Combinatorics Basics

Agenda

- Addition and Multiplication Rule
- Permutation Basics
- Combination Basics & Properties
- Pascal triangle
- Find N^{th} column tile.

Previous PSP	Current PSP	Next target
66.4	67.25 %	70 %

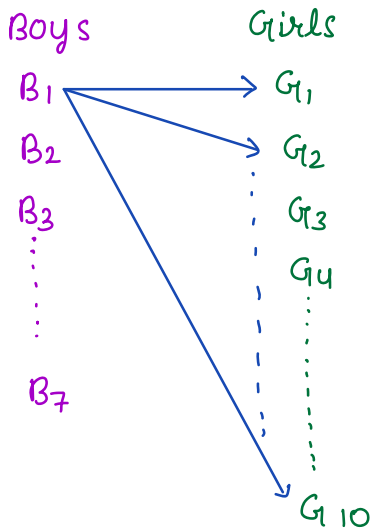
Personal target. $\simeq$ 100% PSP

Live Contest 1 re-attempt 2 4th Feb

Q ⟶ Q tab

A ⟶ Private Chat.

Q> Given 10 girls and 7 boys.
How many different pairs can be formed.

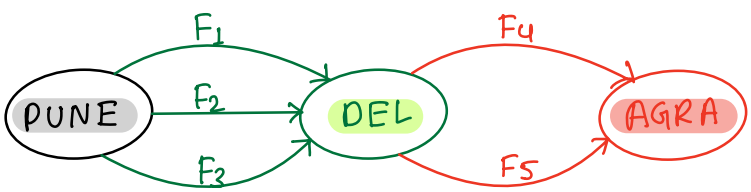


1 G AND 1 B

# of Boys * # of girls = 7 * 10 = 70.

AND ⟶ * { Multiplication Rule }.

Q>



ways to reach Agra from Pune via Delhi.

ways PUNE ⟶ DELHI = 3

ways DELHI ⟶ AGRA = 2

F_1 F_4 F_2 F_4 F_3 F_4

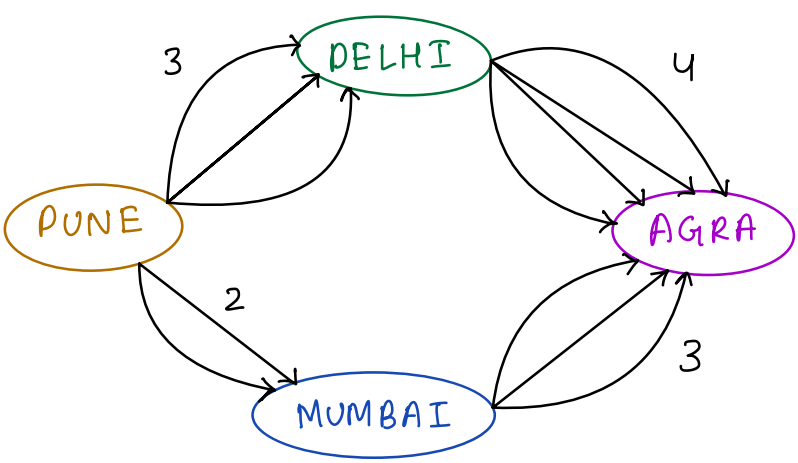
F_1 F_5 F_2 F_5 F_3 F_5

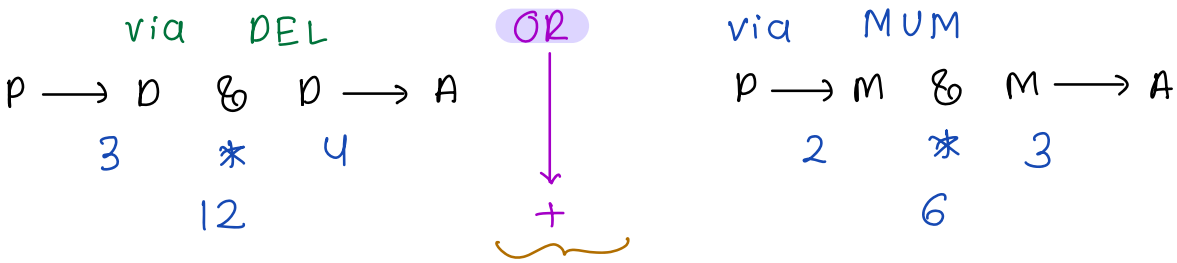
PUNE ⟶ DELHI AND DELHI ⟶ AGRA

3 * 2

6

Quiz 1> # ways of reaching Agra from Pune.





