

# Feature extraction and end analysis

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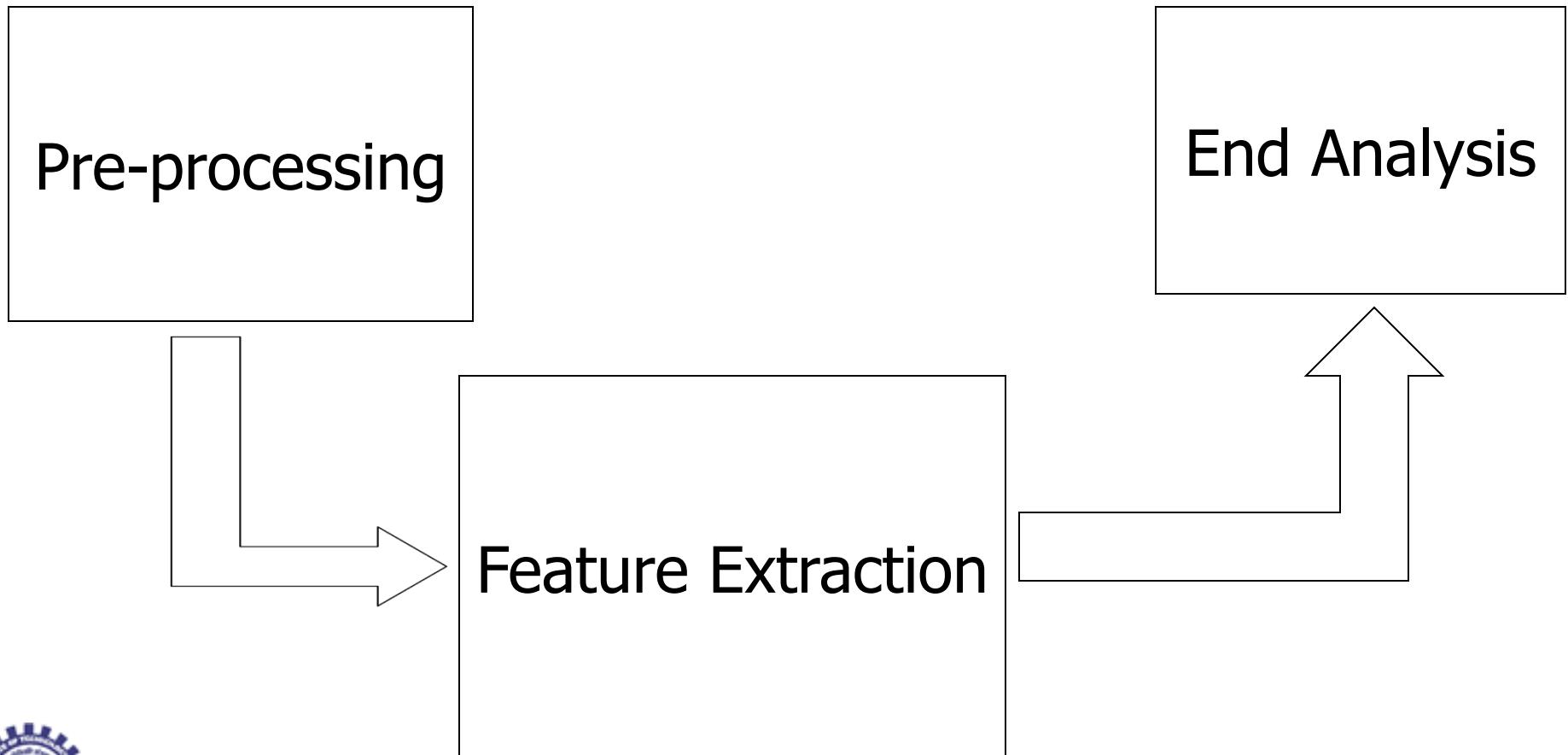


# A feature of an image ?

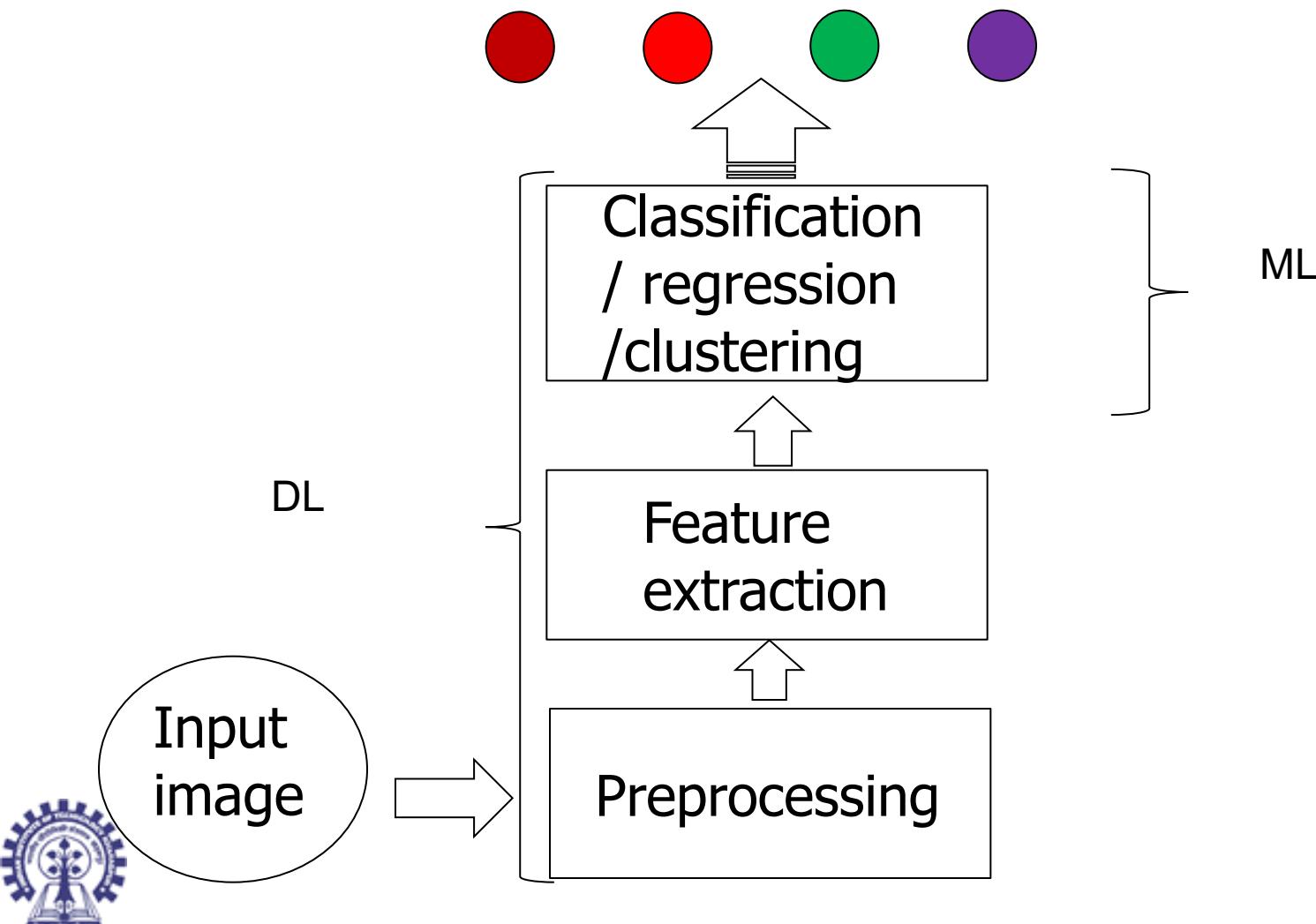
- Characterizes its visual content.
  - A part of an organization on describing a region
  - A higher level organization than a pixel
- A point in a multidimensional space
  - feature vector: n-Dimensional vector:  $x \in \mathbb{R}^n$
- Represents
  - a point within a neighborhood
  - a pattern
  - a patch
  - an object
  - the whole image



# **Role of features in Image Analysis**



# Role of features in learning visual content



# Region descriptors

- Patch descriptors
- Texture descriptors
- Shape descriptors



# Patch Descriptor: Histogram of Gradients (HoG)

- Compute centered horizontal and vertical gradients with no smoothing.
- Compute gradient orientation and magnitudes,
- For color image, pick the color channel with the highest gradient magnitude for each pixel.
- For a 64x128 image, divide the image into 16x16 blocks of 50% overlap.  $\rightarrow 7 \times 15 = 105$  blocks in total.



# Histogram of Gradients (HoG)

- Each block: 2x2 cells with size 8x8.
- Quantize the gradient orientation into 9 bins.
- The vote is the gradient magnitude.
- Interpolate votes between neighboring bin center.
- The vote can also be weighted with Gaussian to down-weight the pixels near the edges of the block.
- Concatenate histograms.
  - Feature dimension:  $105 \times 4 \times 9 = 3,780$



# Object detection with patch descriptors.

- Typical examples:
  - Pedestrian detection
  - Character recognition
- Detection as a classification task.
  - Generate labeled sample feature descriptors.
  - Train a classifier.
    - NN, SVM, Decision Tree, Random Forest .....
  - Label an unknown patch using its descriptor.



# Applications

- Pedestrian Detection



N.Dalal and B. Triggs, Histograms of oriented gradients for human detection, CVPR-2005



# Non-maximal suppression

- Expected to get a high detection score with neighboring overlapping patches.
  - Select the patch with locally maximal score.
- A greedy approach:
  - Select the best scoring window
    - It is expected to cover the target object.
  - Suppress the windows that are too close to the selected window.
  - Search next top-scoring windows out of the rest.

