Algorithms – I (CS29003/203)

Autumn 2022, IIT Kharagpur

Hashing

Resources

- Apart from the book
- UC Davis ECS 36C Course by Prof. Joël Porquet-Lupine

Introduction

- Symbol table
  - Association of a value with a key
    - `table[key] = value`
  - Typical operations
    - `Insert(), Remove(), Contains()/Get()`
  - Other possible operations
    - `Min(), Max(), Rank(), SortedList(), etc.`
- Example #1: frequency of letters
  - Compute the frequency of all the (alphabetical) characters in a text

> *I was born in Tuckahoe, near Hillsborough, and about twelve miles from Easton, in Talbot county, Maryland. I have no accurate knowledge of my age, never having seen any authentic record containing it. By far the larger part of the slaves know as little of their ages as horses know of theirs, and it is the wish of most masters within my knowledge to keep their slaves thus ignorant. I do not remember to have ever met a slave who could tell of his birthday. They seldom come nearer to it than planting-time, harvest-time, cherry-time, spring-time, or fall-time. [...]*

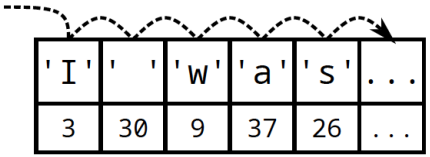
Key	Value
'a'	37
'b'	6
'c'	10
'd'	10
'e'	54
'g'	11
...	...

Source: UC Davis, ECS 36C course, Spring 2020

Naïve Implementations

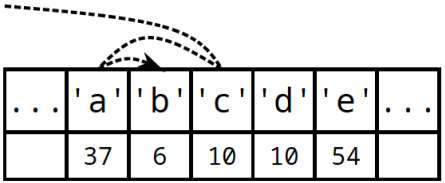
- Unordered array
  - Linear search: Sequentially check each item until match is found

Search	Insert	Delete
$O(N)$	$O(1)$	(search time +) $O(1)$



- Ordered/sorted array
  - Binary search: Can perform binary search to find item

Search	Insert	Delete
$O(\log N)$	$O(N)$	$O(N)$

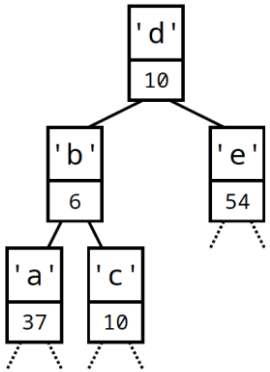


Source: UC Davis, ECS 36C course, Spring 2020

Advanced Implementations

- Binary search tree: Organization of data items in BST, following certain order
  - Any node's key is greater than all keys in its left subtree
  - Any node's key is smaller than all keys in its right subtree
- At most, explore height of tree

Search avg/worst	Insert avg/worst	Delete avg/worst
$O(\log N)/O(N)$	$O(\log N)/O(N)$	$O(\log N)/O(N)$



- AVL tree
  - Balanced BST
  - Height of tree is kept to $O(\log N)$

Search	Insert	Delete
$O(\log N)$	$O(\log N)$	$O(\log N)$

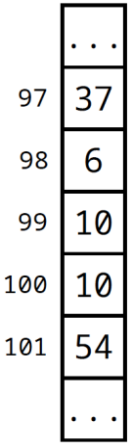
Source: UC Davis, ECS 36C course, Spring 2020

Ideal Data Structure

- Can we achieve constant time on all three main operations?

Search	Insert	Delete
$O(1)$	$O(1)$	$O(1)$

- Using data as index
  - Keys are small integers
  - ASCII is 128 characters
  - Can directly be used as index
    - `table[key] = value`



- Limitations
  - Doesn't (efficiently) support other operations – `min()`, `max()` etc.
  - Wasted memory for unused keys – e.g., ASCII defines about 30 control characters
  - What if the key is not a small integer?

Source: UC Davis, ECS 36C course, Spring 2020

Example #2: frequency of words

- Compute the frequency of all the words in a text

> *I was born in Tuckahoe, near Hillsborough, and about twelve miles from Easton, in Talbot county, Maryland. I have no accurate knowledge of my age, never having seen any authentic record containing it. By far the larger part of the slaves know as little of their ages as horses know of theirs, and it is the wish of most masters within my knowledge to keep their slaves thus ignorant. I do not remember to have ever met a slave who could tell of his birthday. They seldom come nearer to it than planting-time, harvest-time, cherry-time, spring-time, or fall-time. [...]*

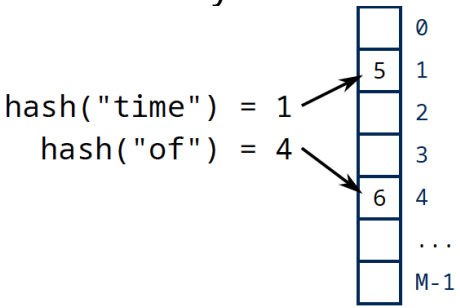
Key	Value
'of'	6
'time'	5
'to'	3
'the'	3
'it'	3
'I'	3
...	...

