Image Restoration

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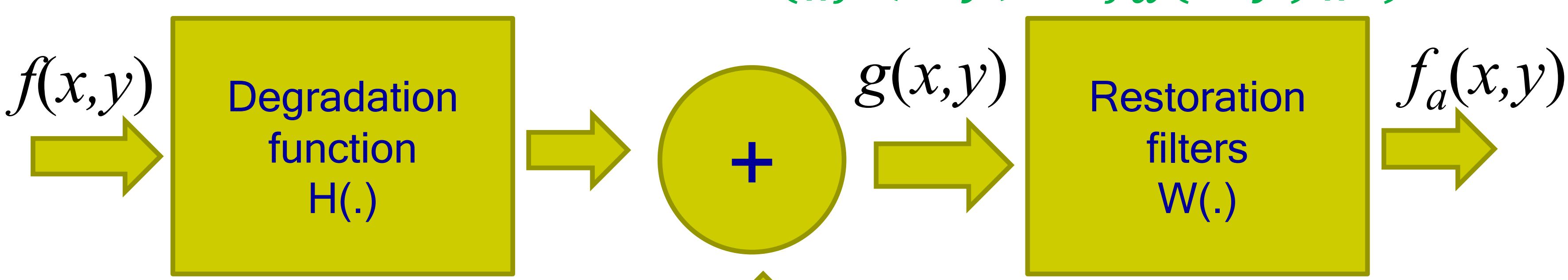
A degradation model and restoration

- Degradation function:
  - Identity (No degradation), Linear filter (Motion Blur), Transformation of a functional value
- Noise model
  - White, Gaussian, Rayleigh,

- Linear degradation model.
  $g(x, y) = h(x, y) * f(x, y) + n\ (x, y)$
- To design a restoration filter w(x,y)
  - such that w(x,y)*g(x,y) = f_a(x,y) close to f(x,y).
- To minimize
  $E(\|f(x, y) - f_a(x, y)\|^2)$



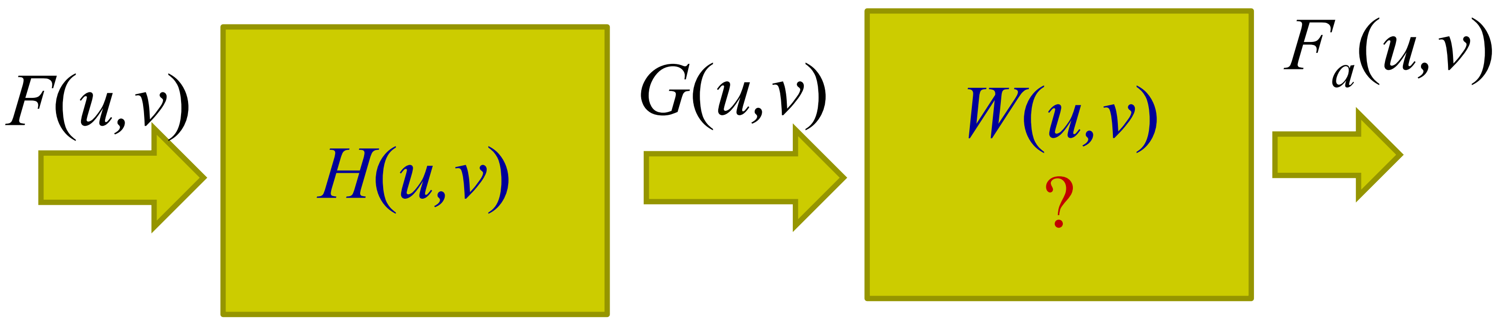
If H(.) is the identity function, the task is simply noise cleaning.

Objective function in Freq. domain?