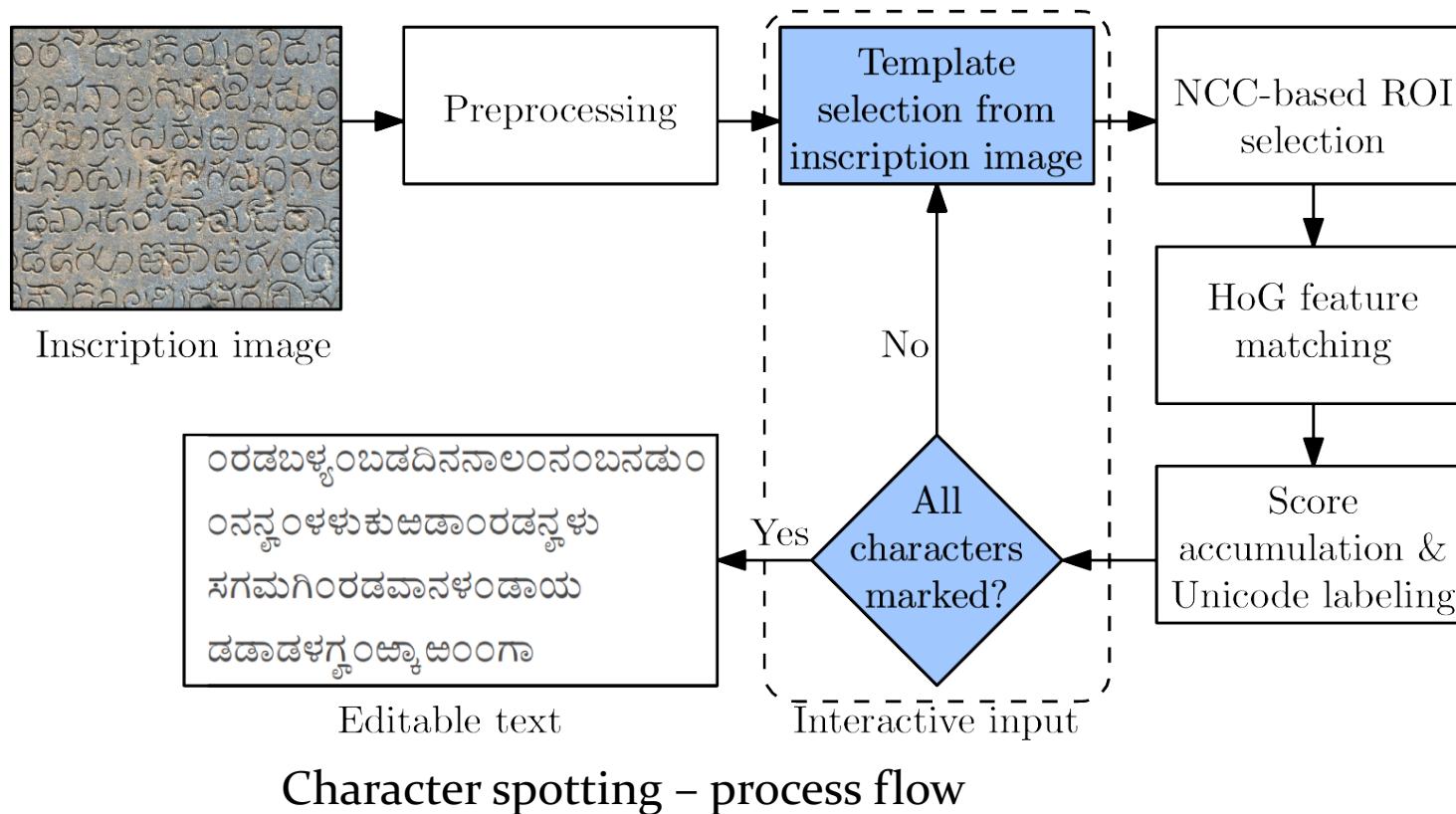


# Character Spotting (E-PURALEKHAK)

- A tool for digital paleography
  - Aids analysis of inscription
  - Converts inscribed substrate to editable text
- Processing pipeline
  - Preprocessing - denoising and normalization ([0 255])
  - Search possible locations of user indicated character by cross correlation
  - Prune the search results by HoG feature matching
  - Parse the Unicode file - editable text

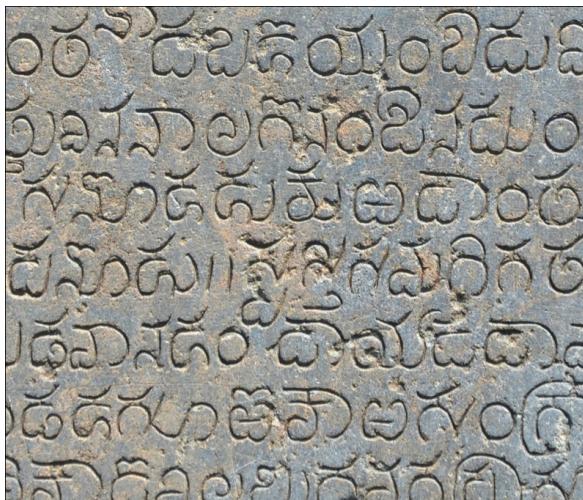


# Character Spotting (E-PURALEKHAK)



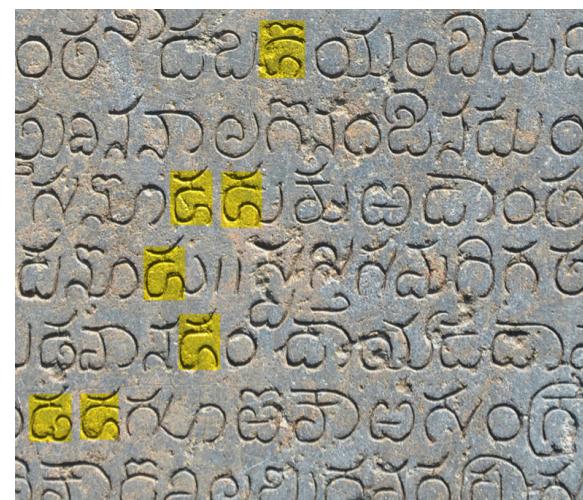
# E-PURALEKHAK

Original  
Inscription  
Image



Unicode  
generation

OCE6	0CB0	0CA1	0CAC	0CB30CCD	0CAF	0CE6	0CAC	0CA1
0CA809CA3	0CA80CBE	0CB2	0CE6	0CAB	0CE6	0CAC	0CA8	0CA10CC1
OCE6	0CA80CCEB	0CB3	0CB30CC1	0C950CC1	0CB1	0CA10CBE	0CE6	0CB
0CA1	0CA80CE1	0CB30CC1	0CB8	0C97	0CAE	0C970CBF	0CE6	0CB
0CA1	0CB50CB	0CA8	0CB3	0CE6	0CA10CBE	0CAF	0CA1	0CA10CBE



Intermediate  
Result

ಂರಡಬಳ್ಳಂಬದದನನಾಲಂನಂಬನಡುಂ  
ಂನನ್ನಂಳಳುಕುಉಡಾಂರಡನ್ನಳು  
ಸಗಮಗಿಂರಡವಾನಳಂಡಾಯ  
ಡಡಾಡಳಗ್ಗಂಟ್ಯಾಂಬಂಗಾ

Editable  
text



# Texture descriptor



- Texture: spatial arrangement of the colors or intensities in an image
  - A quantitative measure of the arrangement of intensities in the region.



# Texture descriptors

- Edge density and direction
- Local Binary Pattern (LBP).
- Co-occurrence Matrix.
- Laws' texture energy features.



# Edge density and direction

- Compute gradient at each pixel.
  - The descriptor: normalized histograms of magnitudes and directions of gradients over a region.
    - $(H_R(\text{mag}), H_R(\text{dir}))$
  - Numbers of bins in histograms kept small (e.g. 10).
  - Use L1 norm between the feature vectors as a distance measure.
- Normalized histogram of magnitudes.
- Normalized histogram of directions.
- Normalized histogram → Area = 1;



# Local Binary Pattern (LBP).

3	2	1
4	<i>c</i>	0
5	6	7

$$b(i) = \begin{cases} 1 & \text{if } (I(i) > I(c)) \\ 0 & \text{Otherwise} \end{cases}$$

$$LBP(c) = \sum_{i=0}^7 b(i)2^i$$

You may have different ordering of neighbors.

- Values range from 0 to 255.
- Obtain normalized histogram over a region.
- Not rotational invariant.
- Invariant to illumination and contrast.



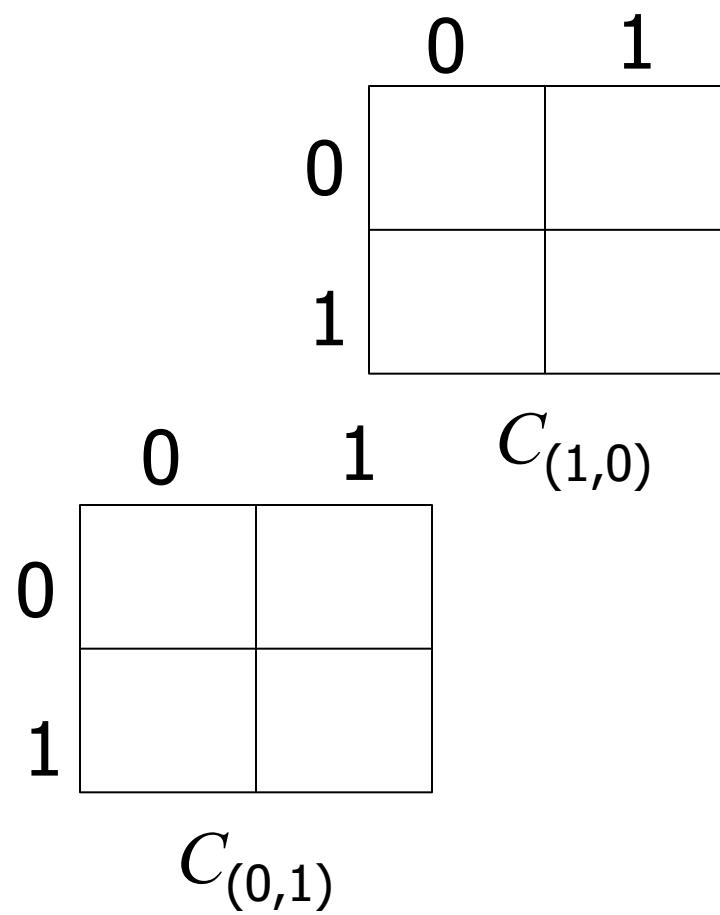
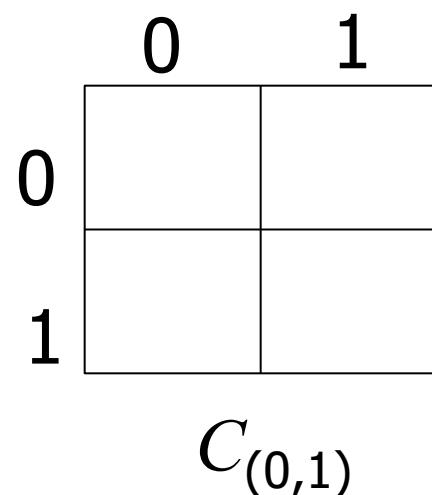
# Co-occurrence Matrix ( $C_r$ )

- $C_r(x,y)$ : How many times elements  $x$  and  $y$  occur at a pair of pixels related spatially (designated by  $r$  in the notation).
  - e.g.  $\mathbf{p} \rightarrow \mathbf{q}$  denotes  $\mathbf{q}$  is shifted from  $\mathbf{p}$  by a translation of  $\mathbf{t}=(a,b)$ , i.e.  $\mathbf{q}=\mathbf{p}+\mathbf{t}$ .
    - $C_{(a,b)}(x,y)$ : Number of cases in an image where  $I(\mathbf{p})=x$  and  $I(\mathbf{p}+\mathbf{t})=y$ .



