INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Time 2 Hrs

Date **Full Marks:** 60

Autumn Semester, 2023-24 **UG students**

Sub No. CS40019 Sub. Image Processing

No. of Students 56

Section A (All questions)

1. Highlight the radiometric factors of image formation at a point by providing the mathematical relationship. 3

   Ans. These three factors are:

   (a) Spectral power distribution of the incident ray $E(\lambda)$,

   (b) Surface reflectance spectrum $R(\lambda)$, and

   (c) Spectral response of the sensor $S(\lambda)$.

   The mathematical relationship is as follows:

$$I(x) = \int_{\lambda} E(\lambda)R(\lambda)S(\lambda)d\lambda$$

2. Provide the expression for City-block distance in 2-D and compute the distance between $(1, 2)$ and $(-3, 4)$. 3

   Ans. The city-block distance $d(.)$ between two points $p = (x_1, y_1)$ and $q = (x_2, y_2)$ is given by:

$$d(p, q) = |x_1 - x_2| + |y_1 - y_2|$$

   The distance between $(1, 2)$ and $(-3, 4)$ is $|1 - (-3)| + |2 - 4|$=6.

3. State whether the following statements are true or false with justification. 3

(a) The saturated colors are at the periphery of the CIE xy chromaticity curve.

Ans. Yes. Achromatic or white point in the CIE xy chromaticity space is at an interior point $(\frac{1}{3}, \frac{1}{3})$. The saturated color has less whiteness and it is radially away from the white point and lies in the periphery of chromaticity curve.

(b) Any color can be represented as a linear mixing of three primary colors.

Ans. Yes. Color is a perceptual phenomena as a result of interaction of light and human visual system. In our visual system there are only three different types of color sensing cells (cones). Any perception is a result of combination of these three sensations. It is observed in nature it follows a linear superposition of different colors. A primary color cannot be perceived by linear mixing of other set primary colors. Hence minimally we require three primary colors to provide equivalent sensation through linear mixing.

(c) It is possible to display any color as superposition of three primary colors.

Ans. No. Due to the characteristics of our human visual system a coefficient of linear combination may also be negative. Hence, through superposition this equivalent sensation cannot be created in this case.

4. Given the value of pixel in CMY space as (128, 94, 211). What will be the representation in RGB space assuming 24-bit representation of a color pixel (8-bit for each color plane)? 2

Ans. C=255-128=127; M=255-94=161; Y=255-211=44

5. How distance transform is used to perform Medial Axis Transform of binary images. 3

Ans. A local maximum of the distance transform of a binary image provides the center and value of the distance less one at that point provides the radius of a medial disk. Thus the set of local maxima provides the Medial Axis Transform of a binary image.

6. For removing unwanted structures in a binary image what morphological operations you need to carry out. State with justification. 3

Ans. The image should be eroded with a structuring element (SE) larger than the unwanted structure and then to be dilated with the same structure for recovering eroded boundary of the foreground regions of the image. This composite operation is named *opening*.

As the SE is the superset of the unwanted structure, erosion will turn all the associated foreground points into background point. However this will also erode the boundary points. To recover them, it is needed to apply dilation operation with the same SE. As the exposed interior points, which have become boundary points due to erosion, will be hit by the SE from a neigboring background point, those points will again be included in the resulting binary image. But complete elimination of structure does not keep any trace of foreground point belonging to them. Hence, they still remain background points even after dilation.

7. What are the mathematical properties that a pixel mapping function should have for its application to r contrast enhancement. 3

Ans. The function should be *monotonically increasing* and *total*.

Section B (Any two)

8. Consider an image of size 4×4 represented in the form of a 2-D matrix A. The coordinates of pixels of the image are given by respective row and column numbers of the matrix. The row index of the first row starts with 1 and similarly the column index also starts from 1.

$$A = \begin{bmatrix} 50 & 50 & 51 & 50 \\ 52 & 58 & 56 & 70 \\ 53 & 50 & 60 & 65 \\ 50 & 54 & 64 & 68 \end{bmatrix}$$

Answer the following:

(a) Write the histogram stretching function so that the dynamic range of the image becomes from 0 to 255. Compute the mapped pixel value at $A(2, 2)$. 4+2

Ans.