$E(\|F(u, v) - F_a(u, v)\|^2)$

Restoration in the absence of noise

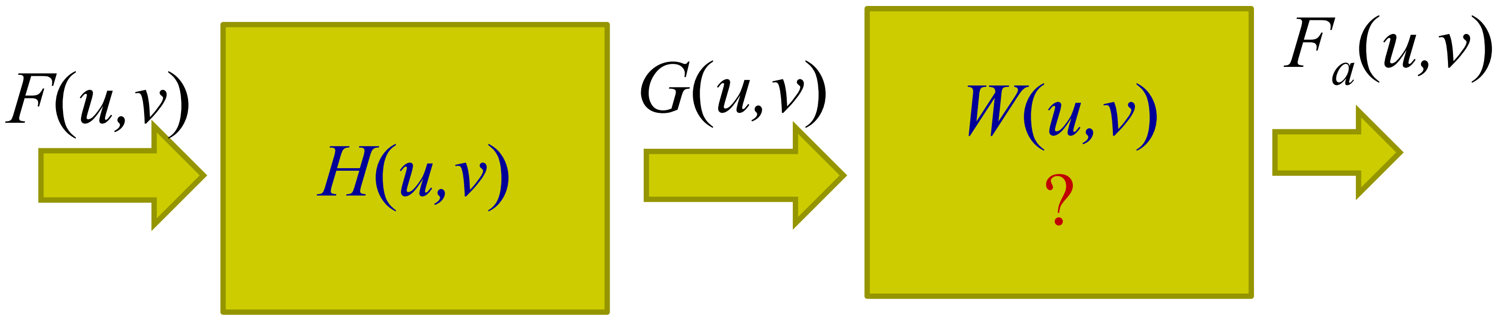
$$g(x,y) = h(x,y) * f(x,y) \Leftrightarrow G(u,v) = H(u,v)\,F(u,v)$$



- Inverse filtering: $W(u,v) = 1/H(u,v)$
  - Problem with zeros and low values in $H(u,v)$
- Minimize $E(||F(u,v) - F_a(u,v)||^2)$

$$= E(||F(u,v) - W(u,v)\,H(u,v)\,F(u,v)||^2)$$

Restoration in the absence of noise



- Power Spectrum of the image: $S_f(u,v)=||F(u,v)||^2=F^*(u,v)F(u,v)$
- To minimize $E_w=E(||F(u,v) - W(u,v)\, H(u,v)\, F(u,v)\, ||^2)$

For convenience, F(u,v) written as F and the same for all others.

$$E_w=||F||^2-(W^*H^*+WH)||F||^2+||W||^2||H||^2||F||^2$$

$$\frac{\partial E_w}{\partial W(u,v)}=0 \Rightarrow -HS_f+W^*||H||^2S_f=0 \Rightarrow W=H^*/||H||^2=1/H$$

Pretending W^* constant for W

Inverse filtering!
The same problem!

Restoration in the presence of noise

$G(u,v)=H(u,v)F(u,v)+N(u,v)$ → $W(u,v)$? → $F_a(u,v)$

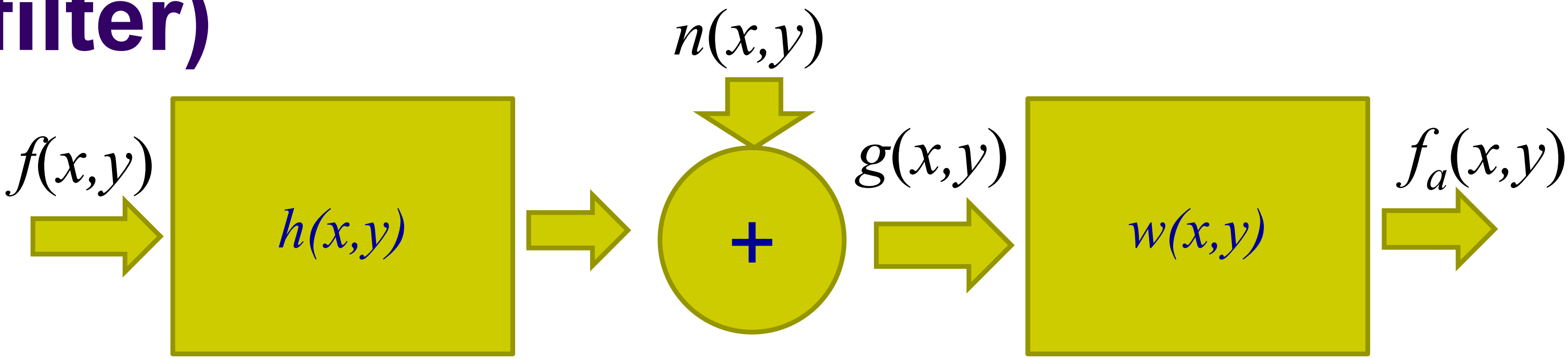
- Power Spectrum of the original image: $S_f= ||F||^2 =F^*F$
- Power Spectrum of the noisy image: $S_g= ||G||^2 =G^*G$
  - $S_g = ||H||^2\ ||F||^2 +H^*F^*N+HFN^*+ ||N||^2$
- As noise assumes to be uncorrelated: $E(F^*N)=E(N^*F)=0$
- Hence, $S_g = ||H||^2\ ||F||^2 +||N||^2 = ||H||^2\ S_f+ S_n$
- To minimize $E_w=E(||F - WG||^2)$