# Co-occurrence Matrix ( $C_r$ )

0	0	1	1
0	0	1	1
1	1	0	0
1	1	0	0



# Co-occurrence Matrix ( $C_r$ )

0	0	1	1
0	0	1	1
1	1	0	0
1	1	0	0

	0	1
0	4	2
1	2	4

$C_{(0,1)}$

	0	1
0	4	2
1	2	4

$C_{(1,0)}$

	0	1
0	2	2
1	2	3

$C_{(1,1)}$



# Normalized Co-occurrence Matrix ( $N_r$ )

Divide by the sum of frequencies in a matrix.

0	0	1	1
0	0	1	1
1	1	0	0
1	1	0	0

	0	1
0	1/3	1/6
1	1/6	1/3

$C_{(0,1)}$

	0	1
0	1/3	1/6
1	1/6	1/3

$C_{(1,0)}$

	0	1
0	2/9	2/9
1	2/9	1/3

$C_{(1,1)}$



# Symmetric Co-occurrence Matrix ( $S_r$ )

$$S_r(x,y) = C_r(x,y) + C_{-r}(x,y)$$

0	0	1	1
0	0	1	1
1	1	0	0
1	1	0	0

		0	1
0	4+4	2+2	
1	2+2	4+4	

$$C_{(0,1)} + C_{(0,-1)}$$

		0	1	$C_{(1,0)} + C_{(-1,0)}$
0	4+4	2+2	4+4	
1	4+4	2+2	2+2	
		0	1	$C_{(1,1)} + C_{(-1,-1)}$
0	2+2	2+2	2+2	
1	2+2	3+3	3+3	



# Features from Normalized Co-occurrence Matrix

$$Energy = \sum_x \sum_y N_r^2(x, y)$$

$$Entropy = - \sum_x \sum_y N_r(x, y) \log_2 N_r(x, y)$$

$$Contrast = \sum_x \sum_y (x - y)^2 N_r(x, y)$$

$$Homogeneity = \sum_x \sum_y \frac{N_r(x, y)}{1 + |x - y|}$$

$$Correlation = \frac{\sum_x \sum_y N_r(x, y) xy - \mu_x \mu_y}{\sigma_x \sigma_y}$$



# Features from Normalized Co-occurrence Matrix

$$\text{Correlation} = \frac{\sum_x \sum_y (x - \mu_x) (y - \mu_y) N_r(x, y)}{\sigma_x \sigma_y}$$

Mean and s.d.  
of row sums      Mean and s.d.  
of column sums

$$f(x) = \sum_y N_r(x, y)$$
$$g(y) = \sum_x N_r(x, y)$$



# Laws' texture energy features

- A set of 9 5x5 masks used to compute texture energy.

L5 (Level):  $[1 \ 4 \ 6 \ 4 \ 1]^T$

E5 (Edge):  $[-1 \ -2 \ 0 \ 2 \ 1]^T$

S5 (Spot):  $[-1 \ 0 \ 2 \ 0 \ -1]^T$

R5 (ripple):  $[1 \ -4 \ 6 \ -4 \ 1]^T$

Computation with mask:  
**Convolution**

A mask: Outer  
product of any pair.  
e.g. E5L5:  $E5 \cdot L5^T$

$$\begin{bmatrix} -1 \\ -2 \\ 0 \\ 2 \\ 1 \end{bmatrix} \cdot [1 \ 4 \ 6 \ 4 \ 1]$$



# Laws' texture energy features

- A set of 9 5x5 masks used to compute texture energy.

L5 (Level):  $[1 \ 4 \ 6 \ 4 \ 1]^T$

E5 (Edge):  $[-1 \ -2 \ 0 \ 2 \ 1]^T$

S5 (Spot):  $[-1 \ 0 \ 2 \ 0 \ -1]^T$

R5 (ripple):  $[1 \ -4 \ 6 \ -4 \ 1]^T$

Take  
average of  
responses  
of two  
masks.

L5E5 and E5L5

L5R5 and R5L5

E5S5 and S5E5

L5S5 and S5L5

E5R5 and R5E5

S5R5 and R5S5

S5S5

R5R5

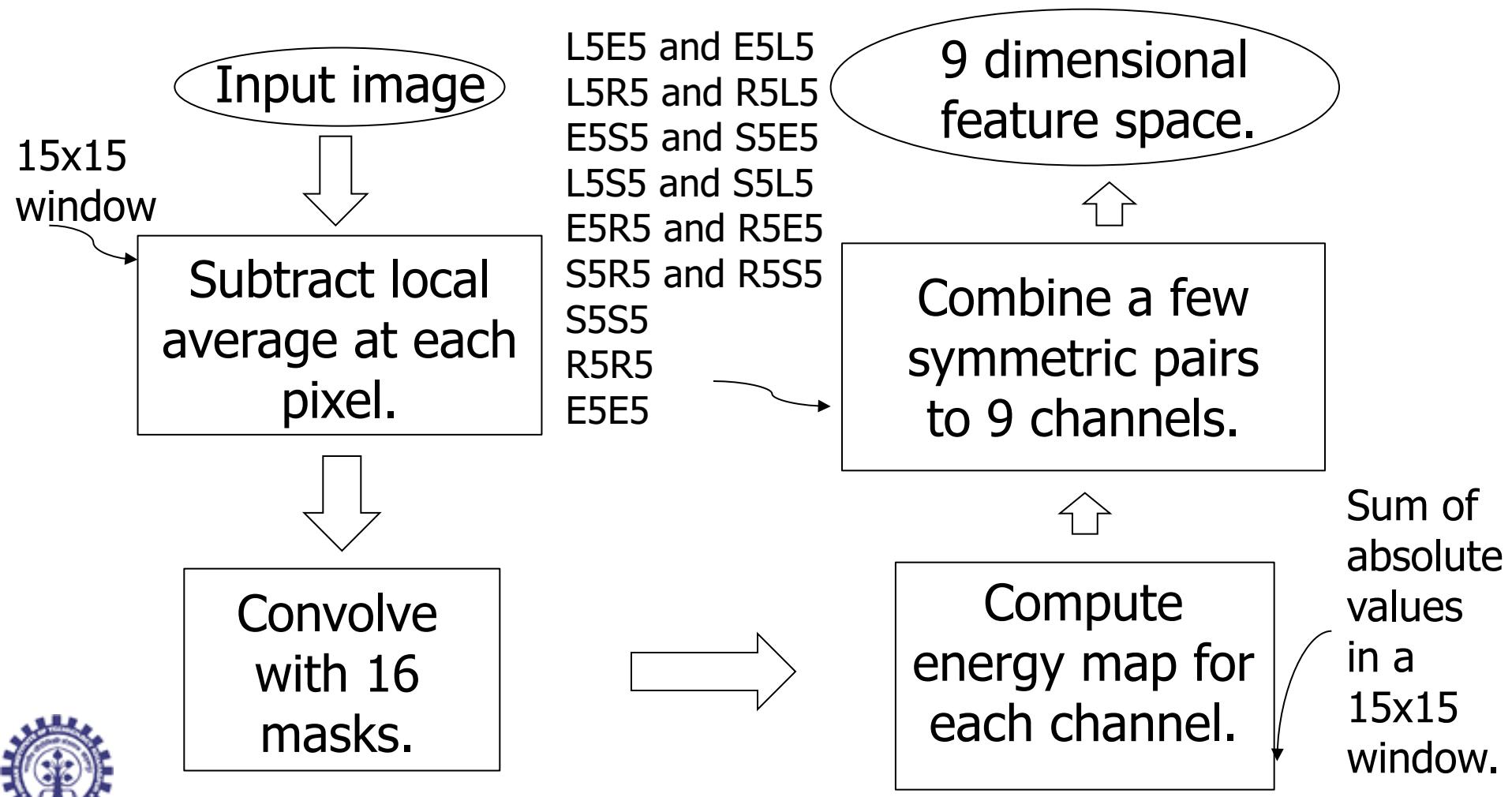
E5E5

16 such masks possible.

Combine a few pairs to make 9 masks.



# Laws' texture energy



# Use of texture descriptors

- Detection of object patches represented by textured patterns.
- Segmentation of images.
- Classification / Matching
  - Generate a Library of labelled feature descriptors.
  - Detection of classes (class labels).
    - Matching to the nearest texture descriptor.



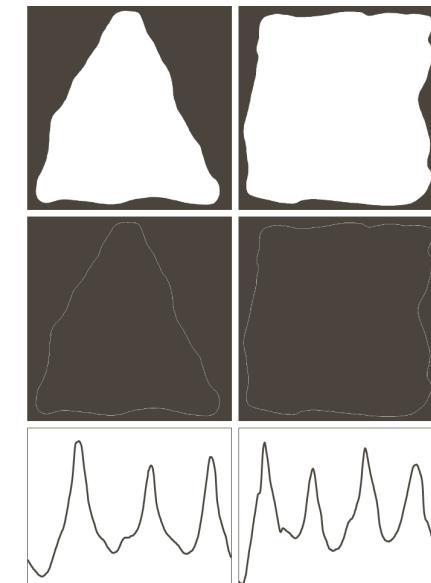
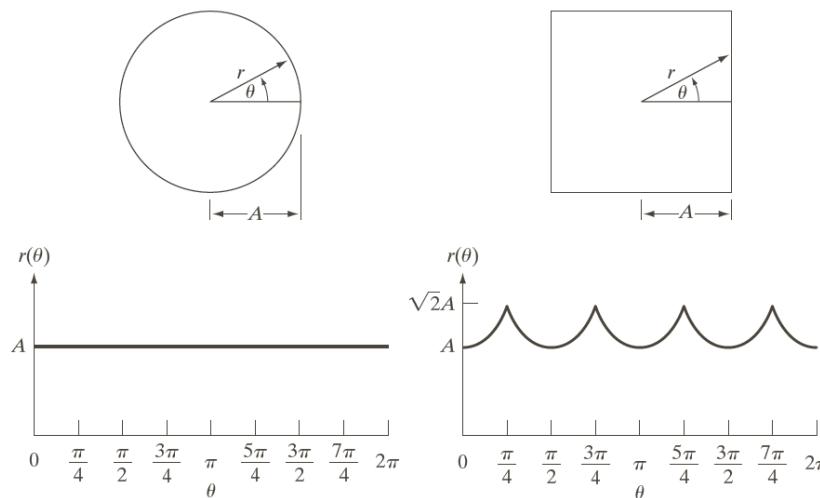
# 2-D shape descriptors

- Signature of a contour
- Slope density function
- Features of boundary segments



# Signature of a contour

- For a signature convert a 2-D boundary into a representative 1-D function
- Plot the distance of the boundary from the centroid as a function of angle



# Signature: Transformation Dependency

- Invariant to location, but will depend on rotation and scaling.
- Starting at the point farthest from the reference point or using the major axis of the region can be used to decrease dependence on rotation.
- Scale invariance can be achieved by either scaling the signature function to fixed amplitude or by dividing the function values by the standard deviation of the function.