- Issues
  - Very large number of words in English
    - Would need a huge table to index by word
    - `int table[171476];`
  - Cannot index an array with a string
    - table["time"] is not a proper C construct
    - Can only index arrays with an integer

Source: UC Davis, ECS 36C course, Spring 2020

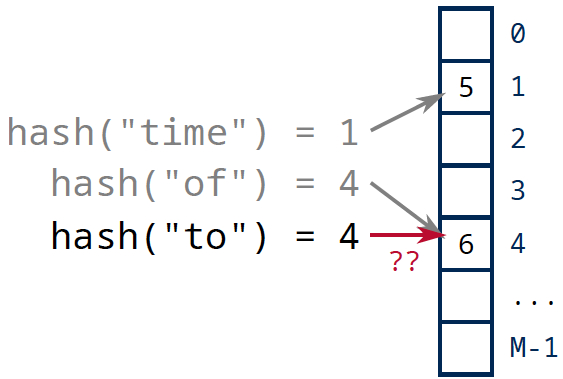
Hashing

- Compute an array index from **any** key
- With direct addressing, a key k is stored in slot k
- With hashing, this element is stored in slot $h(k)$
- we use a hash function h to compute the slot from the key k
- h maps the universe U of keys into the slots of a hash table $T[0, \ldots, M-1]$
- $h: U \rightarrow \{0,1, \ldots, M-1\}$
- The size M of the hash table is typically much less than $|U|$
- We say that an element with key k **hashes to** slot $h(k)$
- And $h(k)$ is the hash value of key k



Hashing

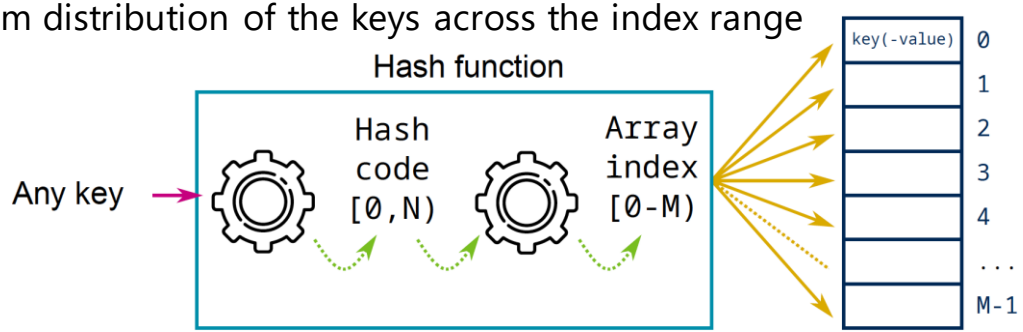
- We will see how to provide hashing for any type of key, e.g., Boolean, integers, floating-point numbers, strings, user-defined objects, etc.
- Two keys may hash to the same slot. Such a situation is called a **collision**
- How to resolve potential collisions properly?



Source: UC Davis, ECS 36C course, Spring 2020

Hashing

- Before going to collisions, lets discuss some hashing techniques
- Required properties:
  - Simple to compute and fast
  - Hash of a key has to be deterministic
    - Equal keys must produce the same hash value
    - `assert(a == b && hash(a) == hash(b));`
  - Uniform distribution of the keys across the index range



- Two implicit steps:
  - Transform any potentially non-integer key into an integer hash code
  - Reduce this hash code to an index range

Source: UC Davis, ECS 36C course, Spring 2020

Some Pitfalls

- Worse (yet functional) hash function

```
def GalaxyHashCode(obj: Any) -> int:
  return 42;
```

- Problem
  - Can only insert one key
  - All the following keys will collide, regardless of their value

```
print("Hash of 'time' => {:d}".format(GalaxyHashCode('time')))
print("Hash of 'of' => {:d}".format(GalaxyHashCode('of')))
print("Hash of '99fdr' => {:d}".format(GalaxyHashCode('99fdr')))
print("Hash of '55' => {:d}".format(GalaxyHashCode(55)))
```

```
Hash of 'time' => 42
Hash of 'of' => 42
Hash of '99fdr' => 42
Hash of '55' => 42
```

