INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Time 3 Hrs

Date **Full Marks:** 100

Autumn Semester, 2023-24 **UG students**

Sub No. CS40019 Sub. Image Processing

No. of Students 56

Section A (All questions)

1. Name the geometric entities sufficient to define parallel and perspective projections, respectively from a 3D real space to a 2D real space. How many independent parameters are required to uniquely define each of these projections. 3+2

2. Consider the following transformation matrix of color spaces (from RGB to XYZ)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.49 & 0.31 & 0.2 \\ 0.18 & .81 & .01 \\ 0 & 0.01 & .99 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

   Answer the following.

   (a) Convert a color vector $(60, 50, 20)$ in the RGB space to a vector in the normalized $x - y$ chromaticity space. 2

   (b) Explain how hue can be measured of the above color vector. 1

   (c) Suppose you have another color vector $(50, 60, 40)$. Which one of these two vectors is more reddish? Justify. 2

3. Consider the octagonal digital distance defined by the periodic neighborhood sequences $\{1, 1, 2\}$, where in the sequence 1 denotes type-1 (or 4-Neighbor) neighborhood and 2 denotes type-2 (or 8-Neighbor) neighborhood. Given a radius of 12 compute the perimeter of the convex polygon formed by the vertices of the disc of this distance function. 5

4. Given a sequence $x(n), n = 0, 1, 2, .., N - 1$, of length N define its Discrete Fourier Transform (DFT). Define the corresponding Inverse

Discrete Fourier Transform (IDFT). Suppose the sequence is circularly shifted by 3 samples, derive the ratio of the kth transform coefficient of the modified sequence with that of $x(n)$ 2+2+2

5. Derive the mask of size 3×3 implementing the Laplacian operator, where the gradients along horizontal and vertical directions are weighed $\sqrt{2}$ times with respect to those in the diagonal directions. Compute the Laplacian operated value at the central pixel (having the value 78 and shown underlined) of 3×3 image block as given in the following. 4+2

85	80	100
140	<u>78</u>	46
130	27	68

6. Suppose that an image $g(x, y)$ of size $M \times N$ has been captured under degradation by a linear filter whose Fourier spectrum is given by $H(u, v), 0 \leq u \leq M - 1, 0 \leq v \leq N - 1$. Express the power spectra of the undegraded image as a function of Fourier Spectra of $g(x, y)$ and the filter $H(u, v)$. Supposing that the system is further contaminated by a random additive noise with the power spectra of $S_\eta(u, v)$, how this relationship would be modified. 3+2

7. Compute the mean shift at the central pixel (shown underlined) assuming that the kernel is a truncated Gaussian function (zero mean and s.d.=1 along both horizontal and vertical directions) with width 2 following the computation of mean shift segmentation algorithm. 8

60	110	12	110	60
50	100	11	100	50
40	90	<u>10</u>	90	40
30	80	9	80	30
20	70	8	70	20

Section B (Any three)

8. Consider an image of size 4×4 represented in the form of a 2-D matrix A. The coordinates of pixels of the image are given by respective row and column numbers of the matrix. The row index of the first row starts with 1 and similarly the column index also starts from 1.

$$A = \begin{bmatrix} 50 & 50 & 51 & 50 \\ 51 & 58 & 58 & 70 \\ 50 & 70 & 60 & 60 \\ 50 & 58 & 51 & 68 \end{bmatrix}$$

Answer the following:

(a) Compute the local binary pattern (LBP) at $A(3, 2)$ assuming the scanning order of neighboring pixels anticlockwise and starts with the leftmost neighbor. Write the algorithm to compute the pattern. 2+3

(b) Explain how a feature vector could be formed to represent the above image using LBPs. State two transformations (with justification) for which this feature vector is invariant. 3+2

(c) Considering formation of the co-occurrence matrix from the above image, where two pixels are at relative offset of a vector $(1, -1)$. List the non-zero bins of the 2-D histogram of the co-occurrence matrix with their frequency of occurrences. 5

(d) Explain how a feature vector could be formed exploiting the statistics of the co-occurrence matrix for representing the image. 5

9. (a) Given a set of N complex basis vectors $B = b_1, b_2, \ldots, b_N$ (called *base*) for linear expansion of a vector of dimension N, form the forward transformation matrix and answer the following. 2

   i. State the .properties related to the basis vectors that required to be satisfied for the the following cases. 5×2

      A. The base is orthogonal.

      B. The base is complete.

      C. The base is non-orthogonal but complete.

      D. The base is orthonormal.

      E. The transformation matrix is invertible.

ii. Prove that, B is complete if B is orthogonal. 2

iii. Explain how computation of inverse transforms gets simplifid if B is orthonormal. 2

(b) Explain how DC coefficients of blocks of images in JPEG compression standard are computed and encoded. 1+3

10. (a) Consider the problem of separating foreground from background in an image and modeling the probability of a pixel value as a mixture of Gaussians, each expressing class likelihood of a pixel value. Formulate the segmentation problem as a Bayesian classification problem. How the parameters of the distributions are computed and updated iteratively using expectation maximization method? 4+6

(b) Consider a color image whose pixels are given in RGB color space. How many possible colors could be there in an image given each component is represented by 8 bit. Write an algorithm by adapting k-means clustering algorithm for quantizing the color vectors in M number of representative colors. Formulate a criteria for choosing an appropriate value of M for an image. 2+5+3

11. (a) Suppose an image is contaminated by a periodic noise of spatial frequency of 5 pixels per cycle in the horizontal direction in an image of size 420×640. Design a zero phase notch filter to restore the image. Provide the computational steps for filtering an image in the transform domain using the filter. 5+5

(b) Describe the computational steps for adaptive restoration of an image using local statistics of noise and image. How these statistics could be computed? 6+4

12. (a) Define morphological dilation and erosion operations on gray scale images using a non-flat structuring element. 6

(b) Describe an algorithm for sharpening images using above morphological operators. 4

(c) Define top hat transformation using dilation and erosion. Explain how it could be used for extracting foreground objects. 5+5