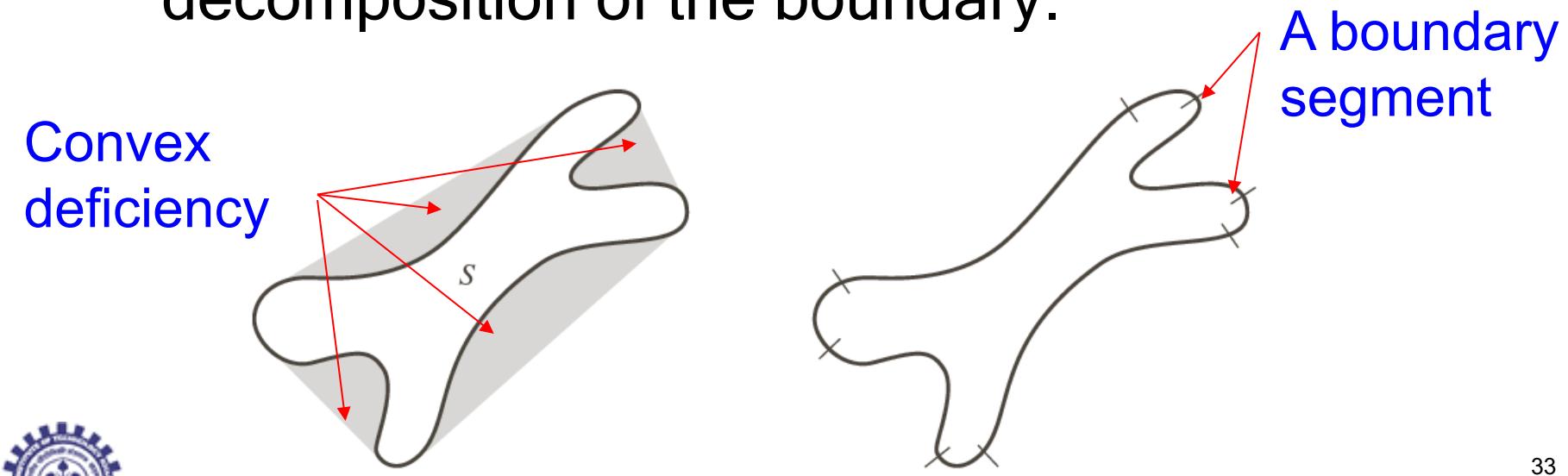
# Contour representation: Slope density function

- Histogram of the slopes (tangent angles) along a contour.
- Orientation of the object can be determined using correlation of slope histograms of model contour with that of an image contour.
- Can be very useful for object recognition.



# Boundary Segments

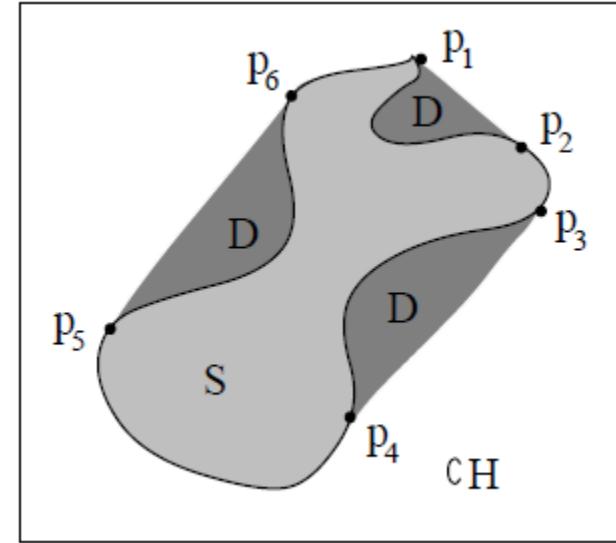
- Boundary segments: decompose a boundary into segments.
- Use of the convex hull of the region enclosed by the boundary is a powerful tool for robust decomposition of the boundary.



# Boundary Segments

- The Convex Hull (CH)  $H$  of a set  $S$  is defined as the smallest convex set that contains  $S$
- We define the set of Convex Deficiencies (CD)
  - $D = H \setminus S$ .
- Follow the boundary and mark the points at which transition is made into or out of a component of  $D$

Apply polygon approximation to find CH.



Small irregularities lead to tiny meaningless convex deficiency components scattered all along the boundary.



# Region characterization

- CH and CD are useful for entire regions
  - Area of the region
  - Area of its convex deficiency
  - Number of components of convex deficiency
  - Relative location of the components of CD



# Learning classes from images: Supervised learning

Supervised learning: exploits knowledge about the classification problem, such as example instances of classes.

- Choice of features for discriminating classes.
- Extracting features from images.
- Form training and test data set with class labels.
- Train classifiers and evaluate performance.
- Use for classification of unknown samples.



# Classification problem

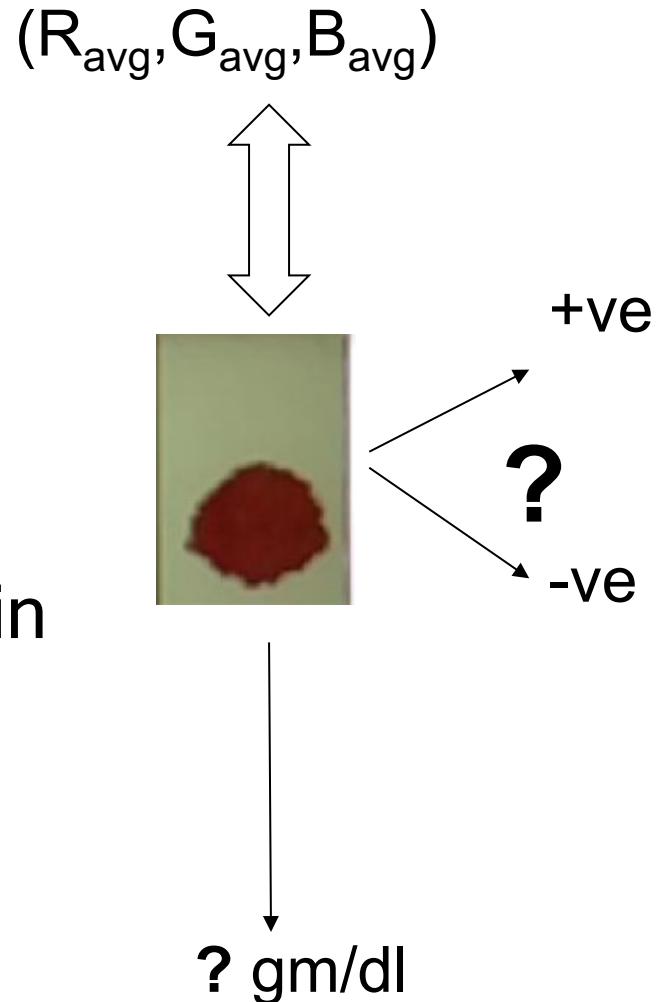
- Given a labelled data sets:  
 $\{(y_i, x_i)\}$ ,  $i=1, 2, \dots, n$  such that  $x_i$  in  $R^n$  and  $y_i$  is the class of  $x_i$ , an element of the finite set of classes.
  - $y_i$  could be +1 or -1 for a two class problem.

Design a classifier  $C$  which assigns class  $y_i$  (output) to  $x_i$  (input).



# Typical examples

- Given images of blood samples determine whether a patient is anemic or not.
  - Classification Problem
- Given images of blood samples determine amount of hemoglobin concentration (in gm/dl).
  - Regression problem.



# Classification approaches

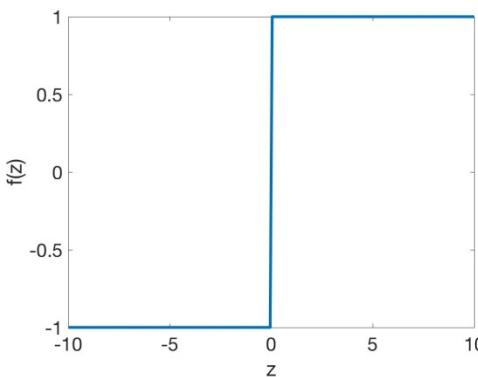
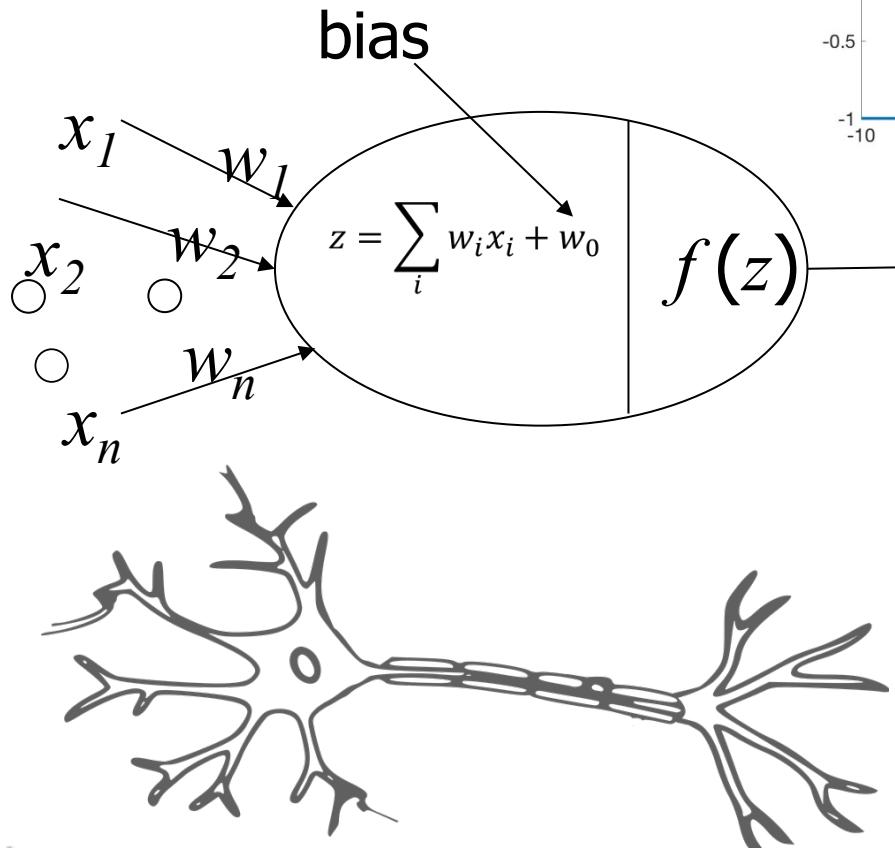
Classification:

Task of assigning a known category or class to an object.