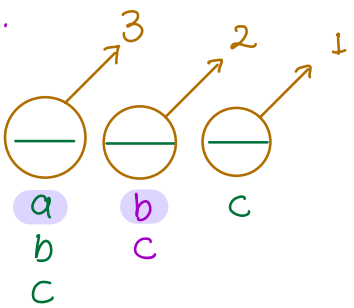
Addition Rule.

12 + 6 = 18.

Permutation Arrangements. } Order matters.

Given 3 distinct characters. How many ways can we arrange them.

a b c



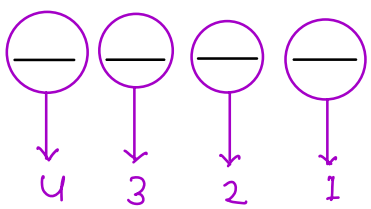
$3 * 2 * 1 = 3!$

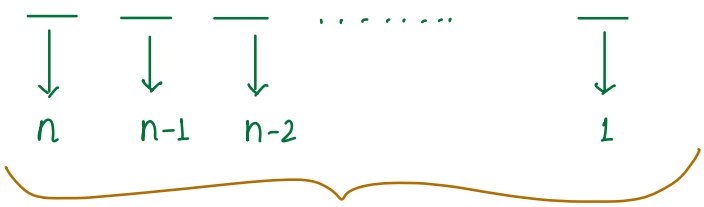
a b c
a c b

b a c
b c a

c a b
c b a

d a t e





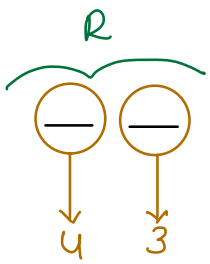
$= n!$

ways to arrange n distinct characters = n !

Given N distinct characters, in how many ways you can arrange

d a t e

N = 4 R = 2



$= 12$

d	a t e	3
a	d t e	3
t	d a e	3
e	d a t	3

$3 + 3 + 3 + 3 = 12$

Given N distinct characters, arrange R characters.

$n \quad n-1 \quad n-2 \quad \ldots\ldots \quad n-(r-1)$

R positions

$$\frac{n * n-1 * n-2 * \ldots\ldots \quad n-(r-1) * n-r * \ldots * 1}{(n-r) * (n-r-1) \ldots * 1}$$

$${}^{n}P_{r} \quad \boxed{\frac{n!}{(n-r)!} = \text{\# ways to arrange } r \text{ objects from N distinct objects}}$$

Combination — selections } order doesn't matter

Given 4 players, # ways to select 3 players.

$\{ P_1 \quad P_2 \quad P_3 \quad P_4 \}$

$P_1 \; P_2 \; P_3$
$P_1 \; P_3 \; P_4$
$P_1 \; P_2 \; P_4$
$P_2 \; P_3 \; P_4$

$P_1 P_2 P_3$	$P_1 P_3 P_4$	$P_1 P_2 P_4$	$P_2 P_3 P_4$
$P_1 P_3 P_2$	$P_1 P_4 P_3$	$P_1 P_4 P_2$	$P_2 P_4 P_3$
$P_2 P_1 P_3$	$P_3 P_1 P_4$	$P_2 P_1 P_4$	$P_3 P_2 P_4$
$P_2 P_3 P_1$	$P_3 P_4 P_1$	$P_2 P_4 P_1$	$P_3 P_4 P_2$
$P_3 P_1 P_2$	$P_4 P_1 P_3$	$P_4 P_1 P_2$	$P_4 P_2 P_3$
$P_3 P_2 P_1$	$P_4 P_3 P_1$	$P_4 P_2 P_1$	$P_4 P_3 P_2$
6 arrangements	6 arrangements	6 arrangements	6 arrangements

Assume no. of ways to select 3 players from 4 = x

ways select * # ways arrange each selection = total arrangements.

$$x * 3! = \frac{4!}{(4-3)!} = 24$$

$$X = \frac{4!}{(4-3)!\ 3!} = \underline{\underline{4}}$$

∴ Similarly,

Assume no. of ways to select r objects from N objects $= X$

ways select * # ways arrange each selection = total arrangements.