The dynamic range of the image is $[50, 70]$. It is to be mapped to the dynamic range of $[0, 255]$. Hence the stretching function is:

$$\begin{aligned} f(x) &= \tfrac{255}{20}(x-50) && \text{for } 50 \le x \le 70 \\ &= 0 && \text{Otherwise} \end{aligned}$$

The value at $A(2,2)$ would be $f(58) = \frac{255}{20}(58-50) = 102$.

(b) Estimate the horizontal and vertical gradient values using the Sobel operator. 3+3

Ans.

This may be shown at any point with suitable assumption in bordering pixels. Give full marks if anyone has explained simply the computation.

Below showing example at $A(2,2)$:

Vertical gradient: $(\frac{1}{8}((50-53)+2x(50-50)+(51-60)) = -1.5$
Horizontal gradient: $(\frac{1}{8}((51-50)+2x(56-52)+(60-53)) = 2.0$

(c) Considering a flat structuring element of a disk of 4-Neighbor distance of radius 1. Compute the processed value at $A(3,2)$ after performing gray scale morphological opening operation with the structuring element. 8

Ans.

Let us ignore pixels outside (otherwise with suitable assumption it needs to be recomputed).

The structuring element is shown as follows

$$SE = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

The opening operation performs first grey level erosion using the flat structuring element (taking the minimum of the neighboring object point). This gives the following processed array:

$$A = \begin{bmatrix} 50 & 50 & 50 & 50 \\ 50 & 50 & 51 & 50 \\ 50 & 50 & 50 & 60 \\ 50 & 50 & 54 & 64 \end{bmatrix}$$

Then compute the dilation (maximum of neighboring points in the structuring element). At $A(3,2)$ the value becomes 50.

9. Consider a bilevel image of size 8×8 represented in the form of a 2-D matrix B. The coordinates of pixels of the image are given by respective row and column numbers of the matrix. The row index of the first row starts with 1 and similarly the column index also starts from 1. The object points are denoted by 1 and the background points by 0.

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Answer the following:

(a) Consider the object points are 4-connected. Explain why back ground pixels require to be 8-connected for satisfying the condition of Jordan's curve theorem in the discrete grid. 6

Ans.

With four connectivity, there are two connected components as shown in the following. Each component is labeled by a number (either by '1' or by '2').

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Since these components are to be enclosed by background point, we need 8-connected neighborhood definition for them. As shown by marking those criticial points in red.

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) Suppose it is required to eliminate structures of width less than 3. What morphological operations are required to be carried out for that purpose? 6

Ans.

The structuring element should be a disk (chess-board distance) of radius half the width i.e. 1. We should carry out *opening* operation for eliminating such structure. It should be followed by an illustration with the sequence of operations.

(c) Give an example of a 8-connected shortest path from $B(3,3)$ to $B(7,6)$. State how many shortest paths are possible with justification. 4+4

Ans.

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & * & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & * & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & * & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

There are two such possibilities. The other path is as follows:

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & * & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & * & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & * & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

10. (a) Consider a surface point P in the 3D space is at $(70, 80, 90)$. Given the optical center of the camera is at $(0, 0, 0)$ and the projection plane is at $Z = 10$, compute the coordinates of the projected image point of P. 10

Ans.

$X = \frac{10}{90} \times 70 = \frac{70}{9}; Y = \frac{10}{90} \times 80 = \frac{80}{9}; Z = \frac{10}{90} \times 90 = 10;$

(b) State the tri-chromacy law of color vision. Explain why this law is applicable for human visual system.

Ans.

Trichromacy law: Any perceived color can be represented by three numbers or as a three vector. Hence, for every perceived color there exists a linear combination of three primary colors.

Explanation from human visual system: Describe the functions of retinal cells, their spectral responses and their interaction in the visual pathway through bipolar and horizontal cells. Both the subtractive and additive combination should be discussed.

5+5