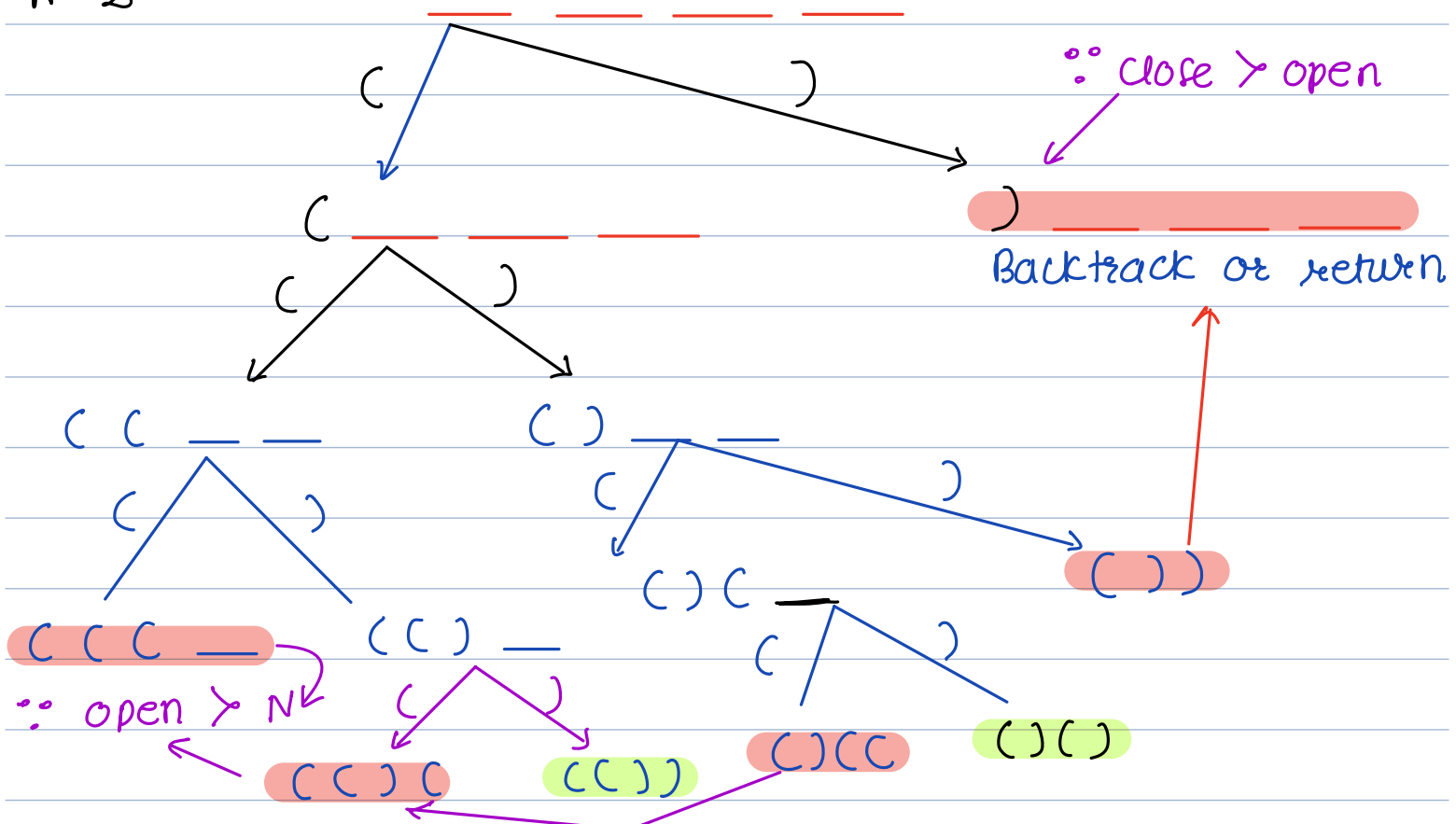
$$N = 2$$

(())