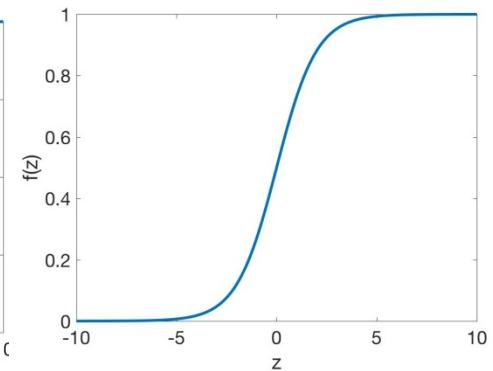
- Probabilistic
  - Bayesian classification
- Distance based
  - K-Nearest neighbor
- Discriminant analysis
  - Linear discriminant analysis (LDA)
- Artificial neural network (ANN)
  - Feed-forward neural network.



# Perceptron modelling a neuron



Signum



Logistic / Sigmoid

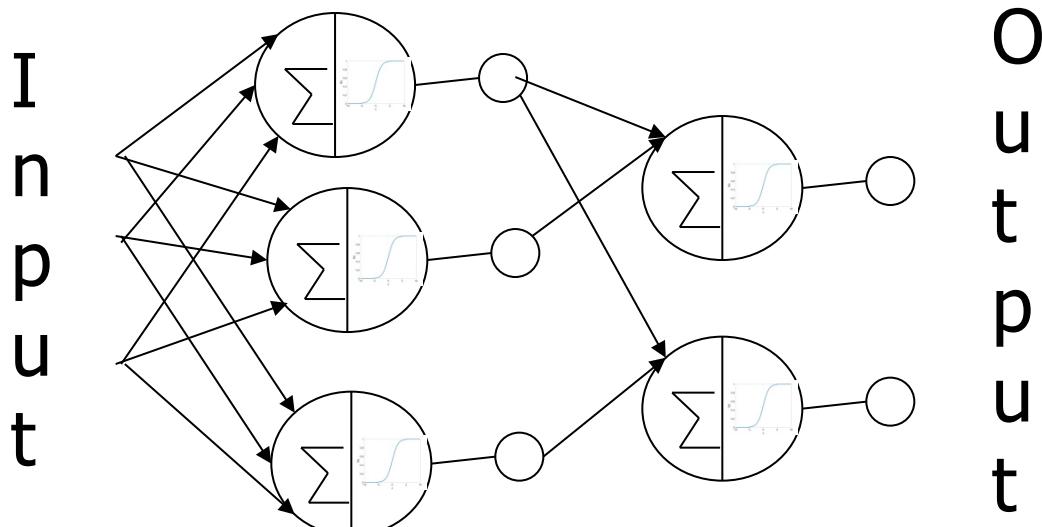
$$f(z) = \frac{1}{1 + e^{-z}}$$

A network of perceptrons provides a powerful model describing input / output relations.



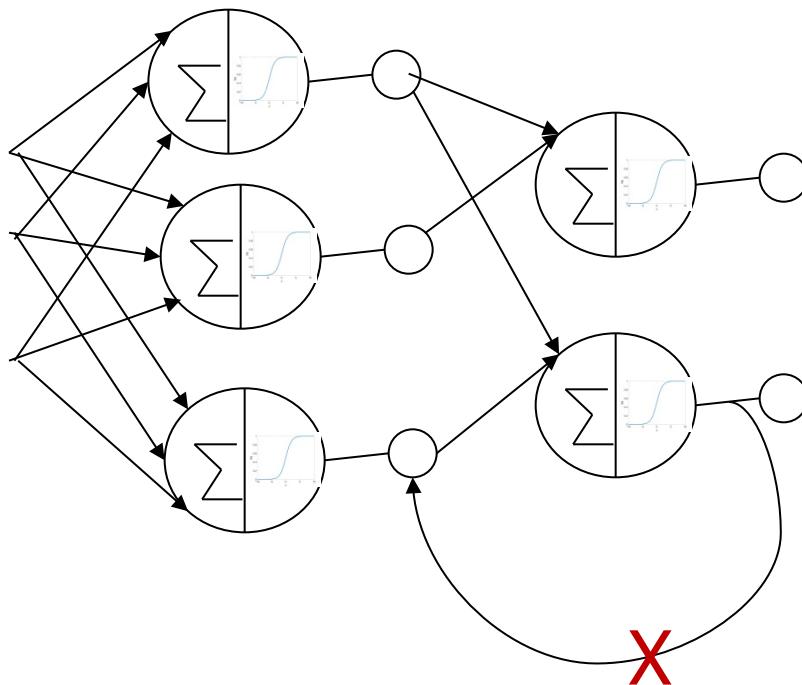
# Artificial Neural Network

- A network of perceptrons.
  - Input: A vector
  - Output: A vector / A scalar

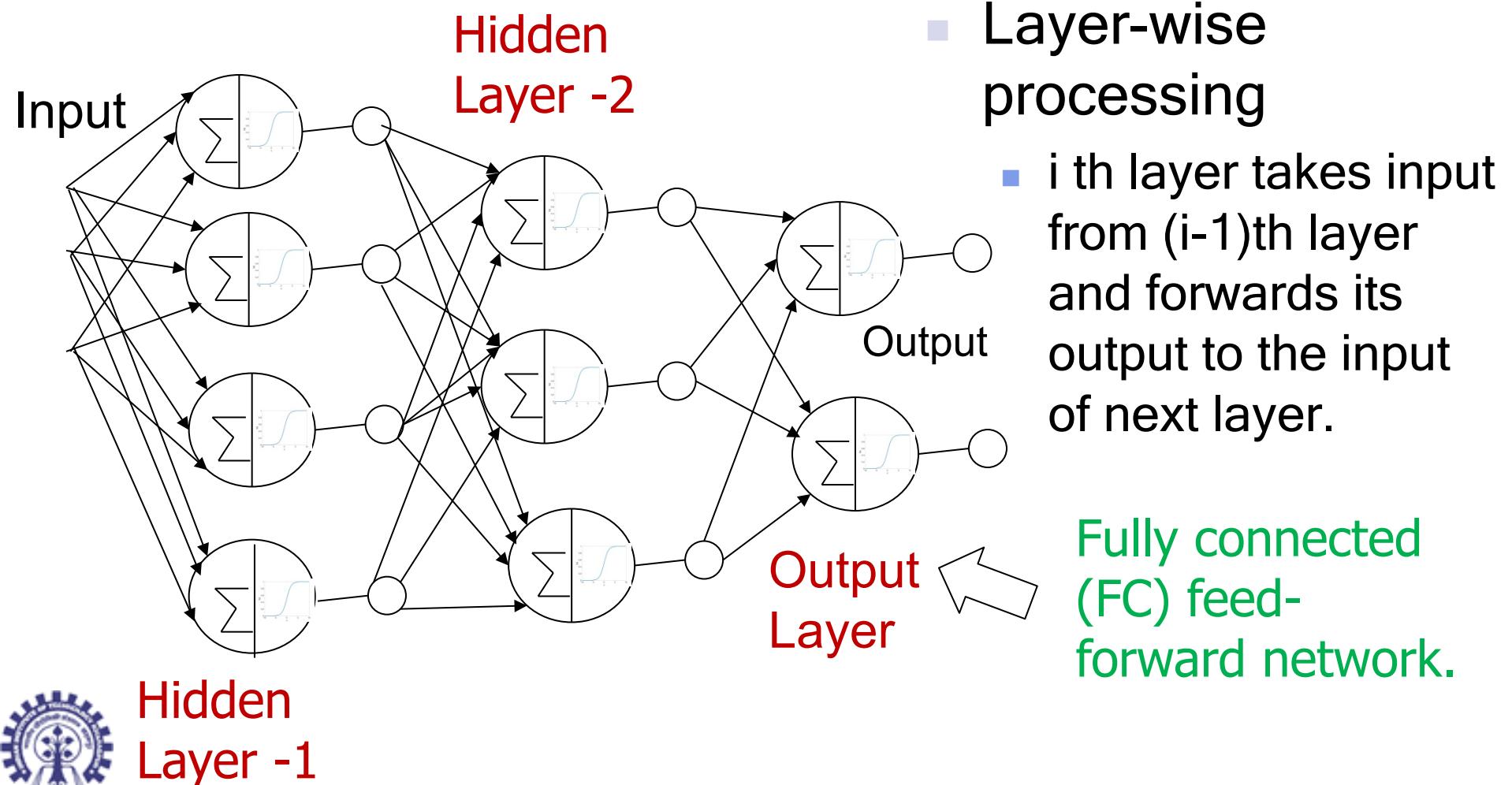


# Feed-forward Network

- No feed back or loop in the network.