$$X * r! = {}^{N}P_{r} = \frac{N!}{(N-r)!}$$

no. of ways to select R objects from N objects

$$= \frac{N!}{(N-r)!\ r!} \qquad {}^{N}C_{r}$$

$0 <= r <= N$

Properties of Combination

ways to select 0 items from n items

$$^{n}C_{0} = 1$$

ways to select n items from n items

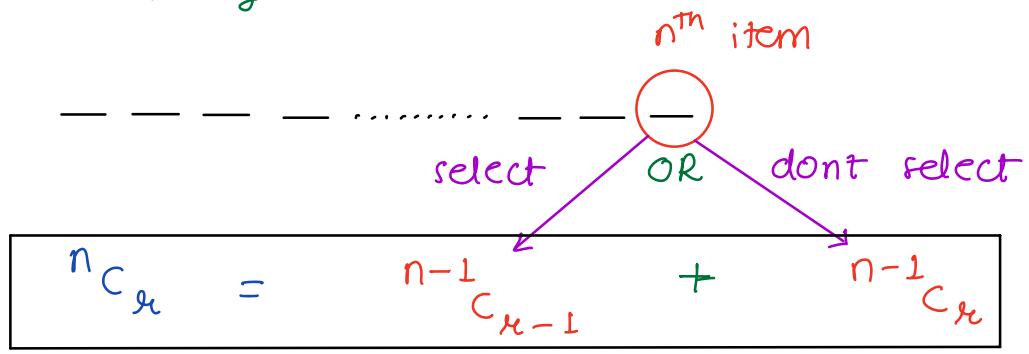
$$^{n}C_{n} = 1$$

ways to select $n-r$ items from n items

$$^{n}C_{n-r} = \frac{n!}{(n-(n-r))!\,(n-r)!} = \frac{n!}{(n-r)!\,(r!)} \rightarrow {}^{n}C_{r}$$

$$^{n}C_{r} = {}^{n}C_{n-r}$$

Special Property



Break ⟶ 22:34

Pascal's Triangle

Generate Pascal's triangle for given value of n.

n = 4

$^0C_0$

$^1C_0$ $^1C_1$

$^2C_0$ $^2C_1$ $^2C_2$

$^3C_0$ $^3C_1$ $^3C_2$ $^3C_3$

$^4C_0$ $^4C_1$ $^4C_2$ $^4C_3$ $^4C_4$

```
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
```

To fill A[r][c] ⟶ ^rC_c

Bruteforce Approach

TC : O(n³)

```
f r ⟶ 0 to n {
|   f c ⟶ 0 to r {
|   |   A[r][c] = rCc } generate
|   3
3
```

TC to calculate n! = O(n)

```
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
```

idea use special property.

```
nCr [N+1] [N+1]          // initially all 0

for n ⟶ 0 to N {                    n ⟶ row idx
    for r ⟶ 0 to n {                r ⟶ col idx
        if (r==0 || r==n) {
            nCr[n][r] = 1
        }
        else {
            nCr[n][r] = nCr[n-1][r] +
                        nCr[n-1][r-1]
            nCr[n][r] %= MOD
        }
    }
}
```

TC : $O(N^2)$

SC : $O(1)$ → ignore output space.

Nth Column title *** {MSFT}

Find the n^{th} column title.

1	2	3	.....	26	27	28	.....	52	53	54	.....	702
A	B	C	.....	Z	AA	AB	.....	AZ	BA	BB	.....	ZZ

Input

n	ans
3	C
30	AD
50	AX

Idea ⟶ Base 26 conversion

A	B	C	.....	X	Y	Z
↓	↓	↓		↓	↓	↓
0	1	2		23	24	25

} our system.

n = 1 ans = A

1	2	3	.....	26
A	B	C	.....	Z

} given system

n = 50

26	50 − 1	23 ⟶ X ↑
	1 − 1	0 ⟶ A
	0	

n = 10000

26	10000 − 1	15 ⟶
	384 − 1	19 ⟶
	14 − 1	13 ⟶

26	52 − 1	25 ⟶ 2
	1 − 1	0 ⟶ A
	0	

s = " " } stringBuilder.

```
while (n>0) {
    n -= 1
    ch = (char)(n % 26) + 'A'
    s = ch + s
    n = n/26
}
```

n = 50

print(s)

TC : $O(\log_{26} N)$

SC : $O(\log_{26} N)$ → because of string concatenation.