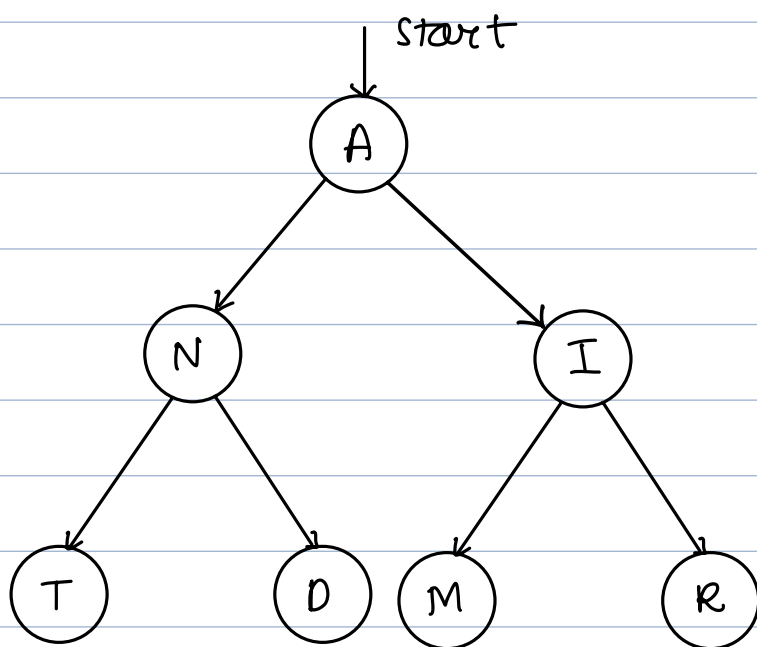
() ()

$$N = 3$$

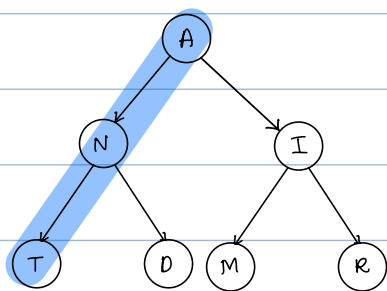
$((())()), ()(())(), (())(), (())()$
 $()()()$

$$N = 2$$


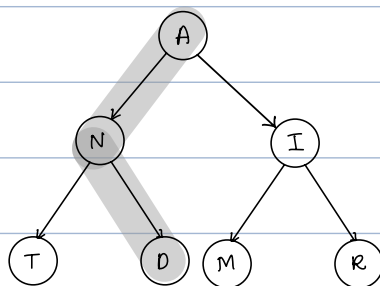
what is Backtracking?



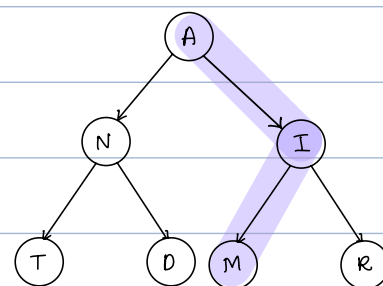
search for AIM



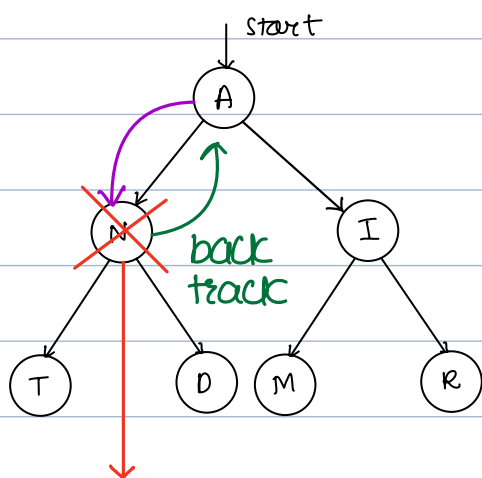
ANT \neq AIM



AND \neq AIM



AIM == AIM



Try out all possibilities to find a solution.

If solⁿ is not possible from a path we exit early {backtrack}

Backtrack is a technique

I can never find AIM, if I doesn't match with N

Pseudocode

```
void valid ( int N , String s , int open , int close ) {  
    // Base condition  
    // opening brackets cannot exceed N  
    if ( open > N ) return  
    // closing bracket > opening  
    if ( close > open ) return  
  
    // condn for valid ans  
    if ( open == N && close == N ) { // s.len = 2N OR  
        print (s)  
        return  
    }  
  
    // Main logic  
    // add opening bracket  
    valid ( N , s + '(', open + 1 , close )  
    // add closing bracket  
    valid ( N , s + ')', open , close + 1 )  
}
```