$W=(H^*S_f)/\ (||H||^2S_f+S_n)$

  - $E_w= E(||F\ ||^2-FW^*G^*-WGF^*+ ||W||^2\ ||G||^2)$
  - $= E(||F\ ||^2-FW^*G^*-WHF^*F-WF^*N+ ||W||^2\ ||G||^2)$

$$\frac{\partial E_w}{\partial W(u,v)} = 0 \Rightarrow -HS_f+W^*\ (||H||^2S_f+S_n)=0$$

Weiner Filter (Least square error filter)



- Solution in frequency domain: $W=(H^*S_f)/(||H||^2S_f+S_n)$

$$W = \frac{H^*}{||H||^2 + \frac{S_n}{S_f}}$$

$$W = \frac{H^*}{||H||^2 + K}$$

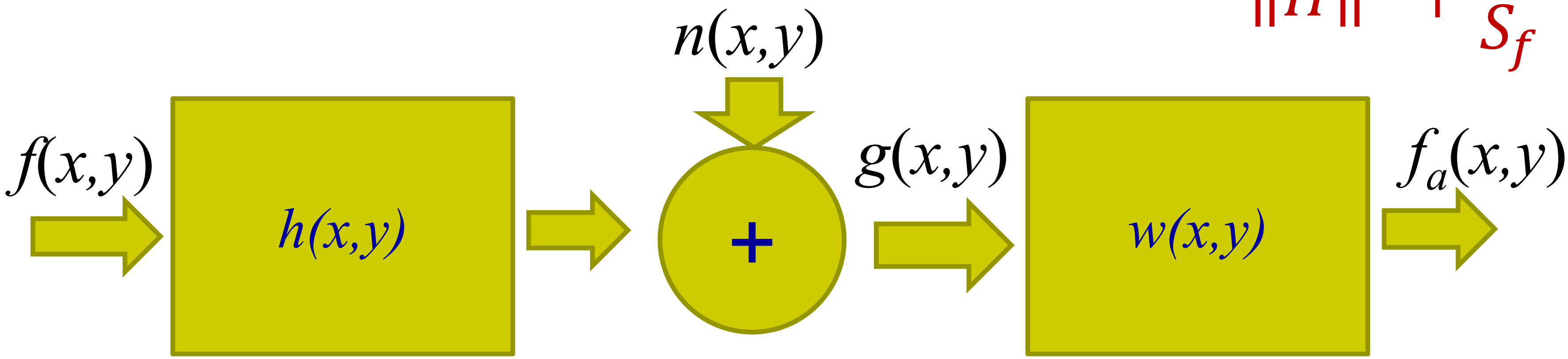
Noise to Signal Ratio

$$W = \frac{1}{H}\frac{||H||^2}{||H||^2 + K}$$

Weighted Inverse filter!

Tasks of restoration

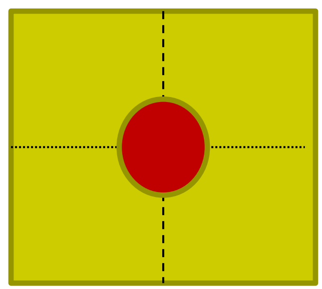
$$W = \frac{H^*}{\|H\|^2 + \frac{S_n}{S_f}}$$



- Model degradation:
  - Design / derive $h(x,y)$
- Model Noise
  - Identify PDF and estimate parameters
- Derive W or $w(x,y)$
- Apply filtering: $f_a(x,y) = w(x,y) * g(x,y)$

Restoring defocused image

- Defocused image: The projected point is not sharp.
- e.g. The projection forms a circle of radius r.
  - Spatial resolution along x: Δx and y: Δy



N: Total number of pixels within the circle

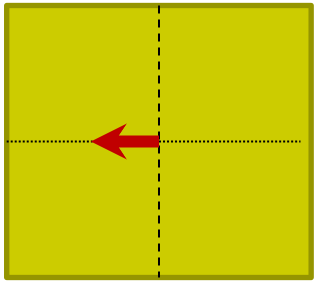
h(I,j)

$$h(i\ ,\ j) = 1/N,\quad (i.\ \Delta x\ )^2+(j.\ \Delta y\ )^2 \le r$$
$$= 0 \qquad \text{Otherwise}$$