- Solution
  - Need to distinguish keys by value

Source: UC Davis, ECS 36C course, Spring 2020

Key Subset Selection

- Possible scenario
  - Keys are phone numbers: e.g., (530) 752-1011
  - Hash table is of size 1000
  - Idea: use first three digits as a direct index?
    - `hash("(530) 752-1011") = 530`
- Problem
  - Phone numbers with same area code will collide
- Solution
  - Select another set of 3 digits that shows better "random" properties (e.g., the last three)
  - Or consider the entire phone number instead

Source: UC Davis, ECS 36C course, Spring 2020

Hashing Positive Integers

- Key is a simple unsigned integer in the range 0..K-1
- Array index is always an unsigned integer in the range 0..M-1
- Hashing is done via modulo operation

```
def HashUInt(obj: np.uint32, M: int) -> int:
  return obj % M
```

```
# Initialize hash table of size 100
ht = np.full((100), False, dtype=bool)

# Uniform random number generator
distr = np.random.permutation(999)[:25]

for i in distr:
  hash = HashUInt(i, ht.shape[0])
  print("\nHash of {:d} => {:d}".format(i,hash), end="")
  if (ht[hash]):
    print(" (collision!)",  end="")
  ht[hash] = True
```

```
Hash of 562 => 62
Hash of 833 => 33
Hash of 764 => 64
Hash of 981 => 81
Hash of 294 => 94
Hash of 572 => 72
Hash of 605 => 5
Hash of 337 => 37
Hash of 854 => 54
Hash of 939 => 39
Hash of 292 => 92
Hash of 356 => 56
Hash of 364 => 64 (collision!)
Hash of 648 => 48
Hash of 990 => 90
Hash of 753 => 53
Hash of 340 => 40
Hash of 442 => 42
Hash of 20 => 20
Hash of 623 => 23
Hash of 31 => 31
Hash of 453 => 53 (collision!)
Hash of 954 => 54 (collision!)
Hash of 545 => 45
Hash of 847 => 47
```

Source: UC Davis, ECS 36C course, Spring 2020

Consideration about table size (1/2)

- Does the size of the hash table matter?
- Same scenario as previous slide
  - Keys are random numbers between 0 and 999
  - Distribution of keys is uniform
  - Hashing of 25 keys
- But hash table can either be of size 100 or size 97

- Conclusion
  - Table size is not critical
  - If distribution of keys is uniform

100	97
461 => 61	461 => 73
741 => 41	741 => 62
412 => 12	412 => 24
482 => 82	482 => 94
522 => 22	522 => 37
19 => 19	19 => 19
941 => 41 (c!)	941 => 68
569 => 69	569 => 84
634 => 34	634 => 52
763 => 63	763 => 84 (c!)
476 => 76	476 => 88
191 => 91	191 => 94 (c!)
898 => 98	898 => 25
796 => 96	796 => 20
583 => 83	583 => 1
370 => 70	370 => 79
331 => 31	331 => 40
883 => 83 (c!)	883 => 10
631 => 31 (c!)	631 => 49
352 => 52	352 => 61
718 => 18	718 => 39
959 => 59	959 => 86
702 => 2	702 => 23
469 => 69 (c!)	469 => 81
409 => 9	409 => 21

Source: UC Davis, ECS 36C course, Spring 2020

Consideration about table size (2/2)

- Does the size of the hash table matter?
- Same scenario as previous slide
  - Keys are random numbers between 0 and 999
  - Distribution of keys is **non-uniform**
  - Hashing of 25 keys
- But hash table can either be of size 100 or size 97

- Conclusion
  - Table size becomes critical
  - If distribution of keys is non-uniform
  - Prime numbers exhibit good properties

100

```
0 => 0
25 => 25
50 => 50
75 => 75
100 => 0 (c!)
125 => 25 (c!)
150 => 50 (c!)
175 => 75 (c!)
200 => 0 (c!)
225 => 25 (c!)
250 => 50 (c!)
275 => 75 (c!)
300 => 0 (c!)
325 => 25 (c!)
350 => 50 (c!)
375 => 75 (c!)
400 => 0 (c!)
425 => 25 (c!)
450 => 50 (c!)
475 => 75 (c!)
500 => 0 (c!)
525 => 25 (c!)
550 => 50 (c!)
575 => 75 (c!)
600 => 0 (c!)
```

