Markov Chains

Deadline: 21.03.25, 23:59

In this hometask you will be asked to perform different sampling exercises on generative adversarial networks treated as energy-based models. The hometask consists of several parts. In both parts of the task you are supposed to sample from some distribution $p_{data}(x)$, where $x \in \mathbb{R}^{28\times 28\times 3}$ are images of the objects from Cifar-10 dataset.

Exercise 1 (50 points + bonus part). Implement the GAN sampling procedure described in the paper [1]. You may use this colab template for a headstart. Consider the following steps:

- Load the DC-GAN, pretrained on Cifar-10. This part is done in the provided template.
- Now you have a pair of neural networks $D(\cdot)$ and $G(\cdot)$, which corresponds to the discriminator and the generator, respectively. Suppose that the discriminator $D(\cdot)$ is optimal. Introduce the notations $p_{data}(x)$ and $p_G(x)$ for the true data density and the density, implicitly induced by the generator G. In this case
$$D(x) = \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}\,. \tag{1}$$
Note that the density $p_G(x)$ is implicit, yet we can sample from it with usual sampling procedure from the model generator $G(\cdot)$. This gives an idea that we can try to sample from $p_{data}(x)$ using the Metropolis-Hastings procedure, using $p_G(x)$ as a proposal. Indeed,
$$\frac{p_{data}(x)}{p_G(x)} = \frac{1}{D^{-1}(x) - 1}\,.$$
Implement the Metropolis-Hastings procedure to sample from $p_{data}(x)$ using p_G as an independent proposal.
- Evaluate the proposed sampler. To do this, generate 1000 parallel chains of length up to 200 (you can try longer chains if you have enough resources). Compute the inception score for each 20-th slice of your chain. Try different starting points for your MCMC: try either random starting points or pictures from training set.
- **Bonus part, up to** 100 **points.** Try to fix generator $G(\cdot)$ and train discriminator D solely. Then try to re-sample from the model (1) with new discriminator D and check, if there are any quality improvements. Any additional discriminator regularisations are allowed.

References

[1] Ryan Turner, Jane Hung, Eric Frank, Yunus Saatci, and Jason Yosinski. Metropolis-hastings generative adversarial networks, 2018. https://arxiv.org/abs/1811.11357.