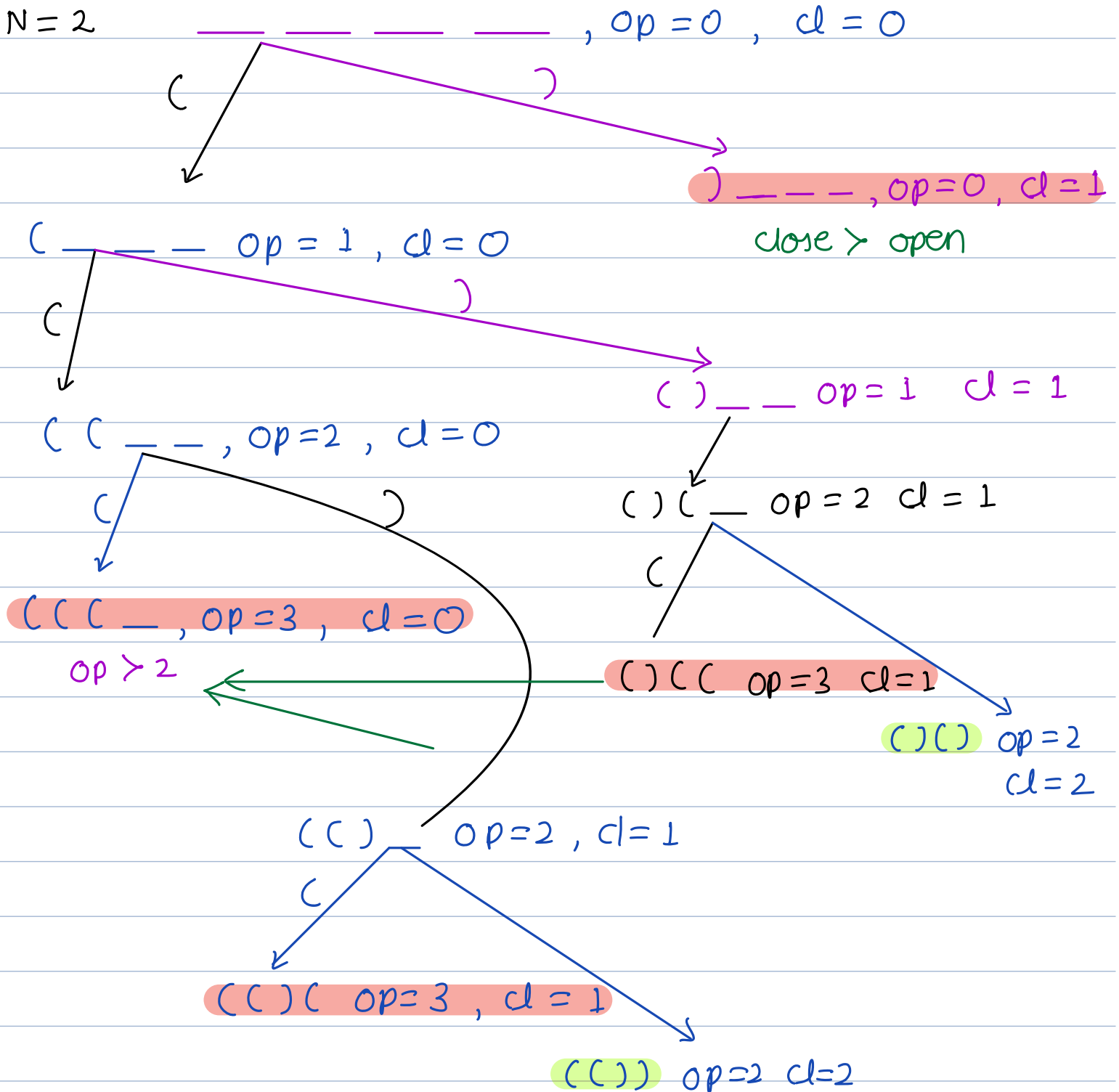
Output

(())

() ()

```
void valid (int N, String s, int open, int close) {
    if (open > N) return
    if (close > open) return
    if (open == N && close == N) {
        print(s)
        return
    }
    valid(N, s + '(', open + 1, close)
    valid(N, s + ')', open, close + 1)
}
```

N = 2



TC for valid Parenthesis.

TC : $O(2^N)$