97

```
0 => 0
25 => 25
50 => 50
75 => 75
100 => 3
125 => 28
150 => 53
175 => 78
200 => 6
225 => 31
250 => 56
275 => 81
300 => 9
325 => 34
350 => 59
375 => 84
400 => 12
425 => 37
450 => 62
475 => 87
500 => 15
525 => 40
550 => 65
575 => 90
600 => 18
```

Source: UC Davis, ECS 36C course, Spring 2020

Hashing Strings

- Naïve approach: Sum ASCII value of each character and then reduce to table size

```
# Hashing strings
def HashStr(key: str, M: int) -> int:
  h = np.uint32(0)
  for c in key:
    h = h + ord(c)
  return h % M
```

- Observations:
  - Average word length is 4.5. 4.5*127 = 576
  - Longest word is 45 letters long. 45*127 = 5715
- Issues:
  - Assuming a hash table of size 10,000
  - Indices up to 500 will be crowded
  - Indices above 5000 will never be used

Source: UC Davis, ECS 36C course, Spring 2020

Hashing Strings

- Experiment

```
WordList = [
    # Some of the most used words
    "the", "and", "that", "have",\
    "for", "not", "with", "you", \
    # Longest word in English
    "pneumonoultramicroscopicsilicovolcanoconiosis"]

for wd in WordList:
  print("Hash({:s}) => {:d}".format(wd,HashStr(wd,10000)))
```

```
Hash(the) => 321
Hash(and) => 307
Hash(that) => 433
Hash(have) => 420
Hash(for) => 327
Hash(not) => 337
Hash(with) => 444
Hash(you) => 349
Hash(pneumonoultramicroscopicsilicovolcanoconiosis) => 4880
```

Source: UC Davis, ECS 36C course, Spring 2020

Hashing Strings

- Better approach: Multiply 31 and add ASCII value of each character and then reduce to table size

```
# Hashing strings
def HashStr(key: str, M: int) -> int:
  h = np.uint32(0)
  for c in key:
    h = (h*31) + ord(c)
  return h % M
```

```
Hash(the) => 4801
Hash(and) => 6727
Hash(that) => 8823
Hash(have) => 5240
Hash(for) => 1577
Hash(not) => 9267
Hash(with) => 9734
Hash(you) => 9839
Hash(pneumonoultramicroscopicsilicovolcanoconiosis) => 2420
```

- Horner's method
  - Consider string of length L as a polynomial
  - $h = key[0] * 31^{L-1} + key[1] * 31^{L-2} + \ ... + key[L-1] * 31^{0}$
  - Makes the distribution more uniform
  - If unsigned hash value gets too big, overflow is controlled
  - 31 is an interesting prime number (Mersenne prime (like 127 or 8191))
  - Multiplying item by 32 (by shifting <<5) and subtract item
  - Experiments shown good distribution of indices overall

Source: UC Davis, ECS 36C course, Spring 2020

Collisions

- Uniform Hashing:
  - We assume that the hash function has a uniform distribution
  - Keys are mapped as evenly as possible over the index range
- Collision Probability:
  - Assuming an array index in the range 0..M-1
  - After how many hashed keys will there be the first collision?

- Experiments:
  - Example:- Output range of 97
  - 10 hashes to observe the first collision
  - Now, what is the output range was 223?
  - Need to restart a simulation.
  - Or need for better mathematical tools!

```
461 => 73
741 => 62
412 => 24
482 => 94
522 => 37
19 => 19
941 => 68
569 => 84
634 => 52
763 => 84 (c!)
```

10

Source: UC Davis, ECS 36C course, Spring 2020

Collisions

- Birthday paradox:
  - In a group of N random people, what is the probability that two people have the same birthdate, assuming a year consists of 365 days?
  - (pigeonhole principle) 100% if group counts 366 people
  - Surprisingly it is 50% if group consists of 23 people only
  - And 99.9% if number of persons is 70
- Collision Probability:
  - Probability of at least two people having same birthdate = 1 – probability of all birthdates are separate
  - Consider $M(= 365)$ containers and assume N balls are to be randomly placed in these containers
  - There are M options for the first ball to sit
  - For the second ball, $M - 1$ options for 2 distinct containers
  - For the third ball, $M - 2$ options for 3 distinct containers
  - Continuing, for N^{th} ball, $M - (N - 1)$ options for N distinct containers
  - Without considering separate containers, the number possibilities is M^N