# Multilayered feed-forward Network



Hidden  
Layer -1

# Mathematical description of the model

- Let  $j$  th neuron of  $i$  th layer be  $ne_j^{(i)}$ .
- Its corresponding weights
  - $W_j^{(i)} = (w_{j1}^{(i)}, w_{j2}^{(i)}, \dots, w_{jn_{(i-1)}}^{(i)})$
  - Bias:  $w_{j0}^{(i)}$
  - $n_{(i-1)}$ : Dimension of input to the neuron
  - $n_i$ : Dimension of output at  $i$  th layer
- Output of the neuron:

$$y_j^{(i)} = f\left(W_j^{(i)T} X^{(i-1)} + w_{j0}^{(i)}\right)$$



# Mathematical description of the model

- Output of j th neuron in i th layer:

$$y_j^{(i)} = f\left(W_j^{(i)T} X^{(i-1)} + w_{j0}^{(i)}\right)$$

- Input output relation in i th layer

$$Z^{(i)} = \begin{bmatrix} W_1^{(i)T} \\ W_2^{(i)T} \\ \vdots \\ W_{n\_i}^{(i)T} \end{bmatrix} X^{(i-1)} + \begin{bmatrix} w_{10}^{(i)} \\ w_{20}^{(i)} \\ \vdots \\ w_{n\_i0}^{(i)} \end{bmatrix}$$

$\mathbf{W}^{(i)}$    $\mathbf{b}^{(i)}$  



# Input output relation

- Output of j th neuron in i th layer:

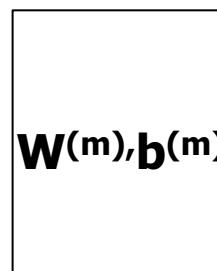
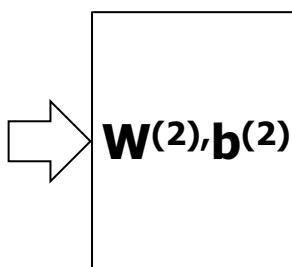
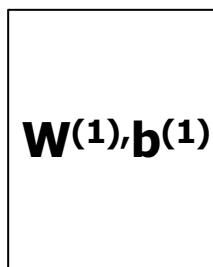
$$y_j^{(i)} = f\left(W_j^{(i)T} X^{(i-1)} + w_{j0}^{(i)}\right)$$

- Input output relation in i th layer

$$Z^{(i)} = \begin{bmatrix} W_1^{(i)T} \\ W_2^{(i)T} \\ \vdots \\ W_{n,j}^{(i)T} \end{bmatrix} X^{(i-1)} + \begin{bmatrix} w_{10}^{(i)} \\ w_{20}^{(i)} \\ \vdots \\ w_{n,j0}^{(i)} \end{bmatrix}$$

$$Y^{(i)} = f(W^{(i)}X^{(i-1)} + b^{(i)}) \equiv \begin{bmatrix} y_1^{(i)} \\ y_2^{(i)} \\ \vdots \\ y_{n,j}^{(i)} \end{bmatrix}$$

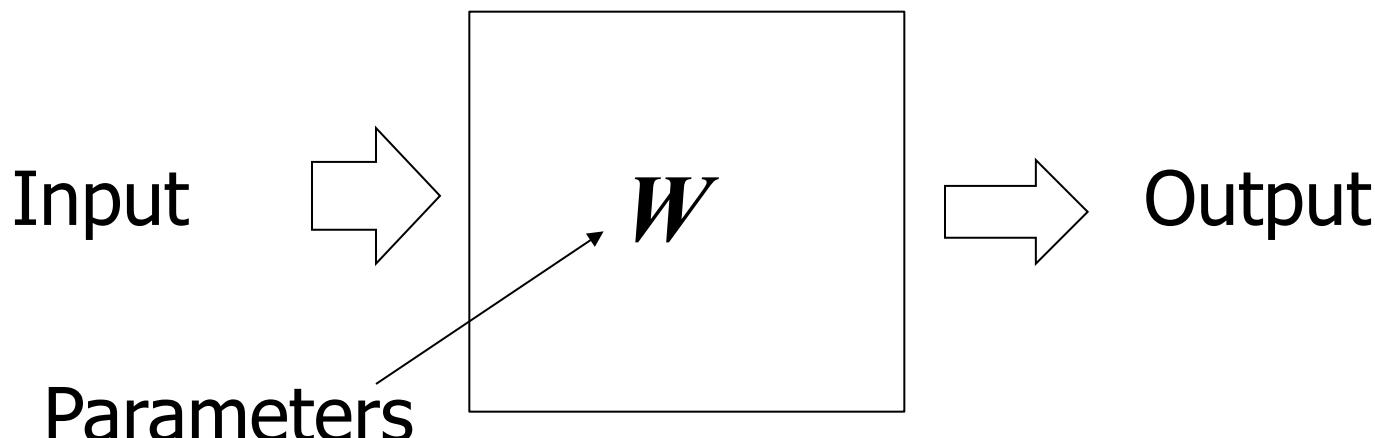
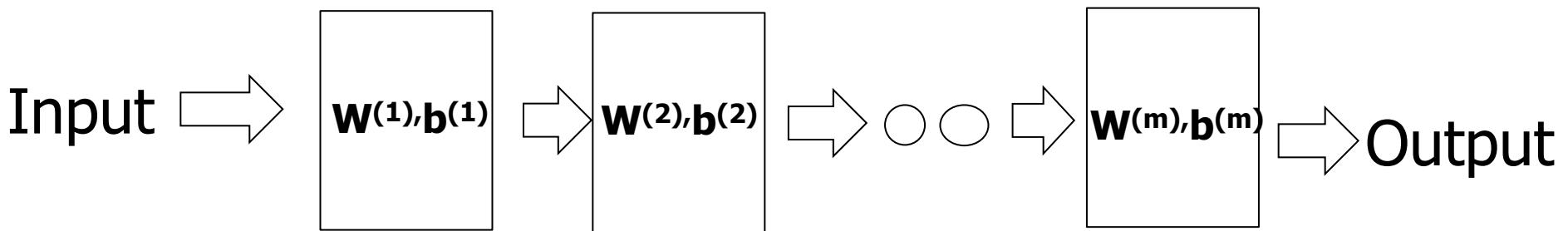
Input



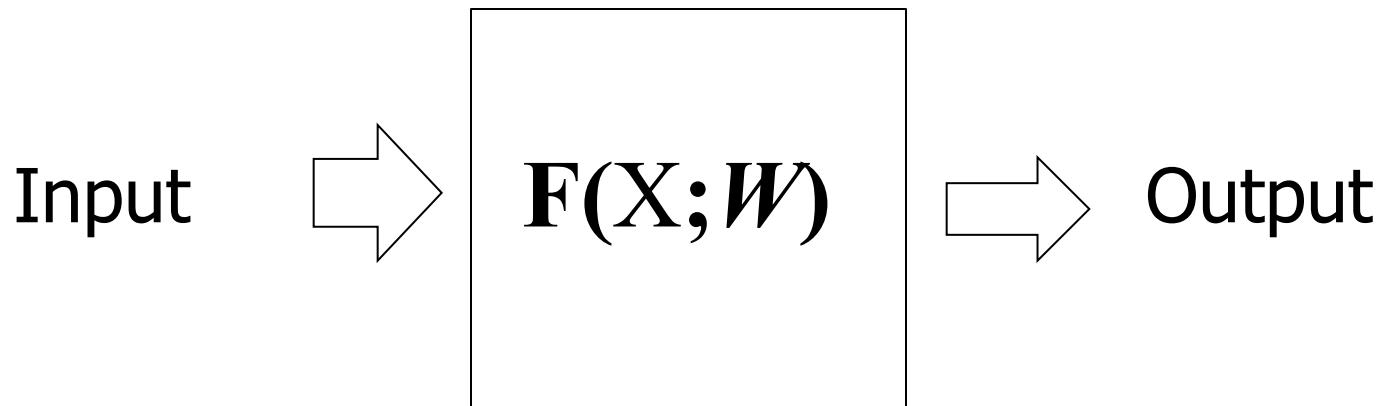
Output

# Input output relation

$$Y^{(i-1)} = f(W^{(i)}X^{(i-1)} + b^{(i)})$$



# Optimization problem



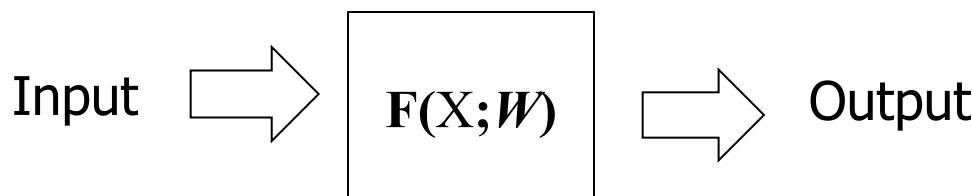
Given  $\{(X_i, O_i)\}$ ,  $i=1,2,\dots,N$ , find  $W$  such that it produces  $O_i$  given input  $X_i$  for all  $i$ .

Minimize:  $J_n(W) = \frac{1}{N} \sum_{i=1}^N \|O_i - F(X_i; W)\|^2$



Apply the same gradient descent procedure to obtain the solution.

# Optimization problem



Minimize:  $J_n(W) = \frac{1}{N} \sum_{i=1}^N \|O_i - F(X_i; W)\|^2$

1. Start with an initial  $W_0$ .
2. Update  $W$  iteratively.

$$W_i = W_{i-1} + \eta(i) \sum_k (O_k - F(X_k; W_{i-1})) \nabla F(X_k; W_{i-1})$$

Stochastic gradient descent:

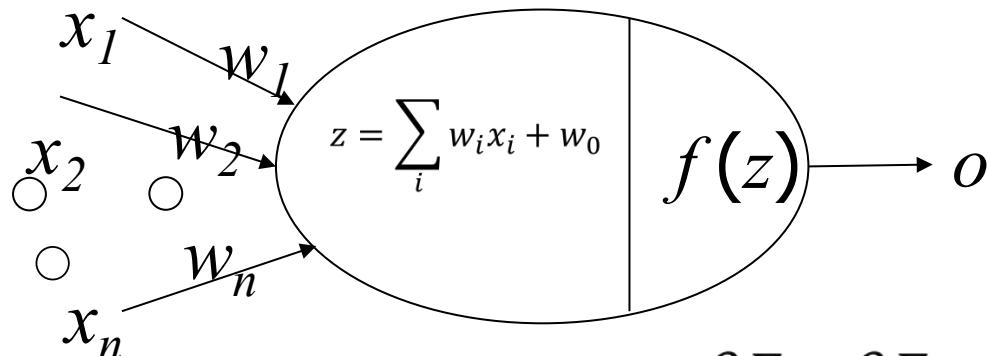
$$W_i = W_{i-1} + \eta(i)(O_k - F(X_k; W_{i-1})) \nabla F(X_k; W_{i-1})$$

Training samples:  
 $\{(X_i, O_i)\}, i=1,2,\dots,N$

Apply the same  
gradient descent  
procedure to obtain  
the solution.



# Chain rule of computing gradient of a single neuron



Target response:  $t$

Error:  
 $E = (t - o)^2$

$$\frac{\partial E}{\partial w_i} = \frac{\partial E}{\partial o} \frac{\partial o}{\partial z} \frac{\partial z}{\partial w_i}$$

$$\frac{\partial E}{\partial w_i} = -2(t - o)f'(z)x_i$$

$$\nabla(W) = \left( \frac{\partial E}{\partial w_0}, \frac{\partial E}{\partial w_1}, \dots, \frac{\partial E}{\partial w_n} \right)$$

$$f(z) = \frac{1}{1 + e^{-z}}$$

$$f'(z) = \frac{e^{-z}}{(1 + e^{-z})^2}$$

Analytical method!  
 Computed given the functional values.

$$\frac{\partial E}{\partial x_i} = \frac{\partial E}{\partial o} \frac{\partial o}{\partial z} \frac{\partial z}{\partial x_i}$$

$$\frac{\partial E}{\partial x_i} = -2(t - o)f'(z)w_i$$

$$\frac{1}{1 + e^{-z}} \left( 1 - \frac{1}{1 + e^{-z}} \right)$$

$$f(z)(1 - f(z))$$



# Computing gradient: Back propagation method

- For multi-layered feed forward network.
- Apply chain rule.
  - From output to toward input.
  - From output layer to toward input layer.
  - Compute partial derivatives of weights at  $(i-1)$ th layer from the  $i$  th layer.