Restoring of motion blur

- Motion blur: Movement of camera, Movement of object.
  - e.g. The camera moving with a velocity v_x in the direction of x
    - Exposure time: t
    - Spatial resolution along x: Δx
    - Number of pixels covered in a shot due to movement: $n=(v_x t) / \Delta x$

$$
\begin{aligned}
h(i,j) &= 1/n, \quad -(n-1) \leq i \leq 0 \\
&= 0 \qquad \text{Otheriwse}
\end{aligned}
$$



h(I,j)

Noise models

- Gaussian:

$$p(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$mean = \mu \quad var = \sigma^2$$

- Rayleigh:

$$p(x) = \begin{cases} \frac{2}{b}(x-a)e^{-\frac{(x-a)^2}{b}} & for \quad x \geq a \\ 0 & for \quad x < a \end{cases}$$

$$mean = a + \sqrt{\frac{\pi b}{4}}$$

$$var = \frac{b(4-\pi)}{4}$$

- Erlang (Gamma):

$$p(x) = \begin{cases} \frac{a^b x^{b-1}}{(b-1)!} e^{-ax} & for\ x \geq 0 \\ 0 & for\ x < a \end{cases}$$

$$mean = \frac{b}{a} \quad var = \frac{b}{a^2}$$

Noise models

- Exponential:

$$mean = \frac{1}{a} \quad var = \frac{1}{a^2}$$

$$p(x) = \begin{cases} ae^{-ax} & for\ x \geq 0 \\ 0 & for\ x < 0 \end{cases}$$

- Uniform:

$$mean = \frac{a+b}{2}$$

$$var = \frac{(b-a)^2}{12}$$

$$p(x) = \begin{cases} \dfrac{1}{b-a} & for\ a \leq x \leq b \\ 0 & Otherwise \end{cases}$$

- Impulse (Salt and Pepper):

Bipolar impulse

$$p(x) = \begin{cases} P_a & for\ x = a \\ P_b & for\ x = b \\ 0 & Otherwise \end{cases}$$

Estimation of noise

- Study relatively flat (constant) region and study the histogram.
- The shape of histogram may indicate appropriate PDF to be chosen.
- Compute mean and variance of the flat region.
- Relate to them to the parameters of the distribution.

Noise removal: linear and nonlinear filters exploiting local statistics

- Arithmetic mean. $\frac{1}{N}\sum_{i=0}^{N-1} x_i$
- Geometric mean. $\left(\prod_{i=0}^{N-1} x_i\right)^{\frac{1}{N}}$
- Harmonic mean. $\frac{1}{\frac{1}{N}\sum_{i=0}^{N-1}\frac{1}{x_i}}$
- Contraharmonic mean. $\frac{\sum x_i^{Q+1}}{\sum x_i^{Q}}$

  Q: Order of filter

  Q= 0 (A.M.), Q= -1 (H.M.)

- Order Statistics.
  - Median
  - Max
  - Min
  - Mid-point
    - Average of Max and Min.
  - Alpha-trimmed mean
    - Mean excluding top (d/2) and bottom (d/2) in the rank order.

Adaptive filter for restoration

- Exploit local statistics
  - Local mean: $\boldsymbol{\mu}$ Local variance: $\boldsymbol{\sigma}^2$
  - Local noise variance: η^2
  - Pixel value: $g(x,y)$
- Desirable
  - If $\eta^2=0$ return $g(x,y)$
  - If $\boldsymbol{\sigma}^2$ high return close to $g(x,y)$
  - If $\eta^2 = \boldsymbol{\sigma}^2$ return local mean $\boldsymbol{\mu}$
- Adaptive expression

$$f_a(x,y) = g(x,y) - \frac{\eta^2}{\boldsymbol{\sigma}^2}(g(x,y) - \boldsymbol{\mu})$$

Summary

- Degradation model
- Weiner (LSE) filter
  - to model degradation filter and noise, and then obtain the LSE filter.
  - Apply Weiner filter on degraded image to restore it.
- Modeling motion blur and defocusing.
- Noise model
- Use of various local statistics for removing noise.
- Adaptive filter exploiting local statistics and variance of noise.

Thank You