Collisions

- Collision probability:
  - $P(no\ collision) = \frac{M(M-1)(M-2)\ldots(M-N+1)}{M^N} = \prod_{i=0}^{N-1}\frac{M-i}{M}$
  - $P(at\ least\ one\ collision) = 1 - \prod_{i=0}^{N-1}\frac{M-i}{M}$
- Approximation
  - Remember that $\log(1+x) \approx x$, for small x
  - $\log\prod_{i=0}^{N-1}\frac{M-i}{M} = \sum_{i=0}^{N-1}\log\left(\frac{M-i}{M}\right) = \sum_{i=0}^{N-1}\log\left(1-\frac{i}{M}\right) \approx \sum_{i=0}^{N-1} -\frac{i}{M} = -\frac{1}{M}\sum_{i=0}^{N-1} i$
  - $\log(P(no\ collision)) = -\frac{1}{M}.\frac{(N-1)N}{2}$
  - $P(no\ collision) = e^{-\frac{1}{M}.\frac{(N-1)N}{2}}$
  - $P(at\ least\ one\ collision) = 1 - e^{-\frac{1}{M}.\frac{(N-1)N}{2}} = 1 - e^{-const.(N^2-N)}$

Collisions

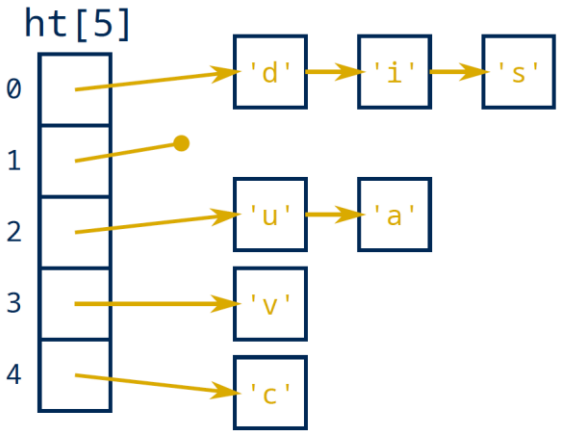
- Collision probability:
  - Number of hashed keys to first collision is $\approx \sqrt{\frac{\pi}{2}M}$
  - For an output range of 97: $\sqrt{\frac{\pi}{2}97} = 12$
  - Our experiments showed first collision at 10th record
  - For an output range of 223: $\sqrt{\frac{\pi}{2}223} = 18$
- Observation:
  - Unless table size is quadratically bigger than necessary it is impossible to avoid collisions
- Conclusion:
  - Need to deal with collisions gracefully
  - Two options:
    - **Separate chaining**: couple the hash table with another container
    - **Open addressing**: put the key at another index in case of collision

Source: UC Davis, ECS 36C course, Spring 2020

Separate Chaining

- Principle:
  - Hash table is an array of linked-lists
  - Keys are still hashed to array index in the range 0..M-1

key	hash
u	2
c	4
d	0
a	2
v	3
i	0
s	0



- **Insertion**: key pushed in the i^{th} chain
- **Search**: only need to search key in the i^{th} chain

Source: UC Davis, ECS 36C course, Spring 2020

Separate Chaining

```
class HTChaining:
  # init function for our hash table
  def __init__(self, Hash, M):
    # Hash is the hash function that maps a
    # universe U to range(n)
    self.Hash = Hash
    # Create M slots, each of which has a linked list.
    # We're just going to implement as python list
    self.ht = [[] for i in range(M)]
```

- Hash table of chains, as a list of lists in python

- Python's internal hash functionality

```
print('Hash of 42 is:', hash(42))
print('Hash of -42 is:', hash(-42))
print('Hash of 42.23 is:', hash(42.23))
print('Hash of -42.23 is:', hash(-42.23))
print('Hash of IITKharagpur is:', hash('IITKharagpur'))
```

```
Hash of 42 is: 42
Hash of -42 is: -42
Hash of 42.23 is: 530343892119142442
Hash of -42.23 is: -530343892119142442
Hash of IITKharagpur is: -6146170003778908 25
```

```
print('Hash of 42 is:', ctypes.c_size_t(hash(42)).value)
print('Hash of -42 is:', ctypes.c_size_t(hash(-42)).value)
print('Hash of 42.23 is:', ctypes.c_size_t(hash(42.23)).value)
print('Hash of -42.23 is:', ctypes.c_size_t(hash(-42.23)).value)
print('Hash of IITKharagpur is:', ctypes.c_size_t(hash('IITKharagpur')).value)
```

```
Hash of 42 is: 42
Hash of -42 is: 18446744073709551574
Hash of 42.23 is: 530343892119142442
Hash of -42.23 is: 17916400181590409174
Hash of IITKharagpur is: 17832127073331660791
```