# ANN training

- Initialize  $W^{(0)}$  .
- For each training sample  $(x_i, o_i)$  do
  - Compute functional values of each neuron in the forward pass.
  - Update weights of each link starting from the output layer using back propagation.
  - Continue till it converges.

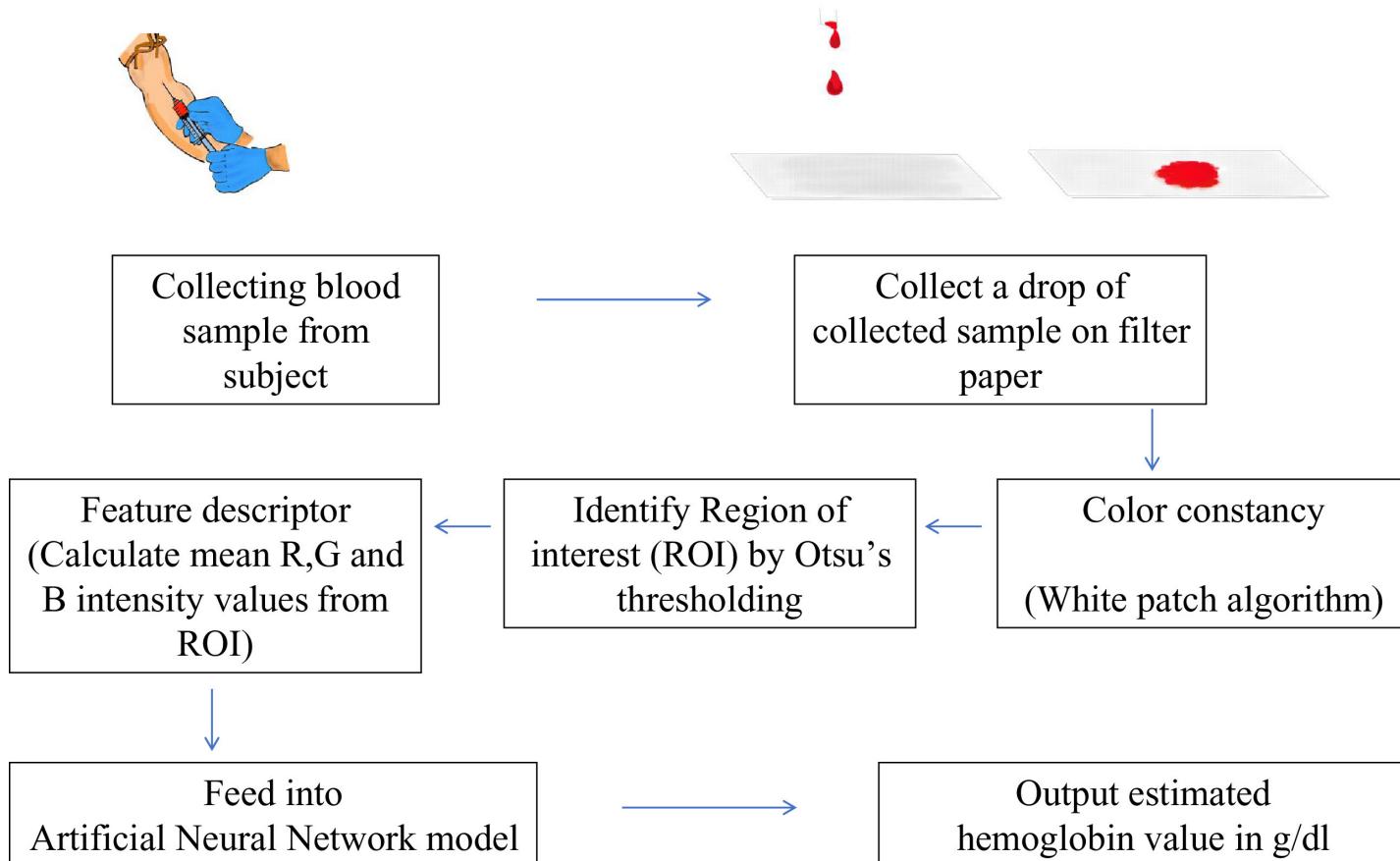


# Classification or regression?

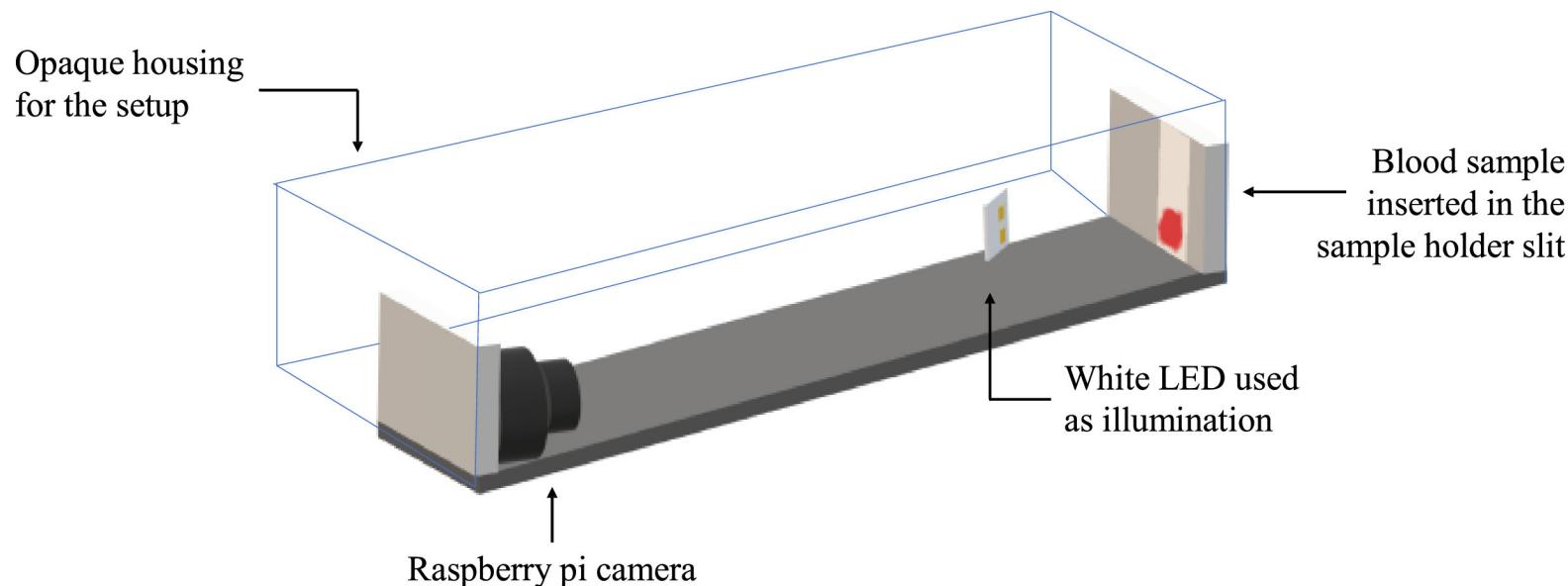
- Primarily a regressor.
  - Build a model to predict functional value  $F(x)$  given input  $x$ .
- Can be converted to a classifier by appropriate encoding of classes (output vector  $o$ ).
  - Two class problem
    - Binary encoding: 0 / 1
    - One hot encoding: (1 0) / (0 1)



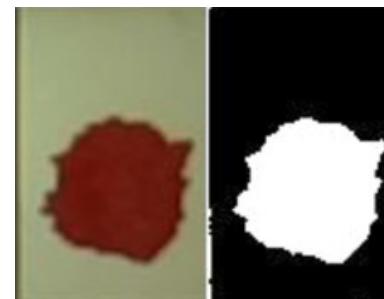
# Measuring hemoglobin from images of blood sample