Separate Chaining

- Defining a custom hash function

```
def HashFn(key: Any, M: int) -> int:
  return ctypes.c_size_t(hash(key)).value % M
```

- Attaching it to the Hash Table object

```
# make a hash table that uses the
# custom defined HashFn
HT = HTChaining(HashFn, 10)
```

- Inserting a key

```
def insert(self, x):
  if self.find(x) is None:
    # Push key into right slot
    self.ht[self.Hash(x,len(self.ht))].append(x)
```

- Finding a key

```
def find(self,x):
  slot = self.ht[self.Hash(x,len(self.ht))]
  # take time O(n) to look for x in the slot
  for i in range(len(slot)):
    if slot[i] == x:
      return slot[i]
  return None
```

Separate Chaining

- Deleting a key from the hash table

```
# delete an item in the hash table, if it's in there
# returns the deleted item, or None if it wasn't found.
def Remove(self,x):
  slot = self.ht[self.Hash(x,len(self.ht))]
  # take time O(n) to look for x in the bucket.
  for i in range(len(slot)):
    if slot[i] == x:
      return slot.pop(i)
  return None
```

- Helper code to print the hash table

```
def __repr__(self) -> str:
  """
  Get the string representation of this hash tabke.
  """
  # return f"Node({self.data})"
  return str([self.ht[i] for i in range(len(self.ht))])
```

Separate Chaining

- Driver code (1/2)

```
# make a hash table that uses the
# custom defined HashFn
HT = HTChaining(HashFn, 10)

x = 1234567
y = 890
# Insert x but not y
HT.insert(x)

# let's make sure that x is there and y isn't.
if HT.find(x) == x:
  print("Successfully found", x, "in the hash table")

if HT.find(y) is None:
  print( y, "is not in the hash table")

# Insert z and remove x
z = 1234
HT.insert(z)
removed_key = HT.Remove(x)
print("Successfully removed", removed_key)
if HT.find(x) is None:
  print( x, "is not in the hash table")
```

```
Successfully found 1234567 in the hash table
890 is not in the hash table
Successfully removed 1234567
1234567 is not in the hash table
```

Separate Chaining

- Driver code (2/2)

```
# Uniform random number generator
distr = np.random.permutation(999)[:15]

# make a hash table that uses the
# custom defined HashFn
HT = HTChaining(HashFn, 10)
for i in distr:
  HT.insert(i)

print(HT)
```

```
[[950], [], [132, 242], [], [854, 864], [], [246, 636, 216, 846], [417, 47, 567], [448, 708, 148], []]
```

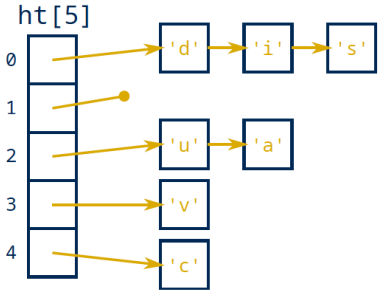
Separate Chaining - Performance analysis

- Hashing key to array index: $O(1)$
- Searching key in chain: $O(length\ of\ chain)$
- If using poor hash function, such as GalaxyHashCode()
  - One chain can end up containing all the items
  - Runtime complexity of operations: $O(N)$!
- If using hash function providing uniform distribution
  - On average, chains will be of length $^{N}/_{M} = L$
  - L is also know as the load factor
  - Runtime complexity of operations: $O(L)$
  - If load factor is maintained to be constant, then complexity also becomes constant
  - Number of items N in hash table cannot be controlled
  - But number of chains M can be ...

Source: UC Davis, ECS 36C course, Spring 2020

Separate Chaining - Resizing

Scenario

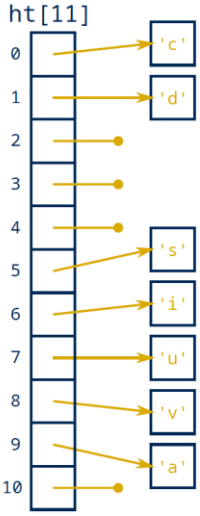


- Load factor = 7/5=1.4

Resizing - Rehashing

- Increase the size of hash table
- **Rehash** every item into new table

Result



- Load factor = 7/11=0.63

Source: UC Davis, ECS 36C course, Spring 2020

Separate Chaining - Resizing

- Implementation

```
class HTChainingWResize:
  # init function for our hash table
  def __init__(self, Hash, M):
    # Hash is the hash function that maps a
    # universe U to range(n)
    self.Hash = Hash
    # Create M slots, each of which has a linked list.
    # We're just going to implement as python list
    self.ht = [[] for i in range(M)]
    # Number of current keys
    self.cur_size = 0

  def insert(self, x):
    if self.find(x) is None:
      # Push key into right slot
      self.ht[self.Hash(x,len(self.ht))].append(x)
      # Update current size
      self.cur_size = self.cur_size + 1
      # Resize hash table if load factor is 1
      if self.cur_size > len(self.ht):
        self.Resize(len(self.ht) * 2)
```

Separate Chaining - Resizing

- Implementation

```
def Resize(self, capacity):
  # Back up existing hash table
  old_ht = copy.deepcopy(self.ht)
  # Create an empty table with new capacity
  self.ht = [[] for i in range(capacity)]
  self.cur_size = 0
  # Reinsert all previous items into new table
  for chain in old_ht:
    for i in chain:
      self.insert(i)
```

```
# delete an item in the hash table, if it's in there
# returns the deleted item, or None if it wasn't found.
def Remove(self,x):
  slot = self.ht[self.Hash(x,len(self.ht))]
  # take time O(n) to look for x in the bucket.
  for i in range(len(slot)):
    if slot[i] == x:
      return slot.pop(i)
      # Update current size
      self.cur_size = self.cur_size - 1
  return None
```

Complexity

- Resizing happens infrequently
- Amortized time complexity: O(1)
- Ideally, resize to another prime number

Separate Chaining - Conclusion

Complexity

- Proportional to load factor. Typically load factor is maintained at 1.0

Search	Insert	Delete
$O(1)$	$O(1)$ *amortized*	$O(1)$

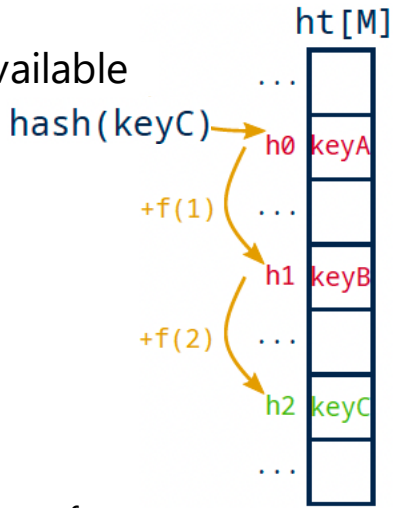
- Pros
  - Very effective if keys are uniformly distributed
- Cons
  - Rely on another data structure, such as lists, to implement the chains
    - Note that the chains could also be implemented with balanced trees!
  - Cost associated to this additional data structure
    - Memory management, traversal, etc.

Source: UC Davis, ECS 36C course, Spring 2020

Open Addressing

Principle

- We are not going to chain collisions
- Maximum number of keys that can fit is same as the hash table size
- Collided keys are inserted at the same flat hash table, but at the next available index
- Formally: Key x is inserted at the first index h_i available
  - $h_i(x) = (hash(x) + f(i)) \; mod \; M$
  - where $f(0) = 0$
- Function f is the collision resolution strategy
  - Linear probing, quadratic probing etc.
- Operations
  - Insert: Keep probing until an empty index is found
  - Search: Keep probing until key is found
    - Or until encountering an empty available index (i.e., key is not found)



Source: UC Davis, ECS 36C course, Spring 2020

Linear Probing

Principle

- Function f is a linear function of i
  - Typically $f(i) = i$
- In case of collision, try next indices sequentially

Example

- Hash table of size M = 17
- Insert sequence of characters

Key	Hash	Index
I	5	5
<	9	9
3	0	0
L	8	8
U	0	1 *(= 0 + 1)*
V	1	2 *(= 1 + 1)*

Observations

- Keys can form clusters of contiguous blocks
- Caused by multiple collisions
- Keys hashed into a cluster might require multiple attempts to be inserted