# Measuring hemoglobin from images of blood sample



Color correction

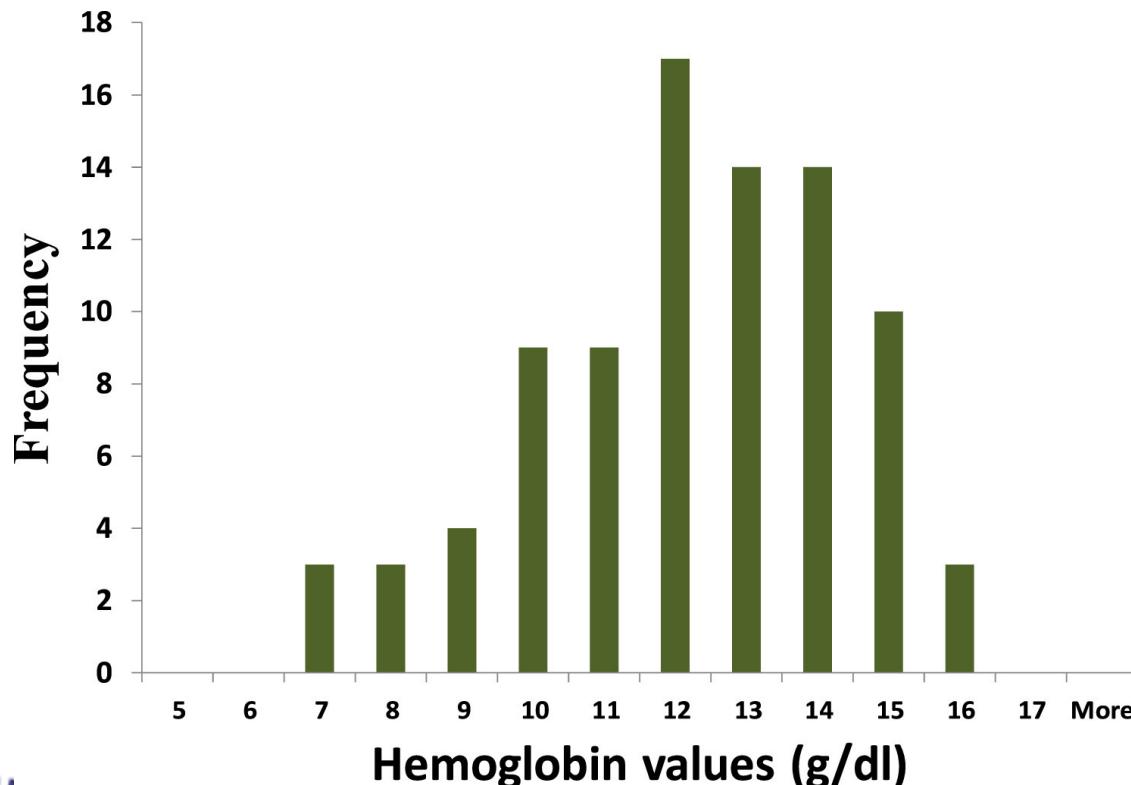


ROI



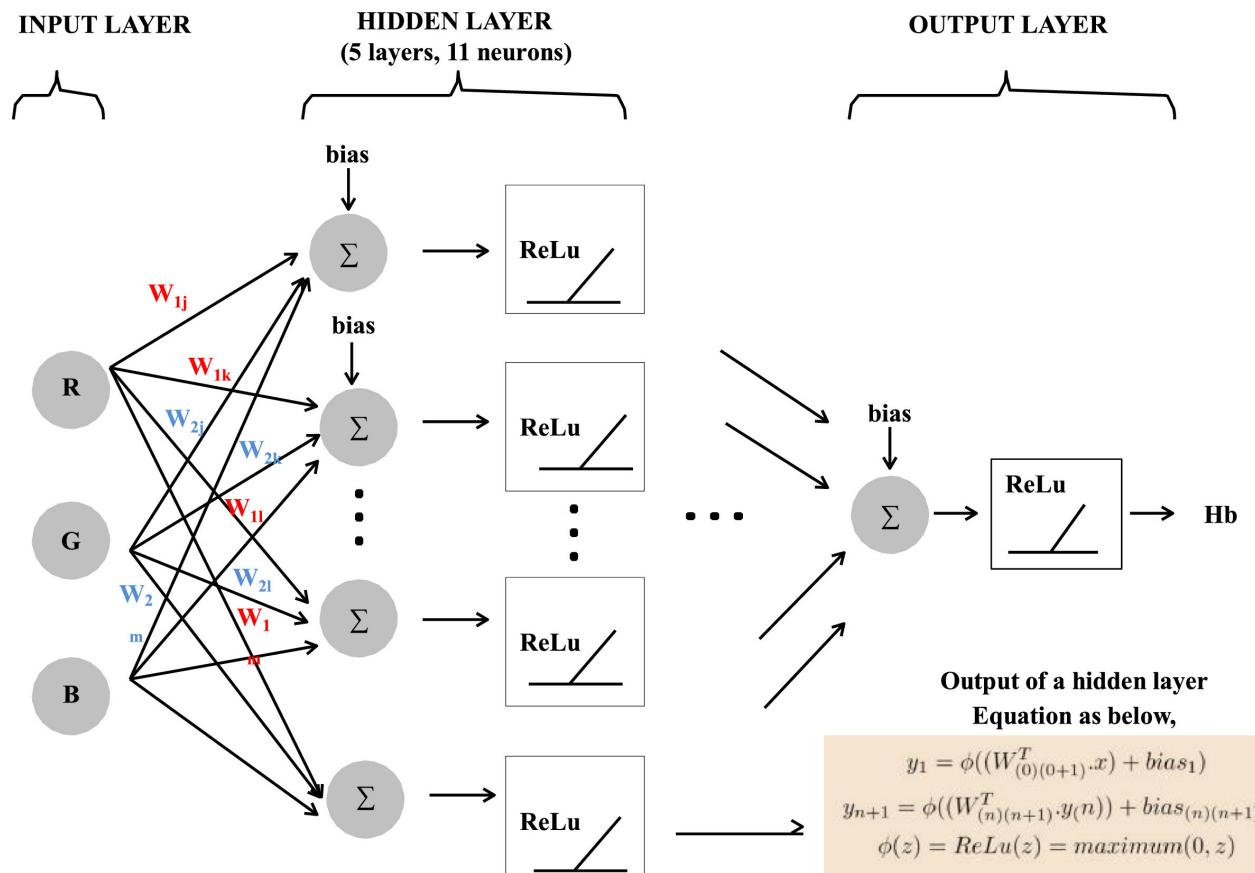
# Measuring hemoglobin from images of blood sample

Data Set: 344 Blood Samples of 86 patients



A. Ghosh, et al, A Low-Cost Test for Anemia Using an Artificial Neural Network,  
Computer Methods and Programs in Biomedicine, Volume 229, 2023, 107251.

# Measuring hemoglobin from images of blood sample

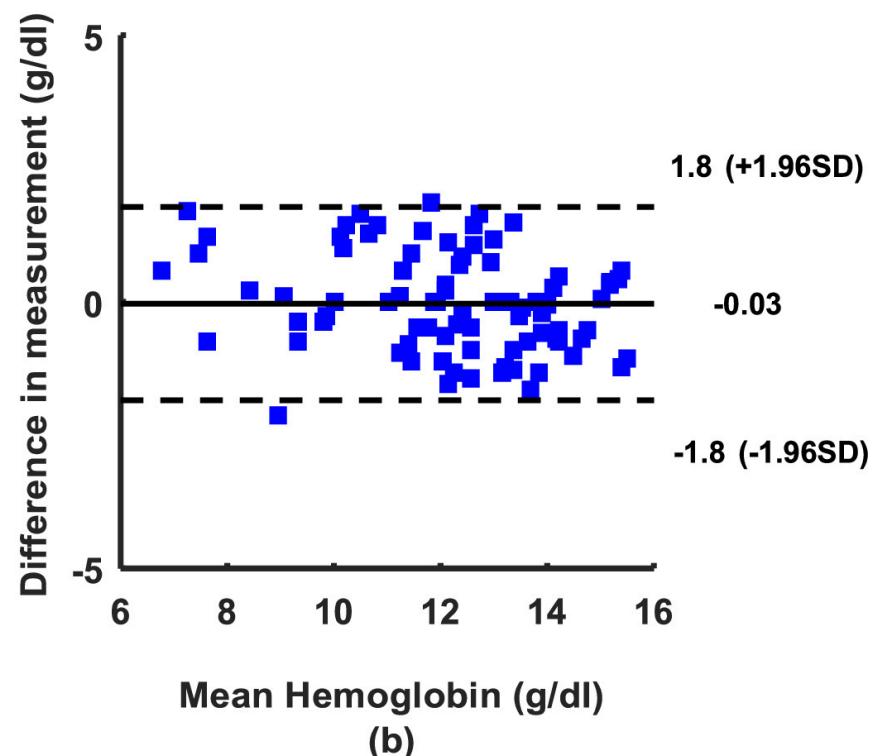
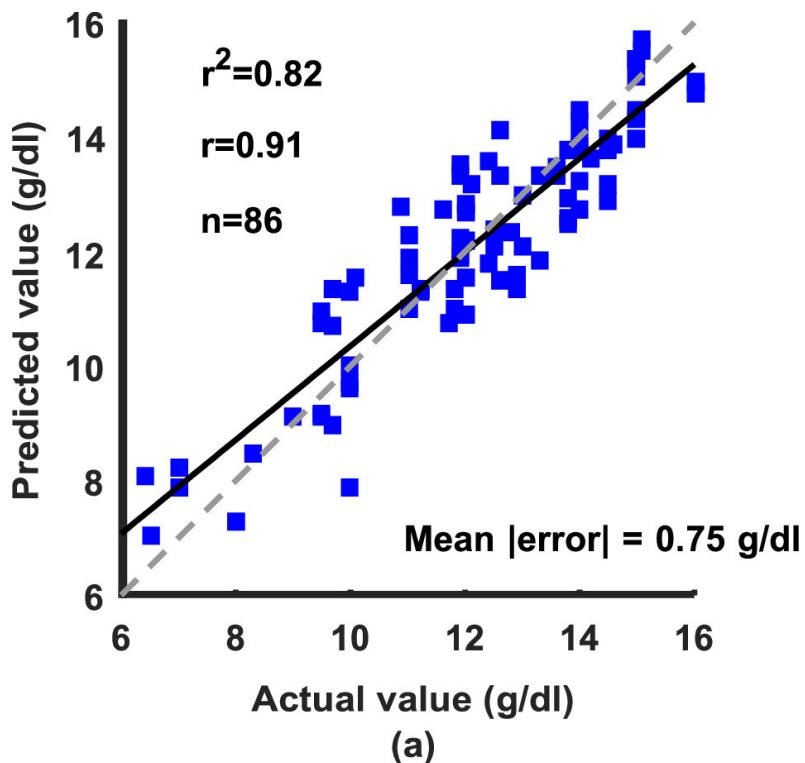


# Performance Analysis

- Multiple observations of the same subject.
  - Reporting mean and Standard Error.
    - S.E. : Estimate of the standard deviation
  - Avg. S.E.: 0.22g/dl (range: 0.02 - 0.75 gm/dl)
- Mean absolute error with respect to the gold standard cyanmethemoglobin measured hemoglobin values: 0.75 gm/dl
  - Correlation Coefficient: 0.91
  - Coefficient of regression: 0.82



# Performance



# Evaluation of a classifier

- Two class problems.

- Positive class and Negative class
- TP: Set of +ve samples predicted +ve.
- FP: Set of -ve samples predicted +ve.
- TN: Set of -ve samples predicted -ve.
- FN: Set of -ve samples predicted +ve.

AP	AN	
PP	TP	FP
PN	FN	TN

Accuracy:  
 $(TP+TN)/\text{Total}$

Precision:  $TP/PP$   
Recall:  $TP/AP$

Sensitivity / Recall:

$TPR = TP/AP$

Specificity:

$TNR = TN/AN$

F-Score:

Harmonic mean of precision and recall

$$F = \frac{2}{\frac{1}{\text{Prec}} + \frac{1}{\text{Recall}}} = \frac{2 \times \text{Prec} \times \text{Recall}}{\text{Prec} + \text{Recall}}$$



# Anemia detection

- Normal Population (< 12.5 gm/dl)
  - Sensitivity: 82% Specificity: 80%
- Pregnant Women (< 11 gm/dl)
  - Sensitivity: 89.4% Specificity: 94%
- On a validation data set of another 64 volunteers
  - MAE: 0.78 gm/dl w.r.t. the gold standard measurement
    - Correlation Coefficient= 0.88, Coefficient of regression = 0.77
  - Normal Population: Sensitivity: 92% Specificity: 85%
  - Pregnant Women: Sensitivity: 93% Specificity: 78%



# Summary of Techniques

- Region and texture descriptors.
  - HoG
  - Edge density
  - LBP
  - Co-occurrence matrix
  - Laws' texture energy
- Features used for
  - classifying objects
  - estimating parameters / various measures.
  - Clustering
- Use of ANN for classification and regression



# Thank You