Source: UC Davis, ECS 36C course, Spring 2020

Linear Probing - Implementation

```
class HTLinear:
  # init function for our hash table
  def __init__(self, Hash, M):
    # Hash is the hash function that maps a
    # universe U to range(M)
    self.Hash = Hash

    @dataclass
    class Node:
      key: int
      taken: bool
    # Create M slots as an array of Nodes. We're
    # going to implement the array as python list
    self.ht = [Node(0, False) for i in range(M)]
    # Number of current keys
    self.cur_size = 0
```

Same hash function as for separate chaining implementation

```
def HashFn(key: Any, M: int) -> int:
  return ctypes.c_size_t(hash(key)).value % M
```

Linear Probing - Implementation

```
def find(self,x):
  idx = self.Hash(x,len(self.ht))
  while (self.ht[idx].taken):
    if (self.ht[idx].key == x):
      return self.ht[idx].key
    idx = (idx + 1) % len(self.ht)
  return None
```

```
def Resize(self, capacity):
  # Back up existing hash table
  old_ht = copy.deepcopy(self.ht)
  # Create an empty table with new capacity
  self.ht = [Node(0, False) for i in range(capacity)]
  self.cur_size = 0
  # Reinsert all previous items into new table
  for nd in old_ht:
    if nd.taken:
      self.insert(nd.key)
```

```
def insert(self, x):
  if self.find(x) is None:
    idx = self.Hash(x,len(self.ht))
    # Find empty spot
    while (self.ht[idx].taken):
      idx = (idx + 1) % len(self.ht)
    # Push key into right spot
    self.ht[idx].key = x
    self.ht[idx].taken = True
    # Update current size
    self.cur_size = self.cur_size + 1
    # Resize hash table if half full
    if self.cur_size > len(self.ht)/2:
      self.Resize(len(self.ht) * 2)
```

Try to maintain load factor of maximum 0.5

Deletion Principle

Key	Hash	Index
I	5	5
<	9	9
3	0	0
L	8	8
U	0	1
V	1	2

Scenario

- Assuming following hash table
  - Cluster of three keys
- How to remove key U?
  - Key V should still be reachable

Problems

- If free index at U, V is not reachable anymore
  - Hash(V) points to available entry
- Same issue for key U if remove key 3

Solution

- Free key's index upon removal
- But rehash every key of cluster located directly after key
  - V would be reinserted at index 1, and stay reachable

Source: UC Davis, ECS 36C course, Spring 2020

Deletion Implementation

```
# delete an item in the hash table, if it's in there
# returns the deleted item, or None if it wasn't found.
def Remove(self,x):
  if self.find(x) is None:
    return None

  # Find key position and remove it
  idx = self.Hash(x,len(self.ht))
  while (self.ht[idx].key != x):
    idx = (idx + 1) % len(self.ht)
  self.ht[idx].taken = False
  ret_val = self.ht[idx].key

  # Rehash the next keys of the same cluster
  idx = (idx + 1) % len(self.ht)
  while (self.ht[idx].taken):
    # Temporarily remove key from its spot, and reinsert it right away
    temp_key = copy.deepcopy(self.ht[idx].key)
    self.ht[idx].taken = False
    # Decrease current size, as it will increased in subsequent insert
    self.cur_size = self.cur_size - 1
    self.insert(nd.key)
    idx = (idx + 1) % len(self.ht)

  # Update current size and resize hash table if 12.5% full
  self.cur_size = self.cur_size - 1
  if self.cur_size > 0 and self.cur_size < len(self.ht)/8:
    self.Resize(len(self.ht)//2)
  # Return
  return ret_val
```

Linear Probing - Conclusion

Complexity

- Difficult complexity analysis
  - Often simplified to $0(1)$

Search hit	Search miss/Insert
$O(\sim \frac{1}{2}\left(1+\frac{1}{1-\alpha}\right))$	$O(\sim \frac{1}{2}\left(1+\frac{1}{(1-\alpha)^2}\right))$

Pros

- No overhead for memory management
- Locality of reference, especially if multiple consecutive items can be in the same cache line

Cons

- Requires low load factor (0.5) compared to separate chaining
  - Bigger hash table
- Causes primary clustering

Source: UC Davis, ECS 36C course, Spring 2020

Open Addressing Strategies

Linear probing

- $f(i) = i$
- If index at i is taken, try $i + 1$, then $i + 2$, … , $i + k$

Quadratic probing

- $f(i) = i^2$
- If index at i is taken, try $i + 1^2$, then $i + 2^2$, … , $i + k^2$

Double hashing

- Use second hash functions to compute intervals between probes
- $f(i) = i * h_2(k)$

Source: UC Davis, ECS 36C course, Spring 2020

Thank You